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C PROGRAM AS A TOOL FOR THE TEACHING OF SECOND ORDER ORDINARY DIFFERENTIAL EQUATION

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ABSTRACT

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Second order ordinary differential equation (ODE) has many applications in science and engineering. Undergraduate students in the science and engineering departments must study this subject to understand other subjects related to real applications they will encounter later. There are many excellent textbooks on differential equations where the students can study the theory and solve the problems. However, a textbook cannot give a quick answer to a problem, particularly when the problem is quite difficult. A good choice is to use software such as Maple or Mathematica. However, this software is not always available for teaching and purchasing. It is usually beyond the ability of a student or even a lecturer. On the other hand, lecturers who want to create problems by themselves will follow the theory of the ODE. While creating the problems may not be difficult, answering them is harder. Problems that are easy to answer are less worthless because they will not increase the student's knowledge. Here comes the solution. A C program has been created to help lecturers create problems and solve them quickly. The program is interactive and can be easily understood by anyone who has a basic theory of ODE. No knowledge of programming is needed; a user runs it and follows the instructions. Students can also use the program to sharpen their knowledge. They can compare the solution of a problem they have solved with the answer given by the program. While commercial software such as Maple and Mathematica is very powerful, it cannot be used without writing the necessary commands to solve a problem.

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1. INTRODUCTION

A short definition of the differential equation (or DE for short) is given by [1]: A *differential equation* is an equation involving an unknown function and its derivatives. From the definition, we know that an DE must contain derivatives $Y', Y", \ldots$. Given a DE, our task is to find the function which fulfills the equation.

A DE can be divided into Ordinary Differential Equation (ODE) and Partial Differential Equation (PDE). An ODE contains one or more functions of one independent variable and the derivatives of those functions. On the other hand, a PDE has two or more independent variables.

An ODE can be further divided into linear and nonlinear. A linear ODE is given by

$$a_0(x)Y + a_1(x)Y' + a_2(x)Y^{"} + \cdots + a_n(x)Y^{(n)} + f(x) = 0 \quad (1)$$

Here, $a_0(x), a_1(x), \ldots, a_n(x)$ are the coefficients of the DE which are differentiable functions and not necessarily linear while $Y', Y^{"}, \ldots, Y^{(n)}$ are the successive derivatives of the unknown function $Y(x)$ and all of them are linear. $f(x)$ is also a differentiable function. The independent variable is x. $Y', Y^{"}, \ldots, Y^{(n)}$ are often written as $\frac{dY}{dx}, \frac{d^2Y}{dx^2}, \ldots, \frac{d^nY}{dx^n}$. Equation (1) denotes an ODE with order n, the highest degree of the derivatives. In this paper we only discuss a second order, linear ODE with constant coefficients which is written as

$$a\frac{d^2Y}{dx^2} + b\frac{dY}{dx} + cY = f(x) \quad (2)$$

The aforementioned ODE is called non homogenous since it contains a term on the right-hand side. If $f(x) = 0$, the ODE is called homogenous. Basic theory on the DE and the terminology used are described in more detailed in textbooks on differential equations or engineering mathematics; see [2], [3], [4], [5] and [6], for example.

Second order ordinary differential equation (ODE) has many applications in science and engineering. Undergraduate students in the science and engineering departments must study this subject to understand other subjects related to real applications they will encounter later. There are many excellent textbooks on differential equations where the students can study the theory and solve the problems; [3] and [6] are two very readable textbooks. Polyanin and Zaitsev [7] provide many exact solutions for ODE; they are very useful if we want to find the correct answer to our problems. Applications of DE are often given in textbooks; however, most of them are usually simple. We will discuss the applications of DE in real life in the next section.

Most textbooks on DE are good in the exposition of the theory. However, they cannot quickly answer a problem, particularly when the problem is quite difficult. A good choice is to use software such as Maple or Mathematica. However, this software is only sometimes available for teaching and purchasing. It is usually beyond the ability of a student or even a lecturer. A good compromise is to have a mathematics laboratory where the students can use mathematical software to practice. Unfortunately, not every faculty has a mathematics laboratory that offers software to students as well as lecturers. This can be understood due to the expensive price of commercial software. For example, Maple undergraduate teaching lab package licenses for 25 students cost around US$ 5300, while for 50 students, they cost around US$ 8300.

