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COMPARISON OF PROJECTED UNIT CREDIT AND ENTRY AGE NORMAL METHODS IN PENSION FUND VASICEK AND COX-INGERSOLL-ROSS MODELS

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ABSTRACT

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The pension program is one of the pension fund's efforts to anticipate the risks that will be experienced by participants in old age. Actuarial calculations help to determine the benefits that participants will receive by considering life chances, interest rates, age when becoming a participant, and normal retirement age. This study aims to determine normal contributions and actuarial liabilities with the Projected Unit Credit and Entry Age Normal methods using stochastic interest rates, namely Vasicek and Cox-Ingersoll-Ross (CIR). The data used in this study are civil servants who work at the Natural Resources Management Office, Bulukumba Regency. The results of the calculation analysis showed that normal cost using the Projected Unit Credit (PUC) method with the Vasicek and Cox-Ingersoll-Ross (CIR) model interest rates increased as the length of service increased, and at the end of the working period the Cox-Ingersoll-Ross (CIR) model interest rate reached Rp14.773.176,- which was higher than Vasicek by Rp3.849.898,-. The results of the calculation of normal cost using the Entry Age Normal (EAN) method with the Vasicek model increase in the period 0-20 years of service, then decrease towards the contribution value at the beginning of the service period of Rp1.499.725,-. At the beginning of the working year, the normal cost using the Entry Age Normal method with the Cox-Ingersoll-Ross (CIR) model interest rate is Rp7.581.593,- then decreases for 24 years of service to Rp5.849.854,- after which it increases again towards the initial contribution value of the working year. The results of the calculation of actuarial liabilities show an increase as the length of service increases, for the Entry Age Normal (EAN) and Projected Unit Credit (PUC) methods with the Cox-Ingersoll-Ross (CIR) interest rate model at the end of the service period, it is found that both are the same value, namely Rp443.195.285,-. By using the Vasicek interest rate model for both methods, the same result is obtained at the end of the service period of Rp115.496.951,-. This shows that the actuarial liabilities for both methods used are affected by interest rates, and the Cox-Ingersoll-Ross (CIR) model is higher than Vasicek.

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1. INTRODUCTION

Civil servants are citizens who are appointed as state civil apparatus with permanent employment status. Employees who will retire in government agencies are uncertain in number, someone can retire due disability, death, resignation and when reaching normal retirement age. The pension program managed by the Pension Fund is a form of compensation to improve employee welfare when facing risks at work. To anticipate risks in old age, the government needs to calculate periodic payments for employees by including them in pension program to get retirement benefits.

Actuarial calculation can help determine pension fund projections in the form of pension benefits to received and normal contributions to be paid by employees or participants. In general, there are two groups of actuarial valuations to determine the amount of normal contributions and actuarial liabilities, namely the

Accrued Benefit Cost Method which includes the Projected Unit Credit and Projected Benefit Cost Method with the Entry Age Normal method. The Projected Unit Credit method divides the total pension benefit at normal retirement age by the total service period allocated to each year of service, while Entry Age Normal assumes that the projection of pension benefits when reaching retirement age begins at entry age.

Interest rates are something that needs to be considered in pension fund calculations because they fluctuate with the economy. Constant interest rates can be used in calculating normal contributions and actuarial liabilities of pension funds, but this article uses stochastic interest rates, namely the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models. The Vasicek interest rate model predicts future interest rates by looking at the movement of previous interest rates and has a mean reverting phenomenon, which always moves towards the equilibrium point [1]. The Vasicek model does not limit the interest rate requirement, so it is possible to obtain negative interest rate results [2]. Unlike the Vasicek model, the Cox-Ingersoll-Ross (CIR) interest rate has mean reversion properties or a tendency to return to the average and is positive [3].

There are several studies that discuss the use of Projected Unit Credit and Entry Age Normal methods with constant interest rate including [4], it was found that the amount of normal contributions using the Projected Unit Credit method continues to increase as salary increases while the Entry Age Normal results remain the same for each year. These results are in line with the results obtained by [5] by providing several examples of cases in civil servants. In addition, the amount of actuarial liabilities for each year is smaller when using the Projected Unit Credit method. In line with previous research, [6] shows that the greater the salary and the level of salary increase, the greater the normal contributions and actuarial liability to pension fund participants. A comparison of the Projected Unit Credit with other methods such as Attained Age Normal is given by [7], every year normal contributions with the Projected Unit Credit method show a considerable increase while the Attained Age Normal method is relatively slow.

