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COMPARISON OF H-INFINITY AND ENSEMBLE KALMAN FILTER FOR ESTIMATING MOTION OF MIDDLE FINGER

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ABSTRACT

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Upper extremity paresis is a condition in which a person experiences muscle weakness in one or both hands. This condition can cause impairment in motor function, hinder daily activities, and affect the life quality of the sufferer. In some cases, paresis can result from nerve injury, neurological disease, or an accident. To help improve the life quality of the sufferer experiencing upper extremity paresis, the development of the Finger Prosthetic Arm Robot, an assistive robotic hand designed to provide assistance in the movement of the finger experiencing paresis, is required. This technology aims to restore its functional ability and the independence of the patient in performing daily activities, such as picking up objects, grasping, and performing other precise movements. The main purpose of this paper, the researcher compared two methods to estimate the motion of the middle finger robot, that is, the H-infinity method and the Ensemble Kalman Filter (EnKF) method. The simulation results show that both methods had almost the same accuracy, and the simulation by generating 800 ensembles was more accurate than that by generating 400 ensembles with an accuracy difference of about 10% above the accuracy rate of 98%.

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1. INTRODUCTION

Hand paresis is characterized by a significant decrease in the range of any movement. You can detect this with a simple handshake. In the absence of an obvious reason, acute paresis of the hand may occur. It is usually accompanied by significant pain (for example, pain that increases when coughing) [1]. Peripheral paresis and its acute development occurs with traumatic injury to several nerves of the hand [2]. The paralysis of the muscles in the distal arm is called Dejerine-Klumpke palsy, usually a birth injury to the brachial plexus. Symptoms such as paralysis of the hand, flexors and extensors of the fingers, as well as paralysis of the small muscles of the hand are marked [3]. Ulnar nerve defeat - weakness, pain of the muscles that straighten the hand and turn it to the side of the elbow, low mobility of the fifth finger, atrophy and hypotenarity occur.

Of the number of post stroke clients, 10% of clients can return to work without weakness, 40% have mildly disabled and 50% have severely disabled. For this reason, post stroke clients need a rehabilitation program [4]. Medical rehabilitation is an effort to restore the client's physical ability to its pre-ill state in the shortest possible time [5]. So, we need a biomedical robotics to help rehabilitation of the post-stroke patients. One such technology is the robotic finger arm assistance. Assistance finger arm robot is one of the solutions to help the recovery process for paresis patients specifically for their finger movements [6].

Finger movement estimation is an important aspect in the development of this technology, for it functions to determine the accuracy and effectiveness of the robot in providing motion assistance to the paresis-affected hand [7]. Two motion estimation methods having a high degree of accuracy are the Ensemble Kalman Filter (EnKF) and H-Infinity methods [8], [9]. The middle finger arm robot model is used as a platform that becomes the real movement of the finger, in this case the middle finger used as a reference is those belonging to the residents of Surabaya, Indonesia with an average middle finger length of around 7.7 – 8.15 cm. The H-infinity and EnKF methods were frequently used for estimating the motion of the forefinger [3] and mobile robot [10]. And, this paper compared the simulation results, that is, comparing the accuracy resulted from those by the EnKF and H-infinity methods by generating 400 and 800 ensembles.

2. RESEARCH METHODS

2.1 Mathematical Model of Finger Arm Robot

Here is the analysis of the 3-joint robot arm [6].

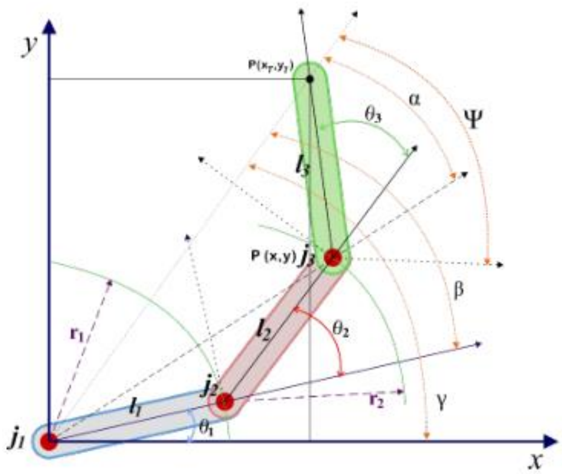


Figure 1. Configuration of 3-joint finger arm robot

Figure 1 shows a 3-joint arm robot using forward kinematics in the x and y coordinates as its equation analysis [3].

