BAREKENG: Journal of Mathematics and Its Applications
March 2024 Volume 18 Issue 1 Page 0163–0170
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol18iss1pp0163-0170

MODELING THE MANY EARTHQUAKES IN SUMATRA USING POISSON HIDDEN MARKOV MODELS AND EXPECTATION MAXIMIZATION ALGORITHM

Muhammad Arib Alwansyah [1*], Ramya Rachmawati[2]

[1,2]*Statistics Study Program Department of Mathematics, Faculty of Mathematics and Natural Science, The University of Bengkulu*
St. WR. Supratman Kandang Limun, Muara Bangkahulu, Bengkulu 38122, Indonesia

*Corresponding author's e-mail: * **muhammadaribalwansyah232@gmail.com***

ABSTRACT

Article History:

Received: 3rd August 2023
Revised: 21st November 2023
Accepted: 23rd December 2023

Keywords:

Akaike Information Criterion; Expectation Maximization Algorithm; Overdispersion; Poisson Distribution; Poisson Hidden Markov Models

The article titled "Modeling the Many Earthquakes in Sumatra Using Poisson Hidden Markov Models and Expectation Maximization Algorithm" presents a comprehensive study on earthquake prediction modeling in Sumatra. This research is crucial considering Sumatra's high seismic activity due to its location at the confluence of three major tectonic plates. Sumatra Island is one of the islands that are prone to earthquakes because it is located at the confluence of three plates, namely the large Indo-Australian plate, the Eurasian plate and the Philippine plate. In general, the number of earthquake events follows the Poisson distribution, but there are cases where there is overdispersion in the Poisson distribution. The Poisson Hidden Markov Models (PHMMs) method is used to overcome overdispersion, and then the Expectation-Maximization Algorithm (EM algorithm) is used in each model to obtain the estimated parameters. From the models obtained, the best model will be selected based on the smallest Akaike Information Criterion (AIC) value. The data used is earthquake event data on Sumatra Island, obtained from the United States Geological Survey (USGS) catalog from January 2000 to December 2022, with a depth of ≤70 Km and a magnitude of ≥4.4 Mw. From the research, the model with m = 3 is the best estimation model with an AIC value of 1503,286. From the best model, estimates are obtained for Poisson Hidden Markov Models with an average occurrence of earthquakes of 5.7633 ≈ 6 events within one month.

How to cite this article:

M. A. Alwansyah and R. Rachmawati., "MODELING THE MANY EARTHQUAKES IN SUMATRA USING POISSON HIDDEN MARKOV MODELS AND EXPECTATION MAXIMIZATION ALGORITHM," *BAREKENG: J. Math. & App.*, vol. 18, iss. 1, pp. 0163-0170, March, 2024.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article • Open Access

1. INTRODUCTION

An earthquake is an event of shaking or shaking of the earth caused by a sudden release of energy, which is marked by the breaking of rock layers in the earth's crust [1]. Earthquakes caused by the movement of tectonic plates are called tectonic earthquakes. Other than that, earthquakes can occur due to volcanic activity which is referred to as volcanic earthquakes. The movement suddenly from the rock layers in the earth produces energy emitted in all directions in the form of earthquake waves or seismic waves. When these waves reach the earth's surface, the vibrations can damage everything on the earth's surface, such as buildings and other infrastructure that can cause casualties and property [2]. Indonesia is a country with a high level of earthquake risk [3]. An earthquake happens because Indonesia is located in the Pacific Ring of Fire, with many active volcanoes, so volcanic earthquakes often occur. Indonesia is also located at the confluence of three plates, namely the Indo-Australian Plate, the Pacific Plate, and the Eurasian Plate, so it is prone to tectonic earthquakes [4]. Sumatra Island is one of the most active tectonic regions in the world. According to [3], 6 of the 25 earthquake-prone areas in Indonesia are on the island of Sumatra, including Aceh, Jambi, Bengkulu, Lampung, West Sumatra and North Sumatra. The high risk of earthquakes on the island of Sumatra is influenced by the geographical conditions of the region, where active faults, volcanic paths and subduction zones traverse the entire area of the island of Sumatra. Several large-scale earthquakes caused damage and even claimed lives, namely the Mentawai earthquake that occurred in 2010 with a magnitude of 7.7 M, the Nias earthquake that occurred in 2005 with a magnitude of 8.6 M, and the Aceh earthquake in 2004, which claimed 250,000 lives and triggering earthquakes and tsunamis in several neighboring countries, namely Thailand, Sri Lanka, and India [3].

