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MODELING OPEN UNEMPLOYMENT RATE IN KALIMANTAN ISLAND USING NONPARAMETRIC REGRESSION WITH FOURIER SERIES ESTIMATOR

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ABSTRACT

Nonparametric regression is a regression approach used to determine the relationship between the response variable and the predictor variable if the shape of the regression curve is unknown. One of the popular estimators used in nonparametric regression is the Fourier series estimator. Fourier series nonparametric regression is generally used when the pattern of the investigated data is unknown and there is a tendency for the pattern to repeat. The purpose of this study is to estimate nonparametric regression using the Fourier series approach and to find out the factors that influence the open unemployment rate on the island of Borneo in 2021. The criteria for the goodness of the model used Generalized Cross Validation (GCV) and the coefficient of determination (R^2). Based on the results, it was found that the best nonparametric regression model for the Fourier series was the model with 5 oscillations, which indicated a minimum GCV of 10.47 and an R^2 of 74.22%. Furthermore, based on the results of parameter significance testing, either simultaneously or partially, it shows that all predictor variables have a significant effect on the open unemployment rate. The predictor variables include the labor force participation rate, the average length of schooling, the percentage of poor people, the economic growth rate, and the total population.

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1. INTRODUCTION

Regression analysis is a statistical method that is commonly used in analyzing and explaining the relationship between variables, namely the response variable and the predictor variable [1], [2]. There are three types of approaches used to estimate the regression curve, namely parametric, nonparametric, and semiparametric regression [3]. In the parametric regression approach, it is assumed that the shape of the regression curve is known and follows a certain pattern, for example, linear, quadratic, cubic, p-degree polynomial, and so on [4]-[6]. In contrast to parametric regression, the nonparametric regression approach is more flexible because it does not depend on a particular curve shape. The data are expected to be able to find their own form of estimation of the regression curve without being influenced by the subjectivity of the research [7], [8]. In addition to parametric and nonparametric regression approaches, there is also a semiparametric regression approach. Semiparametric regression is used if the regression model contains parametric and nonparametric regression components [9], [10]. Many nonparametric regression approaches have been developed; some of the most frequently used regression estimation methods include spline [11], [12], kernel [13], [14], and Fourier series [15], [16].

One of the most popular estimation methods used by researchers is the Fourier series estimator [17]. The Fourier series is a trigonometric polynomial that has flexibility, so that it can adapt effectively to the local nature of the data [18], [19]. The Fourier series is generally used when the data pattern being investigated is unknown, and there is a tendency for the pattern to repeat [20], [21]. The advantage of the Fourier series approach is that it can handle data if the curve shows cosine and sine waves. Research on the Fourier series was first carried out by Bilodeau [15] and then developed by several researchers, including [22]-[25]. One of the important things in nonparametric Fourier series regression modeling is the selection of smoothing parameters. The Fourier series uses the oscillation parameter as a smoothing parameter. Selection of the optimal oscillation needs to be done to get the best Fourier series estimator in nonparametric regression. One of the methods that can be used to determine the optimal oscillation parameters in Fourier series nonparametric regression is the Generalized Cross Validation (GCV) method. The GCV method theoretically has asymptotically optimal properties, the formula does not contain unknown population variance [26], [27].

The Fourier series nonparametric regression approach can be applied to various fields, one of which is the social field. One of the most common social problems is that the majority of the population has a low level of education, which will lead to high levels of unemployment. The Fourier series nonparametric regression approach can be applied to various fields, one of which is the social field. One of the most common social problems is that the majority of the population has a low level of education which will lead to high levels of unemployment. Not only that, there are many factors that affect the unemployment rate in a region, such as poverty, economic growth, and so on. Therefore, unemployment is one of the main problems that must be addressed immediately [28].

Unemployment is a problem faced by almost all regions in Indonesia, including Kalimantan Island. The main indicator used to measure the unemployment rate in the labor force is the open unemployment rate. The open unemployment rate is the percentage of the number of job seekers to the total workforce. The Central Statistics Agency recorded open unemployment rate in Kalimantan in February 2022 of 4.93%, a decrease of 0.23% points compared to February 2021 of 5.16%. Even though this figure has decreased, it is still relatively high compared to the national target of 4.8%. Based on the background that has been described, a nonparametric regression study will be carried out in this study using the Fourier series approach on the open unemployment rate data on the island of Kalimantan.

