

OPTIMAL CONTROL ANALYSIS OF HIV/AIDS DISEASE SPREAD MODEL IN INDONESIA

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ABSTRACT

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The Human Immunodeficiency Virus (HIV) is a contagious virus that weakens the immune system of infected individuals, making them more susceptible to various diseases. These individuals are referred to as those exposed to the AIDS disease, which unfortunately, cannot be cured. To effectively manage AIDS, prevention is crucial in slowing down the spread and growth of the HIV virus. Mathematical modeling can play a significant role in the optimal control of AIDS. In this study, the S, E, I, A, T model with three different optimal controls were employed. Optimal control involves public health education campaigns, screening, and treatment. The goal is to minimize the number of individuals infected with HIV/AIDS using Pontryagin's Maximum Principle. This principle considers various factors, such as population class coefficients, cost weights, and control variables to determine the most effective approach. The simulation results indicate that counseling control in the exposed population class (E) yields better outcomes compared to counseling control in the susceptible class and treatment control in the HIV-infected population class. This implies that focusing on educating and counseling individuals who are exposed to HIV can be more efficient in AIDS control than targeting those already infected or at risk. By applying these optimal control strategies, it may be possible to mitigate the impact of HIV/AIDS and improve public health outcomes.



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1. INTRODUCTION

Human Immunodeficiency Virus (HIV) is the primary cause of acquired immunodeficiency syndrome (AIDS) [1]. HIV is a contagious virus with various transmission sources, including blood, cervical secretions, and breast milk from individuals with HIV [2]. When a person becomes infected with HIV, their immune system weakens making them more susceptible to diseases [3]. While AIDS is an incurable disease, there are treatments available to manage the condition and prolong the lives of affected individuals [4]. The World Health Organization predicts that the number of deaths due to HIV/AIDS in Southeast Asia will reach 380,000 by 2022 [5]. Data from the Directorate General of Disease Prevention and Control, Indonesian Ministry of Health, shows a cumulative total of 9,675 HIV cases and 36,902 AIDS cases in Indonesia by 2021. Additionally, by 2022, there were an additional 10,525 HIV cases and 1,907 AIDS cases. By 2023, there was a rise in the number of HIV cases to 13,279 and an increase in the number of AIDS cases to 149,579. HIV cases are mostly found in the age group of 25 - 49 years. Meanwhile, the age range of 20 - 29 years exhibits the highest proportion of AIDS cases [6].

Controlling the spread of HIV/AIDS is crucial due to its high number of cases. Disease control can be achieved through mathematical modeling, which plays a significant role in managing infectious diseases. Mathematical modeling serves as a means of mitigation and aids in analyzing the dynamics of disease transmission. In specific situations, integrating mathematical modeling into optimal control strategies can lead to even more effective outcomes [7].

Many previous studies have been conducted to investigate optimal control in modeling the transmission of HIV/AIDS disease modeling [8], [9][10] [11], [12], [13]. Previous studies in the field of HIV/AIDS disease modeling have primarily focused on controlling the disease through the administration of Antiretroviral Therapy (ART), with an emphasis on optimizing treatment strategies and associated costs [14]. Additionally, other studies have explored the optimal control of HIV/AIDS spread, using a model that incorporates three variables to represent the spread of HIV. The control measures employed in these studies involve therapy for HIV-infected individuals [15]. Another line of research has investigated controlling the spread of HIV using optimal control techniques with the SIR model, which includes additional variables like Exposed (E) and two separate Infected classes (I_1 and I_2). Control variables are applied to the population classes of Susceptible (S), Exposed (E), Infected (I_1), and Infected (I_2) [16]. The findings of this study indicate that the HIV spread model is locally asymptotically stable and globally asymptotically stable when R_0 (basic reproduction number) exceeds 1. The optimal control problem presented in this research generates state and co-state equations, leading to numerical solutions for five distinct scenarios [17].