Almost all events or incidents that occur in nature are probabilistic or random. A process that can predict or explain these events is called a stochastic process. The Poisson process is a stochastic process whose formation is based on the Poisson distribution and its independence properties. If a random variable has a Poisson distribution, the mean and variance are assumed to be the same [5]. However, in practice, it is more common to find data that experiences overdispersion, namely a situation where the variance of the dependent variable is greater than the average [6]. There is the influence of other variables or sources of diversity from an event that cannot be observed directly, which results in the probability of an event occurring depending on the previous event, which is one of the causes of overdispersion [7]. The causes of these events sometimes form a Markov chain.

The Markov chain is an event process where the conditional probability of the next event X_{t+1} depends only on the time of the current event X_t [8]. One special form of the Markov chain is the hidden Markov model. The hidden Markov model is a discrete-time model consisting of two parts. The first part is the hidden or unobservable cause of the event and forms a Markov chain, while the second part is the observation process or the observed part, which depends on the event's cause [9].

In general, the event characteristics of the many earthquakes in a certain period are approximated by a Poisson distribution. Research on earthquake events using the Poisson distribution in Sulawesi conducted by Pertiwi showed that large earthquakes ($M \geq 5$) in the East Luwu, Morowali, and North Morowali areas did not occur in grouping types [10]. However, there are times when applying the Poisson distribution, especially in the case of earthquakes, when there is a variance value greater than the average (overdispersion), so that the initial assumptions are not fully met and the distribution as a model is inaccurate. Therefore, this research uses an additional method, namely the Expectation Maximization Algorithm, to overcome the problem of overdispersion and the need to know the average and modeling of earthquake events on the island of Sumatra to minimize the negative impact of earthquakes.

Based on the brief description above, the study aims to implement Poisson Hidden Markov Models (PHMMs) combined with the Expectation Maximization (EM) algorithm to model and estimate the average number of earthquakes in Sumatra per month.

2. RESEARCH METHODS

Data for earthquake events from January 2000 to December 2022 was utilized, considering events with a depth of ≤70 Km and a magnitude of ≥4.4 Mw. The study employs PHMMs to address the overdispersion in the Poisson distribution of earthquake occurrences. The EM algorithm is applied to each model to estimate parameters, and the Akaike Information Criterion (AIC) is used for model selection.

2.1 Markov Chain

The Markov chain is a stochastic process that states that if a random variable $\boldsymbol{X}$ is given with a time index $\boldsymbol{t}$ $(\boldsymbol{X_t})$, then the value of $\boldsymbol{X_s}$ for s $> \boldsymbol{t}$ is not affected by the value of $\boldsymbol{X_u}$ for $\boldsymbol{u} < \boldsymbol{t}$, with $\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{u} \in \boldsymbol{N}$ [5]. The following are important definitions of the Markov chain.

1. Homogeneous Markov Chain. According to [11], a Markov chain is said to be homogeneous if the transition probability from the state $\boldsymbol{i}$ at time $\boldsymbol{t}$ to state $\boldsymbol{j}$ at time $\boldsymbol{t+1}$ can be expressed as $\boldsymbol{P(X_{t+1} = j|X_t = i) = P(X_2 = j|X_1 = i) = p_{ij}}$.
2. Aperiodic State. According to [12], a state i is aperiodic if $\boldsymbol{d(i) = 1}$, $\boldsymbol{d(i)}$ is the greatest common factor for $\boldsymbol{n}$ so that $\boldsymbol{P(X_{t+n} = i|X_t = i) > 0}$.
3. Irreducible Markov Chain. According to [11], a Markov chain is irreducible if all states communicate.