Furthermore, studies using Vasicek dan Cox-Ingersoll-Ross (CIR) interest rates are increasingly being studied because the interest rate follows a stochastic process [2], [8]–[11]. Furthermore, in research [12] used the Projected Unit Credit method with stochastic interest rates, namely Vasicek and Cox-Ingersoll-Ross (CIR). Based on these conditions, this research focuses on the comparison of normal contributions and actuarial liabilities using the Projected Unit Credit and Entry Age Normal methods with the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models.

2. RESEARCH METHODS

The data used in this study is a retired civil servant at the Water Resources Management Office in Bulukumba Regency, namely age when registered as a civil servant, gender, normal retirement age, basic salary per month, percentage of salary given for retirement benefits, and accumulated salary for the last year before retirement. The data obtained from pension fund participants is used to determine the opportunity for participants to actively work and the amount of pension benefits that will be obtained based on the accumulated last salary before retirement. The Bank Indonesia monthly interest rate for 2018-2022 is used to determine the value of the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate parameters.

2.1 Survival Function

The survival function describes an employee's life expectancy, specifically it can be used to determine the probability of a person still actively working to reach a predetermined normal retirement age. The probability of an employee aged x years old survives until the age of t year is expressed by [13]:

$$ {}_tp_x = \frac{l_{x+t}}{l_x} \tag{1} $$

with

$_tp_x$: The probability that a person aged x years old can work until the age of $(x + t)$ years
l_{x+t} : Number of people exactly aged $x + t$ years
l_x : Number of people exactly aged x years

2.2 Interest Rate Function

The interest rate is used to determine the deductibility of future payments which is referred to as the present value. If i denotes the interest rate in t year with $t = 1,2, \ldots, n$ then the present value of a unit of money in n year is shown by [14]:

$$ v^n = \frac{1}{(1+i)^n} \tag{2} $$

2.3 Life Annuity Function

A life annuity is a series of payments that a person makes as long as the person is alive, it can be expressed by [15]:

$$ \ddot{a}_x = \frac{N_x}{D_x} \tag{3} $$

with $\ddot{a}_x$ is the initial lifetime annuity. Form N_x and D_x are commutations symbols simplifying calculations in calculating mortality tables, expressed by:

$$ D_x = v^x l_x \tag{4} $$

$$ N_x = D_x + D_{x+1} + D_{x+2} + \cdots + D_\omega = \sum_{i=0}^{\omega-x} D_{x+i} \tag{5} $$

with ω is the highest age reached.
Cash value of term life annuity n year is expressed by [9]:

$$ \ddot{a}_{x:[n]} = \frac{N_x - N_{x+n}}{D_x} \tag{6} $$

2.4 Benefit Function

The benefit function is used to determine the amount of benefits that the company needs to pay for participants when entering retirement age. In this study, the benefit function of participants with retirement age is used r based on the last salary [8]:

$$ B_r = k(r-y)s_{r-1} \tag{7} $$

with

B_r : Accumulated retirement benefits at age r years
k : Percentage of salary that will go towards retirement benefits
r : Normal retirement age
y : Age of entry into the pension fund
s_{r-1} : Last year's salary before retirement

2.5 Present Value of Future Benefits (PVFB)

For example ${}_{r-x}p_x$ denoted the probability of an employee staying employed until the age of r years, B_r state the amount of retirement benefits at age r year, $P(r-x)$ state the expected cash value of payments at time $(r-x)$ year, and $\ddot{a}_r$ denotes the initial lifetime annuity. Present Value of Future Benefit for someone aged x years old and with a normal retirement age r can be expressed [5] :

$$^{r}(PVFB)_x = B_r\ {}_{r-x}p_x P(r-x)\ddot{a}_r \tag{8}$$

2.6 Vasicek and Cox-Ingersoll-Ross Interest Model

The Vasicek interest rate model is a stochastic model that has a tendency to return to an equilibrium point after experiencing a decrease or increase, expressed as follows [10]:

$$dr(t) = \alpha\big(\mu - r(t)\big)dt + \sigma\, dW(t) \tag{9}$$

with
α : Speed of interest rate adjustment towards the long-term interest rate level
μ : Long-term average interest rate
σ : Interest rate volatility
$r(t)$: Interest rate at time t
$W(t)$: Wiener Process at time t

Suppose $P_1(t)$ denote the expectation of the cash value of a one-unit payment when t for interest rates following the Vasicek model:

$$P_1(t) = exp\left(\big(B(t)-t\big)\left(\mu - \frac{\sigma^2}{2\alpha^2}\right) - \frac{\sigma^2 B(t)^2}{4\alpha} - r(0)B(t)\right) \tag{10}$$

with $r(0) = r_0$ and $B(t) = \frac{1-exp(-\alpha t)}{\alpha}$.