2. RESEARCH METHODS

2.1 Nonparametric Regression

Nonparametric regression is a regression method that is not bound by the assumption of a particular regression curve shape; a nonparametric regression approach is used to solve data patterns between response variables and predictor variables that do not form certain patterns. Nonparametric regression provides high flexibility in predicting a model. Therefore, there are many methods that can be used to estimate the nonparametric regression function. The methods that are often used include the Fourier series, spline, kernel, wavelet, k-nearest neighbor, etc.

2.2 Fourier Series Nonparametric Regression

According to Sudiarsa et al. in Adrianingsih et al. [29], the Fourier series is a trigonometric polynomial function that has high flexibility and its curve shows the sine and cosine functions. The Fourier series is one of the estimators that has been widely studied by researchers. The Fourier series estimator used is the sum of a linear function and a trigonometric polynomial function. The multivariable Fourier series nonparametric regression model is completely written in **Equation (1)**.

$$y_i = \frac{1}{2}\alpha_0 + \sum_{j=1}^{p}\left(\gamma_j(x_{ji}) + \sum_{q}^{Q}\alpha_{jq}\cos\ \mathrm{qx}_{ji}\right) + \varepsilon_i, j = 1,2,\ldots,p \tag{1}$$

Equation (1) can be expressed in the matrix form in **Equation (2)**.

$$\vec{\boldsymbol{y}} = \mathbf{X}(\mathbf{Q})\vec{\boldsymbol{\theta}} + \vec{\boldsymbol{\varepsilon}} \tag{2}$$

where $\vec{y}$ is response vector with size $n \times 1$; $\mathbf{X}(\mathrm{Q})$ is matrix of size $n \times (Q+2)$; $\vec{\theta}$ is parameter vector to be estimated with size $(Q+2) \times 1$; and $\vec{\varepsilon}$ is error vector of size $n \times 1$, which is assumed to be identical, independent, and normally distributed with a mean of zero and a variance σ^2.

2.3 Generalized Cross Validation

There are several methods that can be used to determine the optimal oscillation parameters in nonparametric Fourier series regression, one of which is the GCV method. According to Wang in Dani, the GCV method has several advantages compared to other methods, such as Cross Validation (CV) and/or the Unbiased Risk (UBR) method [3]. The GCV method theoretically has asymptotically optimal properties [4], [30], the formula does not contain the unknown population variance $\boldsymbol{\sigma^2}$, and invariance to transformation. The GCV function for selecting optimal oscillations can be shown in **Equation (3)**.

$$GCV(Q) = \frac{MSE(Q)}{(n^{-1}tr(\mathbf{I} - \mathbf{A}(Q)))^2} \tag{3}$$

$$MSE(Q) = n^{-1}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2 \tag{4}$$

where Q is oscillation parameters; $\mathbf{I}$ is identity matrix of size $n \times n$; y_i is response variable to-i; $\hat{y}_i$ is estimator of the response variable and $\mathbf{A}(Q)$ is $\mathbf{X}(Q)[\mathbf{X}(Q)^T\mathbf{X}(Q)]^{-1}\mathbf{X}(Q)^T$

2.4 Model Parameter Testing

Model parameter testing was conducted to determine whether the predictor variable has a significant effect on the response variable. There are two stages in testing model parameters, namely simultaneous testing and then continuing with partial testing [27]. Simultaneous testing uses the F test, while partial testing uses the t-test.

Simultaneous testing is carried out to find out whether all predictor variables have an influence on the response variable. This test is carried out simultaneously or simultaneously with the parameters contained in the model [21]. The following are hypotheses for simultaneous testing:

$H_0: \gamma_1 = \gamma_2 = \ldots = \gamma_p = \alpha_{11} = \ldots = \alpha_{pQ} = 0;$

H_1: there is at least one $\gamma_j = \alpha_{jq} \neq 0; j = 1,2,\ldots,p \;\; q = 1,2,\ldots,Q.$

Test statistics used in parameter testing simultaneously are written in **Equation (5)**.

$$\boldsymbol{F} = \frac{\boldsymbol{MSR}}{\boldsymbol{MSE}} \tag{5}$$

Reject $\boldsymbol{H_0}$ if $\boldsymbol{F > F_{\alpha;(p+pQ,n-(p+pQ)-1)}}$ or $\boldsymbol{p - value < \alpha}$, which means that there is at least one significant parameter of the Fourier series regression model.