The coordinate points of X_T and Y_T as follows:

$$x = x_1 + x_2 + x_3$$

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

dan

$$y = y_1 + y_2 + y_3$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3) \tag{2}$$

So that

$$x_T = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y_T = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Simplified by using trigonometric identity formula:

$$x_T = x - l_3 \cos(\theta_1 + \theta_2 + \theta_3) \tag{3}$$
$$y_T = x - l_3 \sin(\theta_1 + \theta_2 + \theta_3) \tag{4}$$

The formula for forward kinematic equation of 3 joints

$$x_T = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y_T = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

For the invers kinematic, if the coordinates of P(XT, YT) and P(x,y) are known, then θ_1 and θ_2 can be obtained by using the same equation as that applied to two joint arm robot:

$$\theta_2 = cos^{-1}\left(\frac{x^2 + y^2 - {l_1}^2 - {l_2}^2}{2 l_1 l_2}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y(l_1 + l_2 \cos\theta_2) - x.\, l_2 \sin\theta_2}{x(l_1 + l_2 \cos\theta_2) + y.\, l_2 \sin\theta_2}\right)$$

The angle $\psi = \theta_1 + \theta_2 + \theta_3$ can be obtained by using P(XT,YT) and P(x,y) inserted into **Equation (3)** and **Equation (4)**, so that θ_3 can be found out.

By substituting P(XT, YT) and P(x,y) into **Equation (3)**, we get

$$l_3 \cos\psi = 0 \tag{5}$$

Whereas, by substituting P(XT, YT) and P(x,y) into **Equation (4)**, it becomes

$$Y_T = x - l_3 \sin\psi$$
$$l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin\psi = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos\psi - l_3 \sin\psi$$
$$2 l_3 \sin\psi - l_3 \cos\psi = l_1(\cos\theta_1 - \sin\theta_1) + l_2(\cos(\theta_1 + \theta_2) - \sin(\theta_1 + \theta_2))$$

Since in **Equation (5)** $l_3 \cos\psi = 0$, then it is obtained as follows

$$\psi = \theta_1 + \theta_2 + \theta_3 = sin^{-1}\left(\frac{l_1(\cos\theta_1 - \sin\theta_1) + l_2(\cos(\theta_1 + \theta_2) - \sin(\theta_1 + \theta_2))}{2\, l_3}\right)$$

The following is the finger arm robot image.

Figure 2. Finger Arm Robot Image

2.2 Ensemble Kalman Filter

Here is the flow of the Ensemble Kalman Filter algorithm [11]–[13].

