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A MATHEMATICAL MODEL WITH INVENTORY- AND SELLING PRICE-DEPENDENT DEMANDS CONSIDERING ALL-UNITS DISCOUNT AND CARBON EMISSION

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ABSTRACT

Customer satisfaction is a crucial aspect that consistently takes center stage in business operations. In light of this, companies must devise appropriate strategies to fulfill customer demands. Consequently, this study delves into examining various factors that facilitate the supply process, including the application of discounts. Moreover, in line with the advancements in eco-green concepts, businesses have begun considering carbon emission factors concerning storage and distribution, which is further supported by the United Nations Framework Convention on Climate Change (UNFCCC). In this context, the paper presents an enhanced version of the economic order quantity model encompassing all-unit discount and carbon emission factors. The developed model entails inventory management approaches where demand relies on inventory levels, inventory levels coupled with selling prices, time-dependent demand, and exponentially declining demand patterns. The primary objective is to aid companies in optimizing their inventory management by determining the optimal quantity of goods while minimizing overall costs. Sensitivity analysis conducted to observe the influence on the reorder point (T), total inventory cost (TC), and total carbon emission (TE) reveals that lower unit purchase prices, driven by high demand, correspond to larger order quantities. Furthermore, it is worth noting that the higher average carbon emission within warehouses results in increased carbon emissions overall.

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1. INTRODUCTION

Effective inventory management is vital for companies to ensure the seamless continuation of their production processes. Inventories encompass various goods, materials, and assets held by a company for future use or sale. Managing inventories efficiently is crucial in controlling costs and ensuring the optimal supply of goods. However, inventory management can be complex due to several factors. Fluctuating demand patterns, storage limitations, and the risk of value loss over time present challenges for companies in determining when to place orders and how much to order. To address these complexities, various inventory models have been developed to provide solutions that consider factors such as demand fluctuations, storage capacities, and the deterioration of goods over time. Regarding demand patterns, there are many models that have been developed by considering certain types of demand such as inventory-dependent demand ([1], [2], [3]), price-dependent demand [4], time-dependent demand [5], price- and inventory-dependent demand ([6], [7], [8], [9]), certain demand functions ([10], [11], [12]) and probabilistic demand ([13], [14], [15]). Besides demands, there is another factor that needs to be considered, such as deterioration. Deterioration relates to the quality decreasing of the goods after a certain time has elapsed. Many models have been proposed by introducing deterioration factor in addition to demand variation ([7], [8], [12], [16], [17]). Inventory models with deterioration factor and return policy were developed in [8] and [16], while adding the capacity constraint is another development proposed in [18]. The complexity increases when a company deals with a multi-item inventory problem. There are two replenishment policies that can be considered, viz. individual ordering policy and joint ordering policy. Some papers deal with a multi-item inventory problem, such as in [2], [10], [11], [19], [20].

Supplier discounts play a significant role in the production process and can greatly impact inventory management. These discounts enable companies to purchase goods in larger quantities at reduced costs, which can lead to improved inventory management practices. By purchasing goods in optimal quantities, companies can minimize the cost per unit of goods ordered and, in turn, optimize their overall inventory management strategies. Moreover, supplier discounts can enhance a company's competitiveness by allowing it to offer more competitive prices to its customers. Therefore, considering the all-unit discount factor becomes essential in inventory management, as it can provide substantial benefits in terms of cost reduction and market competitiveness.

The current global industrial landscape is experiencing rapid growth and intense competition among companies to meet customer demands and preferences, particularly in terms of product quality and availability. In this context, it is crucial for industries to adopt long-term planning strategies that consider the environmental impact of their operations. Consequently, effective inventory management practices should incorporate factors such as carbon emissions and waste reduction ([21], [22], [23]). By implementing strategies to reduce physical process inefficiencies, adopting cleaner and sustainable energy sources, and implementing environmentally conscious practices, companies can significantly contribute to minimizing their carbon footprint and promoting environmental sustainability ([24], [25]).

In line with these considerations, this paper aims to develop an inventory model based on the model in [26] that not only considers factors such as demand patterns, selling prices, and product deterioration but also accounts for carbon emissions. The incorporation of the inventory- and price-dependent demand, all-units discount, and carbon emission is the contribution and novelty of this paper. By incorporating the carbon emissions factor, the developed models will optimize inventory management practices by determining the ideal timing for reordering, appropriate order quantities, and methods to minimize total costs. This allows businesses to make informed decisions that not only align with their economic objectives but also support broader sustainability goals. This novel aspect not only distinguishes the research but also positions it as a valuable tool for companies seeking to navigate the complex landscape of balancing operational efficiency with environmental responsibility. In this way, the research contributes not only to the theoretical understanding of inventory management but also offers a practical and quantifiable approach that can be applied across various industries to foster sustainable business practices.