Consequently, this study was conducted to explore the optimal control of HIV/AIDS transmission. Moreover, modifications were introduced by incorporating a new class representing the population under HIV/AIDS treatment. The optimal controls in this study, specifically a public health education campaign and treatment interventions. As a result, this research aims to implement control measures within the HIV/AIDS treatment population for experimental purposes.

2. RESEARCH METHODS

The research of mathematical models on the spread of HIV/AIDS disease is quantitative research. The data used in this study is HIV/AIDS data in Indonesia obtained from the Joint United Nations Program on HIV/AIDS (UNAIDS) in 2019. In this study we proposed mathematical model of HIV/AIDS diseases spread without optimal control and mathematical model of HIV/AIDS diseases spread with optimal control to minimize the number of individuals infected with HIV/AIDS.

2.1 Mathematical Model of HIV/AIDS Diseases Spread Without Optimal Control

The HIV/AIDS disease spread model, as derived from the research Stability Analysis and Simulation of Disease Spread Models HIV/AIDS Type SIA (Susceptible, Infected, Abstained), consists of three population classes denoted as S , I , and A [1]. In this model, S represents the susceptible population class, I represent individuals who have been infected with HIV, and A represents the population class of HIV-infected individuals in the AIDS stage. However, for this study, the HIV/AIDS disease spread model is modified by including an additional population class, E , which stands for the exposed individuals. The other research used

the population class S , E , I_1 , I_2 and A [18]. In this study, the model was modified, S represents the susceptible population class that can transition to the E -exposed population class. The E -exposed population class comprises individuals who have encountered those who have a history of HIV/AIDS or individuals exposed to individuals with a history of HIV/AIDS. Model S , E , I , and A explain that the model divides the human population N into four categories: individuals who are potentially infected with the virus (S), individuals who are potentially infected with the virus when in an environment with HIV/AIDS (E), individuals who have been infected with the virus based on HIV/AIDS test (I), and individuals with AIDS (A). Based on these assumptions, the compartment model of S , E , I , and A is formulated as follows.

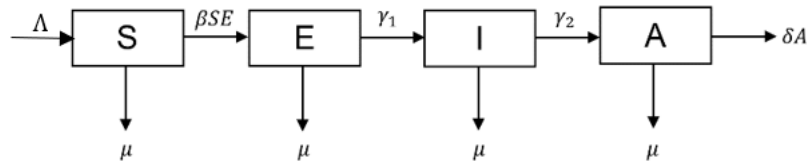


Figure 1. Compartment Diagram of SEIA Model

Based on **Figure 1**, the following equation is obtained:

$$\begin{aligned} S &= \Lambda - \beta SE - \mu S \\ E &= \beta SE - \mu E - \gamma_1 E \\ I &= \gamma_1 E - \gamma_2 I - \mu I \\ A &= \gamma_2 I - \mu A - \delta A \end{aligned} \quad (1)$$

where Λ denotes the rate of susceptible individuals, β denotes the rate at which the number of individuals changes is influenced by the presence of educated susceptible individuals, γ_1 denotes the rate of exposed individuals become HIV infected individuals because of the effect of interacting with HIV infected individuals by HIV test, γ_2 denotes the rate of HIV infected to AIDS infected, μ denotes natural death, δ denotes death due to AIDS disease.

2. 2 Mathematical Model of HIV/AIDS Diseases Spread with Optimal Control

The HIV/AIDS disease spread model with controls is a modified version of the S , E , I , A model. It includes the addition of a treatment population class, denoted as T , and introduces three control variables. The first control variable, u_1 , represents a public health education campaign targeted at the susceptible population class (S). The second control variable, u_2 , represents screening measures applied to the exposed population class (E). Lastly, the third control variable, u_3 , represents treatment interventions for individuals who have been infected with HIV (I). As a result, the HIV/AIDS disease spread model with controls in this study becomes S , E , I , A and T . Based on these assumptions, the compartment model for S , E , I , A , T is formulated as follows.