Partial testing is done to find out the parameters that are partially significant to the model. The hypotheses for partial testing are as follows:

$H_0 : \gamma_j, \alpha_{jq} = 0$

$H_1 : \gamma_j \neq 0$, $\alpha_{jq} \neq 0; j = 1,2,...,5; q = 1,2,...,5$

Test statistics used:

$$t = \frac{\hat{\theta}_j}{SE(\hat{\theta}_j)} \tag{6}$$

where $\boldsymbol{SE(\hat{\theta}_j)}$ is the standard error of $\boldsymbol{\hat{\theta}_j}$

$$SE(\hat{\theta}_j) = \sqrt{\text{var}(\hat{\theta}_j)} \tag{7}$$

Reject H_0 if $|t|$ greater than $t_{\frac{\alpha}{2};(n-(p+pq)-1)}$ or reject H_0 if $p - value < \alpha$.

3. RESULTS AND DISCUSSION

The analysis includes descriptive and nonparametric regression analysis with the Fourier series approach. The research data used is the open unemployment rate on Kalimantan Island in 2021.

3.1 Data Description

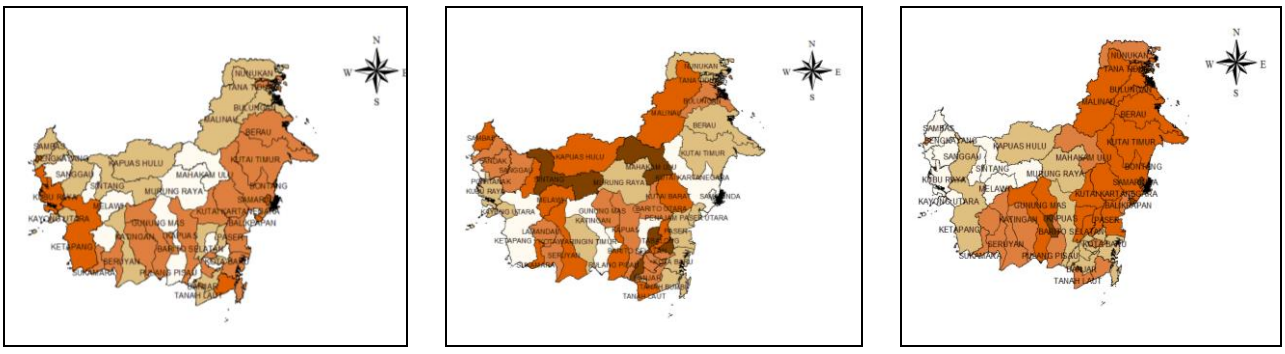
The description of the data is expressed in descriptive statistics which are used to provide an overview of the data. Descriptive statistical analysis used in this study is the average, standard deviation, maximum, and minimum values of each variable used. These variables consist of the open unemployment rate (y), while the independent variables in this study are the labor force participation rate (x_1), the average length of school (x_2), the percentage of poor population (x_3), the economic growth rate (x_4), and the total population (x_5). The results of the descriptive statistical analysis are shown in **Table 1**.

