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ARIMA MODEL VERIFICATION WITH OUTLIER FACTORS USING CONTROL CHART

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ABSTRACT

Control charts are often used in quality control processes, especially in the industrial sector because of their significant benefits in increasing industrial production. However, control charts can also be used in time series modeling to evaluate measures of accuracy represented by a particular time series model. The application of control charts in this research meets the criteria for evaluating accuracy. However, it is not certain that the time series model will have a high level of accuracy. There are various factors that can influence this phenomenon, one of which is the potential for outliers. Therefore, it is very important to perform time series modeling by adding an outlier factor. The residuals of the time series model obtained are used to create a control chart for model verification. The aim of this research is to evaluate the validity of time series models by looking at the influence of outlier characteristics to improve their accuracy. This research studies the accuracy of a time series model built using Gross Domestic Product (GDP) data in Indonesia from 1975 to 2021. There are two different models, namely the ARIMA model without outlier factors and the ARIMA model with outlier factors which are used for research purposes. Both models were performed using the same data set. The results of this study indicate that the ARIMA model with outlier factors has better accuracy than the ARIMA model without outlier factors. This conclusion can be drawn based on the observation that the residual value is within the predetermined control limits, thus indicating that the process is in a state of statistical control.

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1. INTRODUCTION

Control charts are frequently used in Statistical Process Control (SPC) in the industrial sector for tasks such as determining process quality, determining process capability, determining when the process is executed, and determining why product quality criteria are not met [1]. Control charts are also incredibly useful for managing the primary materials that will eventually become finished goods, specifically for improving product quality, reducing waste, increasing manufacturing capacity, minimizing damage, and enhancing consumer values [2]. However, control charts can also be used in time series modelling to assess the model's accuracy.

The Box-Jenkins method, also known as the Autoregressive Integrated Moving Average (ARIMA) technique, is one of the time series modelling techniques [3]. In the 1960s, George Box and Gwilym Jenkins introduced and developed this technique [4]. The ARIMA approach takes into account time series concepts including the stationary test, parameter estimation, and diagnostic tests. Comparatively to other forecasting techniques, the ARIMA method's computation procedure may be viewed as quite complex, and there is no guarantee that the resulting time series model will be highly accurate. There are a variety of factors that can affect it, including the existence of outliers. Consequently, it is necessary to conduct time series modelling and incorporate outlier components [5]. Unexpected or unusual events, such as sudden political or economic crises, strikes, war outbreaks, and even typographical and speculative recording errors, frequently have an impact on time series observations that are outliers [6]. In a time series, there are two categories of outliers: Additive Outliers (AO) and Innovative Outliers (IO) [7]. Verification of the ARIMA model using a control chart was carried out to determine the presence of outliers [8]. It turned out that there were residual values that were out of control because they had extreme values. It can be assumed that the ARIMA (1,0,0) model contains outlier [9].

This study assesses the accuracy of the time series model developed using Indonesian Gross Domestic Product (GDP) data. GDP measures all goods and services produced by a region's production facilities over a given time period [10]. The GDP has always impacted Indonesia's economy [10]. Indonesia's economic growth is influenced by an increase in GDP, and economic growth is closely related to GDP. Rapid GDP growth indicates economic expansion which increases people's purchasing power. GDP can experience drastic changes if there is an extraordinary economic event such as a financial crisis or significant natural disaster [11]. So this study uses two different models: an ARIMA model without outliers and an ARIMA model that includes outliers.

2. RESEARCH METHODS

2.1 Autoregressive Integrated Moving Average (ARIMA)

A Time series model known as ARIMA can also be applied to non-stationary data [12]. The stationary test, a key assumption in analyzing time series models in order to develop a successful model, stipulates that the time series data utilized in the ARIMA model must be stationary [13]. The time series data are stationary when the mean and variance for each latency are constant or fixed over time [14]. The time series data must fulfill these conditions in order to be considered stationary. The stationarity of the time series is determined visually in this study. The plots visually determine whether or not a data set is stationary. If the data is stationary, the plot has a constant trend around a relatively fixed average value, or there is no discernible upward or downward trend [14]. The applied ARIMA model is defined by three orders: p, d, and q, where p is the order of the AR model, q is the order of the MA model, and d is the order of the differentiating process [15]. Therefore, the ARIMA model can be written with ARIMA (p, d, q) with the general form as follows.