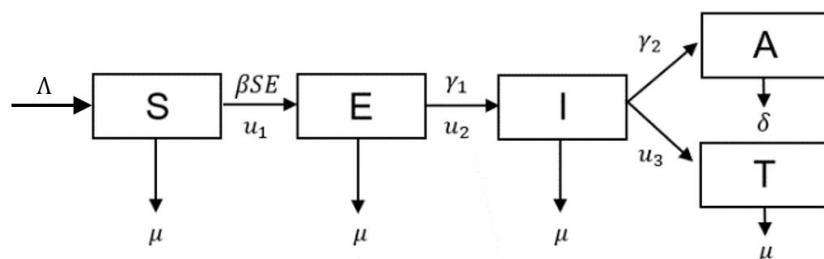


Figure 2. Compartment Diagram of SEIAT Model with Control Optimal

Based on **Figure 2**, the following equation is obtained:

$$\begin{aligned} S &= \Lambda - \beta SE - \mu S - u_1 S \\ E &= \beta SE - \mu E - \gamma_1 E + u_1 S - u_2 E \\ I &= \gamma_1 E - \gamma_2 I - \mu I + u_2 S - u_3 I \\ A &= \gamma_2 I - \delta A \\ T &= u_3 I - \mu T \end{aligned} \quad (2)$$

where Λ denotes the rate of susceptible individuals, β denotes the rate at which the number of individuals changes is influenced by the presence of educated susceptible individuals, γ_1 denotes the rate of exposed individuals become HIV infected individuals because of the effect of interacting with HIV infected individuals by HIV test, γ_2 denotes the rate of HIV infected to AIDS infected, μ denotes natural death, δ denotes death due to AIDS disease, u_1 , u_2 and u_3 respectively state the control. In the model, cases of death u_1S , u_2E , u_3I , and μT can also occur due to natural death or HIV/AIDS.

3. RESULTS AND DISCUSSION

3.1 Objective Function and Hamiltonian Function

The control of HIV/AIDS spread through optimal control involves the initial step of determining the Hamiltonian function. In general, the Hamiltonian function aligns with the objective function and the equation of the HIV/AIDS disease spread model with control. In this study, the objective function aims to minimize the number of individuals infected with HIV/AIDS, utilizing three distinct control variables. Therefore, the objective function derived for this research is as follows:

$$J_{min} = \int_0^t A_1S + A_2E + A_3I + C_1u_1^2 + C_2u_2^2 + C_3u_3^2 \quad (3)$$

where u_1 is the public health education campaign control variable, u_2 is the screening control variable and u_3 is the treatment control variable. Additionally, the variables C_1 , C_2 , and C_3 represent cost weights, and their assumed values are 50, 50, and 100, respectively. Moreover, A_1 , A_2 , and A_3 represent the coefficients associated with the susceptible population class (S), exposed population class (E), and HIV-infected population class (I), respectively.

Based on the **Equation (3)**, the Hamiltonian function is obtained as follows:

$$\begin{aligned} H = & (A_1S + A_1E + A_3I + C_1u_1^2 + C_2u_2^2 + C_3u_3^2) + \lambda_1(\Lambda - \beta SE - \mu S - u_1S) \\ & + \lambda_2(\beta SE - \gamma_1E - \mu E - u_2E + u_1S) + \lambda_3(\gamma_1E - \gamma_2I - \mu I - u_3I + u_2E) \\ & + \lambda_4(\gamma_2I - \delta A) + \lambda_5(u_3I - \mu T) \end{aligned} \quad (4)$$

3.2 State and Co-State Equations

The next step involves deriving the state equation and co-state equation. The state equation is obtained by taking the partial derivative of the Hamiltonian function with respect to the variable λ . The resulting state equation is as follows.

