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STABILITY ANALYSIS OF CELLULAR OPERATING SYSTEM MARKET SHARE IN INDONESIA WITH THE COMPETITIVE LOTKA-VOLTERRA MODEL

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ABSTRACT

The current increase in smartphone users has caused various mobile device operating system companies to compete with each other to create a mobile device operating system that is suitable and acceptable to the public. Competition among mobile operating system market shares can be analyzed using the Lotka-Volterra model. This study aims to reconstruct the Lotka-Volterra model for the cellular operating system market share in Indonesia. In addition, a stability analysis was carried out, which aims to determine the stability of the competitive model for the market share of the operating system in Indonesia. The results of the study show that a competitive Lotka-Volterra model can be built on the Android and iOS operating system market share in Indonesia. In this model, there are four equilibrium points, one of which is unstable, and the other three equilibrium points are conditionally stable.

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1. INTRODUCTION

Smartphones are rapidly being used by humans in this day and age. Smartphones are already widely used in Indonesia. As the number of smartphone users grows, several mobile device operating system businesses compete to build an operating system that is suitable and acceptable to the general public. Because of the growing popularity of smartphones, tablets, and other mobile devices, these operating systems may now control a significant variety of programs and applications. The development of numerous types of operating systems enables rivalry amongst mobile device operating system businesses, often known as competition for operating system market share. This operating system market share competition can be modeled mathematically. A mathematical model can represent a variety of real-world challenges. Mathematical modeling can be utilized to understand the behavior of natural occurrences that occur, anticipate system behavior through time, and create policies [1]. Modeling ecological problems, or the discipline of biology that examines ecosystems, is one use of mathematical modeling. Ecology is a discipline of biology that explores the link between living things and their surroundings [2].

Interaction refers to the relationship that exists between living organisms and their surroundings. Each population in an ecosystem will always interact with other populations, and each of these interactions will result in a predator-prey event. The predator population will always interact, whether it is a mutually beneficial interaction or a competition for prey ([3]–[7]). Alfred Lotka and Vito Volterra ([8], [9]) developed and introduced the basic Lotka-Volterra model for predator-prey modeling in mathematical modeling in 1920. The Lotka-Volterra model was originally designed to predict the rate of development and extinction of a predator-prey population.

The Lotka-Volterra mathematical model has been widely developed. Fitria [10] conducted a study discussing the predator-prey model with the time deceleration of the stability of the equilibrium point of the differential equation system of the predator-prey model. In 2014, Arpa et al. [11] conducted research on model analysis using the perturbation theory, where it is known that the season has a significant influence on the pattern of growth of prey and predator populations. S. Maurer and B. Guberman [12] studied changes in the number of users on a website. Furthermore, there may be intense competition amongst mobile device operating systems if numerous competing systems move. The predator-prey model can depict this competition. Previously performed Lotka-Volterra modelling on the operating system market competition study [13].

Previous research regarding the application of the bifurcation theory to the Lotka-Volterra model was carried out by Sundari [14], where the dynamics of the prey-predator population in this model were divided into four cases. In the second case, there is a change in the stability of the fixed point from a stable spiral to an unstable spiral, accompanied by the appearance of a limit cycle.

The global market share competition for Android and iOS systems was studied in this paper. In addition, this paper also discusses stability at the equilibrium point based on eigenvalue criteria. After analysis, the model is then applied to cases, and the results are interpreted.

2. RESEARCH METHODS

Literature study is used as a research method in this paper by reviewing previous studies such as articles in journals and those related to the Lotka-Volterra model and stability analysis of systems of ordinary differential equations. In this study, the following steps were carried out. The first step is to reconstruct the Lotka-Volterra competitive model on the interaction of Android and iOS systems in Indonesia; second, determine the balance point of the Lotka-Volterra competitive model; third, determine the type of stability at each equilibrium point; fourth, perform bifurcation analysis on the Lotka-Volterra competitive model; and finally, applying data on the percentage of Android and iOS users in Indonesia to the Lotka-Volterra competitive model and interpreting the implementation results Data on the percentage of Android and iOS users in Indonesia comes from Mobile Operating System Market Share Worldwide.