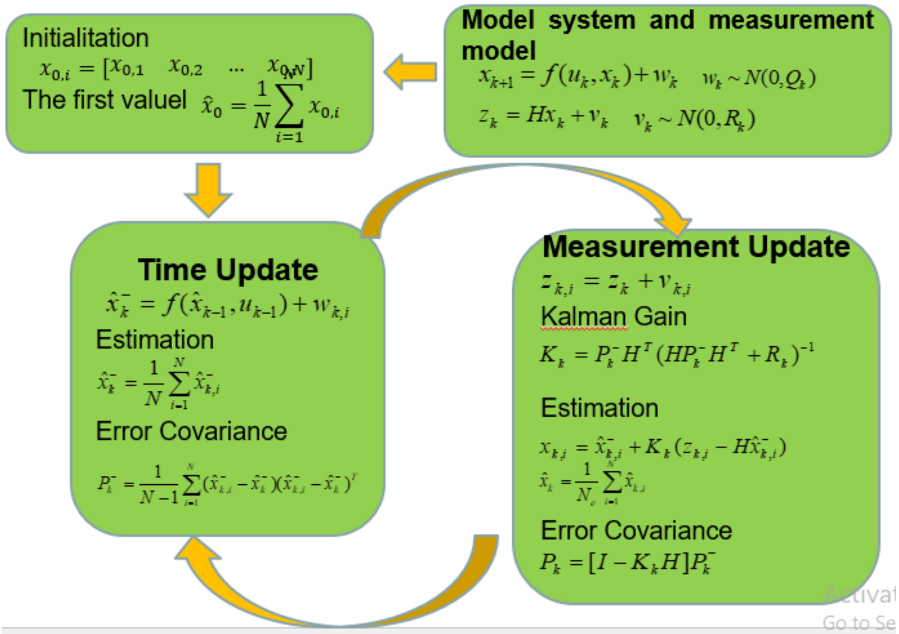


Figure 3. The flow of the ensemble kalman filter algorithm

2.3 Ensemble Kalman Filter

Here is the flow of the H-Infinity algorithm [14], [15].

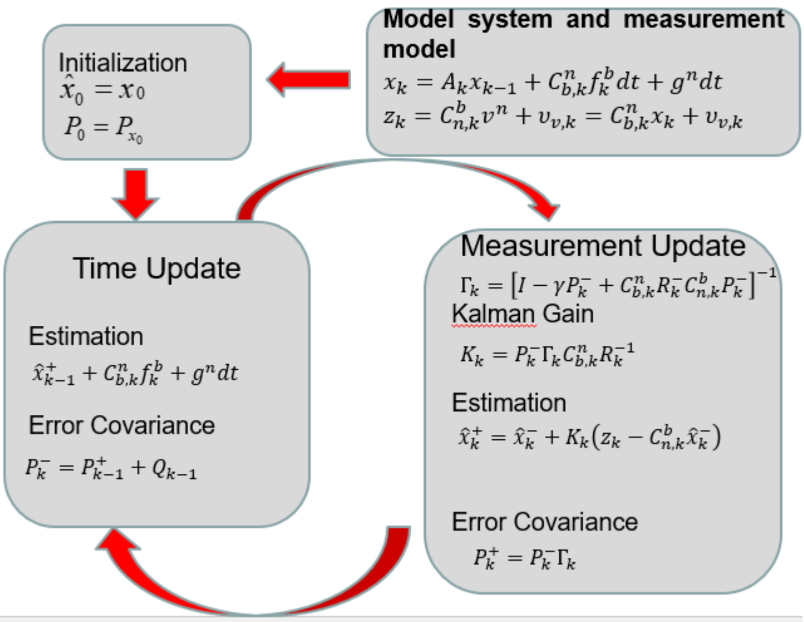


Figure 4. The flow of the h-infinity algorithm

3. RESULTS AND DISCUSSION

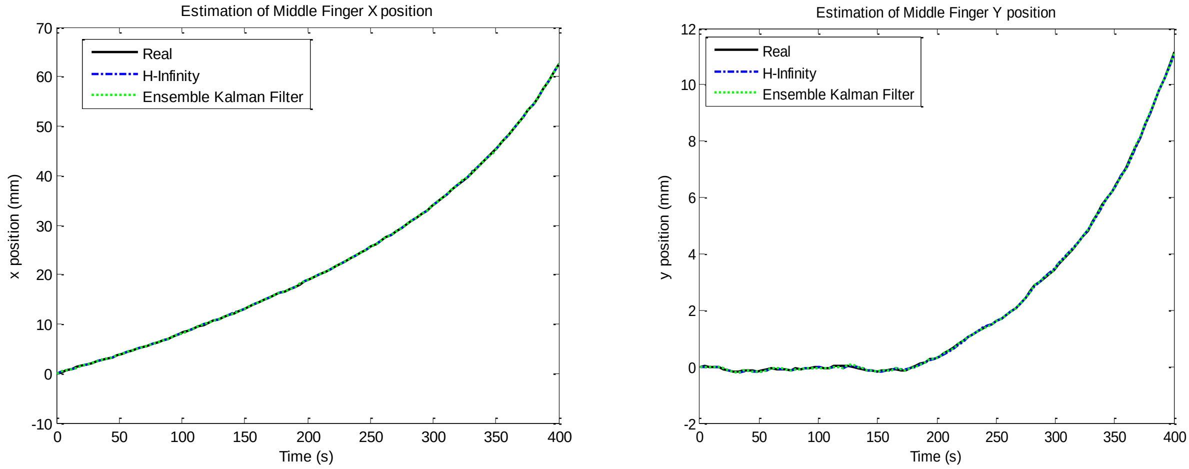
The study of this paper began with forward kinematics modeling to obtain motion in the X and Y axes, after which the inverse kinematic model of the finger prosthetic arm robot was obtained, having 3 joint parts matching the structure of the human finger. The finger prosthetic arm robot aimed to help post-stroke recovery, especially of the finger. The simulation focused on the movement of the middle finger, referring to the average length of those of the Surabaya residents in Indonesia, with a middle finger length range of 7.7 - 8.15 cm.