The Cox-Ingersoll-Ross (CIR) interest rate model guarantees a positive interest rate and a trend back to the mean, expressed as follows [16]:

$$dr(t) = \alpha\big(\mu - r(t)\big)dt + \sigma\sqrt{r(t)}dW(t) \tag{11}$$

with
α : Adjustment speed $r(t)$ to μ
μ : Long-term average interest rate
σ : Interest rate volatility
$r(t)$: Interest rate at time t
$W(t)$: Wiener process at time t

Suppose $P_2(t)$ denote the expected cash value of a one-unit payment at time t for an interest rate that follows the Cox-Ingersoll-Ross (CIR) model:

$$P_2(t) = \left(\frac{2d\exp\left(\frac{\alpha+d}{2}t\right)}{\exp(td)\,(\alpha+d)+d-\alpha}\right)^{\frac{2\alpha\mu}{\sigma^2}} \exp\left(-\frac{2r(0)(\exp(td)-1)}{\exp(td)\,(\alpha+d)+d-\alpha}\right) \tag{12}$$

with $r(0) = r_0$ and $d = \sqrt{\alpha^2 + 2\sigma^2}$.

The value of interest rate parameter of the Vasicek and Cox-Ingersoll-Ross models is calculated using Bank Indonesia interest rate data for 2018-2022, obtained using the following equation [3] :

$$\alpha = \frac{n^2 - 2n + 1 + \sum_{i=1}^{n-1} r_{t+1} \sum_{i=1}^{n-1} \frac{1}{r_t} - \sum_{i=1}^{n-1} r_t \sum_{i=1}^{n-1} \frac{1}{r_t} - (n-1) \sum_{i=1}^{n-1} \frac{r_{t+1}}{r_t}}{\left(n^2 - 2n + 1 - \sum_{i=1}^{n-1} r_t \sum_{i=1}^{n-1} \frac{1}{r_t}\right) \Delta t} \tag{13}$$

$$\mu = \frac{(n-1) \sum_{i=1}^{n-1} r_{t+1} - \sum_{i=1}^{n-1} \frac{r_{t+1}}{r_t} \sum_{i=1}^{n-1} r_t}{\left(n^2 - 2n + 1 + \sum_{i=1}^{n-1} r_{t+1} \sum_{i=1}^{n-1} \frac{1}{r_t} - \sum_{i=1}^{n-1} r_t \sum_{i=1}^{n-1} \frac{1}{r_t} - (n-1) \sum_{i=1}^{n-1} \frac{r_{t+1}}{r_t}\right)} \tag{14}$$

$$\sigma = \sqrt{\frac{1}{n-2} \sum_{i=1}^{n-1} \left(\frac{r_{t+1} - r_t}{\sqrt{r_t}} - \frac{\mu}{\sqrt{r_t}} + \alpha \sqrt{r_t} \right)^2} \tag{15}$$

2.7 Normal Cost (NC)

Normal Cost (NC) is the contribution required in one year to fund part of the present value of pension benefits, allocated in the current year according to the actuarial calculation method used [17]. The normal contribution equation that will be used in this study using the Projected Unit Credit method [8]:

$${}^{PUC}\,{}^{r}(NC)_x = \frac{B_r \; {}_{r-x}p_x P(r-x) \ddot{a}_r}{r-y} = \frac{{}^{r}(PVFB)_x}{r-y} \tag{16}$$

${}^{r}(NC)_x$	: Normal cost of pension fund participants aged x year
B_r	: Accumulated retirement benefits at age r years
$\ddot{a}_r$	: Discrete lifetime annuity starts in r year
v^{r-x}	: Interest rate to $(r-x)$ year
${}_{r-x}p_x$	: Chance of participants aged x years old live up to $(r-x)$ years
${}^{r}(PVFB)_x$	: Cash value of total retirement benefits at age r year
r	: Normal retirement age
y	: Age of entry into the pension fund

Suppose $\ddot{a}_{y:\lceil r-y \rceil}$ denote the amount of the initial term annuity for an employee with the age of y years old at the time of entry into the pension fund, and will retire normally at the age of r year. The normal cost equation using the Entry Age Normal method is expressed as follows [5]:

$${}^{EAN}\,{}^{r}(NC)_x = \frac{P(x-y) \; {}_{x-y}p_y \, {}^{r}(PVFB)_x}{\ddot{a}_{y:\lceil r-y \rceil}} \tag{17}$$

2.8 Actuarial Liability

Actuarial Liability (AL) is the amount of pension plan funds accumulated to pay pension benefits in the future, the actuarial liability equation uses the Projected Unit Credit method [8]:

$${}^{PUC}\,{}^{r}(AL)_x = \frac{(x-y) B_r \; {}_{r-x}p_x P(r-x) \ddot{a}_r}{r-y} = \frac{(x-y)^{r}(PVFB)_x}{r-y} \tag{18}$$