3. RESULTS AND DISCUSSION

3.1 Model Construction

An investigation of the interaction of competition among rival cellular operating systems in Indonesia was carried out in this study. In Indonesia, there are various mobile operating systems, but the mobile operating system rivalry alluded to in this study is the Android and iOS operating systems. This is due to the fact that the two operating systems have a higher user base than other mobile operating systems in Indonesia. Aside from that, the considerable number of people experiencing swings indicates that the rivalry in this operating system is rather intense. Predatory-prey modeling represents the Lotka Volterra model flow for competitive interactions in the mobile device operating system market analysis. The system of differential equations in the Lotka Volterra model is a combination of two nonlinear differential equations[16]. As formulated and introduced by AJ Lotka and Vito Volterra in 1920, the model in equation is a system of differential equations describing the numerical dynamics of two populations of predators and prey.

$$\begin{aligned}\frac{dx}{dt} &= x(a - by)\\ \frac{dy}{dt} &= y(cx - d).\end{aligned} \tag{1}$$

In **Equation (1)**, x and y represent the number of prey and predator populations, a represents the number of prey births without predators, d represents the number of natural deaths from predators, b and c represents interactions between prey-predators. Competitive interaction models are widely used to analyze economic processes and phenomena. Previous research on this topic includes the study of changes in the number of users on a web project[12]. According to the study, the number of users varies not only owing to the appeal of the web project itself but also due to the influence of rival sites offering comparable services.

Nikolaieva et al.[13] conducted subsequent research in which they assessed the prey-predator model and anticipated the global market share of cellular operating systems. At the start of his research, he built a prey-predator model for two global competitors for cellular operating systems, which was later expanded to three global competitors for mobile operating systems. In analyzing competitive interactions in the mobile device operating system market, which includes two operating systems, initial assumptions are used to fulfill a model. In this modeling, the Android operating system is represented by the x variable, and the iOS operating system is represented by the y variable. The growth rate of users of the Android operating system is expressed by α, while the growth rate of users of the iOS operating system is expressed by β.

Every interaction between the two populations will increase the growth of the predator population and hinder the growth of the prey population[17]. The interaction between two populations, namely predator and prey populations, is mathematically described in **Equation (1)**. Furthermore, Based on **Equation (1)**, x and y are used to express the growth rate of the Android operating system at time t. it is believed that the growth rate of users of the two mobile operating systems is restricted by the production capacity of each operating system's products, which is expressed by γ on the Android operating system and by ζ on the iOS operating system. The differential equation for the growth rate of the android operating system at time t is,

$$\frac{dx}{dt} = \alpha y(\gamma - x). \tag{2}$$

In the same way, the differential equation for the growth rate of the iOS operating system at time t is,

$$\frac{dy}{dt} = \beta y(\zeta - y) \tag{3}$$

Furthermore, if there is interaction between users of the Android and iOS mobile operating systems, where product promotions or weaknesses in the operating system itself occur, users of these operating systems may switch to other operating systems or cellular operating system competitors. This can have an impact on the number of Android and iOS users, which are represented by δ and η. Obtained the model for the Android operating system at time t,

$$\frac{dx}{dt} = \alpha y(\gamma - x) - \delta xy. \tag{4}$$

In the same way, the model is obtained for the iOS operating system at time t,

$$\frac{dy}{dt} = \beta y(\zeta - y) - \eta xy. \tag{5}$$

Equation (4) and **Equation (5)** are differential equations that can be written in a single equation system of differential equations in the predator-prey model. Like the following equation,

$$\begin{aligned} \frac{dx}{dt} &= \alpha y(\gamma - x) - \delta xy, \\ \frac{dy}{dt} &= \beta y(\zeta - y) - \eta xy. \end{aligned} \tag{6}$$

with, $t \geq 0$, $\alpha, \beta > 0$, $\delta, \eta, \gamma, \zeta \geq 0$, and initial value $x(t_0) = x_0$, $y(t_0) = y_0$.