The study compared the accuracy rate produced by the numerical simulations of the H-infinity and EnKF methods, that is, by comparing two numerical computation results by generating 400 and 800 ensembles. The H-infinity method was used as optimal control technique to estimate the parameters and state of a dynamic system by minimizing the estimation error and optimizing the control performance. Meanwhile, the Ensemble Kalman Filter (EnKF) was used to solve the state estimation problem in the dynamic systems by using a statistical approach and an "ensemble" of particles representing the state of the system.

The middle finger movement chosen in this simulation was the middle finger movement in the form of a semicircular trajectory because all finger joints can move optimally. With the semi-circular movement having a diameter of about 7.7 - 8.15 cm, physical exercise on the middle finger can be done optimally. The simulation results can be seen in **Figure 5-8**.

Figure 5 and **Figure 6** represent the simulation results of estimating the motion of the middle finger that moves like a semicircle by generating 400 ensembles with 400 iterations, while Figures 7 and 8 use 800 ensembles in the simulation. The ultimate goal of this study is to see how these two methods perform by comparing the numerical computational results of each method with two different ensemble conditions, that is, by using 400 and 800 ensembles.

In **Figure 5** it can be seen that the numerical simulation results on the middle finger motion which is a 2-dimensional movement in the X and Y plane with the required time of 400 seconds, in which this finger movement corresponds to the finger arm robot model in the mathematical equations above. In Figure 5 it can be seen that the EnKF and H-infinity methods have almost the same high rate of accuracy with an error of about 12-14%.



(a) **(b)**

Figure 5. The results of applying the EnKF and H-infinity methods for motion estimation of the middle finger arm robot using 400 ensembles in X and Y-plane
(a) Estimation of middle finger x position, (b) Estimation of middle finger y position

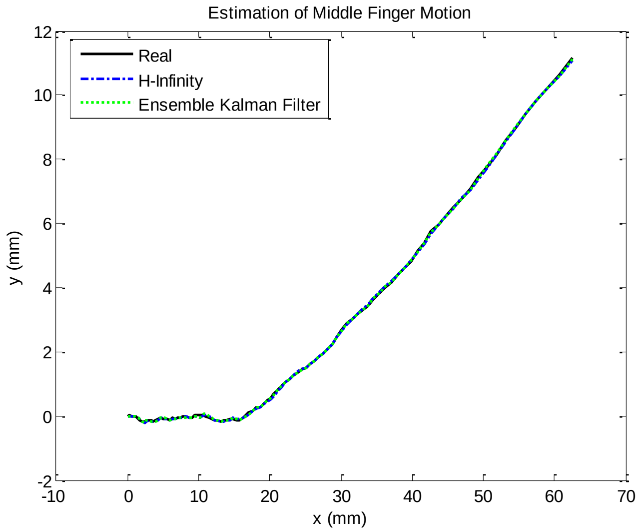


Figure 6. The results of applying the EnKF and H-inifnity methods for motion estimation of the middle finger arm robot using 400 ensembles in XY plane