2.2 Poisson Hidden Markov Models

2.2.1 Hidden Markov Models

The hidden Markov Model is one of the stochastic processes when the future only depends on the present condition and has a hidden state that cannot be observed [13]. The hidden Markov Model stochastic process consists of observable or observable parts $\{X_t : t \epsilon N\}$, otherwise referred to as the "dependent state process," and the unobserved part $\{C_t : t \epsilon N\}$ that satisfies the Markov property. In the observed section, distribution X_t only depends on the current conditions C_t and does not depend on previously observed conditions X_{t-1}. The following is a representation of Hidden Markov Models:

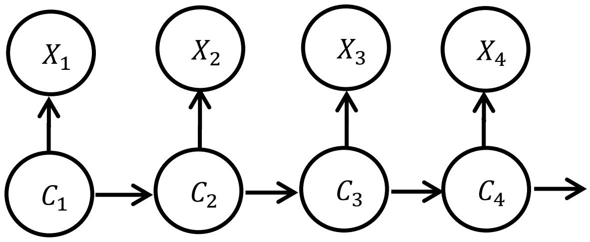

Figure 1. Basic Graph of Hidden Markov Model

Hidden Markov Model $\{X_t : t \epsilon N\}$ is a dependent mixed distribution, with $X^{(t)}$ and $C^{(t)}$ representing past events from time 1 to time t, which the simple model concludes with the equation [14]:

$$P(C_t \mid C^{(t-1)}) = P\ (C_t \mid C_{t-1}), t = 2, 3, \ldots \quad (1)$$

$$P(X_t \mid\ C^{(t-1)}, C^{(t)}) = P\ (X_t \mid C_t), t \in N \quad (2)$$

If the Markov chain $\{C_t\}$ has m hidden states, we say $\{X_t\}$ is a Hidden Markov Model with m states. The following is the definition of the probability mass function X_t if the Markov chain at time t is in state i in the case of discrete observations [14]:

$$pi(x) = Pr(X_t = x \mid C_t = i),\ \ i = 1, 2, \ldots, m \quad (3)$$

Distribution p_i with m hidden states can be said to be state-dependent distributions.

2.2.2 Marginal Distribution and Moment Hidden Markov Models

In the stationary case, the expected value depends on $E(g(X_t))$ and $E(g(X_t, X_{t+k}))$ for each function g is as follows [14]:

$$g(X_t) = \sum_{i=1}^{m} \delta_i \, E(g(X_t)|C_t = i) \tag{4}$$

and

$$\begin{aligned} E(g(X_t, X_{t+k})) &= \sum_{i,j=1}^{m} E(g(X_t, X_{t+k}) \; |C_t = i, C_{t+k} = j))\delta_i \gamma_{ij}(k) \\ &= \sum_{i,j=1}^{m} E(g_1(X_t|C_t = i))E(g_2(X_{t+k}|C_{t+k} = j)\delta_i \gamma_{ij}(k) \end{aligned} \tag{5}$$

Equation (3) and **Equation (4)** are useful for getting the expected value of the Hidden Markov Model. For example, if there are two states of the Hidden Markov Model where the Markov chain is a Poisson distribution and is stationary, then **[4]**:

$$E(X_t) = \delta_1 \lambda_1 + \delta_2 \lambda_2 \tag{6}$$

2.3 Likelihood Scaling

In the case of discrete state-dependent distributions, the element probability values α_t will get smaller as t increases and eventually round off to 0, or the so-called underflow. Likelihood scaling is one way to overcome the underflow problem by calculating the logarithm value of the likelihood, or what is commonly called the opportunity vector scaling forward probability vector scaling α_t. Defined a vector for $t = 0,1, \dots, T$ is:

$$\emptyset_t = \alpha_t / w_t \tag{7}$$

with: $w_t = \sum_i \alpha_t(i) = \alpha_t 1'$.