$$\begin{aligned} M_1 &= \frac{\partial H}{\partial \lambda_1} = \Lambda - \beta SE - \mu - u_1S \\ M_2 &= \frac{\partial H}{\partial \lambda_2} = \beta SE - \gamma_1E - \mu E - u_2E + u_1S \\ M_3 &= \frac{\partial H}{\partial \lambda_3} = \gamma_1E - \gamma_2I - \mu I - u_3I + u_2E \\ M_4 &= \frac{\partial H}{\partial \lambda_4} = \gamma_2I - \delta A \\ M_5 &= \frac{\partial H}{\partial \lambda_5} = u_3I - \mu T \end{aligned} \quad (5)$$

Once the state equation is obtained, the next step is to derive the co-state equation. The co-state equation is the result of taking the partial derivative of the Hamiltonian function with respect to the system's variables. The co-state equation is calculated as follows.

$$N_1 = \frac{\partial H}{\partial S} = A_1 + \lambda_1(-\beta E - \mu - u_1) + \lambda_2(\beta E + u_1) \quad (6)$$

$$\begin{aligned}
 N_2 &= \frac{\partial H}{\partial E} = A_2 + \lambda_1(-\beta E) + \lambda_2(\beta S - \gamma_1 - \mu - u_2) + \lambda_3(\gamma_1 + u_2) \\
 N_3 &= \frac{\partial H}{\partial I} = A_3 + \lambda_3(\gamma_2 - \mu - u_3) + \lambda_4\gamma_2 + \lambda_5u_3 \\
 N_4 &= \frac{\partial H}{\partial A} = -\lambda_4\delta \\
 N_5 &= \frac{\partial H}{\partial T} = -\lambda_5\mu
 \end{aligned}$$

3.3 Optimal Control using Pontryagin's Maximum Principle

After obtaining the state equation and co-state equation, the next step is to determine the control equation. With three control variables involved, the optimal control equation (u^*) for these variables is obtained by minimizing the Hamiltonian function (H) with respect to the control variable u . Therefore, resolving the equation $\frac{\partial H}{\partial u} = 0$. The optimal control equation is derived as follows.

$$\frac{\partial H}{\partial u_1} = 0 \Leftrightarrow 2C_1u_1 - \lambda_1S + \lambda_2S = 0 \Leftrightarrow u_1 = \frac{(\lambda_1 - \lambda_2)S}{2C_1} \quad (7)$$

$$\frac{\partial H}{\partial u_2} = 0 \Leftrightarrow 2C_2u_2 - \lambda_2E + \lambda_3E = 0 \Leftrightarrow u_2 = \frac{(\lambda_2 - \lambda_3)E}{2C_2} \quad (8)$$

$$\frac{\partial H}{\partial u_3} = 0 \Leftrightarrow 2C_3u_3 - \lambda_3I + \lambda_5I = 0 \Leftrightarrow u_3 = \frac{(\lambda_3 - \lambda_5)I}{2C_3} \quad (9)$$

The values of the three control variables, u_1 , u_2 , and u_3 , are constrained within the range of 0 to 1, representing percentages from 0% to 100%. Consequently, the optimal values of u_1 , u_2 , and u_3 are determined as follows.

$$u_1^* = \min \left\{ 1, \max \left\{ 0, \frac{(\lambda_1 - \lambda_2)S}{2C_1} \right\} \right\} \quad (10)$$

$$u_2^* = \min \left\{ 1, \max \left\{ 0, \frac{(\lambda_2 - \lambda_3)E}{2C_2} \right\} \right\} \quad (11)$$

$$u_3^* = \min \left\{ 1, \max \left\{ 0, \frac{(\lambda_3 - \lambda_5)I}{2C_3} \right\} \right\} \quad (12)$$

3.4 Parameter of HIV/AIDS Model

The parameter details of the HIV/AIDS model are presented in **Table 1**. Coefficients A_1 , A_1 and A_1 are assumed to be 1 while the coefficients C_1 , C_2 and C_3 are assumed to be 50, 50 and 100 respectively.