Many lecturers depend on the problems in textbooks when they want to give assignments or exams. Some lecturers who want to create the problems by themselves will follow the theory of the ODE. While creating the problems may not be difficult, answering them is harder. Problems that are very easy to answer are less worthless because they will not increase the student's knowledge. Here comes the solution. A C program has been created to help lecturers create problems and solve them quickly. A lecturer can vary the difficulty of the problems he or she wants to run the program several times until he or she feels satisfied that the problems fulfill the objective of the lecture.

2. RESEARCH METHODS

Differential equations have many applications in engineering, physics, and science. Many phenomena in our lives can be described in differential equations. One popular example is the Newton's law of cooling. Many phenomena in physics and engineering can be written in second-order ODE, which provides solutions for special functions. When an elastic spring is stretched and then released, the DE for the problem can be written as

$$\boldsymbol{X}'' + \boldsymbol{\omega}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{t}) \tag{3}$$

where $\boldsymbol{f}(\boldsymbol{t})$ is the external force acting on the spring. In **Equation (3)**, t is the independent variable which denotes time. In many applications, $\boldsymbol{f}(\boldsymbol{t})$ is usually a sinusoidal function, $\mathbf{sin}\, \boldsymbol{\omega_1 t}$. Solutions of **Equation (3)** are found in resonance or beats; see Tahir-Kelli [8] the treatment of these phenomena.

Second-order ODE also has applications in heat transfer. Most textbooks on heat transfer describe heat transfer in an extended surface or, usually called a fin; see Holman [9], for example, for basic treatment of this subject. One-dimensional conduction and convection in a rectangular fin can be described by

$$\frac{d^2\theta}{dx^2} - \frac{hP}{kA} = 0 \tag{4}$$

Complete treatment of the heat transfer in extended surfaces is given by Kraus, Azis, and Welty [10] where they analyze various types of fins. The case of a longitudinal fin of the triangular profile is given by the following DE.

$$x\frac{d^2\theta}{dx^2} + \frac{d\theta}{dx} - m^2 b\theta = 0 \tag{5}$$

While for a radial fin of radial profile the DE is given by

$$r^2\frac{d^2\theta}{dr^2} + r\frac{d\theta}{dx} - m^2 r^2\theta = 0 \tag{6}$$

Solutions of **Equation (5)** and **Equation (6)** involve Bessel functions. See also Agarwal and O'Regan [1] for the treatment of ODE and PDE with special functions.

The last application we will mention is in the field of control systems. Since the literature on this subject is very vast, we will only mention two widely known textbooks by Golnaraghi and Kuo [11] and Ogata [12]. Mathematical modeling of mechanical and electrical systems, fluids systems, and thermal systems are discussed in detail. Solutions to this kind of problem are usually solved using MATLAB, a very powerful software for control systems. Apart from MATLAB, we can also use Octave and Scilab, which can be freely downloaded from the Internet.

3. RESULT AND DISCUSSION

A lot of mathematical software for solving DE is available. Apart from commercial software such as Maple and Mathematica (which are very expensive), there is also free software that can be searched on Google. However, either commercial or free, the software cannot be used directly without writing the necessary commands to solve a problem. Often, the answer is given in a long-expression that does not resemble the theory you know. You may have to use various commands to simplify the solution. As an example, suppose that we want to solve an ODE as follows:

$$\frac{d^2Y}{dx^2} + 4\frac{dY}{dx} + 13Y = 0$$

If you use Maple, you must write the following (for Maple V)

```
> dsolve(diff(y(x),x$) + 4diff(y(x),x) + 13*y(x)=0,y(x))
```