The actuarial liability equation using the Entry Age Normal method is given as follows [5]:

$${}^{EAN}\,{}^{r}(AL)_x = \frac{\ddot{a}_{y:\lceil x-y \rceil} \, {}^{r}(PVFB)_x}{\ddot{a}_{y:\lceil r-y \rceil}} \tag{19}$$

3. RESULTS AND DISCUSSION

The application of the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods with the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models used in this study is a retired civil servant at the Bulukumba regency water resources management office. Aged when registered as a civil servant is 26 years ($y = 26$), male gender, normal retirement age 56 years ($r = 56$), and with a basic monthly salary of Rp$2.200.000,-$. It was assumed that the percentage of salary given for retirement benefits is equal to 2,25% and the accumulated salary during the last year before retirement is Rp$26.400.000,-$.

3.1 Pension Benefit Amount

The value of retirement benefits is calculated using the assumption of salary for the last year before retirement of $s_{r-1} = Rp26.400.000$, $k = 2{,}25\%$, $r = 56$, and $y = 26$ as follows [8]:

$$B_r = k(r-y)s_{r-1}$$
$$B_{56} = 2{,}25\%(56-26)(26.400.000) = Rp17.820.000,-$$

So, a civil servant who works at the Bulukumba regency water resources management office from the age of 26 will receive a pension benefit of $Rp17.820.000,-$.

3.2 Life Annuity Value

Life annuity is a series of payments made periodically in monthly, quarterly, semi-annual, and annual periods. To calculate a life annuity, it is necessary to expect the cash value of payments of one unit at a certain time following the Vasicek interest rate using **Equation (10)** and the Cox-Ingersoll-Ross interest rate in **Equation (12)**. Parameter values α, μ, and σ in **Equation (13)**-**(15)** are calculated using monthly data on the Bank Indonesia interest rate for 2018-2022, the parameter estimation results are shown in **Tabel 1** below:

Table 1. Vasicek and Cox-Ingersoll-Ross (CIR) Interest Rate Parameter Estimation Results

Parameters	Estimation Results
α	0,008196205
μ	0,070322289
σ	0,131469885

Based on the data used, it is known that the age of becoming a civil servant is $y = 26$, normal retirement age $r = 56$, and x states the age at the time of actuarial valuation. Expected cash value of payments based on the Vasicek interest rate model is calculated using **Equation (10)** and the Cox-Ingersoll-Ross (CIR) interest rate is calculated using **Equation (12)** and $r(0) = 0{,}055$ the results obtained in **Figure 1**

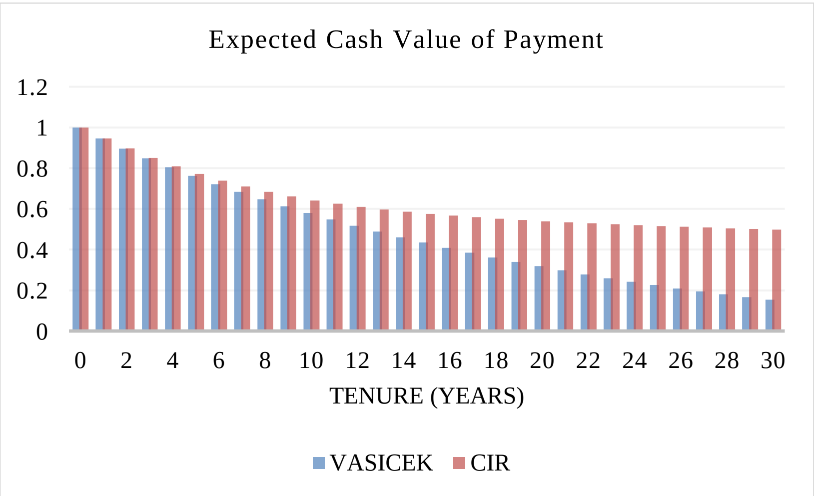

Figure 1. Expected Cash Value of Payments During Working Life

Lifetime annuity value at normal retirement age $r = 56$ year based on the Vasicek interest rate:

$$\ddot{a}_{56} = \frac{N_{56}}{D_{56}} = \frac{5156{,}175007}{795{,}5451437} = 6{,}481310392$$

Based in the Cox-Ingersoll-Ross (CIR) interest rate:

$$\ddot{a}_{56} = \frac{N_{56}}{D_{56}} = \frac{961221{,}9972}{38648{,}8227} = 24{,}87$$

Term life annuity values for $r = 56$ and $y = 26$ based on the Vasicek interest rate is obtained $\ddot{a}_{y:[r-y]} = \ddot{a}_{26:[30]} = 10{,}88096278$ and with the CIR model obtained $\ddot{a}_{y:[r-y]} = \ddot{a}_{26:[30]} = 26{,}82809096$. In full, the value of term life annuities following the Vasicek and CIR models for different values of x which are different $\ddot{a}_{26:[x-26]}$ is given in **Figure 2**.