Table 1. Parameter of HIV/AIDS Model

Parameter	Value
S	19000
E	2200
I	3570
A	420
T	0
Λ	0.55
β	0.0003
γ_1	0.5
γ_2	0.4
μ	0.025
δ	0.47

3.5 Numerical Simulation of Susceptible Population

Based on the simulation results in **Figure 3**, it can be inferred that all three control scenarios exhibit an increase in population annually. In scenario (a), where public health education campaigns are implemented for susceptible individuals, the population rises primarily due to the birth rate. In scenario (b), with control u_2 focusing on screening exposed individuals, this allows interactions between susceptible individuals and

undetected exposed individuals, leading to transitions from the susceptible population class to the exposed population class. As for scenario (c), with control u_3 targeting the treatment of HIV-infected individuals, the population also experiences growth, although not as much as in scenarios with u_1 , u_2 , or no control. The relatively lower growth in the scenario with u_3 control might be attributed to its emphasis on HIV-infected individuals, which results in a considerable number of transitions from the susceptible population class to the exposed population class, accompanied by natural deaths.

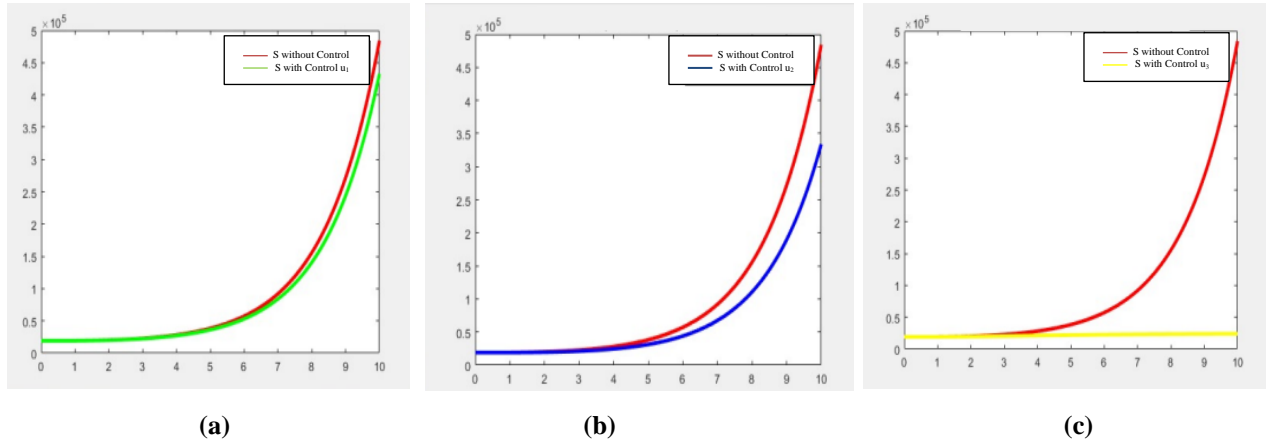


Figure 3. Scenario Comparison of Susceptible (S) Population Class

- (a) Comparison of Susceptible (S) Population Class Without Optimal Control and With Optimal Control u_1 ,
 (b) Comparison of Susceptible (S) Population Class Without Optimal Control and With Optimal Control u_2 ,
 (c) Comparison of Susceptible (S) Population Class Without Optimal Control and With Optimal Control u_3

3.6 Numerical Simulation of Exposed Population

Based on **Figure 4** in scenario (a), with control u_1 , there was a decrease in the first year, but the reduction observed was not significant. This outcome was influenced by natural deaths and transitions of the population class into HIV infection. In scenario (b), with control u_2 , the population declined similarly to the simulation without any control. However, in the scenario with u_2 control, the decline was even more substantial. This can be attributed to control u_2 causing numerous individuals in the exposed population class to transition into the infected population class. On the other hand, in the population scenario (c) with u_3 control, the results were the same as the population scenario without any control. This is because the curved lines without control and with control overlap. This is a result of the u_3 control variable's focus on treatment for the HIV-infected population class, rendering the population scenario with u_3 control identical to the one without any control.