Table 1. Descriptive Statistics

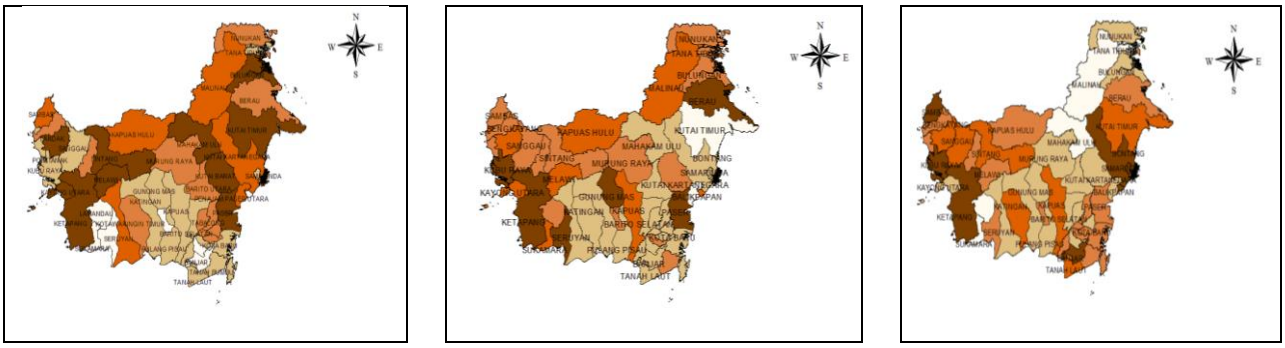
Variable	Mean	Standard Deviation	Maximum	Minimum
$\boldsymbol{y}$	4.95	2.06	12.38	2.30
$\boldsymbol{x_1}$	68.82	3.97	78.4	60.86
$\boldsymbol{x_2}$	8.41	1.22	11.53	6.02
$\boldsymbol{x_3}$	6.30	2.30	12.01	2.89
$\boldsymbol{x_4}$	3.66	1.38	5.61	-1.69
$\boldsymbol{x_5}$	298526.40	195069.10	859250	26000

Based on **Table 1**, it can be seen that the average for the open unemployment rate (y) on Kalimantan Island in 2021 is 4.95% with a standard deviation of 2.06%, which illustrates the distribution of data. The highest open unemployment rate is 12.34% in Pontianak and the lowest open unemployment rate is 2.30% in Lamandau.

After obtaining the average, standard deviation, maximum, and minimum values from the open unemployment rate data and the variables that are thought to have an effect, furthermore, visually the distribution of data from each variable is shown on the spatial mapping in **Figure 1**, where the grouping of each variable is presented in 5 color degradation categories.



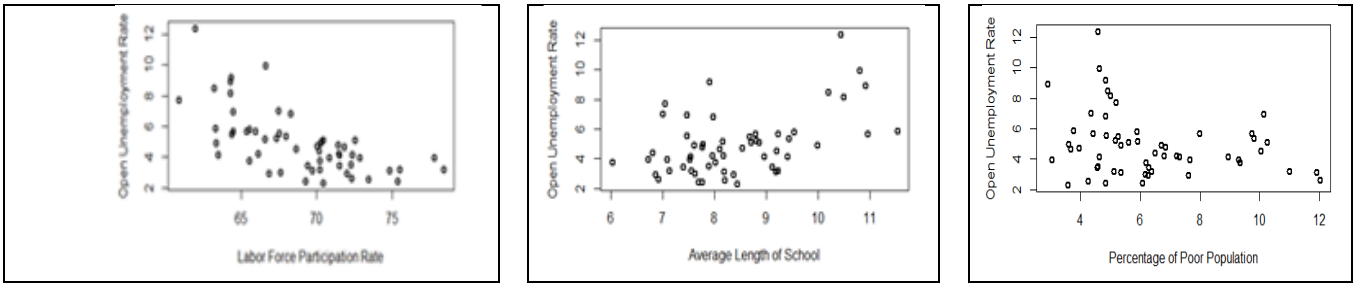
a. Open Unemployment Rate b. Labor Force Participation Rate c. Average Length of School



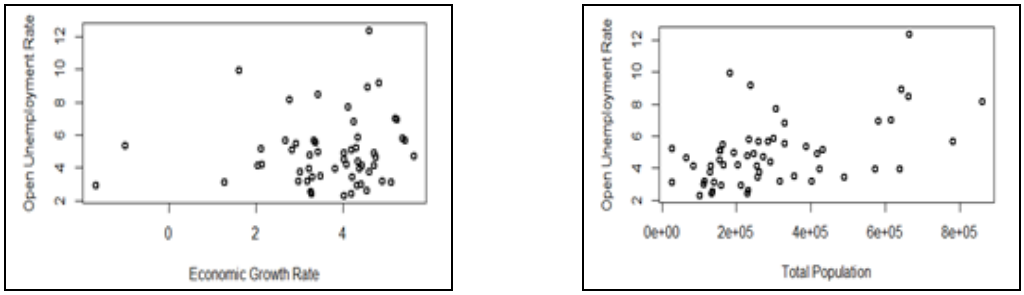
d. Percentage of Poor Population e. Economic Growth Rate f. Total Population

