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EARTHQUAKE FREQUENCY DATA MODELING IN MENTAWAI USING FUZZY TIME SERIES LEE AND FUZZY TIME SERIES TSAUR

Septri Damayanti [1*], Jose Rizal[2], Siska Yosmar[3], Nur Afandi[4], Vivin Acnesya[5]

[1,2,3,4,5]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Bengkulu
WR. Supratman Street, Kandang Limun, Muara Bangka Hulu, Bengkulu, 38371, Indonesia

*Corresponding author's e-mail: *septridamayanti@unib.ac.id*

ABSTRACT

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The Fuzzy Time Series (FTS) was first studied by Song and Chissom based on the theory of fuzzy sets and the concept of linguistic variables and their applications discovered by Zadeh. FTS has several models, namely FTS Lee, FTS Tsaur, and so on. In this study, we will model earthquake frequency data in Mentawai using FTS Lee and FTS Tsaur. The seismicity data used in this study is earthquake frequency data in the Mentawai which are calculated from 1960 to 2022. Additionally, the seismicity data source is taken from the U.S. Geological Survey catalog. Based on MAPE and MSE, the results obtained on the FTS Lee and FTS Tsaur models are MAPE values of 37,511% and 27,051%. And the MSE values obtained were 27,073 and 11,671. Thus, the best model used in modeling data on the frequency of earthquake occurrences in the Mentawai Islands is the FTS Tsaur model.

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1. INTRODUCTION

Geographically, Indonesia is an archipelago country located between the confluence of three large world plates: the Eurasian, Australian, and Pacific. The three plates collide with each other. This causes Indonesia to become a country that has the potential for enormous natural disasters, especially earthquakes [1]. One of the areas that often experiences earthquakes in Indonesia is the Mentawai Islands.

Starting from the results of research by a geologist of the California Institute of Technology, Kerry Sieh, in 1994, and a geologist of the Indonesian Institute of Sciences (LIPI), Danny Hilman Natawidjaja, the Mentawai segment, which is located on the western side of the outer island of Siberut, has the potential for an earthquake of M_w is 8.9 [2]. This is in line with the probability model conducted by [3], namely that after the large earthquake that occurred in the Aceh Andaman and Nias Simelue segments, the greatest opportunity for the next large earthquake to occur is around the Mentawai Siberut segment. Furthermore, in 2010, Mentawai experienced a number of earthquakes, namely an earthquake measuring M_w is 6,8 on March 5, 2010, followed by M_w is 6.5 on May 5, 2010, and finally an earthquake measuring M_w is 7,8 on October 25 2010 which was followed by a tsunami disaster [4]. The Mentawai Islands are located in one of the districts in West Sumatra Province. Geographically, this area is between $98^0 35'$ - $100^0 45'$ East Longitude and $0^0 55'$ - $03^0 33'$ South Latitude. The Mentawai Islands are included in a potential area that is prone to earthquake (tectonic) disasters and large tsunami waves because the Mentawai Islands are flanked by two earthquake sources, both originating in the west and east regions. [5]

According to [6], The subduction route of the India-Australia and Eurasia tectonic plates in Indonesia extends from the west coast of Sumatra to the south of Nusa Tenggara. The India-Australia plate is subducting beneath the Eurasian continental plate at a speed of ±50-60 mm/year. The boundary between these 2 (two) plates is a shallow subduction zone or what is known as the "Sumatra Megathrust Subduction," which is currently of concern to the public because it is predicted that there is still potential for an earthquake with a magnitude of M_w is 8.9 in this zone which is popularly known as the Mentawai Megathrust.

The impacts of earthquakes include building damage and loss of life. To minimize those impacts, efforts to increase sustainable disaster mitigation are needed. One effort that can be made is to form a forecast. Forecasting is a technique used to obtain some information (can be in the form of value) in the future [7]. There are several statistical methods that can be used for forecasting purposes, namely the Fuzzy Time Series (FTS), mixture [8], Copula [9] [10], and hidden Markov models [11] [12], just to name a few. In this paper, we provide an alternative probability model that can be used for forecasting purposes, namely the FTS model. We provide an explanation regarding this method in the second section of this paper.