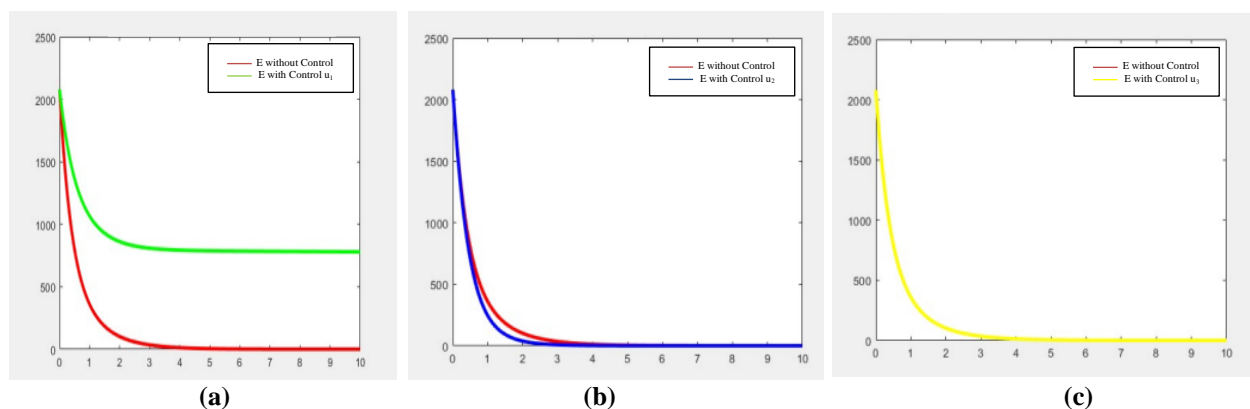


Figure 4. Scenario Comparison of Exposed (E) Population Class

- (a) Comparison of Exposed (E) Population Class Without Optimal Control and With Optimal Control u_1 ,
 (b) Comparison of Exposed (E) Population Class Without Optimal Control and With Optimal Control u_2 ,
 (c) Comparison of Exposed (E) Population Class Without Optimal Control and With Optimal Control u_3

3.7 Numerical Simulation of Infected Population

Based on **Figure 5**, it can be inferred that all three scenarios experienced a decrease in the first and second years, except for the scenario with control u_2 . In the u_2 control scenario (b), there was an initial increase in the first and second years, followed by a decrease from the third to the sixth year. The increase in the first year was a result of a decrease in the exposed population class, leading to an increase in individuals transitioning to the HIV-infected population class. In the population scenario with u_1 control (a), there was a decline from the first to the fourth year. However, the reduction in this scenario was less significant compared to the population scenario without any control. Additionally, the transition to the AIDS population class and natural mortality also played a role in the results of the scenario with u_1 control. Meanwhile, scenario (c) had nearly the same results as the scenario without any control. This was due to various factors, such as natural deaths in HIV-infected individuals and the implementation of control in the form of treatment to suppress the HIV virus in those who have been infected. Consequently, individuals in this scenario switched to the treatment population class.