Figure 1. Spatial Mapping of Variable Data

The next step is to do before modeling using nonparametric Fourier series regression, namely the need to know the relationship pattern between the response variable and each predictor variable that is thought to influence it. The following is the pattern of the relationship that is formed between the response variable open unemployment rate and each predictor variable that is thought to have an influence, including the labor force participation rate, the average years of schooling, the percentage of poor population, the economic growth rate, and the total population.



a. Scatterplot of (y) and (x_1) b. Scatterplot of (y) and (x_2) c. Scatterplot of (y) and (x_3)



d. Scatterplot of (y) and (x_4) e. Scatterplot of (y) and (x_5)

Figure 2. *Scatterplot* of Open Unemployment and Predictor Variables

Based on the explanation in **Figure 2**, the scatter plot shows that the pattern of relationship between the open unemployment rate (y) and the labor force participation rate (x_1), the average length of school (x_2), the percentage of poor population (x_3), the economic growth rate (x_4), and the total population (x_5), do not form a specific pattern, so that the model used to model the open unemployment rate in Kalimantan Island is a nonparametric regression model. The estimator used in this study is the Fourier series.

3.2 Determining Optimal Oscillation

Fourier series nonparametric regression modeling is highly dependent on the oscillation parameter (Q). Selection of the optimal oscillation parameters needs to be done to obtain the best Fourier series estimator in nonparametric regression. The number of oscillations in this study will be limited to 1 to 5 oscillations. The selection of optimal oscillation parameters is carried out using the GCV method whose formula can be seen in **Equation (2)**. The optimal oscillation value chosen is the oscillation value that has the minimum GCV. The minimum GCV value and the maximum $R^2$are the optimal oscillation criteria that will be used **[28]**. The results of the analysis for oscillation values 1 to 5 are shown in **Table 2**.

Table 2. GCV and R^2

Oscillation (Q)	GCV	R^2
1	47.75	63.49%
2	22.44	66.37%
3	13.11	67.53%
4	13.50	67.86%
5	10.47	74.22%

Table 2 shows that the minimum GCV value and the greatest coefficient of determination are obtained when the number of oscillations is 5 with a GCV of 10.47 and a coefficient of determination of 74.22%.

3.3 Fourier Series Nonparametric Regression Model

The best Fourier series nonparametric regression model is obtained by using optimal oscillations. Based on the selection of optimal oscillations that have been carried out, the best regression model is the model with 5 oscillations. The estimation of the model of the open unemployment rate in Kalimantan Island with the nonparametric Fourier series regression is as follows:

$$\begin{aligned}\hat{y}_i = {} & 20.12 - 0.12x_{1i} + 44.02\cos x_{1i} + 4.74\cos 2x_{1i} - 32.68\cos 3x_{1i} - 1.54\cos 4x_{1i} - \\ & 21.53\cos 5x_{1i} + 0.75x_{2i} + 74.27\cos x_{2i} + 96.98\cos 2x_{2i} - 5.77\cos 3x_{2i} - \\ & 10.56\cos 4x_{2i} + 13.71\cos 5x_{2i} + 0.14x_{3i} - 0.01\cos x_{3i} - 29.09\cos 2x_{3i} - \\ & 34.13\cos 3x_{3i} - 37.34\cos 4x_{3i} + 12.97\cos 5x_{3i} + 0.17x_{4i} + 59.67\cos x_{4i} - \\ & 59.47\cos 2x_{4i} + 1.25\cos 3x_{4i} - 89.46\cos 4x_{4i} - 10.13\cos 5x_{4i} + 0.00x_{5i} - \\ & 37.09\cos x_{5i} - 1.37\cos 2x_{5i} - 37.09\,3\cos x_{5i} - 1.37\cos 4x_{5i} - 37.09\cos 5x_{5i}\end{aligned}$$

Based on **Table 2** it can be seen that the optimal oscillation value (Q) is as many as 5 oscillations. This is because the value of Q=5 has the smallest GCV value compared to the oscillation values 1, 2, 3, 4, and 5. So, the above model is obtained as the best model. The number of oscillation values tested can actually be more than 5 oscillations, but in this study, it is limited to 5 oscillations because this will affect the number of parameters to be estimated, and the resulting model is not parsimony. The nonparametric regression model with the Fourier series approach can be used to predict the value of the response variable based on the predictor variable. Visually, the comparison between y and yˆ from the Fourier series nonparametric regression model is shown in **Figure 3**.