Therefore, an inventory model will be developed, specifically focusing on models with demand functions dependent on inventory level and selling price. The decision variable in this model is to determine the appropriate reorder interval. Based on these decision variables, the optimal order quantity and total inventory cost will be determined by selecting the right ordering policy.

2. RESEARCH METHODS

The mathematical model presented in this paper is developed based on the following notations and assumptions. In this paper, only the development of models with demand functions dependent on inventory level and selling price with two different functions $\boldsymbol{r}(\boldsymbol{p})$ will be discussed. In this paper, only the development of models with demand functions dependent on inventory level and selling price, using two different functions $\boldsymbol{r}(\boldsymbol{p})$, will be discussed. The calculations for these models are conducted using MAPLE software.

2.1 Notations

The notations used in the development of this model are:

C_p : total purchase cost per cycle,
C_o : total ordering cost per cycle,
C_s : total storage cost per cycle,
C_t : total transportation cost per cycle,
E_e : carbon emissions resulting from the production process,
F_e : fuel emission standards,
c_1 : fuel consumption when the vehicle is empty,
c_2 : additional fuel consumption per unit load,
e_1 : transportation carbon emission costs,
e_2 : additional cost of transport carbon emission,
t_f : fixed costs for the process of shipping goods,
t_v : variable costs for the process of shipping goods,
t_x : carbon emission tax price,
p_i : selling cost per unit,
w : warehouse average carbon emission,
l : item weight,
T : reorder time,
S : ordering cost for each order placed,
d : mileage from the supplier,
h : fraction of goods stored per unit,
α : initial demand factor,
β : the factor of increasing demand for items depends on inventory and price,
$I(t)$: inventory level at time $t \in [0, T]$,
$r_1(p)$: linear demand function, $r_1(p)$ =10.000-0,05p
$r_2(p)$: logarithmic demand function, $r_2(p) = 10.000 - 150 \ln(p)$
Q : order quantity,
TC : total cost,
TE : total carbon emission.

2.2 Assumptions

The following assumptions are used in this model.

1. There is no lead time for ordering items, which means that inventory will be replenished immediately upon placing an order when the inventory runs out.
2. Inventory-dependent demand function is expressed as follows:

$$D(t) = r(p)(a + \beta I(t)) \tag{1}$$

where $a > 0$ and $\beta > 0$.

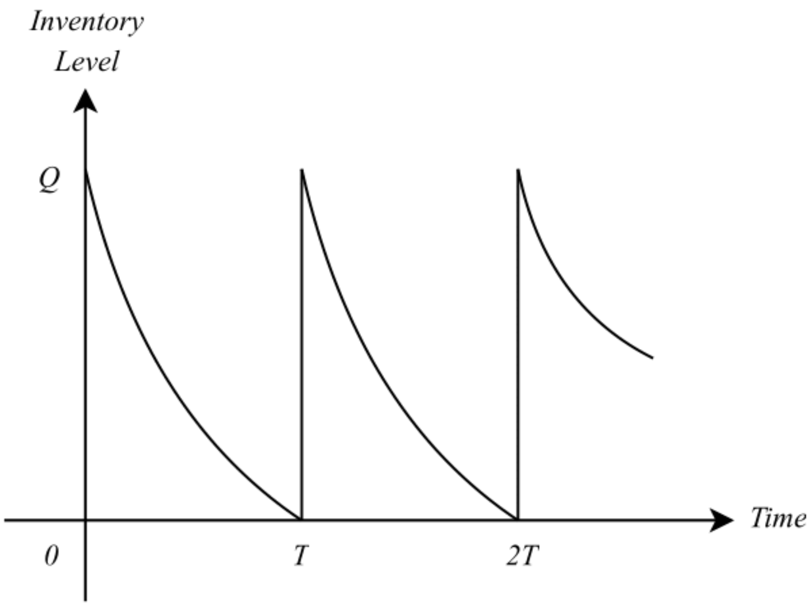

Figure 1. Inventory Model with Demand Dependent on Inventory Level

According to the image shown in **Figure 1.** It can be interpreted that at the initial state ($t = 0$), the company has an inventory of Q units of goods. As time goes by, the stock will diminish based on the price- and inventory-dependent demand at that time (t). This means that if the company holds a substantial inventory, it will decrease more rapidly due to higher demand (and inventory-dependent demand). Once the stock is depleted at time $t = T$, a reorder is initiated for Q units of products, ensuring that the inventory is replenished instantly without any waiting period (inter-order time). Then, the cycle repeats.

2.3 Model Development

The inventory model, which incorporates demand dependence on both inventory levels and selling prices, is a sophisticated mathematical tool aimed at optimizing product inventory. This model acknowledges the reality that consumers not only assess the quantity of available items but also consider the offered selling prices. Consequently, its purpose is to identify the optimal inventory level that maximizes the company's profits while minimizing associated inventory costs.