2.4 Estimation of Expectation Maximization Algorithm

The Estimation Maximization Algorithm method estimates parameter in the Hidden Markov Model. In the context of the Hidden Markov Model, the Estimation Maximization Algorithm is known as the Baum-Welch algorithm, where the Markov chain in the Hidden Markov Model is homogeneous and does not have to be stationary. The parameters of the Hidden Markov Model that are estimated using the Estimation Maximization Algorithm are the state distribution depending on p_i, the probability transition matrix, and the initial distribution δ. In its application, the Estimation Maximization Algorithm requires tools, namely forward opportunities and backward opportunities, where both opportunities can be used for state prediction **[15]**.

1. Forward Opportunities
   Forward opportunities α_t for $t = 1,2, \dots ,T$ defined as a row vector:

$$\alpha_t = \delta P(x_1)\Gamma P(x_2) \dots \Gamma P(x_t) = \delta P(x_1) \prod_{s=2}^{t} \Gamma(x_s) \tag{8}$$

   Where δ is the initial Markov chain distribution. Based on the definition of future opportunities above, $t = 1, 2, \dots, T - 1$ can be written $\alpha_{t+1} = \alpha_t \Gamma P(x_{t+1})$, or $\alpha_{t+1}(j) = \left(\sum_{i=1}^{N} \alpha_t(i)\gamma_{ij}\right) p_j(x_{+1})$ means $\alpha_t(j)$ where j is a component α_t joint opportunities Pr $(X_1 = x_1, X_2 = x_2, \dots, X_t = x_t, C_t = j)$

2. Reverse Opportunity
   Reverse Opportunity β_t for $t = 1, 2, \dots, T$ defined as a row vector:

$$\beta'_t = \Gamma P(x_1)\Gamma P(x_2) \dots \Gamma P(x_T)1' = \left(\prod_{s=t+1}^{t} \Gamma \mathrm{P}\,(x_s) \right) 1' \tag{9}$$

Where to $t = T, \beta_T = 1$. Based on the definition of backward opportunity above, $t = 1,2, \dots, T-1$, can be written $\beta'_t = \Gamma P(x_1)\beta'_{t1}$.

3. Forward Opportunity and Backward Opportunity

A combination of forward and backward opportunities α_t and β_t can be used to calculate $P(X^{(T)} = x^{(T)}, C_t = i)$. Where the combination of these opportunities is needed in the Expectation Maximization Algorithm in the Hidden Markov Model.

In its application, it takes two properties of them:

a. $t = 1, 2, \dots, T$,

$$P(C_t = j | X^{(T)} = x^{(T)}) = \alpha_t(j)\, \beta_t(j)/LT \tag{10}$$

b. $t = 2, \dots, T$.

$$P(C_{t-1} = j, C_t = k | (X^{(T)} = x^{(T)}) = \alpha_{t-1}(j)\gamma_{jk}p_k(x_t)\beta_t(k)/L_T \tag{11}$$

Unobserved Markov chain state sequences may have missing data, resulting in incomplete data. The Expectation-Maximization Algorithm, as an iterative method, functions in calculating the maximum likelihood estimate for incomplete data. In each iteration of the Expectation Maximization Algorithm, there are E (Expectation) and M (Maximization) stages [16].

1. Expectation Stage

Replace all values v_{jk} and $u_j(t)$ with their conditional expectations if it is given an observation $x^{(T)}$.

$$\hat{u}_j(t) = P(C_t = j | \boldsymbol{x}^{(T)}) = \alpha_t(j)\beta_t(j)/LT \tag{12}$$

and

$$\hat{v}_{jk}(t) = P(C_{t-1} = j, C_t = k | \boldsymbol{x}^{(T)}) = \alpha_{t-1}(j)\gamma_{jk}p_k(x_t)\beta_t(k)/L_T \tag{13}$$