The FTS model was first studied by Song and Chissom based on fuzzy set theory and the concept of linguistic variables, and their applications was discovered by Zadeh [13]. It works in forecasting by implementing patterns based on past data to predict future data. One of the advantages of the FTS method compared to other time series methods is that the FTS method does not depend on the type of historical data pattern, so data with any pattern can be used. Therefore, FTS is suitable for forecasting earthquakes, especially in the Mentawai Islands. FTS has several models, including FTS Chen, FTS Cheng, FTS Lee, FTS Tsaur. In this paper, the models to be used are FTS Lee and FTS Tsaur. FTS Lee is a model used for short-term forecasting with stationary and non-stationary data patterns. FTS Tsaur is a new research concept from Tsaur that combines Fuzzy Time Series with Markov Chain. Based on the previous explanation, research was carried out with the title "Earthquake Frequency Data Modeling in Mentawai Using FTS Lee and FTS Tsaur".

2. RESEARCH METHODS

Fuzzy Time Series was first studied by Song and Chissom in 1993. FTS is a data forecasting method that uses fuzzy principles as its basis. The forecasting system with FTS gets patterns from past data then used to project data in the future. This model is used by researchers to solve forecasting problems. The thing that distinguishes between FTS and conventional Time Series is that the values used in forecasting are a fuzzy set of real numbers over a given set of universes [14].

2.1 Reference Review

This research is a case study that compares FTS Lee and FTS Tsaur to model earthquake frequency data in Mentawai. The data used in this research is earthquake frequency data in Mentawai from 1960 to 2022.

One of the applications of FTS Lee is in the research of [15], entitled Application of Lee's Average Based Fuzzy Time Series Method for Forecasting Gold Prices at PT. X. The results of these forecasts show gold price data using average based FST Lee results in a gold price of IDR 872,500 and has an error rate based on the MAPE (Mean Absolute Percentage Error) value of 0.4364%. These results include very good criteria.

Admirani researched the FTS Tsaur model for predicting new student enrollment [16]. The error calculation results using MAPE obtained a value of 8.48%. From this research, it can be seen that the FTS Tsaur model can be used for long-term predictions even though it has a limited data sample.

2.2 Data Analysis

2.2.1 Fuzzy Time Series Lee

Fuzzy Time Series Lee is a forecasting method used when the amount of historical data available is small and does not require certain assumptions to be met. This method uses historical data in the form of fuzzy sets originating from real numbers over the universe set in actual data. FTS Lee is a development of FTS Song and Chissom, FTS Cheng, and FTS Chen [17].

The steps in using FTS Lee model are as follows:

a. Formation of the universe (U)

Formation of the universe (U), can be done using the following formula:

$$U = [X_{min} - D_1; X_{max} + D_2] \tag{1}$$

where:

U : universe set
D_1 dan D_2 : constant value determined by the researcher
X_{min} : the smallest data from the actual data
X_{max} : the largest data from the actual data

b. The formation of the interval length of the fuzzy set uses the average-based length method with the following steps:

1) Determine the length of the interval (R) with the formula,

$$R = (X_{max} + D_2) - (X_{min} - D_1) \tag{2}$$

2) Determine the average value of the difference (lag) absolute

$$av = \frac{\sum_{t=1}^{n} |D_{t+1} - D_t|}{n-1} \tag{3}$$

with an explanation of the variables contained in Equation (3) is as follows:

av : average value
n : number of intervals
D_{t+1} : data to $t + 1$
D_t : data to t

3) Defines the basis interval (L)

$$L = \frac{av}{2} \tag{4}$$

c. Define z atau or the number of fuzzy sets using the following formula:

$$z = \frac{R}{l} \tag{5}$$

d. Determine the middle value of the fuzzy set using the following formula:

$$m_i = \frac{(ba_i + bb_i)}{2} \tag{6}$$

where:
m_i : middle value
i : 1, 2, 3, …, n
ba_i : upper limit to -i
bb_i : lower limit to -i

e. Next is the actual data fuzzification. According to [18], fuzzification is a relationship where an objective problem turns into a fuzzy concept. The use of fuzzification must be in accordance with the input and output that shows the relationship between languages or linguistics. If fuzzification is the process of making real values vague, then defuzzification is the opposite, namely changing vague values into real values. For examples $A_1, A_2, \ldots, A_n$ is a fuzzy set that has a linguistic value from a linguistic variable. The definition of the fuzzy set $A_1, A_2, \ldots, A_n$ on the universal set (U) is as follows:

$$A_1 = {}^{1}/_{u_1} + {}^{0,5}/_{u_2} + {}^{0}/_{u_3} + {}^{0}/_{u_4} + \cdots + {}^{0}/_{u_n}$$
$$A_2 = {}^{0,5}/_{u_1} + {}^{1}/_{u_2} + {}^{0,5}/_{u_3} + {}^{0}/_{u_4} + \cdots + {}^{0}/_{u_n}$$
$$A_3 = {}^{0}/_{u_1} + {}^{0,5}/_{u_2} + {}^{1}/_{u_3} + {}^{0}/_{u_4} + \cdots + {}^{0}/_{u_n}$$
$$\vdots$$
$$A_n = {}^{0}/_{u_1} + {}^{0}/_{u_2} + {}^{0}/_{u_3} + \cdots + {}^{0,5}/_{u_{n-1}} + {}^{1}/_{u_n}$$

where u_i $(i = 1,2, \ldots, n)$ is an element of the universal set (U) and the number marked with the symbol "/" denotes the membership degree of $\mu_{A_i}(u_i)$ to A_i $(i = 1,2, \ldots, n)$, where the value is 0, 0.5 or 1. Can be determined as follows:

$$\mu_{A_i}(u_i) = \begin{cases} 1 & if\ i = i \\ 0,5 & if\ i = i - 1\ or\ i = i + 1 \\ 0 & others \end{cases} \tag{7}$$

f. Determine Fuzzy Logical Relationship (FLR) based on actual data. At this stage determine the fuzzy logic relations, namely $A_i \to A_j$. A_i is the current state at time D_{t-1} and A_j is the next state at time D_t.

g. Determine the Fuzzy Logical Relationship Group (FLRG) by grouping fuzzifications that have the same current state and then grouping them into one group in the next state. In Lee's Fuzzy Time Series, all FLRs are grouped into interconnected FLRGs. For example, there are 3 Fuzzy Logical Relationships (FLR) where 2 of them are the same, namely $A_1 \to A_2, A_1 \to A_2$ and $A_1 \to A_3$ forming FLRG $A_1 \to A_2, A_2, A_3$. According to Lee, $A_1 \to A_2, A_1 \to A_2$ must be calculated, because it can affect the model results.

h. Defuzzification. According to [18], defuzzification is a method of mapping fuzzy set values into crisp values. The rules for defuzzifying FTS Lee are [15]:

Rule 1 : if FLRG A_i changes to the empty set $A_i \to \emptyset$, then the result of forecasting F_t is m_i or the middle value of U_i

$$F_t = m_i \tag{8}$$

Rule 2 : if FLRG A_i changes from one to one ($A_i \rightarrow A_k$ with $P_{ij} = 0$ and $P_{ik} = 1, j \neq k$) then the result of forecasting F_t is m_k or the middle value of U_i.

$$F_t = m_k \tag{9}$$

Rule 3 : if FLRG A_i changes from one to many ($A_i \rightarrow A_1, A_2, \dots, A_n, i = 1,2, \dots, n$), say $A_i \rightarrow A_j, A_j, A_k, A_k, \dots, A_l$ where $A_j, A_j, A_k, A_k, \dots, A_l$ is a fuzzification and the maximum value of membership values $A_j, A_j, A_k, A_k, \dots, A_l$ is in the interval $U_j, U_j, U_k, U_k, \dots, U_l$ and $m_j, m_j, m_k, m_k, \dots, m_l$ is the middle value. Then the results of forecasting F_t are:

$$F_t = \frac{2}{q} m_j + \frac{2}{q} m_k + \dots + \frac{1}{q} m_l \tag{10}$$

2.2.2 Fuzzy Time Series Tsaur

Forecasting steps using the Fuzzy Time Series Tsaur model are as follows [19]:

a. The formation of the universal set (U) uses Equation (1).

b. The formation of the interval length of the fuzzy set uses the average based length method with the following steps:
   1) Determine the length of the interval (R) using Equation (2).
   2) Determine the average absolute value of the difference (lag) using Equation (3).
   3) Determine the interval basis using Equation (4).

c. Determine the number of fuzzy sets using Equation (5).

d. Next is fuzzification. The fuzzification stage is determined based on linguistic values based on predetermined fuzzy sets.

e. Determine FLR.

f. Define FLRG.

g. Calculates the transition probability matrix

$$P_{ij} = (M_{ij})/M_i \tag{11}$$

where:
i, j : $1,2, \dots, n$
P_{ij} : the transition probabilities of the states A_i to A_j
M_{ij} : the number of transitions from the state A_i to A_j
M_i : the amount of data included in state A_i.
The transition probability matrix (T) can be written as follows:

$$T = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix} \tag{12}$$

h. Define defuzzification.
   1) Calculate the initial forecast

Rule 1 : if FLRG A_i changes to the empty set $A_i \rightarrow \emptyset$, then the result of forecasting F_t is m_i or the middle value of U_i.