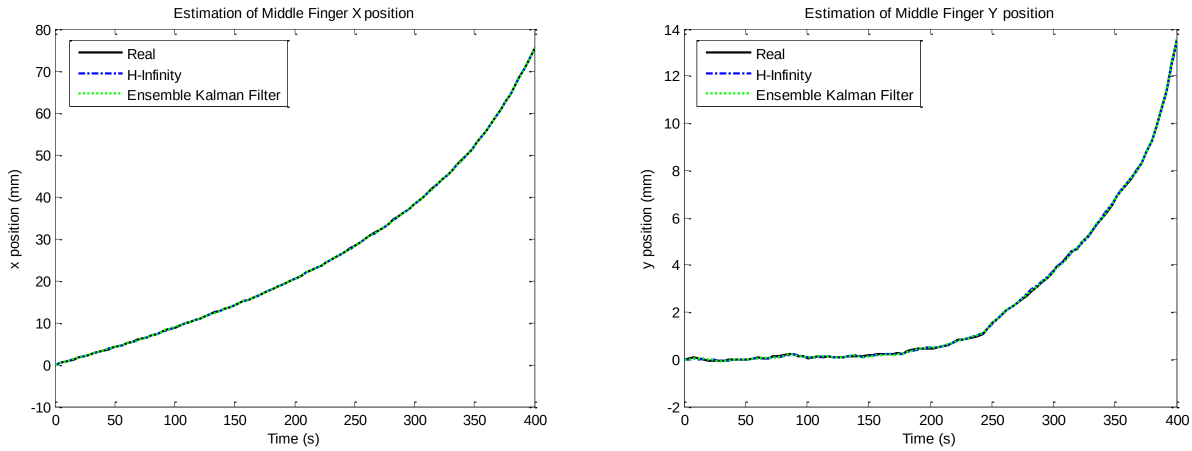
In this study, the numerical simulations were carried out for both methods (H-infinity and EnKF) by observing the accuracy of the results. Two different ensemble conditions were used, that is, 400 and 800 ensembles. The number of ensembles refers to the number of particles in the EnKF approach, affecting the accuracy of the resulting estimates.

Figure 6 shows the simulation results by the H-infnity and EnKF methods using 400 iterations and 400 ensembles. By the EnKF method, the resulted motion resembles a semicircle with a diameter of 1.5 mm. $\sqrt{6.5^2+1.2^2}=\sqrt{43.56+1.44}=\sqrt{45}=6.71$ cm, so that overall when viewed from a diameter of about 7.7 cm - 8.15 cm, the error value is about 12.8%, while the H-infinity method produces motion with a diameter of $\sqrt{6.5^2+1.2^2}=\sqrt{42.25+1.44}=\sqrt{43.69}=6.6$ cm so that overall when viewed from a diameter of about 7.7 cm - 8.15 cm, the error value is about 14.2%.

Table 1. The Accuracy of The Middle Finger Motion Estimation By The H-Infinity and EnKF Methods By Generating 400 Ensembles

	EnKF	H-infinity
XY Motion	87.2%	85.8 %
Time Simulation	10.29 s	8.16 s

When viewed from **Table 1**, it is clear Ki that the application of the Ensemble Kalman Filter method produces high accuracy than that of the H-infinity, the difference in accuracy between the two methods is about 2%, but when viewed from the simulation time, it is faster by the H-infinity method.



(a) **(b)**

Figure 7. The result of applying the EnKF and H-inifnity methods for the motion estimation of the middle finger arm robot using 800 ensembles in X and Y-plane
(a) Estimation of middle finger x position, (b) Estimation of middle finger y position

In **Figure 7**, it can be seen that the numerical simulation results on the middle finger motion which is a 2-dimensional movement in the X and Y plane with the required time of 400 seconds, in which the finger movement corresponds to the finger arm robot model in the mathematical equations above. In **Figure 7** it can be seen that the EnKF and H-infinity methods have almost the same high accuracy with an error of about 2-3%.