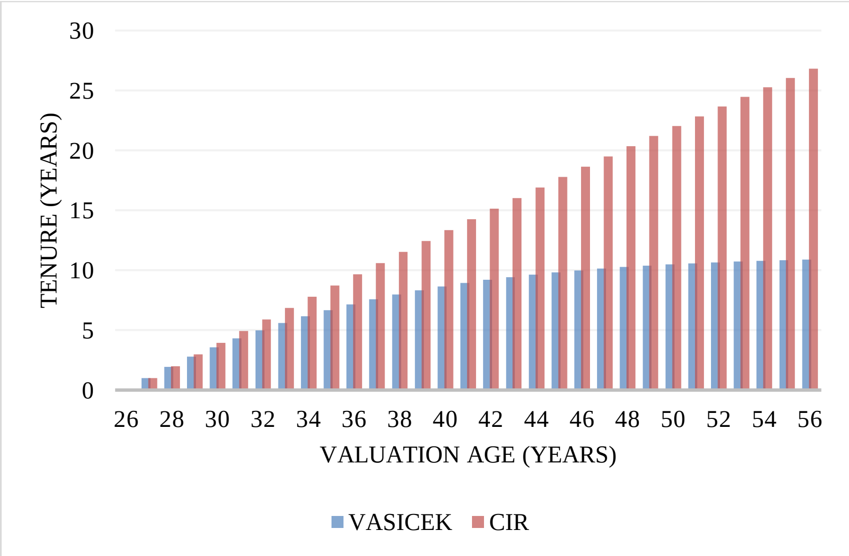

Figure 2. Term Life Annuity $\ddot{a}_{y:[x-y]}$ Vasicek and CIR Models

3.3 Present Value Future Benefit (FVFB)

The present value of pension benefits is influenced by the life expectancy value obtained from the 2019 Mortality Table for males, the value of pension benefits with retirement age r, the expected cash value of payments following the Vasicek and Cox-Ingersoll-Ross model, and the lifetime annuity. The PVFB value is calculated using the equation:

$$^{56}(PVFB)_x = B_{56\ 56-x}p_x P(56-x)\ddot{a}_{56}$$

Vasicek interest rate model for valuation age $x = 26, 27,$ and 28 years, the results are as follows:

valuation age $x = 26$

$$^{56}(PVFB)_{26} = B_{56\ 30}p_{26}P_1(30)\ddot{a}_{56}$$

$$^{56}(PVFB)_{26} = (17.820.000)(0{,}921342632)(0{,}153351247)(6{,}481310392) = 16318453{,}5$$

valuation age $x = 27$

$$^{56}(PVFB)_{27} = B_{56\ 29}p_{27}P_1(29)\ddot{a}_{56}$$

$$^{56}(PVFB)_{27} = (17.820.000)(0{,}921849649)(0{,}166223414)(6{,}481310392) = 17697943{,}8$$

valuation age $x = 28$

$$^{56}(PVFB)_{28} = B_{56\ 28}p_{28}P_1(28)\ddot{a}_{56}$$

$$^{56}(PVFB)_{28} = (17.820.000)(0{,}922403091)(0{,}179846774)(6{,}481310392) = 19159930{,}15$$

Cox-Ingersoll-Ross (CIR) interest rate model for valuation age $x = 26, 27$, and 28 years obtained the following results:

valuation age $x = 26$

$$^{56}(PVFB)_{26} = B_{56\ 30}p_{26}P_2(30)\ddot{a}_{56}$$

$$^{56}(PVFB)_{26} = (17.820.000)(0{,}921342632)(0{,}498119977)(24{,}87) = 203.399.676$$

valuation age $x = 27$

$$^{56}(PVFB)_{27} = B_{56\ 29}p_{27}P_2(29)\ddot{a}_{56}$$

$$^{56}(PVFB)_{27} = (17.820.000)(0{,}921849649)(0{,}501475436)(24{,}87) = 204.882.512$$

valuation age $x = 28$

$$^{56}(PVFB)_{28} = B_{56\ 28}p_{28}P_2(28)\ddot{a}_{56}$$

$$^{56}(PVFB)_{28} = (17.820.000)(0{,}922403091)(0{,}504933413)(24{,}87) = 206.419.153$$

The complete value of present value future benefit for the valuation age of $x = 26 - 56$ is given in Figure 3.