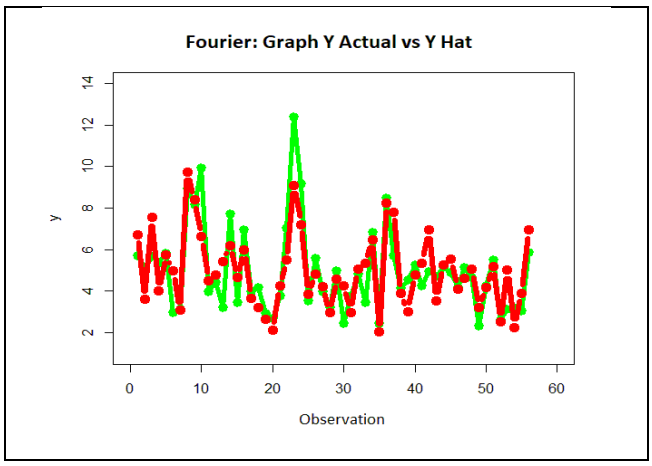


Figure 3. Visualization Comparison of Actual Data with Predicted Data

Based on **Figure 3**, it is known that the prediction results using the best Fourier series nonparametric regression model tend to follow the actual data pattern, which means that the model obtained can be used as a consideration for making decisions in the future.

3.4 Model Parameter Significance Testing

After estimating the model parameters, the next step is to test the significance of the model parameters. This test was carried out in 2 stages, namely, simultaneous and partial testing. Testing the significance of model parameters simultaneously has the following hypotheses:

$$H_0: \gamma_1 = \gamma_2 = \ldots = \gamma_p = \alpha_{11} = \ldots = \alpha_{pQ} = 0$$

$$H_1: \text{there is at least one } \gamma_j = \alpha_{jq} \neq 0; j = 1,2,\ldots,p \;\; q = 1,2,\ldots,Q$$

Based on the results of the analysis of variance in Table 4.4, it can be seen that the F = 3,86 and the p-value is 0.00 with $F_{0,05;(30,25)}$ = 1,92 and a significance level of 0,05. This value indicates that the F value is more than the $F_{0,05;(30,25)}$ value and the p-value is less than α (0,05), so it can be concluded that H_0 is rejected, which means that simultaneously the labor force participation rate variable, average length of schooling, percentage of poor people , the rate of economic growth, and the total population have no effect on the open unemployment rate on Kalimantan Island in 2021.

Furthermore, testing the significance of model parameters partially has the following hypotheses:

$$H_0: \gamma_j, \alpha_{jq} = 0$$
$$H_1: \gamma_j \neq 0\,, \alpha_{jq} \neq 0; j = 1,2,\ldots,5; q = 1,2,\ldots,5$$

Based on the partial test results, it is known that after comparing the p-value with a significance level of 0.05, it is known that all parameter significant. These parameters are considered significant if the p-value is less than the significance level.

4. CONCLUSIONS

Based on the results and discussion that has been obtained, the following conclusions can be drawn:

1. Based on nonparametric regression modeling with the best Fourier series approach on the open unemployment rate data on Kalimantan Island, it can be concluded that the optimal (Q) oscillations obtained are as many as 5 oscillations with a minimum GCV of 10.47
2. The best nonparametric regression model with the Fourier series approach is obtained with Q = 5 as follows:

$$\hat{y}_i = 20.12 - 0.12x_{1i} + 44.02\cos x_{1i} + 4.74\cos 2x_{1i} - 32.68\cos 3x_{1i} - 1.54\cos 4x_{1i} - 21.53\cos 5x_{1i} + 0.75x_{2i} + 74.27\cos x_{2i} + 96.98\cos 2x_{2i} - 5.77\cos 3x_{2i} -$$

$$10.56\cos 4x_{2i} + 13.71\cos 5x_{2i} + 0.14x_{3i} - 0.01\cos x_{3i} - 29.09\cos 2x_{3i} - 34.13\cos 3x_{3i} - 37.34\cos 4x_{3i} + 12.97\cos 5x_{3i} + 0.17x_{4i} + 59.67\cos x_{4i} - 59.47\cos 2x_{4i} + 1.25\cos 3x_{4i} - 89.46\cos 4x_{4i} - 10.13\cos 5x_{4i} + 0.00x_{5i} - 37.09\cos x_{5i} - 1.37\cos 2x_{5i} - 37.09\,3\cos x_{5i} - 1.37\cos 4x_{5i} - 37.09\cos 5x_{5i}$$

3. Based on the parameter significance test, it is found that all predictor variables have a significant effect on the open unemployment rate. These predictor variables include the labor force participation rate, the average length of schooling, the percentage of poor people, the rate of economic growth, and the total population.