3.2 Equilibrium Point and Stability Analysis

The system of differential equations' equilibrium point can be established if the instantaneous changes from users of the Android and iOS operating systems no longer increase and have reached stability, which can be written as

$$\begin{aligned} f_1(x,y) &= \alpha y(\gamma - x) - \delta xy, \\ f_2(x,y) &= \beta y(\zeta - y) - \eta xy. \end{aligned} \tag{7}$$

The equilibrium point is obtained by solving the system of equations $f_1(x,y) = 0$ and $f_2(x,y) = 0$. Therefore, the system of differential equations in **Equation (7)** has 4 equilibrium points, namely $E_1(0,0)$, $E_2(\gamma, 0)$, $E_3(0, \zeta)$, and $E_4\left(\frac{\beta(\alpha\gamma - \zeta\delta)}{\alpha\beta - \delta\eta}, \frac{\alpha(\beta\zeta - \gamma\eta)}{\alpha\beta - \delta\eta}\right)$. If a stability analysis is performed using the eigenvalues of the Jacobian matrix, the Jacobian matrix is obtained from linearization at the equilibrium point,

$$J = \begin{bmatrix} (\gamma - 2x)\alpha - \delta y & -\delta x \\ \eta y & (\zeta - 2y)\beta - \eta x \end{bmatrix}.$$

A stability test can be used to determine the nature of each equilibrium point based on the equilibrium point achieved. By linearizing the eigenvalues and substituting each equilibrium point, the stability test may be performed. Braun**[18]** defined stable equilibrium as the eigenvalue in the linearization of the differential equation at the equilibrium point being $\lambda < 0$. Then, the eigenvalues of the Jacobian matrix are obtained from each equilibrium point, as shown in **Table 1** with $P = \frac{\alpha\beta(\delta\zeta - \alpha\gamma)}{\alpha\beta - \delta\eta}$, $Q = \frac{\beta\delta(\delta\zeta - \alpha\gamma)}{\alpha\beta - \delta\eta}$, $R = \frac{\alpha\eta(\gamma\eta - \beta\zeta)}{\alpha\beta - \delta\eta}$, and $R = \frac{\alpha\beta(\gamma\eta - \beta\zeta)}{\alpha\beta - \delta\eta}$.

Table 1. Jacobian Matrix Eigenvalues and Types of Stability of the Equilibrium Point

Equilibrium Point	Eigen Values	Stability
$E_1(0,0)$	$\lambda_1 = \alpha\gamma$ $\lambda_2 = \beta\zeta$	Unstable
$E_2(\gamma, 0)$	$\lambda_1 = -\alpha\gamma$ $\lambda_2 = \beta\zeta + \gamma\eta$	Stable if $\frac{\zeta}{\gamma} > \frac{\eta}{\beta}$
$E_3(0, \zeta)$	$\lambda_1 = \alpha\gamma - \delta\zeta$ $\lambda_2 = -\beta\zeta$	Stable if $\frac{\gamma}{\zeta} > \frac{\delta}{\alpha}$
$E_4\left(\frac{\beta(\alpha\gamma - \zeta\delta)}{\alpha\beta - \delta\eta}, \frac{\alpha(\beta\zeta - \gamma\eta)}{\alpha\beta - \delta\eta}\right)$	$\lambda_{1,2} = \frac{(P+S) \pm \sqrt{(P+S)^2 - 4(PS - Qr)}}{2}$	Stable if $\frac{\beta}{\eta} > \frac{\gamma}{\zeta} > \frac{\delta}{\alpha}$

According to **Table 1**, the point E_1 is unstable, and the other points will be stable if specific constraints on the coefficients of the system of **Equations (8)** are met. The condition that can cause the equilibrium point to be stable is

$$\frac{\zeta}{\gamma} > \frac{\eta}{\beta}, \quad \frac{\gamma}{\zeta} > \frac{\delta}{\alpha}, \quad \frac{\beta}{\eta} > \frac{\gamma}{\zeta} > \frac{\delta}{\alpha}. \tag{8}$$

The phase plane of **Figure 1** depicts the stability of the equilibrium point with the horizontal axis as x and the vertical axis as y.