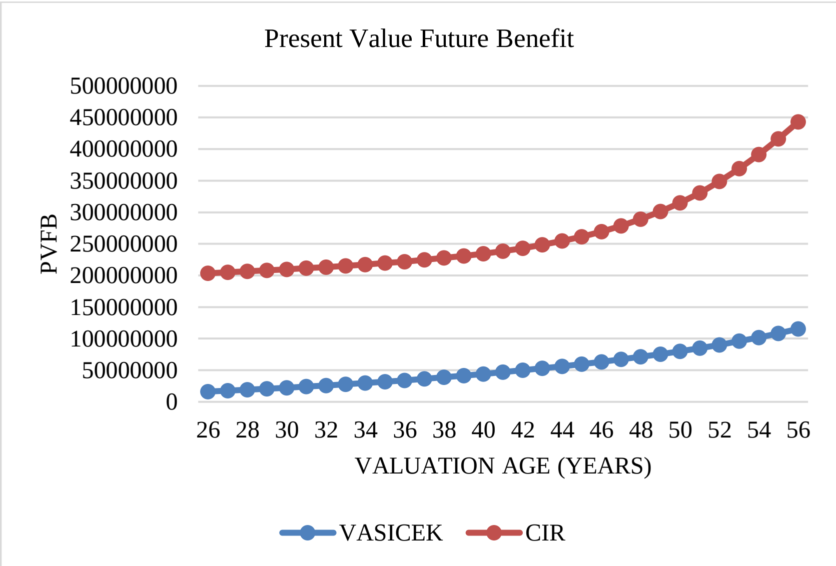

Figure 3. Present Value Future Benefit (PVFB)

3.4 Normal Cost (NC)

To get the Present Value Future Benefit (FVFB), a pension fund participant must make normal cost. The amount of normal cost to be paid by participants is calculated using two methods, namely Projected Unit Credit (PUC) and Entry Age Normal (EAN), with each method using two interest rate model, namely Vasicek and Cox-Ingersoll-Ross (CIR). Normal cost using the Projected Unit Credit method are calculated using the equation

$$^{PUC\ 56}(NC)_x = \frac{B_{56\ 56-x}p_x P(56-x)\ddot{a}_{56}}{30} = \frac{^{56}(PVFB)_x}{30}$$

valuation age $x = 26$ and $x = 27$ Vasicek interest rate model obtained

$$^{PUC\ 56}(NC)_{26} = \frac{^{56}(PVFB)_{26}}{30} = \frac{16318453{,}5}{30} = 543948{,}4499$$

$$^{PUC\ 56}(NC)_{27} = \frac{^{56}(PVFB)_{27}}{30} = \frac{17697943{,}8}{30} = 589931{,}4599$$

valuation age $x = 26$ and $x = 27$ Cox-Ingersoll-Ross (CIR) interest rate model obtained

$$^{PUC\,56}(NC)_{26} = \frac{^{56}(PVFB)_{26}}{30} = \frac{203.399.676}{30} = 6.779.989$$
$$^{PUC\,56}(NC)_{27} = \frac{^{56}(PVFB)_{27}}{30} = \frac{204.882.512}{30} = 6.829.417$$

Normal cost using the Entry Age Normal method are calculated by the equation

$$^{EAN\,56}(NC)_x = \frac{P(x-26)\ \ _{x-26}p_{26}\ ^{56}(PVFB)_x}{\ddot{a}_{26:[30]}}$$

valuation age $x = 26$ and $x = 27$ Vasicek interest rate model obtained

$$^{EAN\,56}(NC)_{26} = \frac{P_1(0)\ \ _0p_{26}\ ^{56}(PVFB)_{26}}{\ddot{a}_{26:[30]}} = \frac{16318453{,}5}{10{,}88096278} = 1499725{,}147$$
$$^{EAN\,56}(NC)_{27} = \frac{P_1(1)\ \ _1p_{26}\ ^{56}(PVFB)_{27}}{\ddot{a}_{26:[30]}} = \frac{16745081{,}25}{10{,}88096278} = 1538933{,}787$$

valuation age $x = 26$ and $x = 27$ Cox-Ingersoll-Ross (CIR) interest rate model obtained

$$^{EAN\,56}(NC)_{26} = \frac{P_2(0)\ \ _0p_{26}\ ^{56}(PVFB)_{26}}{\ddot{a}_{26:[30]}} = \frac{203399676{,}3}{26{,}82809096} = 7.581.593$$
$$^{EAN\,56}(NC)_{27} = \frac{P_2(1)\ \ _1p_{26}\ ^{56}(PVFB)_{27}}{\ddot{a}_{26:[30]}} = \frac{193829897{,}9}{26{,}82809096} = 7.224.886$$

The complete normalized cost with Vasicek and Cox-Ingersoll-Ross (CIR) interest rates are given in **Figure 4**.