The price- and inventory-dependent demand used in developing the model takes the following form.

$$D(t) = r(p)\big(\alpha + \beta I(t)\big), \quad 0 \le t \le T$$

where $a > 0$ and $\beta > 0$.

Thus, the rate of change in the amount of inventory from time to time can be modeled using the differential equation as follows:

$$\frac{dI(t)}{dt} = -\alpha r(p) - \beta r(p) I(t), \quad 0 \le t \le T$$

with the respective boundary conditions and initial conditions, namely $I(t) = 0$ and $I(0) = Q$.

Therefore, changes in the amount of inventory from time to time can be solved with the following steps:

$$\frac{dI(t)}{dt} = -\alpha r(p) - \beta r(p) I(t)$$
$$\frac{dI(t)}{-\alpha r(p) - \beta r(p) I(t)} = dt$$
$$\int \frac{dI(t)}{-\alpha r(p) - \beta r(p) I(t)} = \int dt$$

$$-\frac{1}{\beta r(p)}\ln\big(\alpha r(p)-\beta r(p)I(t)\big)=t+C_1$$
$$\ln\big(\alpha r(p)+\beta r(p)I(t)\big)=-\beta r(p)t+C_2$$
$$\alpha r(p)+\beta r(p)I(t)=C_3e^{-\beta r(p)t}$$
$$\beta r(p)I(t)=-\alpha r(p)+C_3e^{-\beta r(p)t}$$

So, the general solution is obtained from the following equation:

$$I(t)=-\frac{\alpha}{\beta}+ce^{-\beta r(p)t} \tag{2}$$

After getting the general solution, a special solution will be determined based on the boundary conditions, namely by finding a constant value c.

To find the value of c, we will substitute the boundary conditions $I(T)=0$ in **Equation (2)**. Found,

$$0=-\frac{\alpha}{\beta}+ce^{-\beta r(p)t}$$
$$c=\frac{\alpha e^{-\beta r(p)T}}{\beta}$$

Next, substitute the value of c, into the **Equation (2)** to get the specific solution of the equation, i.e.

$$0=-\frac{\alpha}{\beta}+\frac{\alpha}{\beta}(e^{-\beta r(p)(T-t)})$$
$$=\frac{\alpha}{\beta}(e^{-\beta r(p)(T-t)}-1)$$

Substitute the initial condition $I(0)=Q$ into **Equation (2)** to obtain the maximum inventory quantity (Q). It is obtained

$$I(0)=Q=\frac{\alpha}{\beta}(e^{-\beta r(p)T}-1) \tag{3}$$

There are four cost components for the total cost of inventory for one period of time, which is purchase cost (C_p), order cost (C_o), storage cost (C_s), and transportation cost (C_t), which each are given below.

1. **Purchase Cost (C_p)**

The Purchase Cost (C_p) is the cost incurred for acquiring a particular item. The magnitude of the purchase cost over a specific period can be expressed as follows

$$C_p=\frac{\alpha}{\beta T}(-1+e^{-\beta r(p)T})\cdot P_i$$

2. **Order Cost (C_o)**

The Ordering Cost (C_o) is the cost incurred when a purchase order is placed. The amount of ordering cost during a single time period can be expressed as follows

$$C_o=\frac{S}{T}$$

3. **Storage Cost (C_s)**

Storage Cost (C_s) is the cost incurred for maintaining goods during their storage period. The magnitude of the storage cost over a specific time period can be expressed as follows

$$C_s=(h\cdot P_i+w\cdot E_e\cdot T_x)\cdot\left[-\frac{\alpha}{\beta}-\frac{\alpha(1-e^{-\beta r(p)T})}{r(p)T\beta^2}\right]$$

4. **Transportation Cost (C_t)**

Transportation Cost (C_t) is a combination of cost components that include fixed costs, variable costs, and carbon emission costs resulting from vehicle usage. The total transportation cost over a specific period can be expressed as follows

$$C_t = \frac{1}{T}\left(t_f + \left(2dc_1t_v + dc_2l\frac{\alpha}{\beta}\left(-1+e^{\beta r(p)T}\right)t_v\right) + \left(2de_1 + de_2\frac{\alpha}{\beta}\left(-1+e^{\beta r(p)T}\right)\right)\right)$$

Total Cost of Inventory for One Period of Time

Therefore, the total inventory cost can be determined, which is the sum of purchase costs, ordering costs, holding costs, and transportation costs, namely

$$TC(T) = \frac{\alpha}{\beta T}\left(-1+e^{-\beta r(p)T}\right)\cdot P_i + \frac{S}{T} + (h\cdot P_i + w\cdot E_e\cdot T_x)\cdot\left[-\frac{\alpha}{\beta} - \frac{\alpha\left(1-e^{-\beta r(p)T}\right)}{r(p)T\beta^2}\right] + \frac{1}{T}\left(t_f + \left(2dc_1t_v + dc_2l\frac{\alpha}{\beta}\left(-1+e^{\beta r(p)T}\right)t_v\right) + \left(2de_1 + de_2\frac{\alpha}{\beta}\left(-1+e^{\beta r(p)T}\right)\right)\right). \quad (4)$$

With the calculation of the total carbon emissions as follows:

$$TE(T) = \frac{\alpha}{2\beta}\left(-1+e^{-\beta r(p)T}\right)\cdot(wE_e) + \frac{1}{T}\left(2dc_1F_e + dc_2lF_e\frac{\alpha}{\beta}\left(-1+e^{\beta r(p)T}\right)\right)$$

The first term in $TE(T)$ is expressing storage emissions and the second term is for transportation emissions.