$$F_t = m_i \tag{13}$$

Rule 2 : if FLRG A_i changes from one to one ($A_i \rightarrow A_k$ with $P_{ij} = 0$ and $P_{ik} = 1, j \neq k$) then the result of forecasting F_t is m_k or the middle value of U_i.

$$F_t = m_k \quad (14)$$

Rule 3 : if FLRG A_i changes from one to many $(A_i \rightarrow A_1, A_2, \dots, A_n, i = 1,2, \dots, n)$ and data group X_{t-1} when $t-1$ is A_i. Then the result of forecasting F_t is worth,

$$F_t = m_1 P_{i1} + \cdots + m_{(i-1)} P_{i(i-1)} + X_{t-1} P_{ii} + m_{(i+1)} P_{i(i+1)} \quad (15)$$

where X_{t-1} is the actual data at $t-1$.

2) Set the trend of forecasting values.

Rule 1 : if state A_i communicates with A_j, starting from state A_i at time t–1 as $X_{t-1} = A_i$ and making an upward transition to state A_j at time *t*, where $(i < j)$ then the formula for setting the trend value is:

$$D_t = \left(\frac{l \times s}{2}\right) \quad (16)$$

where:

l : the length of the fuzzy set interval

s : many forward transitions

Rule 2 : if state A_i communicates with A_j, starting from state A_i at time t–1 as $X_{t-1} = A_i$ and making an upward transition to state A_j at time *t* where $(i > j)$ then the trend setting formula is:

$$D_t = -\left(\frac{l \times r}{2}\right) \quad (17)$$

where:

r : many backward transitions

Rule 3 : if state A_i communicates with A_j, where $i = j$ then the trend value of the result of forecasting is $D_t = 0$.

3) Calculating the final forecasting results obtained from the sum of the initial forecasting results and the trend of forecasting values. The general form of FM_t final forecast is,

$$FM_t = F_t + D_t \quad (18)$$

where $\boldsymbol{FM_t}$ is the final result of forecasting at time *t*.

2.3 Forecasting Accuracy Measurement

Forecasting accuracy will be measured using the MSE and MAPE functions.

a. Mean Square Error (MSE)

Mean square error (MSE) is one of the forecasting test calculations that is used by calculating the average rank residual [20]. Here's the formula for calculating MSE:

$$MSE = \frac{1}{N}\sum_{t=1}^{N}\left(X_t - \hat{X}_t\right)^2 \quad (19)$$

where:

N : lots of data

X_t : actual data to *t*

$\hat{X}_t$: forecast data to *t*

b. Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is the average of the first percentage errors from several periods. Forecasting accuracy using MAPE is formulated as follows [21] :

$$\text{MAPE} = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{X_t - \hat{X}_t}{X_t}\right| \times 100\% \quad (20)$$

MAPE values can be interpreted into 4 categories, namely:

a. ≤ 10% : The forecasting results are very accurate

b. 10% - 20% : Good forecasting results

c. 20% - 50% : Forecasting results are decent (pretty good)

d. > 50% : Inaccurate forecasting results.

A small MAPE value indicates that the forecast is more accurate, in other words, the smaller the MAPE value, the closer the forecast is to the actual value.

2.4 Analytical Method

The stages in this research are as follows:

a. Obtain earthquake frequency data in Mentawai from 1960 to 2022 from the USGS catalog. Then use the MATLAB 2018a and Zmap7 applications to process the data.

b. Perform calculations with FTS Lee.

c. Perform calculations with FTS Tsaur.

d. Testing the level of prediction accuracy used to determine the level of accuracy of FTS Lee and FTS Tsaur. Testing the level of accuracy using the criteria for calculating the MSE and MAPE.

e. Comparing the forecasting results from FTS Lee and FTS Tsaur.

f. Conclusion.

The analysis method for calculations of FTS Lee and FTS Tsaur can also be described with the following flowchart:

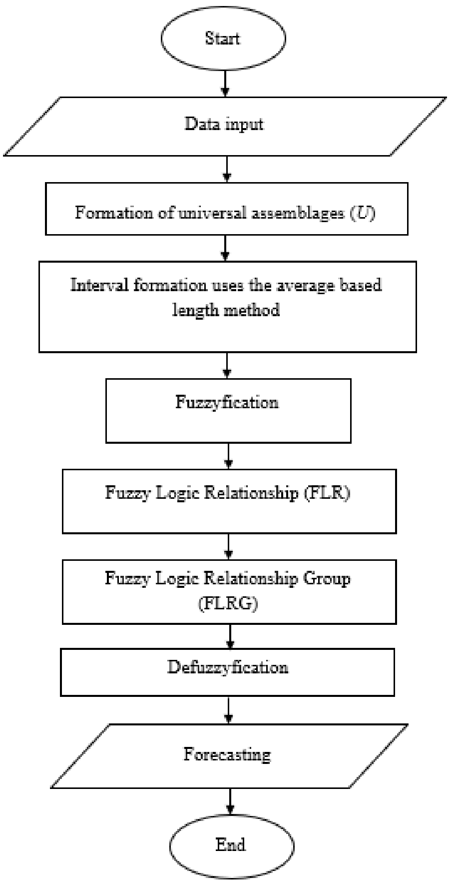

Figure 1a. Flowchart of Fuzzy Time Series Lee

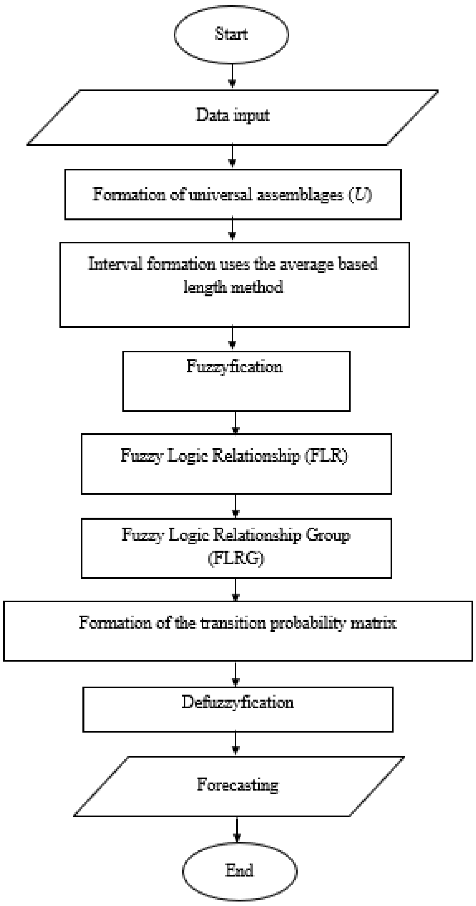

Figure 1b. Flowchart of Fuzzy Time Series Tsaur

3. RESULTS AND DISCUSSION

3.1 Data Description

The data used in this study are the earthquake frequency data in Mentawai, calculated from 1960 to 2022, with a total of 63 data. The results obtained using data from 1960 to 2022 have several obstacles, including the calculation of the FTS Lee and FTS Tsaur models, the results of measuring forecasting accuracy using the MAPE function cannot be calculated. This is because there is data with a value of 0 in 1960-1966 and 1968.

So, the alternative taken by the researchers is to use data from 1970 to 2022 because, in that period, there is no data with a value of 0. Thus, the MAPE value in the FTS Lee and FTS Tsaur models can be measured/calculated.

Table 1. Summary of Earthquake Frequency Data in Mentawai

Data	*N*	*Min*	*Max*	St. Dev	*Mean*
Actual annual data on the frequency of earthquakes in the Mentawai Islands	53	1	39	10,131	13,566

3.2 Forecasting Earthquake Frequency Data in Mentawai using Fuzzy Time Series

Forecasting of earthquake frequency data in Mentawai uses the FTS Lee and FTS Tsaur models. The first step is to form a set of universes. The values of D_1 and $D_2$are constant values determined by the researcher. So, the following results are obtained:

$$U = [X_{min} - D_1; X_{max} + D_2]$$
$$= [1; 39]$$

The formation of the interval length of the fuzzy set uses the average based length method with the following steps:

a. Specifies the length of the interval (R)

$$R = (X_{max} + D_2) - (X_{min} - D_1)$$
$$= 38$$

b. Determines the average value

Table 2. Average Value of Actual Absolute Data

No.	Year	Frequency	$\lvert D_{t+1} - D_t \rvert$
1.	1970	1	0
2.	1971	1	2
3.	1972	3	0
4.	1973	3	3
⋮	⋮	⋮	⋮
50.	2019	29	3
51.	2020	26	3
52.	2021	23	2
53.	2022	25	25
	Amount		**273**

$$av = \frac{\sum_{t=1}^{n} \lvert D_{t+1} - D_t \rvert}{n-1}$$
$$= 5,25$$

c. Determine the basis interval

$$L = \frac{av}{2}$$
$$= 2,625$$

Based on the calculation results obtained, the basis interval value is 2.625. It is based on the interval basis table included in the 1st interval basis with rounding off to 2.6. So that the length of the fuzzy set interval is 2.6.