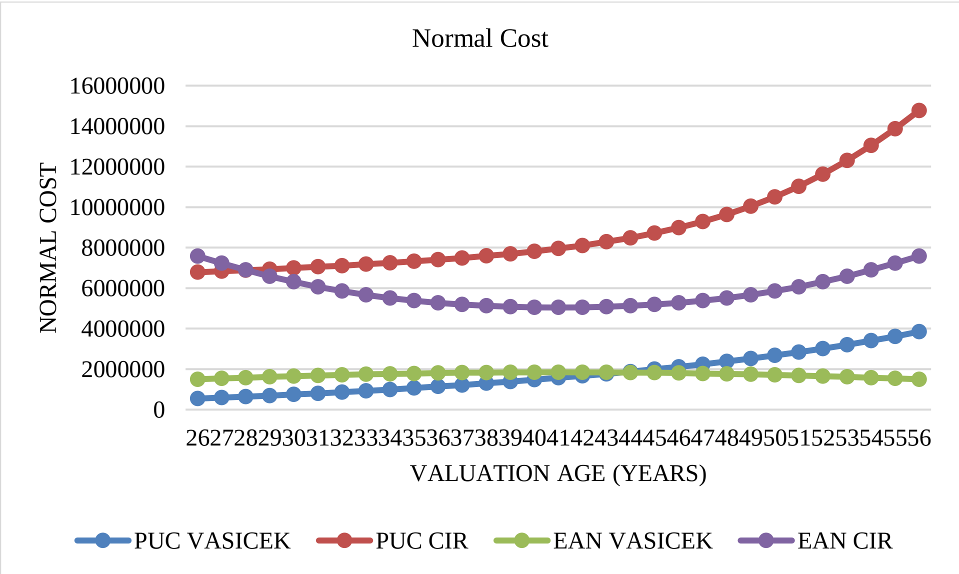

Figure 4. Normal Cost (NC)

Based on **Figure 4**, it can be seen that in general, in the 0-2 years of services period, the normal cost using the Projected Unit Credit (PUC) method is smaller than the Entry Age Normal (EAN) method. In the third year, the normal cost with the CIR interest rate of the Projected Unit Credit (PUC) method is the highest and continues to increase until the working period reaches 30 years by $Rp14.773.176$ every year. While the Projected Unit Credit (PUC) method of the Vasicek interest rate model is smaller than Entry Age Normal with the same interest rate until the working age reaches 17 years. The at 18 years of service, the Projected Unit Credit (PUC) method with the Vasicek interest rate continues to increase and the value is always higher than EAN method of the Vasicek model. It can be seen that using these two methods, the normal cost every year for the CIR interest rate is always greater than Vasicek. Normal cost using the Projected Unit Credit (PUC) method tend to always increase as the working period increases. Whereas with the Entry Age Normal (EAN method, the Vasicek model initially increases and then decreases as the normal retirement age approaches, and the CIR model initially decreases but increases again at the end of the working period with the amount of contributions equivalent to the initial contribution of the working period.

3.5 Actuarial Liability (AL)

The actuarial liability using the Projected Unit Credit (PUC) method is calculated using the equation

$$^{PUC\ 56}(AL)_x = \frac{(x-y)^{56}(PVFB)_x}{30}$$

valuation age $x = 26$ and $x = 27$ Vasicek interest rate model obtained

$$^{PUC\ 56}(AL)_{26} = \frac{(0)^{56}(PVFB)_{26}}{30} = 0$$
$$^{PUC\ 56}(AL)_{27} = \frac{(1)^{56}(PVFB)_{27}}{30} = \frac{17697943{,}8}{30} = 589931{,}4599$$

valuation age $x = 26$ and $x = 27$ Cox-Ingersoll-Ross (CIR) interest rate model obtained

$$^{PUC\ 56}(AL)_{26} = \frac{(0)^{56}(PVFB)_{26}}{30} = 0$$
$$^{PUC\ 56}(AL)_{27} = \frac{(1)^{56}(PVFB)_{27}}{30} = \frac{204.882.512}{30} = 6829417{,}074$$