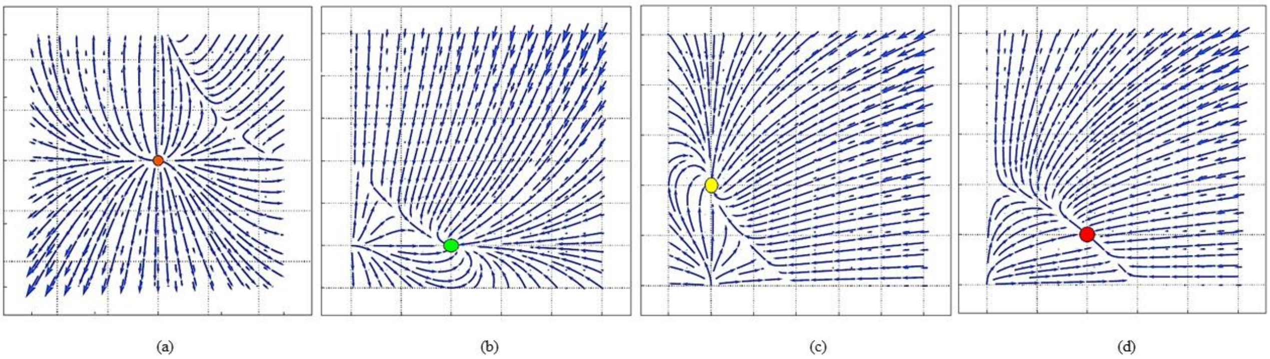

Figure 1. Phase Plane at The Equilibrium Point (a) E_1, (b) E_2, (c) E_3, and (d) E_4

3.3 Application of Cases

This analysis uses of statistics on the percentage of Android and iOS users in Indonesia[20]. The information used ranges from July 2021 to December 2022. The Lotka-Volterra model is simulated using Minitab software, while numerical simulations are performed using Wolfram Mathematica software. **Table 2** displays data on the percentage of users of the Android and iOS operating systems from July 2021 to December 2022.

Table 2. The Percentage of Android and iOS Users in Indonesia

Period	Operating Systems Users		Period	Operating Systems Users	
	Android (%)	iOS(%)		Android (%)	iOS(%)
Jul 2021	90.87	8.89	Apr 2022	90.67	9.23
Aug 2021	90.99	8.79	May 2022	91.57	8.31
Sep 2021	90.69	9.14	Jun 2022	90.83	9.06
Oct 2021	90.82	9.05	Jul 2022	89.97	9.92
Nov 2021	91.78	9.09	Aug 2022	89.42	10.46
Dec 2021	91.25	8.64	Sep 2022	89.79	10.10
Jan 2022	91.43	8.46	Oct 2022	89.77	10.12

Period	Operating Systems Users		Period	Operating Systems Users	
	Android (%)	iOS(%)		Android (%)	iOS(%)
Feb 2022	91.38	8.51	Nov 2022	89.81	10.09
Mar 2022	91.24	8.65	Dec 2022	89.29	10.61

Data source: https://gs.statcounter.com/os-market-share/mobile/worldwide/#monthly-202107-202203

Using the data in **Table 3**, the differential equation system and estimated parameter values of the model were derived using nonlinear regression in Minitab Software. Then, at time t, a differential equation is generated that describes the instantaneous change in the percentage of Android and iOS mobile operating system users as follows.

$$\frac{dx}{dt} = 0.4748x - 0.00489x^2 - 0.00348xy,$$
$$\frac{dy}{dt} = 0.08137y - 0.00067y^2 - 0.00207xy. \quad (9)$$

According to the system **Equation (9)**, the Android mobile operating system and the iOS mobile operating system compete. These two competitions have distinct effects on one another, with the effect of utilizing the Android mobile operating system on the iOS mobile operating system being quite large, particularly $\beta < \delta = 0.00348$. The effect of utilizing the iOS operating system on the Android operating system, on the other hand, is relatively tiny, equaling only $\alpha > \eta = 0.00207$. It is demonstrated that condition (9) satisfies the coefficients in the differential equation system (10).