After obtaining the interval length value of the fuzzy set, then the number of fuzzy sets will be determined.

$$z = \frac{R}{l}$$
$$= 14{,}615 \qquad \text{(rounded up to 15)}$$

Then determine the middle value (m_i) by adding up the interval limit which is then divided by two.

$$m_1 = \frac{(ba_1 + bb_1)}{2}$$
$$= 2{,}3$$

By calculating using the same method, the complete mid value of $m_1, m_2, m_3, \dots m_{15}$ can be seen in the following table.

Table 3. Middle Value of Fuzzy Sets

Intervals (U_i)	Lover Limit (bb_i)	Upper Limit (ba_i)	Middle Value (m_i)
U_1	1	3,6	2,3
U_2	3,6	6,2	4,9
U_3	6,2	8,8	7,5
U_4	8,8	11,4	10,1
U_5	11,4	14	12,7
U_6	14	16,6	15,3
U_7	16,6	19,2	17,9
U_8	19,2	21,8	20,5
U_9	21,8	24,4	23,1
U_{10}	24,4	27	25,7
U_{11}	27	29,6	28,3
U_{12}	29,6	32,2	30,9
U_{13}	32,2	34,8	33,5
U_{14}	34,8	37,4	36,1
U_{15}	37,4	40	38,7

The fuzzification results can be seen in Table 4 below.

Table 4. Fuzzification Results

No.	Year	Frequency	Fuzzification
1.	1970	1	A_1
2.	1971	1	A_1
3.	1972	3	A_1
4.	1973	3	A_1
⋮	⋮	⋮	⋮
50.	2019	29	A_{11}
51.	2020	26	A_{10}
52.	2021	23	A_9
53.	2022	25	A_{10}

The formation of FLR and FLRG is identified based on actual data that has been previously fuzzified. If a time series variable $D_{(t-1)}$ or current state has a fuzzification form as A_i and a time series variable D_t or next state as A_j, then A_i has a relationship with A_j or it can be said that A_i can predict the data in A_j. A relationship like this can be written with the notation $A_i \rightarrow A_j$ where A_i is data on the frequency of earthquake events at the current time (current state) and A_j is data on the frequency of earthquake events at a later time than now (next state).

Table 5. Fuzzy Logic Relationship Results

No.	Year	Fuzzification	FLR
1.	1970	A_1	-
2.	1971	A_1	$A_1 \rightarrow A_1$
3.	1972	A_1	$A_1 \rightarrow A_1$
4.	1963	A_1	$A_1 \rightarrow A_1$
⋮	⋮	⋮	⋮
50.	2019	A_{11}	$A_{11} \rightarrow A_{11}$
51.	2020	A_{10}	$A_{11} \rightarrow A_{10}$
52.	2021	A_9	$A_{10} \rightarrow A_9$
53.	2022	A_{10}	$A_9 \rightarrow A_{10}$

The formation of FLRG is determined by grouping the same current state into one group. The following are the FLRG groups that were formed:

Table 6. Fuzzy Logic Relationship Group

No.	Group
1.	$A_1 \rightarrow A_1, A_1, A_1, A_2, A_2$
2.	$A_2 \rightarrow A_2, A_2, A_2, A_2, A_2, A_2, A_3, A_3, A_3, A_4, A_4, A_4 A_6$
3.	$A_3 \rightarrow A_1, A_2, A_2, A_3, A_3, A_4$
4.	$A_4 \rightarrow A_2, A_2, A_3, A_4, A_4, A_4, A_7, A_7$
5.	$A_5 \rightarrow \emptyset$
6.	$A_6 \rightarrow A_2, A_{15}$
7.	$A_7 \rightarrow A_4, A_7, A_9, A_{10}, A_{14}$
8.	$A_8 \rightarrow A_7, A_{11}$
9.	$A_9 \rightarrow A_{10}, A_{12}$
10.	$A_{10} \rightarrow A_6, A_9$
11.	$A_{11} \rightarrow A_{10}, A_{11}$
12.	$A_{12} \rightarrow A_8, A_{13}$
13.	$A_{13} \rightarrow A_{12}$
14.	$A_{14} \rightarrow A_8$
15.	$A_{15} \rightarrow A_7$

3.2.1 Forecasting Using FTS Lee

Defuzzification is used to convert the fuzzy output into a firm value based on a representative predetermined membership function.