In order to obtain a value of T that minimizes the total cost, the conditions that must be met are

$$\frac{dTC(T)}{dT} = 0 \text{ and } \frac{d^2TC(T)}{dT^2} > 0$$

We can find the optimal value of T satisfying the two conditions above, using Maple software.

3. RESULTS AND DISCUSSION

3.1 Results

Below, an example for calculation will be provided to illustrate the influence of changes in the purchase price of goods due to the all-unit discount factor on the quantity of goods and the inter-order time (T) in the inventory model dependent on inventory level and selling price.

Table 1. Parameter Values for Inventory Model

Definition	Notation	Value
Fraction of goods stored per unit	h	4%
Mileage from the supplier	d	100 km
Ordering cost for each order placed	S	Rp10.000
Warehouse average carbon emission	w	1.14 kWh/unit/period
Carbon emissions from the production process	E_e	0,005 tonCO_2/kWh
Carbon emission tax price	t_x	Rp30.000/ tonCO_2
Fuel consumption when the vehicle is empty	c_1	0.18 L/km
Additional fuel consumption per unit load	c_2	0.057 L/km/unit
Item weight	l	0,01 ton/unit
Fixed costs for the process of shipping goods	t_f	Rp1,000/delivery
Variable costs for the process of shipping goods	t_v	Rp100/L
Transportation carbon emission costs	e_1	Rp150/km
Additional cost of transport carbon emission	e_2	Rp100/unit/km
Fuel emission standards	F_e	0.0026 tonCO_2/L
Initial demand factor	α	0.013
The factor of inventory- and price dependent demand	β	0.00002
Selling cost per unit	p_i	$120\%\cdot P_i$

With an all-unit discount applied to the listed prices in the following table.

Table 2. Price Selling Terms Influenced by All-unit Discount

P_i	q_i	**Interval** q_i
Rp40,000	0	≤ 25
Rp32,500	26	26 - 39
Rp25,000	40	≥ 40

Using the parameter values in **Table 1** and **Table 2**, substituting the optimal value of T into **Equation (2)**, we obtain the quantity of items to be ordered at the beginning of the period, and substituting it to **Equation (1)** the minimum total cost is obtained. The results can be seen in **Table 3** and **Table 4**.

Table 3. Price Selling Terms Influenced by All-unit Discount

i	P_i	T	Q_i	q_i	$TC(T)$	**Note**
1	Rp40,000	0.3056	31	≤ 25	Rp5,230,205	Q_i Not Valid
2	Rp32,500	0.3148	34	26 - 39	Rp4,729,196	Q_i Valid
3	Rp25,000	0.3304	38	≥ 40	Rp4,135,595	Q_i Not Valid

From **Table 3**, by using $r_1(p)$ =10,000-0.05p, it is obtained that, to achieve the optimal total cost, the process of restocking the goods should be done every 0.3148 years, with 34 units ordered in each period, and the optimum total cost is Rp4,729,196. Using the logarithmic demand function of $r_2(p)$ =10,000 – 150 ln(p), in Table 4, the replenishment interval is shorter, every 0.3023 years with higher optimum total cost of Rp4,942,098, and the same order quantity of 34 units.

Table 4. Price Selling Terms Influenced by All-units Discount

i	P_i	T	Q_i	q_i	$TC(T)$	**Note**
1	Rp40,000	0.2794	31	≤ 25	Rp5,766,351	Q_i Not Valid
2	Rp32,500	0.3023	34	26 - 39	Rp4,942,098	Q_i Valid
3	Rp25,000	0.3320	38	≥ 40	Rp4,113,166	Q_i Not Valid

3.2 Sensitivity Analysis

In the previous discussion, numerical simulations were conducted for each type of inventory model. Furthermore, in this section, sensitivity analysis is performed, aiming to provide an overview of the impact parameter value changes and their interpretations. Sensitivity analysis in this subsection is conducted for the second model, which is the inventory model with a demand function dependent on inventory levels and selling prices.

3.2.1 The influence of changing values of α, β, and S for $r_1(p) = 10,000 - 0.05p$

The influence of changes in the values of α, β, and S for $r_1(p) = 10,000 - 0.05p$ on inter-order time (T), total cost (TC), and total carbon emissions (TE) can be seen in **Table 5**.