2. Maximization Stage

After changing the value v_{jk} and $u_j(t)$ with $\hat{u}_j(t)$ and $\hat{v}_{jk}$, then the continuous-time double layered levy (CDLL) maximization is carried out, which is related to three parameters, namely the parameter of the state-dependent distribution $(\lambda_1, \lambda_2, \dots, \lambda_3)$, initial distribution δ, and the transition opportunity matrix. The CDLL log-likelihood contains a series of observations $x_1, x_2, \dots, x_T$ as well as lost data $c_1, c_2, \dots, c_T$ shown in the following equation:

$$\log\left(\mathrm{P}\left(X^{(T)}, c^{(T)}\right)\right) = \sum_{j=1}^{m} u_j\,(1) log\delta_j + \sum_{j=1}^{m}\sum_{k=1}^{m}\left(\sum_{t=2}^{T} v_{jk}(t)\right) log \sum_{j=1}^{m} u_j + \sum_{j=1}^{m}\sum_{t=1}^{T} u_j(t) log p_j(x_t) \tag{14}$$

2.5 Model Selection based on Akaike Information Criterion (AIC)

The criteria for selecting the best estimation model in this study are based on the AIC (Akaike Information Criterion) value. The AIC equation in selecting the model proposed by Akaike is [14]:

$$AIC = -2\, log\ L + 2p \tag{15}$$

Where log L is the log-likelihood value of each model and p is the number of parameters in the model, this study will the EM algorithm method to find 3 estimation models, namely the model with a hidden state $m = (2,3,4)$. The best estimation model based on the AIC standard is the model with the smallest AIC value [4].

2.6 Earthquake

An earthquake is when the earth vibrates or shakes due to the sudden movement or shift of the rock layers in the earth's crust due to the movement of tectonic plates. The energy accumulation that causes earthquakes results from the movement of tectonic plates. The resulting energy is emitted in all directions in earthquake waves so that the effects can be felt up to the earth's surface. Earthquakes are one of the natural phenomena that cannot be avoided or prevented. Therefore an earthquake is one of the biggest disasters because of the risks it can cause [17].

3. RESULTS AND DISCUSSION

The article could benefit from a more thorough discussion of the implications of these findings for earthquake preparedness and risk management in Sumatra.

3.1 Data Types and Sources

The data used is earthquake event data on Sumatra Island, obtained from the United States Geological Survey (USGS) catalog from January 2000 to December 2022, with a depth of ≤ 70 Km and a magnitude of ≥ 4.4 Mw. The data screening criteria are due to these conditions. Earthquakes can be felt and have the potential to damage the surrounding area.

3.2 Data Analysis Method

The analysis process in this study uses R studio Version 4.2.2 software. The data analysis method used in this study is as follows:

1. Descriptive analysis, used to determine overdispersion in earthquake data.
2. Poisson Hidden Markov Models are used to obtain the input parameters, which are then estimated from these parameters.
3. Estimating the Expectation Maximization-Algorithm, used to obtain the best model from a given hidden state by looking at the smallest Akaike Information Criterion value.

3.3 Data Overdispersion Check

Checking for overdispersion was carried out on data on the number of earthquake events on the island of Sumatra that occurred with a depth of ≤ 70 Km and a magnitude of ≥ 4.4 Mw monthly. The following is data on the number of earthquake events:

Table 1. Data on the Many Earthquakes

Month	Number of Earthquakes
Jan-2000	4
Feb-2000	1
Mar-2000	6
Apr-2000	4
⋮	⋮
Nov-2022	12
Dec-2022	10

Based on **Table 1**, in January 2000, there were 4 earthquakes. In February 2000, there was 1 earthquake. In March 2000, there were 6 earthquakes. In April 2000, there were 4 earthquakes, until in December 2022, there will be 10 earthquakes. The characteristics of the data follow the Poisson distributed data. The following results of the calculation of the average and variance are shown in **Table 2** below.

Table 2. Descriptive Statistics of Earthquake Event Data

N	$\bar{x}$	s^2	Maximum	Minimum
264	9.1014	21.4224	39	0

Based on **Table 2** shows that the average value is 9.1014, the variant value is 21.4224, the maximum value is 39 and the minimum value is 0. Because the variant value is greater than the average value, it can be said that the data on the number of earthquake events occur in overdispersion to the Poisson distribution.