$$\phi_p(B)(1-B)^d X_t = \theta_q(B)\, e_t \tag{1}$$

where,

ϕ_p : p – parameter coefficient for autoregressive
θ_q : q – parameter coefficient for moving average
X_t : observations at t-time with $t = 1, 2, \cdots, n$
e_t : residual at t-time with $t = 1,2,\cdots,n$
$(1-B)^d$: d-order differentiation
B : backshift operator

2.2 Time Series Control Chart

A control chart is a tool used to determine whether a process is statistically under control to address problems and improve quality [16]. On the control chart, there are three lines: the Centre Line (CL), the Upper Control Limit (UCL), and the Lower Control Limit (LCL) [17]. CL represents the mean value of quality attributes, while UCL and LCL are control limits [18].

The control chart's underlying assumptions are independence and the absence of correlation between observations [16]. The observations utilized by the time series control chart are residuals because the residual assumption in the time series model is white noise, which is autocorrelated and independent of time. The residual model only takes into account a single variable [19]. Control charts for time series models can be applied to any model, assuming that residuals are white noise. If these conditions are satisfied, the calculations can be represented on an IMR control chart [16]. As a control chart, this study employs the IMR control chart. IMR control charts are used for continuous data with a single group size, and the calculations are as follows [16]:

$$Individual\ plot: \begin{cases} UCL = \bar{x} + 3\dfrac{\overline{MR}}{d_2} \\ CL = 0 \\ LCL = \bar{x} - 3\dfrac{\overline{MR}}{d_2} \end{cases} \quad (2)$$

and

$$Movingrange\ plot: \begin{cases} UCL = D_4 \times \overline{MR} \\ CL = \overline{MR} \\ LCL = 0 \end{cases} \quad (3)$$

where $\overline{MR}$ is the average of MR_t with $MR_t = |x_t - x_{t-1}|$, x_t is t-th observation, d_2 and $D_4$are constants.

2.3 Innovative Outlier (IO)

Outliers are random and unexpected observations unrelated to specific occurrences, such as political policies and natural disasters [20]. Time series data has the ability to detect outliers [5]. The presence of outliers can lead to biassed parameters and invalid results [21]. Detecting outliers becomes an essential stage that must also be performed on time series data to mitigate this effect. Outlier detection was first introduced by Fox (1972) and Wei (2006) [22]. Fox (1972) introduced AO (Additive Outlier) and IO (Innovative Outlier), while Tsay (1988) introduced LS (Level Shift) and TC (Temporary Change) [23]. Type AO impacts only the T observation. While the IO, LS, and TC types are outliers that influence the T, T+1, T+2, ... observations [24]. In this paper, the outlier discussed is IO. The following is the general form of the outlier factors

$$Y_t = \sum_{j=1}^{k} \omega_j \nu_j(B) I_t^{(T_j)} + \frac{\theta(B)}{\phi(B)} e_t \quad (4)$$

where $\nu_j(B) = \frac{\theta(B)}{\phi(B)}$ for an IO at time T_j [25]. And IO detection on time series as follows:

$$Y_t = \frac{\theta(B)}{\phi(B)} e_t + \frac{\theta(B)}{\phi(B)} \omega I_t^{(T)} = \frac{\theta(B)}{\phi(B)} \left(e_t + \omega I_t^{(T)}\right) \quad (5)$$

On modelling data suspected of containing IO, a procedure for iterative outlier detection will be applied. Following is the iterative procedure [26], [27] :

1. Assume the data $\boldsymbol{X_t}$ has no outlier. Then calculate the residual from the estimated model as follows:

$$\boldsymbol{\hat{e}_t = X_t - \hat{X}_t} \quad (6)$$

and calculate the variance of residuals $\boldsymbol{\hat{\sigma}_e^2}$ such that:

$$\boldsymbol{\hat{\sigma}_e^2 = \frac{1}{n} \sum_{t=1}^{n} \hat{e}_t^2} \quad (7)$$