Table 5. Effect of Changes in Ordering Cost for Joint Order Policy

Parameter	**Value**	**% Change**	Q	T	T **% Change**	**TC**	**TC % Change**	**Carbon Emission**
α	0.0104	−20	30	0.3512	11.5628	Rp3,810,139	-19.4336	0.3654
	0.0117	−10	32	0.3315	5.3049	Rp4,270,077	-9.7081	0.3880
	0.0130	0	33	0.3148	0	Rp4,729,196	0	0.4095
	0.0143	+10	35	0.3003	-4.6060	Rp5,187,616	9.6934	0.4300
	0.0156	+20	37	0.2877	-8.6086	Rp5,645,431	19.3740	0.4497
β	0.000016	−20	37	0.3451	9.6251	Rp4,704,760	-0.5167	0.3923
	0.000018	−10	35	0.3289	4.4760	Rp4,717,247	-0.2526	0.4009
	0.000020	0	33	0.3148	0	Rp4,729,196	0	0.4095
	0.000022	+10	32	0.3023	-3.9707	Rp4,740,672	0.2426	0.4181
	0.000024	+20	31	0.2911	-7.5285	Rp4,751,728	0.4764	0.4267
S	8,000	−20	33	0.3077	-2.2554	Rp4,722,771	-0.1358	0.4141
	9,000	−10	33	0.3113	-1.1118	Rp4,726,002	-0.0675	0.4117
	10,000	0	33	0.3148	0	Rp4,729,196	0	0.4095
	11,000	+10	34	0.3182	1.0800	Rp4,732,355	0.0667	0.4074
	12,000	+20	35	0.3216	2.1601	Rp4,735,481	0.1328	0.4053

The primary demand factor (α) exerts the most substantial influence on carbon emissions compared to the parameters β and S, as it significantly affects both the production and consumption levels of goods. The connection between primary demand and carbon emissions becomes apparent in situations where an upsurge in demand for goods leads to heightened production and consumption, subsequently increasing reliance on fossil energy and, consequently, carbon emissions.

A larger β value corresponds to a reduced order quantity required to meet the desired inventory level. This is due to the influence of β on the responsiveness of inventory levels to changes in demand. A higher β value indicates that the company tends to place smaller but more frequent orders, emphasizing a preference for frequent ordering in smaller quantities.

In this scenario, the advantage of placing smaller, more frequent orders lies in lower ordering costs. Although each order incurs higher costs, the overall storage costs diminish as the company maintains smaller inventory quantities. From a carbon emissions standpoint, the frequent ordering and increased volume of stored goods in the warehouse significantly contribute to an overall rise in carbon emissions.

Within inventory models, striking a balance between ordering costs and holding costs is crucial. When ordering costs are elevated, companies tend to reduce order frequency and opt for larger quantities, resulting in longer reorder intervals. This delicate balance aims to optimize both cost efficiency and environmental impact in inventory management.

3.2.2 The influence of changing values of w and c_1 for $r_1(p) = 10,000 - 0.05p$

The influence of changes in the values of w and c_1 for $r_1(p) = 10{,}000 - 0.05p$ on inter-order time (T), total cost (TC), and total carbon emissions (TE) can be seen in **Table 6**.

Table 6. Effect of Changes in Ordering Cost for Joint Order Policy

Parameter	Value	% Change	*Q*	*T*	*T* % Change	TC	*TC* % Change	Carbon Emission
	0.912	−20	34	0.3154	0.1905	Rp4,728,622	-0.0121	0.3898
	1.026	−10	34	0.3151	0.0952	Rp4,728,909	-0.0060	0.3997
w	1.140	0	34	0.3148	0	Rp4,729,196	0	0.4095
	1.254	+10	34	0.3144	-0.1270	Rp4,729,482	0.0060	0.4193
	1.368	+20	34	0.3141	-0.2223	Rp4,729,768	0.0121	0.4291
	0.144	−20	34	0.3122	-0.8259	Rp4,726,900	-00485	0.3512
	0.162	−10	34	0.3135	-0.4129	Rp4,728,050	-0.0242	0.3804
c_1	0.180	0	34	0.3148	0	Rp4,729,196	0	0.4095
	0.198	+10	34	0.3160	0.3160	Rp4,730,337	0.0241	0.4383
	0.216	+20	34	0.3172	0.7623	Rp4,731,474	0.0481	0.4669

Parameter w represents the average carbon emissions in the warehouse. An escalation in the average carbon emissions, accompanied by an increase in emissions from the production process (E_e), contributes to a growing total carbon footprint. On the other hand, parameter c_1 denotes the fuel consumption when the vehicle is empty. A higher fuel consumption during periods of vehicle emptiness leads to increased transportation costs, potentially inflating the overall inventory expenses. Conversely, for parameter T, the fuel consumption when the vehicle is empty does not exert a direct influence on the reorder interval.