REFERENCES

[1] S. Sifriyani, A. T. R. Dani, M. Fauziyah, M. N. Hayati, S. Wahyuningsih, and S. Prangga, "Spline and Kernel Mixed Estimators in Multivariable Nonparametric Regression for Dengue Hemorrhagic Fever Model," *Commun. Math. Biol. Neurosci.*, vol. 2023, pp. 1–15, 2023.

[2] H. Nurcahayani, N. Budiantara, and I. Zain, "The Curve Estimation of Combined Truncated Spline and Fourier Series Estimators for Multiresponse Nonparametric Regression," *Mathematics*, vol. 9, no. 1141, pp. 1–22, 2021.

[3] V. Ratnasari, I. N. Budiantara, and A. T. R. Dani, "Nonparametric Regression Mixed Estimators of Truncated Spline and Gaussian Kernel based on Cross-Validation (CV), Generalized Cross-Validation (GCV), and Unbiased Risk (UBR) Methods," *Int J Adv Sci Eng Inf Technol*, vol. 11, no. 6, pp. 2400–2406, 2021.

[4] G. Wahba, "Improper Priors, Spline Smoothing and the Problem of Guarding Against Model Errors in Regression," *Journal of the Royal Statistical Society: Series B (Methodological)*, vol. 40, no. 3, pp. 364–372, 1978.

[5] J. Opsomer, Y. Wang, and Y. Yang, "Nonparametric Regression with Correlated Errors," *Statistical Science*, vol. 16, no. 2, pp. 134–153, 2001.

[6] S. Barber and G. P. Nason, "Real nonparametric regression using complex wavelets," *J R Stat Soc Series B Stat Methodol*, vol. 66, no. 4, pp. 927–939, 2004.

[7] N. Y. Adrianingsih, I. N. Budiantara, and J. D. T. Purnomo, "Mixture Model Nonparametric Regression and Its Application," in *Journal of Physics: Conference Series*, IOP Publishing Ltd, Mar. 2021.

[8] C. Gu, "Diagnostics for nonparametric regression models with additive terms," *J Am Stat Assoc*, vol. 87, no. 420, pp. 1051–1058, 1992.

[9] J. Harezlak, D. Ruppert, and M. P. Wand, *Semiparametric Regression with R*. New York: Springer, 2018.

[10] N. Afifah, I. N. Budiantara, and I. N. Latra, "Mixed Estimator of Kernel and Fourier Series in Semiparametric Regression," in *Journal of Physics: Conference Series*, Institute of Physics Publishing, Jun. 2017.

[11] X. Lin and D. Zhang, "Inference in generalized additive mixed models by using smoothing splines," *J R Stat Soc Series B Stat Methodol*, vol. 61, no. 2, pp. 381–400, 1999.

[12] H. Becher, G. Kauerniann, P. Khomski, and B. Kouyaté, "Using penalized splines to model age and season-of-birth-dependent effects of childhood mortality risk factors in rural burkina faso," *Biometrical Journal*, vol. 51, no. 1, pp. 110–122, 2009.

[13] R. Hidayat, I. N. Budiantara, B. W. Otok, and V. Ratnasari, "Kernel-Spline Estimation of Additive Nonparametric Regression Model," in *IOP Conference Series: Materials Science and Engineering*, Institute of Physics Publishing, Jul. 2019.

[14] A. T. R. Dani, V. Ratnasari, and I. N. Budiantara, "Optimal Knots Point and Bandwidth Selection in Modeling Mixed Estimator Nonparametric Regression," *IOP Conf Ser Mater Sci Eng*, vol. 1115, no. 1, p. 012020, Mar. 2021.

[15] M. Bilodeau, "Fourier smoother and additive models," *Canadian Journal of Statistics*, vol. 20, no. 3, pp. 257–269, 1992.