2. Calculate $\boldsymbol{\hat{\lambda}_{1,T}}$ as outlier detection parameter, such that $\boldsymbol{\hat{\lambda}_{1,T}} = (\boldsymbol{\hat{\sigma}_e})^{-1}\boldsymbol{\hat{e}_T}$ where T show the maximum occurrence time. If $\boldsymbol{\eta_t} = \left|\boldsymbol{\hat{\lambda}_{1,T}}\right| > \boldsymbol{C}$, then there is an IO at time $\boldsymbol{T}$. Then modifying the residual according to the type of detected outlier (IO)

$$\tilde{e}_t = \hat{e}_t - \hat{\omega}_{IT} \tag{8}$$

where, $\hat{\omega}_{IT} = \hat{e}_t$ for IO [28].
For $\tilde{\sigma}_e^2$ calculated through the modified residual

3. Recalculate $\hat{\lambda}_{1,T}$ based on the new residual value and $\tilde{\sigma}_e^2$, then repeat the second step and stop the iteration until no outliers can be identified.

2.4 Modeling Procedure

In this study, data analysis is performed by constructing an ARIMA model from time series data. Then, the optimal model is determined using ARIMA, and the model's residuals are used to determine the presence of outliers. A control chart is used to detect outliers; if it is out of control, then there are outliers. Figure 1 provides additional information about this study's methodology. Figure 1 also describes the outlier procedure. For more details, the procedures for this research can be seen in Figure 1.