3.4Modeling Using Poisson Hidden Markov Models

3.4.1 Determination of Input Parameters

The determination of the input parameters is a step in finding the initial parameter values for each model, namely finding the average parameter for the number of earthquakes $\lambda_i = (\lambda_1, \ldots, \lambda_m)$ where every λ_i has a Poisson distribution criterion with the initial probability of the event δ and the transition probability matrix Γ.

Table 3. Calculation Results of Input Parameters in Each State

Model	i	λ	δ	Γ			
				1	**2**	**3**	**4**
$m = 2$	1	8.765	0.986	0.989	0.011		
	2	32	0.014	0.750	0.250		
$m = 3$	1	7.746	0.841	0.866	0.126	0.009	
	2	14.675	0.145	0.725	0.25	0.025	
	3	32	0.014	0.5	0.25	0.25	
$m = 4$	1	6.256	0.580	0.675	0.313	0.006	0.006
	2	12.348	0.406	0.463	0.528	0	0.009
	3	27	0.004	0	0	0	1
	4	33.677	0.011	0.667	0.333	0	0

3.4.2 Poisson Hidden Markov Model Parameter Estimation with Expectation Maximization Algorithm

The next step is to calculate the estimated parameter values $\hat{\lambda}$, $\hat{\delta}$, and $\hat{\Gamma}$ for each model using the Expectation Maximization algorithm.

Table 4. Parameter Estimation Results of the Expectation Maximization Algorithm in Each Poisson Hidden Markov Model

Model	**AIC**	i	$\hat{\lambda}$	$\hat{\delta}$	$\hat{\Gamma}$			
					1	**2**	**3**	**4**
$m = 2$	1584.895	1	8.759	1	0.9889	0.011		
		2	31.346	0	0.738	0.262		
$m = 3$	1503.286	1	5.7633	1	0.941	0.059	0	
		2	10.006	0	0.0101	0.974	0.016	
		3	31.833	0	0.444	0.301	0.254	
$m = 4$	1515.821	1	6.224	1	0.936	0.034	0.020	0.0103
		2	10.189	0	0.013	0.986	0	0
		3	28.438	0	0	0	0	1
		4	29.410	0	1	0	0	0

Based on **Table 4**, the parameter estimation results in each hidden state use Poisson Hidden Markov Models with the Expectation Maximization Algorithm. From the results obtained, the smallest AIC value is 1503.286, so the model with 3 hidden states $(m = 3)$ is the best model compared to $m = 2$ and $m = 4$. The following are the best parameter estimation results for Poisson Hidden Markov Models with 3 hidden states:

$$\hat{\boldsymbol{\lambda}} = (5.7633, 10.006, 31.833)$$
$$\hat{\boldsymbol{\delta}} = (1\ ,0\ ,0)$$
$$\hat{\boldsymbol{\Gamma}} = \begin{bmatrix} 0.941 & 0.059 & 0 \\ 0.010 & 0.974 & 0.016 \\ 0.444 & 0.301 & 0.254 \end{bmatrix}$$

With the expected value and variance of Poisson Hidden Markov Models, namely:

$$\begin{aligned} E(X_t) &= \sum_{i=1}^{3} \delta_i \lambda_i \\ &= \delta_1\lambda_1 + \delta_2\lambda_2 + \delta_3\lambda_3 \\ &= (5.7633 \times 1) + (10.006 \times 0) + (31.833 \times 0) \\ &= 5.7633 \end{aligned}$$
$$Var(X_t) = E(X_t) = 5.7633$$

So, it can be concluded that of the three models for estimating the number of earthquake events in Sumatra, the model with 3 hidden states is the best model for estimating the number of earthquake events with an estimated parameter value of the average number of earthquakes that have occurred as many as $5.7633 \approx 6$ events within a period one month.

4. CONCLUSIONS

The model with three hidden states $(m = 3)$ was found to be the best estimation model based on the smallest AIC value of 1503,286. The best model estimates an average of approximately 6 earthquake events in Sumatra within one month.