[16] Suparti, R. Santoso, A. Prahutama, A. R. Devi, and Sudargo, "Modeling longitudinal data based on Fourier regression," in *Journal of Physics: Conference Series*, Institute of Physics Publishing, Jun. 2019.

[17] I. Wayan Sudiarsa, I. Nyoman Budiantara, S. Suhartono, and S. W. Purnami, "Combined estimator Fourier series and spline truncated in multivariable nonparametric regression," *Applied Mathematical Sciences*, vol. 9, no. 97–100, pp. 4997–5010, 2015.

[18] Suparti, R. Santoso, A. Prahutama, and A. R. Devi, "Indonesia's Inflation Analysis Using Hybrid Fourier - Wavelet Multiscale Autoregressive Method," in *Journal of Physics: Conference Series*, Institute of Physics Publishing, Sep. 2019.

[19] M. A. D. Octavanny, I. N. Budiantara, H. Kuswanto, and D. P. Rahmawati, "Modeling of Children Ever Born in Indonesia Using Fourier Series Nonparametric Regression," in *Journal of Physics: Conference Series*, IOP Publishing Ltd, Feb. 2021.

[20] N. P. A. M. Mariati, I. N. Budiantara, and V. Ratnasari, "Modeling Poverty Percentages in the Papua Islands using Fourier Series in Nonparametric Regression Multivariable," in *Journal of Physics: Conference Series*, Institute of Physics Publishing, Dec. 2019.

[21] H. Husain, I. N. Budiantara, and I. Zain, "Mixed Estimator of Spline Truncated, Fourier Series, and Kernel in Biresponse Semiparametric Regression Model," *IOP Conf Ser Earth Environ Sci*, vol. 880, no. 1, 2021.

[22] A. T. R. Dani and N. Y. Adrianingsih, "Pemodelan Regresi Nonparametrik dengan Estimator Spline Truncated vs Deret Fourier," *Jambura Journal of Mathematics*, vol. 3, no. 1, pp. 26–36, 2021.

[23] A. T. R. Dani, A. F. Dewi, and L. Ni'matuzzahroh, "Studi Simulasi dan Aplikasi: Estimator Deret Fourier pada Pemodelan Regresi Nonparametrik," in *Prosiding Seminar Nasional Matematika, Statistika, dan Aplikasinya*, 2022, pp. 279–288.

[24] M. A. D. Octavanny, I. N. Budiantara, H. Kuswanto, and D. P. Rahmawati, "Nonparametric Regression Model for Longitudinal Data with Mixed Truncated Spline and Fourier Series," *Abstract and Applied Analysis*, vol. 2020, 2020.

[25] L. Laome, I. N. Budiantara, and V. Ratnasari, "Estimation Curve of Mixed Spline Truncated and Fourier Series Estimator for Geographically Weighted Nonparametric Regression," *Mathematics*, vol. 11, pp. 1–13, 2023.

[26] Sifriyani, S. H. Kartiko, I. N. Budiantara, and Gunardi, "Development of Nonparametric Geographically Weighted Regression Using Truncated Spline Approach," *Songklanakarin Journal of Science and Technology*, vol. 40, no. 4, pp. 909–920, 2018.
[27] Sifriyani, I. N. Budiantara, S. H. Kartiko, and Gunardi, "A New Method of Hypothesis Test for Truncated Spline Nonparametric Regression Influenced by Spatial Heterogeneity and Application," *Abstract and Applied Analysis*, vol. 2018, 2018.
[28] Sifriyani, H. Ilmi, and Z. Mar'Ah, "Application of Nonparametric Geographically Weighted Spline Regression Model for Spatial Mapping of Open Unemployment Rate in Kalimantan," *J Phys Conf Ser*, vol. 2123, no. 1, 2021.
[29] N. Y. Adrianingsih, I. N. Budiantara, and J. D. T. Purnomo, "Modeling with Mixed Kernel, Spline Truncated and Fourier Series on Human Development Index in East Java," *IOP Conf Ser Mater Sci Eng*, vol. 1115, no. 1, p. 012024, Mar. 2021.
[30] G. Wahba and Y. Wang, "Spline Function," *Encyclopedia of Statistical Sciences*, pp. 1–27, 2014.