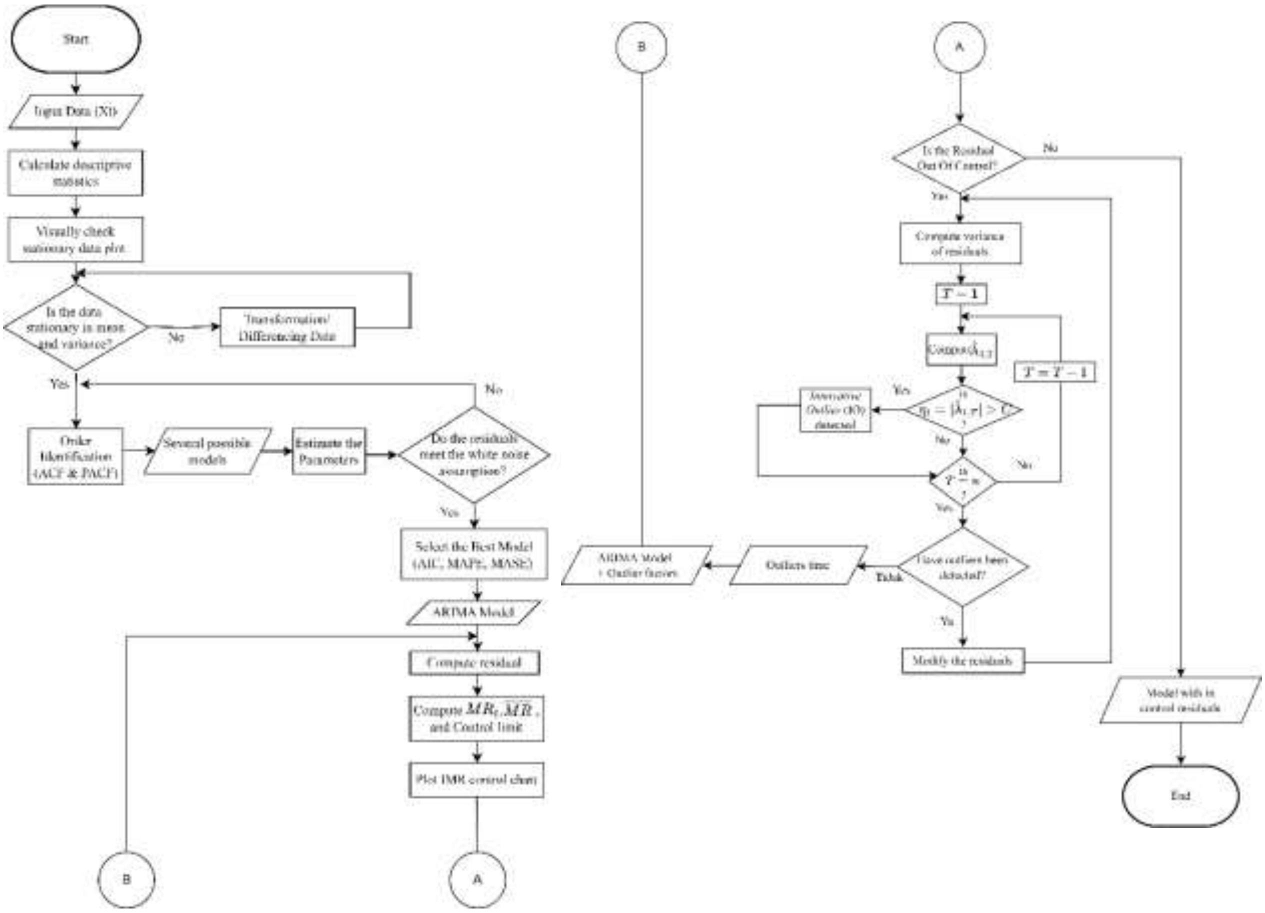


Figure 1. Flowchart Verification ARIMA Model with Outlier Factor Using Control Chart

3. RESULTS AND DISCUSSION

In this research, secondary data on the growth of Indonesia's Gross Domestic Product (GDP) in the form of percentages obtained from the World Bank website is employed. The size of the data used in this study is 47 from 1975 until 2021. These data are used to develop a time series model, from which two models are derived: the ARIMA model with outlier factors and the ARIMA model without outlier factors. Figure 2 below presents a plot of the data used.

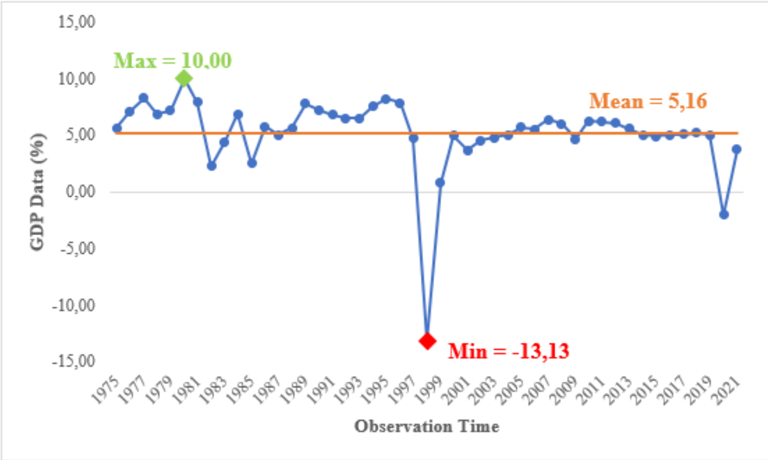
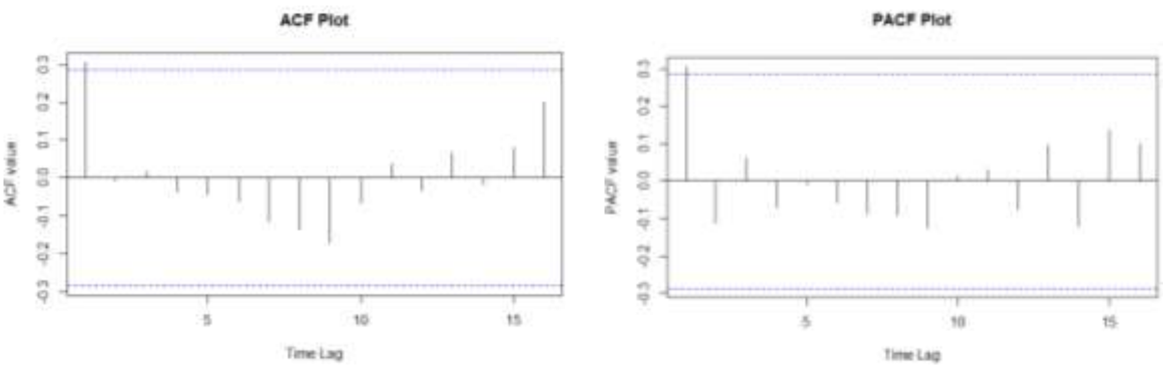