Maple displays the solution quickly

$$y(x) = C_1 e^{-2x}\cos(3x) + C_2 e^{-2x}\sin(3x)$$

When you add a term on the right-side of the equation as

$$\frac{d^2Y}{dx^2} + 4\frac{dY}{dx} + 13Y = 3\sin(3x) - 5\cos(3x)$$

which in Maple will be written as

```
> dsolve(diff(y(x),x$2) + 4*diff(y(x),x) + 13*y(x) = 3*sin(3*x) − 5*cos(3*x),y(x));
```

you may be shocked to see the answer given by Maple. You expect that the particular solution will be A sin(3x) + B cos(3x) where A and B are constants. However, the particular solution given by Maple has 16 terms. However, you can simplify the long-expression and combine the trigonometric terms so you will get the final answer.

$$\boldsymbol{Y_p(x) = -\frac{3}{10}\sin(3x) - \frac{7}{20}\cos(3x)}$$

Unless you have experienced in using Maple, you may not be able to obtain this result! This situation is very common for symbolic computation software; see Cohen [13], for example. See also Gander and J. Hrebicek [14] and Shingareva and Lizarraga-Celaya [15] for Maple solutions for some engineering problems.

This example does not intend to undermine the power of Maple or any other mathematical software. We want to show you that using software may not be as easy as you expect. This is the reason why we offer an alternative solution as the purpose of our paper.

A good program created as a tool for teaching must be easy to use and, if possible, can be accessed freely by the students and lecturers. Easy to use means a user just enters simple inputs to run it. The program should be interactive and must validate any input before executing it. The output should be nicely displayed; this requirement, however, may not be satisfactorily fulfilled due to various constraints faced by the developers of the program.

The C language is chosen as a tool for the teaching of second-order ODE because there are many C compilers that are free to use. Even though they may not produce very nice outputs, they are still very valuable as long as they can display the solutions moderately well.

Before a C program can be run (or used by a user), it must be first compiled. All errors must be corrected. There are five types of errors: syntax, run-time, linker, logical, and semantic errors. Even if a program has been successfully compiled, it may not produce correct output due to semantic error. For example, area of a triangle is $\frac{bt}{2}$ where b is base and h is height. But if you write it as bt, the compiler does not know that it is wrong! So, it is the responsibility of the user to ensure that his or her program is free from semantic error. For an explanation of these errors, refer to any computer programming book; see Olsson [16] for syntax reference in C. See also Chavan [17], Kalicharan [18], and Prinz [19] for examples of C programs. Lee and Schisser [20] provide routines for ODE and PDE in C and other programming languages. Malik [21] presents problem analysis before a program is designed. Tselikis and Tselikas [22] are also very useful.

Before proceeding to the programs developed for this paper, we will summarize basic info for an ODE. Without loss of generality, we write **Equation (2)** as:

$$\frac{d^2Y}{dx^2} + b\frac{dY}{dx} + cY = f(x) \tag{7}$$

The solution for **Equation (7)** is:

$$Y(x) = Y_c(x) + Y_p(x) \tag{8}$$

$Y_c(x)$ is called complimentary solution which is found from its characteristic equation while $Y_p(x)$ is determined from $f(x)$ which fulfils the DE in the left-hand side of the equation.

Characteristic equation of **Equation (7)** is given by

$$m^2 + bm + c = 0 \tag{8}$$

whose roots are m_1 and m_2.

Discriminant of **Equation (8)** is given by

$$D = b^2 - 4ac \tag{9}$$

We will use the terminology in the control systems to describe the nature of the discriminant. There are three possible solutions for **Equation (8)**:

1) $D = 0$; this called critically damped. The roots are the same or double roots $\left(m_1 = m_2 = -\frac{b}{2}\right)$.

$$Y_c(x) = e^{m_2 x}(C_1 x + C_2)$$

2) $D > 0$; this is called overdamped. The roots are real and different $(m_1 \neq m_2)$ where $m_{1,2} = \frac{(-b \pm \sqrt{D})}{2a}$

$$Yc(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

3) $D < 0$; this is called underdamped. The roots are imaginary $\left(m_{1,2} = \alpha \pm \beta i\right)$ where $\alpha = -\frac{b}{2a}$ and $\beta = \frac{\sqrt{|D|}}{2a}$.