3.2.3 The influence of changing values of α, β, and S for $r_2(p) = 10,000 - 150 \ln(p)$

The influence of changes in the values of α, β, and S for $r_2(p) = 10{,}000 - 150 \ln(p)$ on inter-order time (T), total cost (TC), and total carbon emissions (TE) can be seen in **Table 7**.

Table 7. Effect of Changes in Ordering Cost for Joint Order Policy

Parameter	Value	% Change	*Q*	*T*	*T* % Change	TC	*TC* % Change	Carbon Emission
	0.0104	−20	30	0.3373	11.5779	Rp3,981,567	-19.4356	0.3773
	0.0117	−10	32	0.3183	5.2927	Rp4,462,259	-9.7092	0.4007
α	0.0130	0	34	0.3023	0	Rp4,942,098	0	0.4229
	0.0143	+10	36	0.2884	-4.5980	Rp5,421,210	9.6945	0.4441
	0.0156	+20	38	0.2763	-8.6007	Rp5,899,691	19.3762	0.4644
	0.000016	−20	37	0.3317	9.7254	Rp4,916,450	-0.5189	0.4045
	0.000018	−10	35	0.3160	4.5319	Rp4,929,559	-0.2537	0.4137
β	0.000020	0	33	0.3023	0	Rp4,942,098	0	0.4229
	0.000022	+10	32	0.2902	-4.0026	Rp4,954,137	0.2436	0.4321
	0.000024	+20	31	0.2763	-7.5752	Rp4,965,733	0.4782	0.4412
	8,000	−20	33	0.2955	-2.2494	Rp4,935,408	-0.1353	0.4277
	9,000	−10	33	0.2989	-1.1247	Rp4,938,772	-0.0672	0.4252
S	10,000	0	33	0.3023	0	Rp4,942,098	0	0.4229
	11,000	+10	34	0.3056	1.0916	Rp4,945,388	0.0665	0.4206
	12,000	+20	35	0.3089	2.1832	Rp4,948,642	0.1324	0.4184

The influence of changing values of $\alpha, \beta,$ and S as depicted in **Table 7** is similar with the one in **Table 5**. The changes on inter-order time (T), total cost (TC) and carbon emissions in **Table 7** is slightly higher than in **Table 5** due to different demand function used.

3.2.4 The influence of changing values of w and c_1 for $r_2(p) = 10,000 - 150\ln(p)$

The influence of changes in the values of w and c_1 for $r_2(p) = 10{,}000 - 150\ln(p)$ on inter-order time (T), total cost (TC), and total carbon emissions (TE) can be seen in **Table 8**.

Table 8. Effect of Changes in Ordering Cost for Joint Order Policy

Parameter	Value	% Change	*Q*	*T*	*T* % Change	TC	*TC* % Change	Carbon Emission
	0.912	−20	34	0.3029	0.1984	Rp4,941,522	-0.0116	0.4031
	1.026	−10	34	0.3026	0.0992	Rp4,941,810	-0.0058	0.4130
w	1.140	0	34	0.3023	0	Rp4,942,098	0	0.4229
	1.254	+10	34	0.3020	-0.0992	Rp4,942,386	0.0058	0.4327
	1.368	+20	34	0.3017	-0.1984	Rp4,942,673	0.0116	0.4426
	0.144	−20	34	0.2999	-0.7939	Rp4,939,707	-0.0483	0.3621
	0.162	−10	34	0.3011	-0.3969	Rp4,940,905	-0.0241	0.3926
c_1	0.180	0	34	0.3023	0	Rp4,942,098	0	0.4229
	0.198	+10	34	0.3035	0.3969	Rp4,943,287	0.0240	0.4529
	0.216	+20	34	0.3047	0.7939	Rp4,944,470	0.0479	0.4827

Results depicted in **Table 8** have a similar tendency in the direction of changes on inter-order time (T), total cost (TC) and carbon emissions due the changes in parameter w and c_1 with the one in **Table 6**, although the magnitude is a little higher.

3.3 Discussion

3.3.1 Main Demand (α)

The main demand ($\boldsymbol{\alpha}$) has the greatest impact on carbon emissions compared to the parameters β and S because it is a factor that influences the level of production and consumption of goods. The connection between primary demand and carbon emissions can be understood from the condition where higher demand for goods leads to an increase in the production and consumption of those goods. As a result, during this process, more fossil energy will be used, resulting in an increase in carbon emissions.

3.3.2 The Factor of Increasing Demand (β)

The larger the value of $\boldsymbol{\beta}$, the smaller the quantity of orders needed to achieve the desired inventory level. This is because b affects the responsiveness of inventory level changes to demand changes. With a larger $\boldsymbol{\beta}$ value, a company can respond to demand changes with smaller but more frequent orders. Additionally, the parameter $\boldsymbol{\beta}$ also reflects the demand elasticity with respect to inventory levels. Demand elasticity measures the sensitivity of demand to changes in inventory levels.