Figure 2. Plot Data GDP in Indonesia

In 1980, the GDP in Indonesia had the highest growth value when compared to other years, namely 10% (Observations are marked in green in **Figure 2**) which indicated the maximum value in the data because exports and imports in Indonesia increased in 1980 and caused GDP in Indonesia increased in that year. Otherwise, the value of GDP growth in Indonesia was lowest in 1998, namely -13.13% (Observations are marked in red in **Figure 2**) which indicated the minimum value in the data due to the monetary crisis in 1998, which made the country's economy very weak and harmed GDP growth in Indonesia. It can be seen from the orange line on the plot that the mean value of the data is quite far when compared to the maximum and minimum values in the data, namely 5.16 (Observations are marked in the orange line in **Figure 2**). It, raises suspicions of outliers in the data. In this study, the stationarity test was carried out visually. This can be seen from the time series data plot because if you look at the plot, the data tends to be constant around the average value.

3.1. Autoregressive Integrated Moving Average (ARIMA) Model

In this study, the GDP data were visually stationary in variance and mean, so plot of stationary test was sufficient for this investigation. The stationary data are used to determine the p and q orders. Order identification is accomplished by examining the cut-off latency or tail-off lag in the ACF and PACF plots.

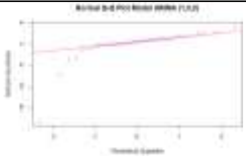
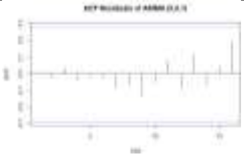
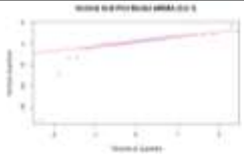
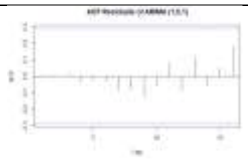
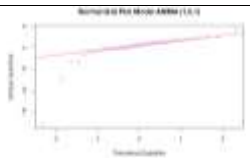


(a) **(b)**

Figure 3. (a) ACF Plot from Data Used, (b) PACF Plot from Data Used

The ACF and PACF plots cut-off after the first lag. Because it is stationary, the appropriate model assumptions in this case study are ARIMA (1,0,0), ARIMA (0,0,1), and ARIMA (1,0,1). The estimated parameters are shown in **Table 1**.

Table 1. Parameter Estimation for Each Model

Model	Parameter		Residual Plot for Diagnostic Test		Model Accuration		
					AIC	MAPE	MASE
ARIMA (1,0,0)	ϕ	0.30	 Pass Diagnostic Tests	 Pass Diagnostic Tests	248.46	33.01	0.76
ARIMA (0,0,1)	θ	0.36	 Pass Diagnostic Tests	 Pass Diagnostic Tests	247.67	35.51	0.77
ARIMA (1,0,1)	θ ϕ	-0.13 0.48	 Pass Diagnostic Tests	 Pass Diagnostic Tests	249.59	34.80	0.78

A diagnostic test has been carried out on the residuals in each model, which consists of an independence test by looking at the ACF plot of the residual model and the normality test. The purpose of the diagnostic test is to ensure that the results of the study are unbiased, consistent, and accurate in estimation. From Table 1 it is known that the three ARIMA models are within the significance line, meaning that the residuals of the three models are independent of each other between time lags and visually the residuals of the three models can be said to be normal. This can be seen from most of scatter points of the residuals of the three models following the reference line. In Table 1 there are red numbers which mean the smallest values of AIC, MAPE, and MPE for each model. Because the time lag is within the significance line and the residuals are normally distributed, the three models pass the residual diagnostic test. The last step in forming the ARIMA model is selecting the best model which is also presented in Table 1. Seen from the AIC, MAPE, and MPE values of the three models, the best model was obtained, namely the ARIMA model (1,0,0) because it has the smallest MAPE and MPE values compared to the other two models.

3.2. Time Series Control Chart

The best model that has been tested for diagnostic testing is the ARIMA model (1,0,0), the model is then verified using a control chart to see whether the model is really good by plotting the residuals on the control chart. In this investigation, only individual plots from the control chart were utilized, while the moving range plot was disregarded. Based on Equation (2) the control chart is obtained as follows.