The Lotka-Volterra model for operating system market share competition is completed using the estimated parameter values in **Table 2** with an initial value of $t = 0$ or the first data, namely in July 2021, and a simulation graph is obtained from the percentage of operating system users android and iOS at time t, as shown in **Figure 2**.

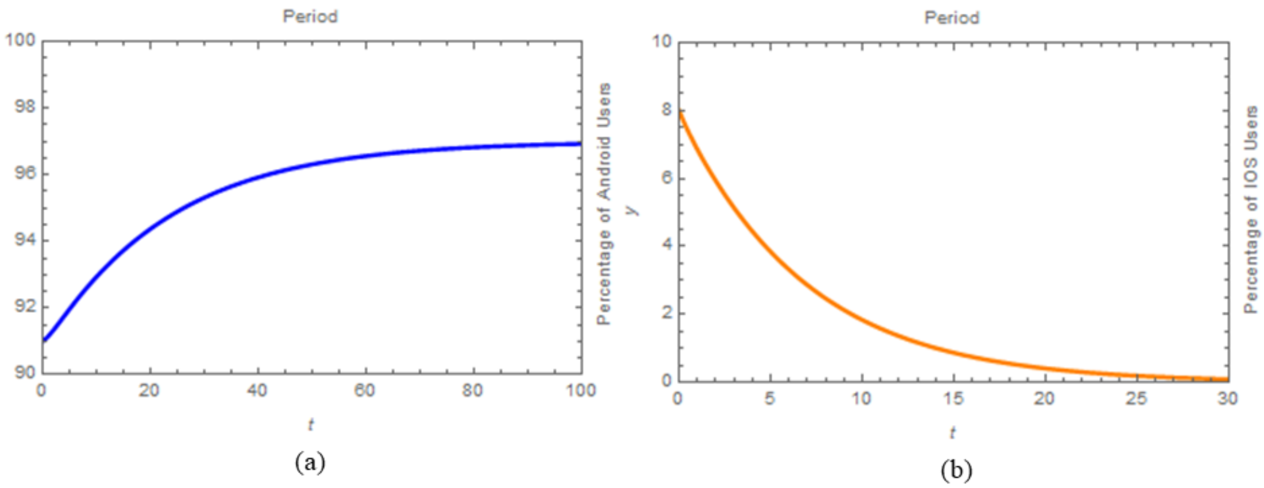

Figure 2. Grow Rate of (a) Android and (b) iOS Operating System Users at *t*

According to **Figure 2**(a), the percentage of Android operating system users increased at the start of its expansion. This occurs as a result of the interaction with the iOS operating system. Then, at $t = 33$, it achieves stability with a user percentage of 90.464%. In **Figure 2**(b), the percentage of iOS operating system users continues to fall due to contact with the Android operating system, and stability has been attained at $t = 63$ with a user percentage of 14.894%. Based on this, it is clear that the Android and iOS operating systems are competing for market share.

4. CONCLUSIONS

The Lotka-Volterra model in this study is obtained based on the data and discussion. The system of differential equations in the Lotka Volterra model is a combination of two nonlinear differential equations. In the model, there are four equilibrium points, one of which is unstable, while the other three are conditionally stable. Data on the percentage of Android and iOS users in Indonesia are used to estimate parameter values in the competitive Lotka-Volterra model. Equation (9) gives the differential equation for the percentage of Android and iOS operating system users in Indonesia at time t. The percentage of Android operating system users climbed during the start of its growth period, while the iOS operating system continued to drop. This occurs as a result of the interaction between the Android and iOS operating systems, resulting in a competition in the market share of the Android and iOS operating systems in Indonesia. Future work could develop the Lotka-Volterra model by considering diffusion and advection processes. Next, the developed model is subjected to bifurcation analysis.

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