If the $\boldsymbol{\beta}$ value is positive, it indicates a positive relationship between inventory levels and demand, meaning that demand tends to increase when inventory levels increase. Conversely, if the $\boldsymbol{\beta}$ value is negative, it indicates a negative relationship between inventory levels and demand, meaning that demand tends to decrease when inventory levels increase. Thus, the $\boldsymbol{\beta}$ value in an inventory model with time and price-dependent demand provides an insight into demand elasticity, the required order quantity, and the time between orders needed to achieve the desired inventory level.

3.3.3 Ordering Cost for Each Order Placed (S)

The larger the value of $\boldsymbol{\beta}$, the smaller the quantity of orders needed to achieve the desired inventory level. This is because b affects the responsiveness of inventory level changes to demand changes. With a larger $\boldsymbol{\beta}$ value, a company can respond to demand changes with smaller but more frequent orders. Additionally, the parameter $\boldsymbol{\beta}$ also reflects the demand elasticity with respect to inventory levels. Demand elasticity measures the sensitivity of demand to changes in inventory levels.

3.3.4 Average Carbon Emissions in The Warehouse (w)

The increase in average carbon emissions in the warehouse, along with the increase in emissions generated from the production process (E_e), will result in a larger total carbon emission.

3.3.5 Fuel Consumption When the Vehicle is Empty (c_1)

The larger the fuel consumption when the vehicle is empty, the higher the transportation cost generated, which can consequently result in an increase in the total inventory cost. On the other hand, for T, the fuel consumption when the vehicle is empty does not have a direct impact on the inter-order time.

4. CONCLUSIONS

From our analysis in the previous sections, we can draw the following conclusions.

1. Based on the results obtained from Table 3 and Table 4, utilizing the demand functions $r_1(p)$= 10,000 - 0.05p and $r_2(p) = 10{,}000 - 150\ \ln(p)$ respectively, the optimal restocking strategy differs. For $r_1(p)$, the optimal total cost is achieved by restocking every 0.3148 years, ordering 34 units in each period, resulting in an optimum total cost of Rp4,729,196. On the other hand, utilizing the logarithmic demand function $r_2(p)$, the replenishment interval is shorter at every 0.3023 years, with the same order quantity of 34 units, but a higher optimum total cost of Rp4,942,098. These findings highlight the sensitivity of the optimal restocking strategy to the choice of demand function, with varying implications for total cost and replenishment frequency.
2. Overall, it is found that the quantity of goods ordered is influenced by the inter-order time and purchase price. The cheaper the purchase price per unit supported by high demand, the greater quantity of goods ordered.
3. Additionally, a longer inter-order time allows for more time to produce larger quantities of the order at a lower cost.
4. Furthermore, based on the sensitivity analysis results, it is known that an increase in the main demand (α) and an increase in the ordering cost (S) have a greater impact compared to other tested parameters on total inventory cost (TC) and inter-order time (T).

5. For future research, exploring dynamic models that consider evolving market conditions, variable demand patterns, and real-time pricing fluctuations to achieve a more nuanced understanding of inventory management dynamics.

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REFERENCES

[1] B. C. Giri, S. Pala, A. Goswami and K. Chaudhuri, "An inventory model for deteriorating items with stock-dependent demand rate," *European Journal of Operational Research,* vol. 95, pp. 604-610, 1996.

[2] N. Tashakkor, S. H. Mirmohammadi and M. Iranpoor, "Joint optimization of dynamic pricing and replenishment cycle considering variable non-instantaneous deterioration and stock-dependent demand," *Computers & Industrial Engineering,* vol. 123, pp. 232-241, 2018.

[3] N. Loedy, D. Lesmono and T. Limansyah, "An Inventory-Dependent Demand Model with Deterioration, All-Units Discount, and Return," *IOP Conf Series: Journal of Physics,* vol. 1108, p. 012010, 2018.

[4] V. Vora and U. Gothi, "Deterministic Inventory Model for deteriorating Items with Price Dependent Demand Rate and Inflation," *International Journal of Scientific Research in Mathematical and Statistical Sciences,* vol. 7, no. 6, pp. 20-28, 2020.

[5] J.I. Ichwanto, J.D. Lesmono and T. Limansyah, "An inventory model for estimation of deterioration with time-dependent demand and storage cost," *IOP Conf. Series: Journal of Physics*, vol. 1940, p.012012, 2021.

[6] W. Adiani, D. Lesmono and T. Limansyah, "Model persediaan dengan permintaan bergantung pada harga jual dan tingkat persediaan dengan faktor deteriorasi," *Jurnal Ilmiah Teknik Industri*, vol. 18, pp. 183-191, 2019.