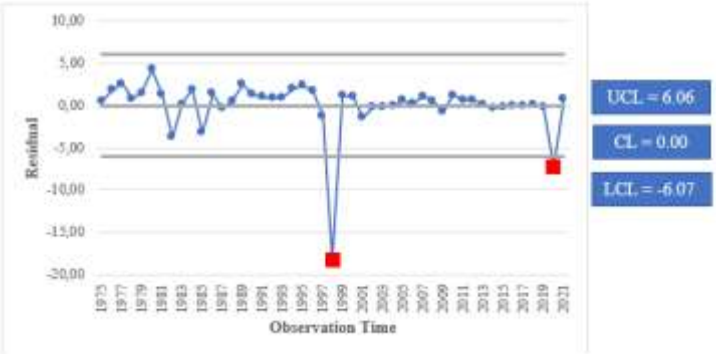


Figure 4. Control Chart of ARIMA Model Residual (1,0,0)

Figure 4 demonstrates the residuals of the ARIMA model (1,0,0), which are out of control due to multiple out-of-control points presence. A residual value outside the control limit indicates a substantial difference between the residual value and other residual values. Thus, the disparity between the observed data and the estimated data becomes substantial. It is because of the presence of outliers in the data. To assess the ARIMA (1,0,0) model, an outlier factor is added.

3.3. Autoregressive Integrated Moving Average (ARIMA) with Outlier Factors

Verification of the ARIMA model using a control chart was carried out to determine the presence of outliers. It turned out that there are residual values that are out of control because they had extreme values. It can be assumed that the ARIMA (1,0,0) model contains outliers. In this study, outlier detection was carried out by an iterative procedure. Each addition of outliers is checked with a control chart until the iteration process will stop when no outliers are detected. The following is a model formed through the outlier detection process using an iterative procedure:

1. ARIMA Model (1,0,0) without outlier factors

$$X_t = 0.30X_{t-1} + e_t \tag{9}$$

2. ARIMA Model (1,0,0) with outlier factors in the first iteration :
   There are two outliers detected in the first iteration, namely at the 24th and 46th times. Then the outliers are added to the ARIMA model (1,0,0) marked in blue.

$$Y_t = 2\phi_1 Y_{t-1} - (\phi_1)^2\, Y_{t-2} + e_t - \phi_1 e_{t-1} + \omega_1 I_t^{(24)} + \omega_2 I_t^{(46)} \tag{10}$$

   After that, parameter estimation is carried out so that **Equation (10)** becomes **Equation (11)**

$$Y_t = 0.58Y_{t-1} - 0.08Y_{t-2} + e_t - 0.29e_{t-1} + \omega_1 I_t^{(24)} + \omega_2 I_t^{(46)} \tag{11}$$

   with the following control chart

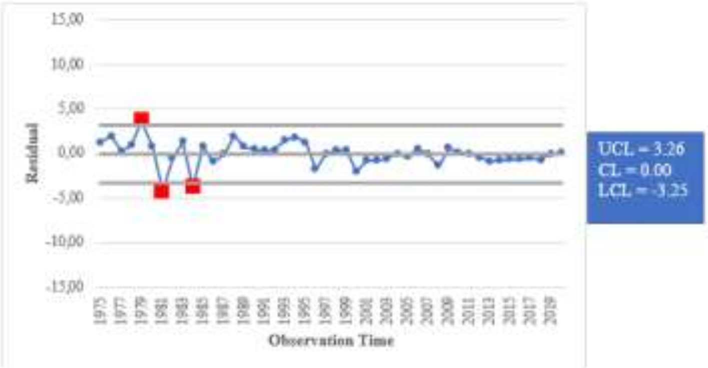


Figure 5. Addition of Outlier Factors in the First Iteration

3. ARIMA Model (1,0,0) with outlier factors in the second iteration :
   In the second iteration there is one outlier detected, namely at the 8th time which is then added to **Equation (12)**

$$Y_t = 2\phi_1 Y_{t-1} - (\phi_1)^2 Y_{t-2} + e_t - \phi_1 e_{t-1} + \omega_1 I_t^{(24)} + \omega_2 I_t^{(46)} + \omega_3 I_t^{(8)} \tag{12}$$