$$Y_c(x) = e^{\alpha x}[C_1 \cos(\beta x) + C_2 \sin(\beta x)]$$

In this section, we will show just two parts of the programs that we have developed. Inputs are integers, and they will be validated before they are executed. The programs have been written in such a way that any user can run them easily. Just click the name of each program, and it is ready to run. It is planned to combine all programs so they become an integrated program that can be used easily. After the program is run, it will display general info on the screen for the user to use the program correctly; see **Figure 1**.

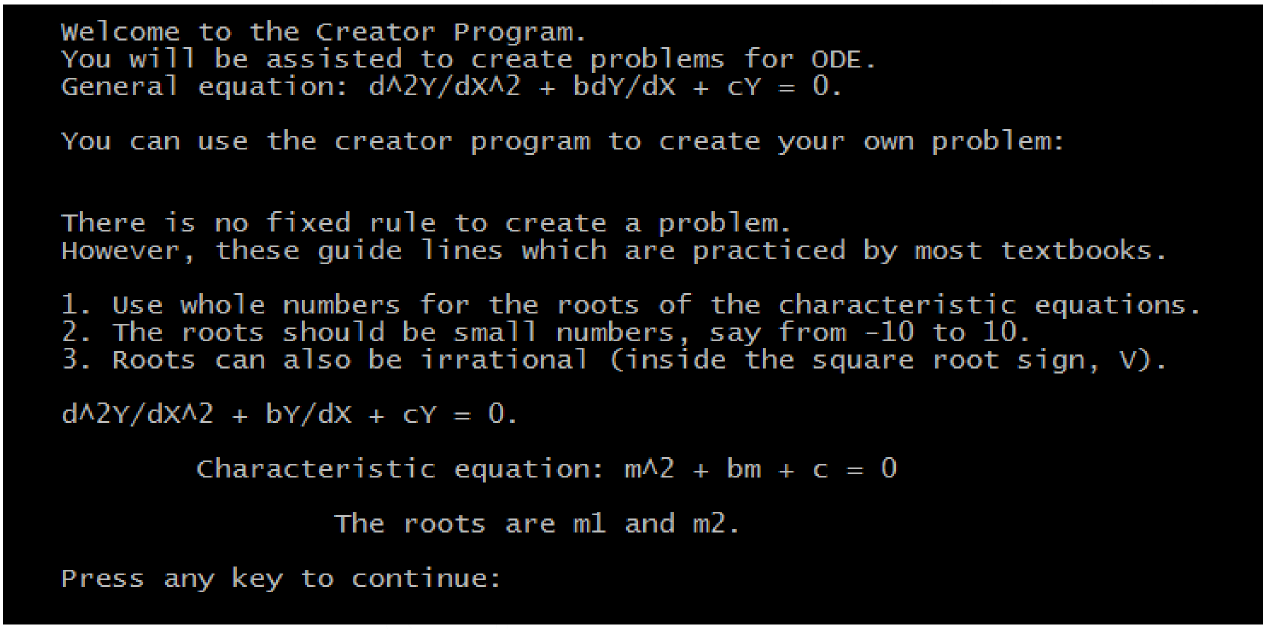

Figure 1. Info page for the user

When the user types any key, another screen is shown, instructing him or her what to do. Here, the user is asked to create his or her own problem. The info for doing this is clearly given. So, the user will be carefully guided step by step to use the program correctly. If the response from the user is not valid, the command will be repeated; see **Figure 2**.