REFERENCES

[1] S. Mutiah, H. Pratiwi, and S. Handajani, "Penerapan Model Epidemic Type Aftershock Sequence (ETAS) pada Data Gempa Bumi Sulawesi dan Jawa," *Prosiding-M28*, 2019.

[2] M. A. Alwansyah, N. Hidayat, D. I. Fadli, D. S. Rini, and P. Jana, "Spatial zone analysis of earthquake in Sumatra," *Proceeding 7Th Int. Conf. Sci. Technol. Interdiscip. Res. (Ic-Star 2021)*, vol. 2601, p. 020009, 2023.

[3] R. Metrikasari and A. Choiruddin, "Pemodelan Risiko Gempa Bumi di Pulau Sumatera Menggunakan Model Inhomogeneous Neyman-Scott Cox Process," *J. Sains dan Seni ITS*, vol. 9, no. 2, 2021.

[4] R. Suryandaru, "Estimasi Model Terbaik Banyaknya Gempa Bumi Menggunakan Poisson Hidden Markov Models Dan Algoritma EM," Universitas Islam Negeri Syarif Hidayatullah Jakarta, 2015.

[5] H. M.Taylor and S. Karlin, *Introduction to Stochastic Modeling*. 2011. doi: 10.1201/b11089-8.

[6] Y. Widyaningsih, G. P. Arum, and K. Prawira, "Aplikasi K-Fold Cross Validation Dalam Penentuan Model Regresi Binomial Negatif Terbaik," *BAREKENG J. Ilmu Mat. dan Terap.*, vol. 15, no. 2, pp. 315–322, 2021.

[7] S. J. Long, *Regression Models for Categorical and Limited Dependent Variables*. California: International Education and Professional Publisher, 1997.

[8] M. Abdy, W. Sanusi, and R. Rahmawati, "Aplikasi Model Rantai Markov Untuk Menganalisis Tingkat Kenyamanan Di Kota Majene Berdasarkan Temperature Humidity Index (Thi)," *BAREKENG J. Ilmu Mat. dan Terap.*, vol. 15, no. 1, pp. 009–014, 2021.

[9] V. Koerniawan, N. Sunusi, and Raupong, "Estimasi Parameter Model Poisson Hidden Markov Pada Data Banyaknya Kedatangan Klaim Asuransi Jiwa," vol. 1, no. 2, pp. 65–73, 2020.

[10] I. I. Pertiwi, "Analisis Statistik Distribusi Kejadian Gempabumi Di Luwu Timur, Morowali, dan Morowali Utara, Sulawesi," *J. Geofis.*, vol. 20, no. 01, pp. 1–7, 2022.

[11] G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes*, Fourth Edit. Oxford University Press, 2020.

[12] V. Capasso and D. Bakstein, *An Introdusction to Continuousi-Time Stochastic Processes*, Fourth Edi. Birkhauser, 2021.

[13] Y. A. Pradana, D. A. Azka, A. C. Aji, and I. M. Fauzi, "Analysis of Weather Changes for Estimation of Shallot Crops Fluctuation Using Hidden Markov," *BAREKENG J. Ilmu Mat. dan Terap.*, vol. 16, no. 1, pp. 333–342, 2022.

[14] W. Zucchini, I. L. MacDonald and R. Langrock, *Hidden Markov Models for Time Series*, Second Edit. London: CRC Press, 2016.

[15] A. Irawan, "Implementasi Poisson Hidden Markov Models & Expectation Maximization Algorithm Untuk Estimasi Kejadian Gempa Bumi Di Indonesia," Universitas Islam Indonesia, 2021.

[16] Irina Hidayati, "Implementasi Poisson Hidden Markov Models Dan Expectation Maximization Algorithm Dalam Penentuan Model Terbaik," Universitas Islam Indonesia, 2018.

[17] M. Irsyam *et al.*, *Peta Sumber dan Bahaya Gempa Indonesia Tahun 2017*, First edit. Kementrian Pekerjaan Umum dan Perumahan Rakyat, 2017.