   After that, parameter estimation is carried out so that **Equation (12)** becomes **Equation (13)**

$$Y_t = 0.62Y_{t-1} - 0.09Y_{t-2} + e_t - 0.31e_{t-1} + \omega_1 I_t^{(24)} + \omega_2 I_t^{(46)} + \omega_3 I_t^{(8)} \tag{13}$$

   with the following control chart

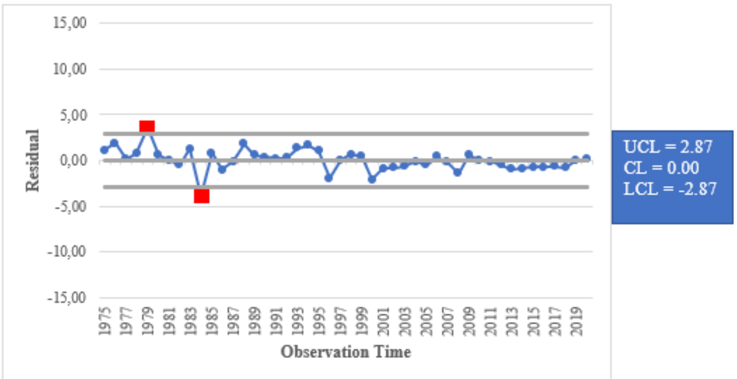


Figure 6. Addition of Outlier Factors in the Second Iteration

4. ARIMA Model (1,0,0) with outlier factors in the third iteration :
   In the third iteration there is two outlier detected, namely at the 6th and 11th times which is then added to **Equation (14)**

$$Y_t = 2\phi_1 Y_{t-1} - (\phi_1)^2 Y_{t-2} + e_t - \phi_1 e_{t-1} + \omega_1 I_t^{(24)} + \omega_2 I_t^{(46)} + \omega_3 I_t^{(8)} + \omega_4 I_t^{(6)} + \omega_5 I_t^{(11)} \tag{14}$$

   After that, parameter estimation is carried out so that **Equation (14)** becomes **Equation (15)**

$$Y_t = 0.62Y_{t-1} - 0.09Y_{t-2} + e_t - 0.31e_{t-1} + \omega_1 I_t^{(24)} + \omega_2 I_t^{(46)} + \omega_3 I_t^{(8)} + \omega_4 I_t^{(6)} + \omega_5 I_t^{(11)} \tag{15}$$

with the following control chart

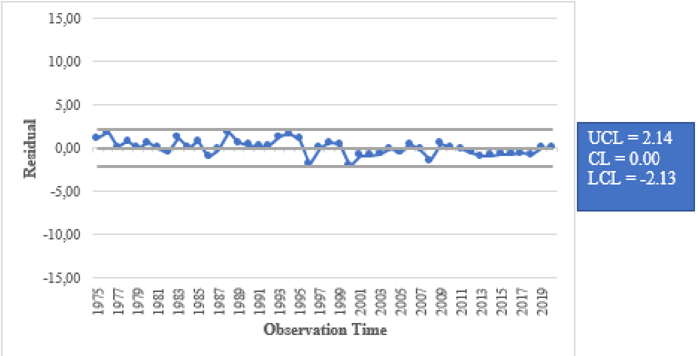
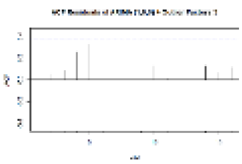
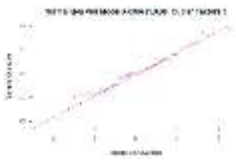


Figure 7. Addition of Outlier Factors in the Second Iteration

After adding the outlier factor to the ARIMA model (1,0,0), it was formed in the third iteration, then outlier detection was carried out in the fourth iteration. It turned out that in the fourth iteration no outliers were detected, thus using an iterative procedure three iterations were carried out to detect and add outlier factors. Meanwhile, the number of outliers detected was five outliers, namely at the 24th, 46th, 8th, 6th and 11th time.