Table 7. Defuzzification Results

No.	Year	Frequency	FLR	Forecasting
1.	1970	1	-	-
2.	1971	1	$A_1 \rightarrow A_1$	3,34
3.	1972	3	$A_1 \rightarrow A_1$	3,34
4.	1973	3	$A_1 \rightarrow A_1$	3,34
⋮	⋮	⋮	⋮	⋮
50.	2019	29	$A_{11} \rightarrow A_{11}$	27
51.	2020	26	$A_{11} \rightarrow A_{10}$	27
52.	2021	23	$A_{10} \rightarrow A_9$	19,2
53.	2022	25	$A_9 \rightarrow A_{10}$	28,3

3.2.2 Forecasting Using FTS Tsaur

The formation of the transition probability matrix can be seen based on FLRG data. At this stage, a transition probability matrix or transfer from state A_i to state A_j is formed with order 15×15.

$$T = \begin{bmatrix}
3/5 & 2/5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 6/13 & 3/13 & 3/13 & 0 & 1/13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/6 & 2/6 & 2/6 & 1/6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2/8 & 1/8 & 3/8 & 0 & 0 & 2/8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 \\
0 & 0 & 0 & 1/5 & 0 & 0 & 1/5 & 0 & 1/5 & 1/5 & 0 & 0 & 0 & 1/5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 & 1/2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Based on the probability values contained in the transition probability matrix, the initial forecasting value can be calculated. The results of the initial forecasting values can be seen in **Table 8**

Table 8. Preliminary Forecasting Results of FTS Tsaur

No.	Year	Frequency	FLR	Initial Forecast
1.	1970	1	-	-
2.	1971	1	$A_1 \rightarrow A_1$	2,56
3.	1972	3	$A_1 \rightarrow A_1$	2,56
4.	1973	3	$A_1 \rightarrow A_1$	3,76
⋮	⋮	⋮	⋮	⋮
50.	2019	29	$A_{11} \rightarrow A_{11}$	26,85
51.	2020	26	$A_{11} \rightarrow A_{10}$	27,35
52.	2021	23	$A_{10} \rightarrow A_9$	19,2
53.	2022	25	$A_9 \rightarrow A_{10}$	28,3

Then set the trend of forecasting values. Then the results are obtained as in **Table 9**.

Table 9. Value Data D_t

No.	Year	Frequency	FLR	Initial Forecast	D_t
1.	1970	1	-	-	-
2.	1971	1	$A_1 \rightarrow A_1$	2,56	0
3.	1972	3	$A_1 \rightarrow A_1$	2,56	0
4.	1973	3	$A_1 \rightarrow A_1$	3,76	0
⋮	⋮	⋮	⋮	⋮	⋮
50.	2019	29	$A_{11} \rightarrow A_{11}$	26,85	0
51.	2020	26	$A_{11} \rightarrow A_{10}$	27,35	-1,3
52.	2021	23	$A_{10} \rightarrow A_9$	19,2	-1,3
53.	2022	25	$A_9 \rightarrow A_{10}$	28,3	1,3

Then, determine the final forecasting value obtained from the initial forecasting stage and the trend of forecasting values.

Table 10. FTS Tsaur Final Forecasting Results

No.	Year	Frequency	FLR	Initial Forecast	D_t	Final Forecast
1.	1970	1	-	-	-	-
2.	1971	1	$A_1 \rightarrow A_1$	2,56	0	2,56
3.	1972	3	$A_1 \rightarrow A_1$	2,56	0	2,56
4.	1973	3	$A_1 \rightarrow A_1$	3,76	0	3,76
⋮	⋮	⋮	⋮	⋮	⋮	⋮
50.	2019	29	$A_{11} \rightarrow A_{11}$	26,85	0	26,85
51.	2020	26	$A_{11} \rightarrow A_{10}$	27,35	-1,3	26,05
52.	2021	23	$A_{10} \rightarrow A_9$	19,2	-1,3	17,9
53.	2022	25	$A_9 \rightarrow A_{10}$	28,3	1,3	29,6

3.3 Forecasting Accuracy

The measurement of forecasting accuracy is the stage of the forecasting value that has been obtained and placed on each fuzzy set. **Table 11** is a comparison of forecasting values at FTS Lee and FTS Tsaur with actual data on earthquake frequency data in Mentawai, as well as the results of calculating error values using MAPE and MSE.