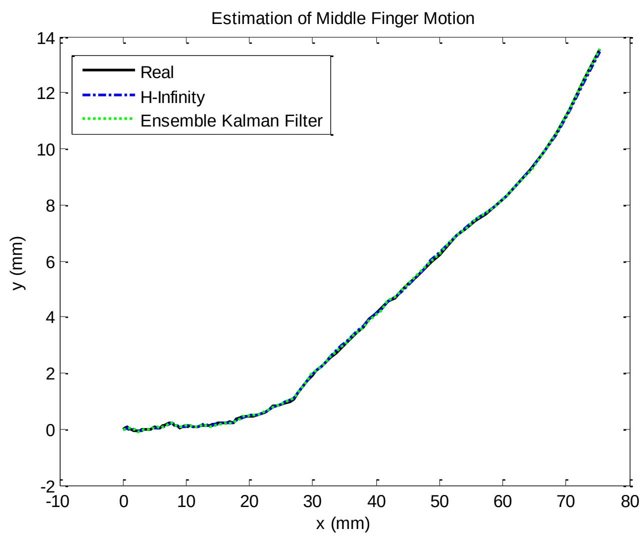


Figure 8. The result of applying the EnKF and H-inifnity methods for motion estimation of the middle finger arm robot using 800 ensembles in XY plane

In this study, the H-infinity and EnKF algorithms were applied in a numerical simulation through which the accuracy of the two methods would be compared. Two different ensemble conditions were used, that is, 400 and 800 ensembles. The number of ensembles refers to the number of particles in the EnKF approach, affecting the accuracy of the resulting estimates.

Figure 8 shows the simulation results by the H-infnity and EnKF methods using 400 iterations and 400 ensembles. The H-infinity method produces a motion resembling a semicircle with a diameter of $\sqrt{7.5^2 + 1.3^2} = \sqrt{56.25 + 1.69} = \sqrt{57.94} = 7.61$ cm, so that overall when viewed from a diameter of about 7.7 cm - 8.15 cm, the error value is about 1, 16%.

Meanwhile, the EnKF method produces a motion with a diameter of $\sqrt{7.6^2 + 1.3^2} = \sqrt{57.76 + 1.69} = \sqrt{59.45} = 7.71$ cm, so that overall, when viewed from a diameter of around 7.7 cm - 8.15 cm, it is within the range of the average length of the middle finger of Surabaya residents, in other words, there is no error or 100% accurate.

Table 2. The Accuracy of The Middle Finger Motion Estimation By The H-Infinity and EnKF Methods With 800 Ensembles

	EnKF	H-infinity
XY Motion	100 %	98.84 %
Time Simulation	18.33 s	15.21 s

When viewed from **Table 2**, it is clear that the application of the Ensemble Kalman Filter algorithm is more accurate than that of the H-infinity one for the movement of the middle finger in the XY plane, the difference in accuracy between the two methods is about 1.2%, but when viewed from the simulation time it is faster by the H-infinity method.

Table 3. The Accuracy of The Middle Finger Motion Estimation By The H-Infinity and EnKF Methods With 400 and 800 Ensembles

	EnKF with 400 Ensembles	H-infinity	EnKF with 800 Ensembles	H-infinity
XY Motion	87.2%	85.8 %	**100 %**	**98,84 %**
Time Simulation	10.29 s	8.16 s	18.33 s	15.21 s

When viewed from **Table 3**, it is clear that the application of the Ensemble Kalman Filter is more accurate than that of the H-infinity, having an accuracy rate of above 98% for the simulations that generate 800 ensembles. When viewed based on the comparison of the number of ensembles, then generating 400 ensembles produces a greater error than generating 800 ensembles, and it can be seen clearly in **Table 3**. Overall, when viewed from the accuracy rate of above 98%, the two methods can be said to be a reliable estimation method and can be used for estimating the motion of the assisted finger

4. CONCLUSIONS

Based on the results of the simulation and analysis above, it is concluded that the EnKF and H-infinity methods are reliable estimation methods and can be effectively used for estimating the motion of the assisted finger arm robot by having an accuracy of about above 98%, whereas the simulations by generating 400 ensembles have a greater error than those by 800 ensembles. And the Ensemble Kalman Filter algorithm is more accurate than the H-infinity one for the movement of the middle finger in the XY plane, the difference in accuracy between the two methods is about 1.2%. Overall, when viewed from the accuracy rate of above 98%, the two methods can be said to be a reliable estimation method and can be used for estimating the motion of the assisted finger

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