[7] N. H. Shah, K. Rabari and E. Patel, "A deteriorating inventory model under overtime production and credit policy for stock-and price sensitive demand function," *Operational Research in Engineering Sciences: Theory and Applications,* vol. 5, no. 2, pp. 85-98, 2022.

[8] M. Onal, A. Yenipazarli and O. E. Kundakcioglu, "A mathematical model for perishable products with price- and displayed-stock- dependent demand," *Computers & Industrial Engineering,* vol. 102, pp. 246-258, 2016

[9] T. Limansyah and D. Lesmono, "A mathematical model for inventory and price-dependent demand with discount," *IOP Conf. Series: Journal of Physics,* vol. 1490, pp. 12-51, 2020.

[10] T. Joviani, D. Lesmono and T. Limansyah, "A Multi-item Inventory Model with Various Demand Functions Considering Deterioration and Partial Backlogging," *BAREKENG: Jurnal Ilmu Matematika dan Terapan* vol.17, pp. 1069-1080, 2023.

[11] D. Lesmono, T. Limansyah and N. Loedy, "A Joint Return Policy for a Multi-Item Perishable Inventory Model with Deterministic Demands, Return and All-Units Discount," *International Journal of Mathematical, Engineering and Management Sciences* Vol. 5, No. 3, 416-431, 2020..

[12] R.I.P. Setiawan, J.D. Lesmono and T. Limansyah. "Inventory Control Problems with Exponential and Quadratic Demand considering Weibull Deterioration," *IOP Conf. Series: Journal of Physics,* vol. 1821 p. 0120571, 2021.

[13] D. Lesmono, T. Limansyah and H. Koswara, "Optimal replenishment policy for multi-item probabilistic inventory model with all-units discount," *IOP Conf. Series: Journal of Physics,* vol. 1218, p.012036, 2019.

[14] T. Limansyah and D. Lesmono, "Probabilistic Inventory Model with Expiration Date and All-Units Discount," *IOP Conf. Series: Materials Science and Engineering*, vol. 546, p.052042, 2019.

[15] D. Lesmono and T. Limansyah, "A probabilistic inventory model with deterioration factor and continuous discount function", *AIP Conference Proceedings,* vol. 2192, p. 060013, 2019.

[16] T. Limansyah, D. Lesmono and I. Sandy, "Economic order quantity model with deterioration factor and all-unit discount," *IOP Cond: Series: Journal of Physics*, vol. 1490, pp. 12-52, 2020.

[17] A. Duary, S. Das, M. G. Arif and K. M. Abualnaja, "Advance and delay in payments with the price-discount inventory model for deteriorating items under capacity constraint and partially backlogged shortages," *Alexandria Engineering Journal,* vol. 61, pp. 1735-1845, 2022.

[18] D. Lesmono, T. Limansyah and S. W. Setiawan, "A Perishable Inventory Model with Return," *IOP Conf.Series: Materials Science and Engineering*, pp. 1735-1845, 2022.

[19] J. Sicilia, L. A. San Jose and D. Alcade-Lopez-de-Pablo, "Optimal policy for multi-item systems with stochastic demands, backlogged shortages and limited storage capacity," *Applied Mathematical Modelling*, vol. 108, pp. 236-257, 2022.

[20] R. Y. H. Silitonga, L. R. Kristiana and T. A. Parley, "A Multi-Item Probabilistic Inventory Model that Considers Expiration Factor, All-unit Discount Policy and Warehouse Capacity Constrains," *Jurnal Teknik Industri*, vol. 23, no. 2, pp. 139-148, 2021.

[21] C.-J. Lu, M. Gu and T.-S. Y. C.-T. Lee, "Impact of carbon emission policy combinations on the optimal production-inventory decisions for deteriorating items," *Expert Systems With Applications*, vol. 201, pp. 117-234, 2022.

[22] C. Yu, Z. Qu, T. W. Archibald and Z. Luan, "An inventory model of a deteriorating product considering carbon emissions," *Computers & Industrial Engineering*, vol. 148, pp. 106-694, 2020.

[23] Y.-S. Huang, C.-C. Fang and Y.-A. Lin, "Inventory management in supply chains with consideration of logistics, green investment and different carbon emissions policies," *Computer & Industrial Engineering*, vol. 139, pp. 106-207, 2020.

[24] S. Benjaafar and M. Daskin, "Carbon footprint and the management of supply chain : Insights from simple models," *IEEE transactions on automation science and engineering*, vol. 10, pp. 99-116, 2012.

[25] X. Chen, S. Benjaafar and A. Elomri, "The carbon-constrained EOQ," *Operation Research Letters*, vol. 41, pp. 172-179, 2013.

[26] I. M. Kristiyani and Y. Daryanto, "An inventory model considering all-unit discount and carbon emissions," *International Journal of Industrial Engineering and Engineering Management*, vol. 1, pp. 43-50, 2019.