Table 11. MAPE and MSE results on FTS Lee and FTS Tsaur

No.	Year	Frequency	Forecast FTS *Lee*	APE	SE	Forecast FTS *Tsaur*	APE	SE
1.	1970	1	-	-	-	-	-	-
2.	1971	1	3,34	234	5.476	2,56	156	2.434
3.	1972	3	3,34	11,333	0.116	2,56	14,667	0.194
4.	1973	3	3,34	11,333	0.116	3,76	25,333	0.578
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
50.	2019	29	27	6,897	4.000	26,85	7,414	4.622
51.	2020	26	27	3,846	1.000	26,05	0,192	0.003
52.	2021	23	19,2	16,5222	14.440	17,9	22,174	26.010
53.	2022	25	28,3	13,2	10.890	29,6	18,4	21.160
MAPE & MSE				37,511	27,073		27,051	11,671

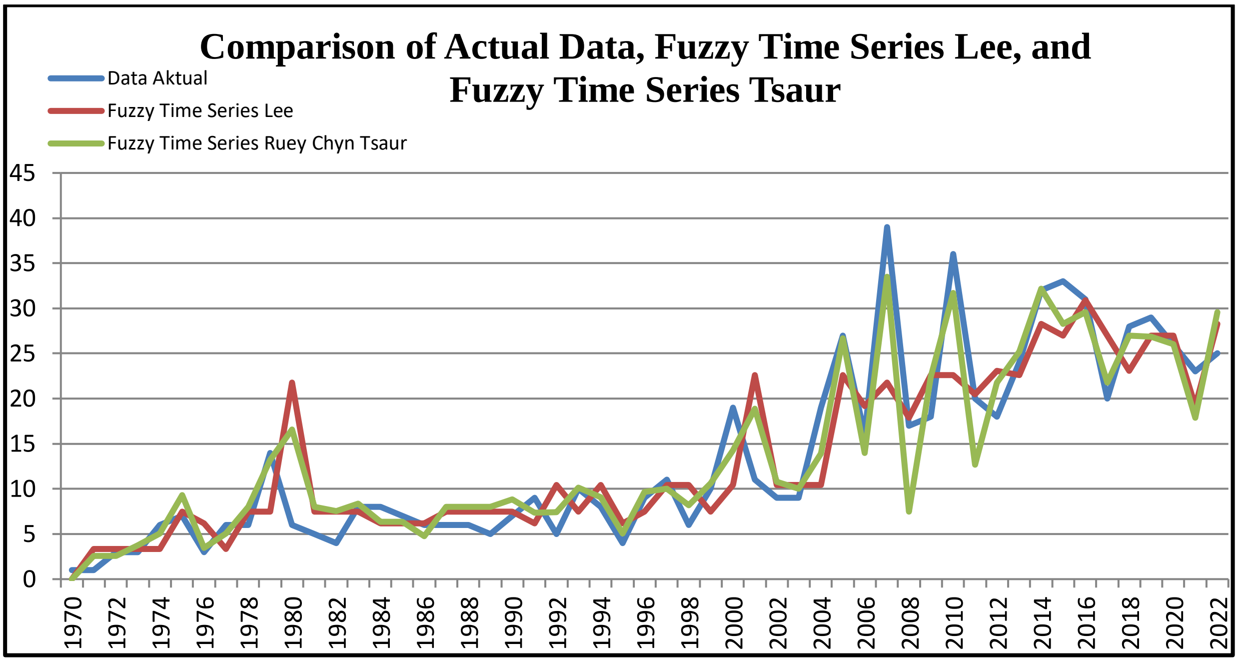

Figure 2. Comparison of Actual Data, FTS Lee, and FTS Tsaur

From the table above, it can be seen that the MAPE value in the FTS Lee model is 37,511% and FTS Tsaur is 27,051%. It can be seen that the two models include an interpretation between 20% -50%, meaning that the forecasting results are said to be quite good. Then in the MSE function the results for the FTS Lee model were 27,073 and the FTS Tsaur was 11,671. So, it can be concluded that the FTS Tsaur model is better than the FTS Lee model. Because the MAPE and MSE values of FTS Tsaur are smaller than the FTS Lee. And then based on **Figure 2** also describe actual data, FTS Lee forecasting data, and FTS Tsaur. The figure shows that FTS Tsaur's forecast is closer to the actual data compared to FTS Lee. Therefore, it can be said that FTS Tsaur is better than FTS Lee for earthquake data in the Mentawai Islands.

4. CONCLUSIONS

From the results and discussion, it can be concluded as follows: the FTS Lee model has a MAPE of 37.511% and an MSE of 27.073, and the FTS Tsaur has a MAPE of 27.051% and an MSE of 11.671. Thus, FTS Tsaur model is better than FTS Lee for modeling earthquake frequency data in Mentawai due to the error value of FTS Tsaur is smaller than that of FTS Lee.

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