Actuarial liabilities using the Entry Age Normal (EAN) method are calculated using the equation

$$^{EAN\ r}(AL)_x = \frac{\ddot{a}_{y:\lceil x-y \rceil}{}^{r}(PVFB)_x}{\ddot{a}_{y:\lceil r-y \rceil}}$$

valuation age $x = 26$ and $x = 27$ Vasicek interest rate model obtained

$$^{EAN\ 56}(AL)_{26} = \frac{\ddot{a}_{26:\lceil 0 \rceil}{}^{56}(PVFB)_{26}}{\ddot{a}_{26:\lceil 30 \rceil}} = \frac{0}{10{,}88096278} = 0$$
$$^{EAN\ 56}(AL)_{27} = \frac{\ddot{a}_{26:\lceil 1 \rceil}{}^{56}(PVFB)_{27}}{\ddot{a}_{26:\lceil 30 \rceil}} = \frac{17697943{,}8}{10{,}88096278} = 1626505{,}316$$

valuation age $x = 26$ and $x = 27$ Cox-Ingersoll-Ross (CIR) interest rate model obtained

$$^{EAN\ 56}(AL)_{26} = \frac{\ddot{a}_{26:\lceil 0 \rceil}{}^{56}(PVFB)_{26}}{\ddot{a}_{26:\lceil 30 \rceil}} = \frac{0}{26{,}82809096} = 0$$
$$^{EAN\ 56}(AL)_{27} = \frac{\ddot{a}_{26:\lceil 1 \rceil}{}^{56}(PVFB)_{27}}{\ddot{a}_{26:\lceil 30 \rceil}} = \frac{204882512{,}2}{26{,}82809096} = 7636865{,}125$$

The complete actuarial liabilities using the Projected Unit Credit (PUC) and Entry Age Normal (EAN) methods are given **Figure 5**.

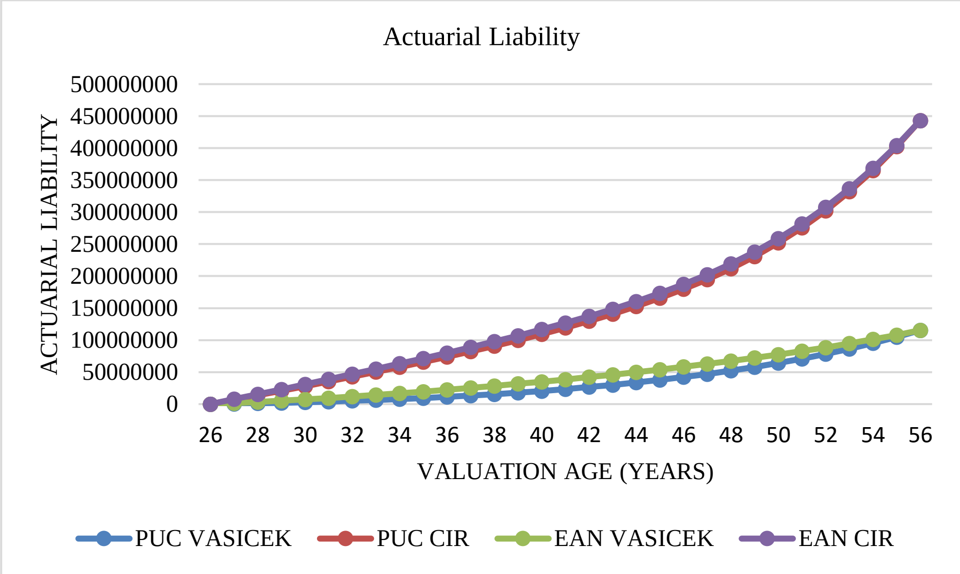

Figure 5. Actuarial Liability

The actuarial liability for 0 years of services is $Rp0,-$ because at that time participants had just started contributions so they did not have a benefit reserve. It can be seen that actuarial liabilities continue to increase every year for each method and interest rate. The actuarial liability of the Projected Unit Credit (PUC) method with the Vasicek interest rate is annually smaller than the Entry Age Normal (EAN) method as well as for the CIR model interest rate, but the same value at the end of the working period. In addition, the actuarial liability of the Entry Age Normal (EAN) method with the CIR interest rate has the highest value since the beginning of the year.

4. CONCLUSIONS

The calculation of normal cost and actuarial liabilities using the Projected Unit Credit (PUC) method with the Vasicek and Cox-Ingersoll-Ross (CIR) interest rate models increases as the length of service increases. Unlike the Projected Unit Credit (PUC) method, the normal cost of the Entry Age Normal (EAN) method of the Vasicek interest rate model initially increases and then decreases towards the initial contributions value of the working period. Meanwhile, the Entry Age Normal (EAN) method of the Cox-Ingersoll-Ross (CIR) model starts with a fairly high initial contribution then decreases but rises again towards the initial contribution of the working period. As for the actuarial liabilities of the Entry Age Normal (EAN) method for both interest rate models, it always increases every year. Thus, it can be concluded that the Entry Age Normal method with the Vasicek model interest rate is better for participants because it has a lower normal cost value. Meanwhile, for pension funds, the Projected Unit Credit method with the Cox-Ingersoll-Ross (CIR) model interest rate is more profitable.

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