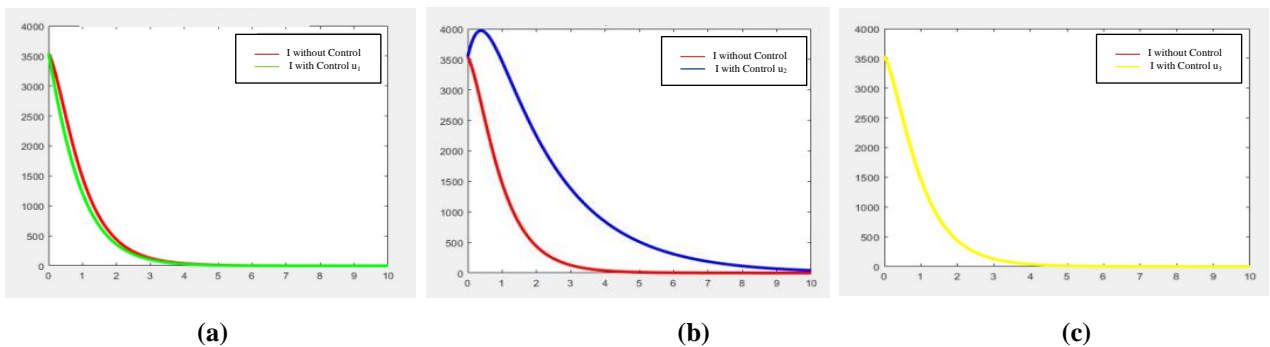


Figure 5. Scenario Comparison of Infected (I) Population Class

- (a) Comparison of Infected (I) Population Class Without Optimal Control and With Optimal Control u_1 ,
- (b) Comparison of Infected (I) Population Class Without Optimal Control and With Optimal Control u_2 ,
- (c) Comparison of Infected (I) Population Class Without Optimal Control and With Optimal Control u_3

3.8 Numerical Simulation of AIDS Population

Based on the simulation results from the three different scenarios in **Figure 6**, it can be concluded that only the scenario (c) with control u_3 exhibited a significant decline, whereas the two scenarios with other controls showed an increase. The decline observed in the u_3 control scenario can be attributed to the treatment control applied to HIV-infected individuals, leading to many individuals transitioning to the treatment population class. This is because the treatment effectively prevents the incubation of HIV. On the other hand, in the u_1 and u_2 control scenarios (a)(b), an increase was observed starting from the fourth year. This increase occurred due to the absence of treatment control, allowing the incubation process to take place in HIV-infected individuals. Consequently, over the next 2 to 10 years, a considerable number of individuals progressed to the AIDS stage.

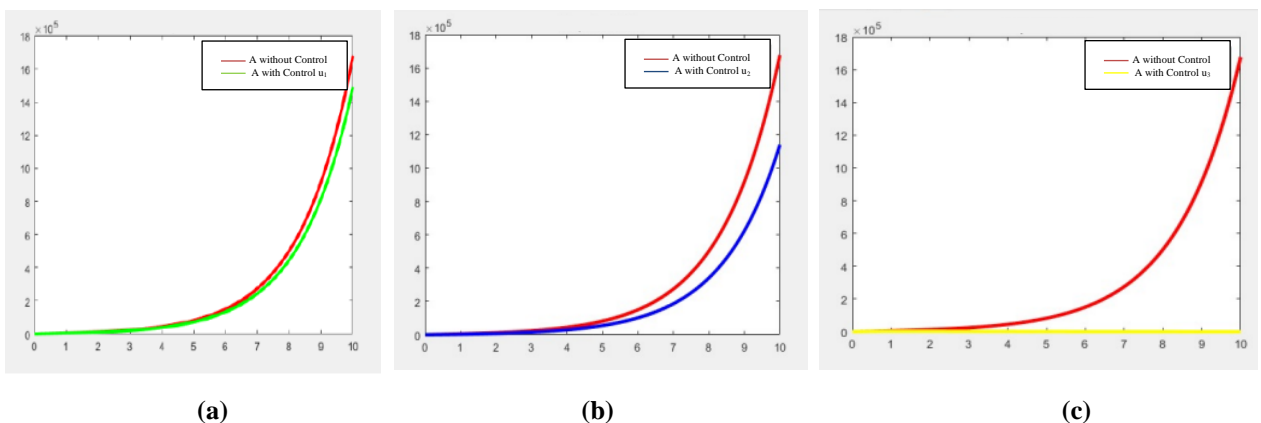


Figure 6. Scenario Comparison of AIDS (A) Population Class

- (a) Comparison of AIDS (A) Population Class Without Optimal Control and With Optimal Control u_1 ,

- (b) Comparison of AIDS (A) Population Class Without Optimal Control and With Optimal Control u_2 ,
 (c) Comparison of AIDS (A) Population Class Without Optimal Control and With Optimal Control u_3

3.9 Numerical Simulation of Treatment Population

The outcomes from the three different scenarios in **Figure 7** lead to the conclusion that only the u_3 control scenario experiences an annual increase. This is because the u_3 control scenario (c) includes treatment for HIV-infected individuals, effectively suppressing the incubation process that could progress to the AIDS stage. On the other hand, in the u_1 and u_2 control scenarios (a)(b), this effect is not observed as they primarily emphasize public health education campaigns and screening measures, respectively.