Checking the residuals from each iteration uses a control chart as shown in **Figure 5**, **Figure 6**, and **Figure 7**. Based on **Figure 7**, there are no points that are outside the control limits on each plot or the residuals are within the control limits. Therefore, the ARIMA model (1,0,0) with outlier factors at the 24th, 46th, 8th, 6th and 11th times is an accurate model for predicting the future period. Alternatively, it can be stated that this model is an optimal model because the residuals are statistically controlled. The next step is to carry out a diagnostic test on the residuals from the ARIMA model (1,0,0) with outlier factors (24,46,8,6,11). The diagnostic tests carried out are the same as those in **Table 2**.

Table 2. Diagnostic Test

Model	Residual Plot for Diagnostic Test	
ARIMA (1,0,0) + Outlier Factors (24, 46, 8, 6, 11)		

Based on **Table 2** demonstrates that residuals are independent of time latency and have a normal distribution. This shows that adding an outlier factor to the ARIMA Model (1,0,0) can make up for outliers in the residuals. This means that the ARIMA Model (1,0,0) with outlier factors can be used. **Figure 8** has plots that show comparisons between real data, ARIMA model estimation data (1,0,0) without outliers, and ARIMA model estimation data (1,0,0) with outlier factors. It can be seen that there are differences in the movement of ARIMA data patterns without outliers and ARIMA with outlier factors. A comparison between actual data, ARIMA model estimation data (1,0,0) without outlier factors, and ARIMA model estimation data (1,0,0) with outlier factors is shown in plot form in **Figure 8**.

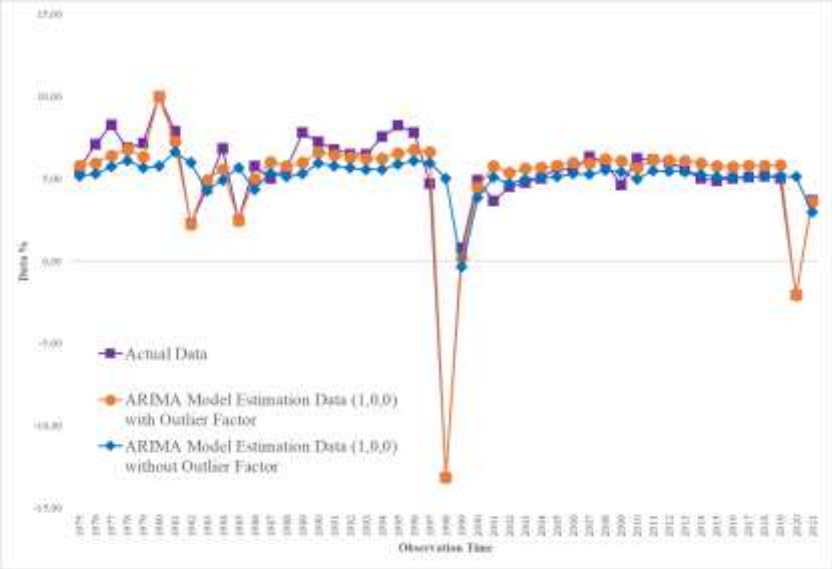


Figure 8. Comparison Plot between Actual Data, ARIMA Model Estimation Data (1,0,0) without Outlier Factors, and ARIMA Model Estimation Data (1,0,0) with Outlier Factors

Figure 8 shows the difference in movement of ARIMA data patterns without outlier factors and ARIMA with outlier factors. Based on **Figure 8**, it can be seen that the ARIMA model which has not added an outlier factor does not yet have a pattern that matches the actual data pattern (observations are marked with a blue line). Therefore, an outlier factor was added because it was suspected that there was an outlier so that the ARIMA model pattern could match the actual data pattern. After adding the outlier factor, it was seen that the ARIMA model already had a pattern that matched the actual data (observations marked with an orange line)

4. CONCLUSIONS

The accuracy of the time series model derived from Indonesia's GDP data is evaluated using the ARIMA model without outliers and the ARIMA model with outliers. Both models are executed using the same set of data. The results of this study indicate that the ARIMA model with outlier components produces a high degree of accuracy. The residual values are within the control limits (in control), which demonstrates this conclusion quite plainly.

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