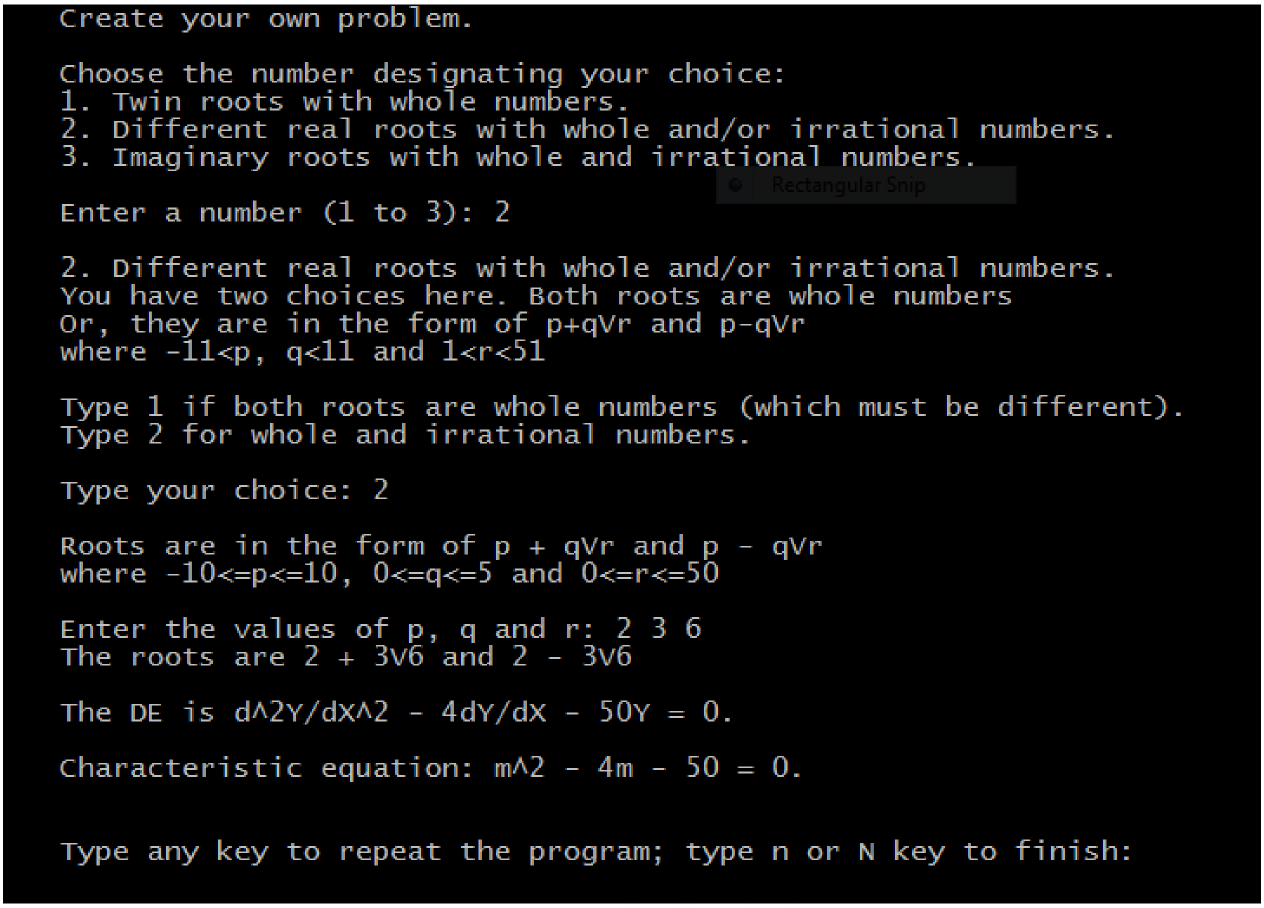

Figure 2. Page for Creating the User's Own Problem

The user can then repeat the program as many times as he or she likes. He or she just follows the instructions displayed on the screen.

4. CONCLUSIONS

A C program as a tool for the teaching of second-order ordinary differential equations has been developed. It has been successful in accomplishing its task. The program can be used to help lecturers create their own problems or solve problems. The program is interactive and can be used very easily. No basic programming is needed. A user just follows the instructions displayed on the screen to use it. Students can also benefit from the program by doing exercises and comparing their results with the results from the program.

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