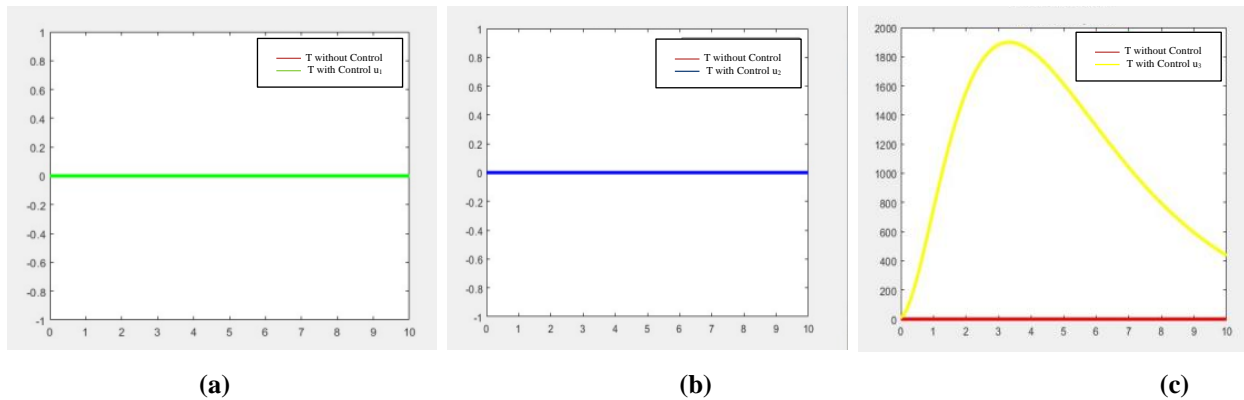


Figure 7. Scenario Comparison of Treatment (T) Population Class

- (a) Comparison of Treatment (T) Population Class Without Optimal Control and With Optimal Control u_1 ,
 (b) Comparison of Treatment (T) Population Class Without Optimal Control and With Optimal Control u_2 ,
 (c) Comparison of Treatment (T) Population Class Without Optimal Control and With Optimal Control u_3

4. CONCLUSIONS

Based on the results and discussion of optimal control analysis of HIV/AIDS disease spread model in Indonesia research, it is concluded that:

1. The mathematical model of HIV/AIDS diseases spread S , E , I , A , and T disease spread model involves three control measures public health education campaigns (u_1) targeting susceptible populations, screening (u_2) measures for exposed populations, and treatment interventions (u_3) for infected populations. The results of the optimal control solution to the HIV/AIDS disease spread model are as follows at **Equation (10), (11), (12)**.
2. The numerical solutions' results lead to the conclusion that implementing control measures such as counseling for the susceptible population (S), control in the exposed population class (E), and treatment interventions for the HIV-infected population (I) have a significant impact on the HIV/AIDS diseases spread model. Control u_1 leads to a significant decrease in population classes (E) and (I), and it slows down the growth of population class A . On the other hand, control u_2 results in a substantial decrease in population classes (E) and (I) and slows down the growth of population class A even more effectively than control u_1 . Control u_3 leads to a decrease in population classes (I) and (A) while increasing the size of the treatment population class (T). Based on these results, it can be concluded that control u_2 , counseling in the exposed population class, is more optimal than control u_1 , counseling in the susceptible class, and control u_3 , treatment in the HIV-infected population class.

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