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REGRESSION OF SURVIVAL DAGUM 3 PARAMETER *LINK FUNCTION* IN HIV/AIDS PATIENTS IN EAST JAVA

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ABSTRACT

AIDS (Acquired Immuno Deficiency Syndrome) is a symptom of blood cell disease which results in decreased immunity in humans. HIV/AIDS causes a decline in economic productivity, health, and environmental fields. Therefore, treatment is required to reduce the spread of the disease in East Java. The spread of the HIV/AIDS virus is growing rapidly both in terms of region and pattern of spread. Therefore, it is necessary to have policies from the government to prevent the spread of HIV/AIDS through mathematical modeling to determine the actual and comprehensive factors that influence HIV/AIDS care. policies needed to handle and reduce HIV/AIDS in East Java. This study aims to determine the factors that influence the care of people with HIV/AIDS with Survival Regression modeling Dagum 3 Parameter link function. This model uses parameter estimation with a Bayesian analysis approach with the results that the factors that significantly influence the length of HIV/AIDS care in East Java are marital status (X_5), absolute CD4 levels (X_7), Suffering Stage (X_8), Functional State (X_9), and Therapy adherence (X_{10}). These results indicate that marital status and condition characteristics of HIV/AIDS sufferers are important information in the socialization program for reducing HIV/AIDS in East Java.

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1. INTRODUCTION

AIDS *(Acquired Immuno Deficiency Syndrome)* is a type of disease symptom caused by the body's immunity being attacked by HIV (*Human Immuno Deficiency Virus*) [1]. HIV/AIDS is a series of viruses that infect white blood cells, resulting in decreased immunity in humans [2]. HIV infection can be controlled with lifelong antiretroviral (ARV) drugs [3]. HIV/AIDS virus is a disease that reduces economic productivity in East Java, such as the supply of ARV drugs, especially in the city of Surabaya [4]. The HIV/AIDS virus can be prevented if an early screening program is carried out at a hospital providing ARV therapy [5]. Factors that influence HIV/AIDS are education, marital status, and the risk of transmission [6]. This makes it an interesting object to study and study further [7].

Soetomo General Hospital is a hospital that provides good medical records regarding the screening of people with HIV/AIDS with ARV therapy [4] [8]**.** Provider of ARV therapy services for the general public and as a referral hospital in the East Java region with one of the services provided is HIV/AIDS care for homosexual and transgender sufferers [8]. ARV therapy has a major role in improving the quality of life for sufferers of HIV/AIDS [8]. Research on antiretroviral drug delivery programs has shown lower mortality rates in the first six months of starting treatment [9]. Administration of these drugs requires high costs from the government in curing HIV/AIDS [9] [10].

The survival model is a type of mathematical model that is applied in the health [11]. Survival modeling can be used to identify factors risk for incident and treat situations when factors risk change over time [12]. If a researcher has a goal to determine the factors that influence the occurrence of a thing or event [13]. With the factor risk for incident over period, the survival modeling of a tool will be more adequate [14].

Survival analysis is a statistical procedure for analyzing data that produces response variables over time until an event occurs [15]. Survival analysis purpose to estimate the probability of survival modelling, relapse, mortality, and other events a specified time [16]. The period in question is years, months, weeks, or days from the initial time of observation until an event happen [17]. The events referred to are mortality, disease occurrence, relapse, recovery, and other events that may occur to an individual [18].

The research conducted [1] applied survival analysis modeling to health sciences, namely the modeling of HIV/AIDS sufferers at Prof. Dr. Soekandar, Mojokerto Regency. This modeling resulted in a significant relationship between ARV therapy and the survival of HIV/AIDS sufferers at Prof. Hospital. Dr. Soekandar, Mojokerto Regency. The average survival time for people with HIV/AIDS is 13 months. The application of the survival model in health sciences has an impact on actual death cases. A similar study was also conducted by [4] applying a survival regression model with a lognormal 3 parameter distribution in cases of HIV/AIDS in East Java. The results of indicate that the factors that significantly influence the care of HIV/AIDS sufferers are education, marital status, patient's stage, adherence to therapy, opportunistic infections, and the risk of HIV/AIDS transmission. A study was also undertaken [10] to examine the implications for development of NADM in British Columbia in terms of cancer incidence and of people living with HIV/AIDS.

This study shows that the factors to influence HIV/AIDS sufferers are gender, age, CD4 levels, and adherence to therapy. Based on research conducted [7] using Bayesian Spatial Survival analysis on the HIV/AIDS event process in East Java using a lognormal 3 parameter distribution *of link function*. Data that follows a certain distribution produces factors that influence HIV/AIDS care, namely gender, education, and absolute CD4 levels. The distribution of survival regression data does not all represent a clear distribution [19]. So this study examines the survival regression with the distribution of the Dagum 3 parameter link function with the Bayesian MCMC-Gibbs Sampling approach. This method is used to estimate the parameters of the survival regression model and the factors that influence the care of HIV/AIDS sufferers in East Java. East Java is the object of research because it has the second largest number of HIV/AIDS sufferers after West Java [3]. The consideration of using survival analysis with Dagum 3 parameters distributions link function can be used as an optimal treatment step for HIV/AIDS. This modeling is also useful in socializing materials about the management of HIV/AIDS and the factors that influence HIV/AIDS care with ARV therapy. ARV therapy can be used as a basis for consideration by the Ministry of Health of East Java in making policies to formulate strategic steps to accelerate the treatment of HIV/AIDS sufferers in East Java.

2. RESEARCH METHODS

This study used secondary data from outpatient medical recap regarding the characteristics of the condition of treating HIV/AIDS patients at Soetomo Hospital, Surabaya. The data taken is treatment data for people with HIV/AIDS with complete screening until declared dead, stopped, and missed follow-up/last visit which is referred to as *a failure event* in 2019. The response variable is the time of treatment for HIV/AIDS sufferers (Y) and the predictor variable is categorically consisting of Sex (X_1), Age (X_2), Education (X_3), Employment Status (X_4), Marital Status (X_5), Weight (X_6), Absolute CD4 levels (X_7), Suffering Stage (X_8), Functional State (X_9), Therapy adherence (X_{10}) Opportunistic infections (X_{11}), Status TB (Tuberculosis) (X_{12}), Transmission Risk Factors (X_{13}), and Companion taking medication (X_{14}) **[1] [3] [4]**. Distribution of data on HIV/AIDS sufferers follows the Dagum 3 Parameter link function. The method in this research is 3 parameter survival regression with a Bayesian approach.

In this study, the distribution Dagum 3 Parameter of length of stay for HIV/AIDS patients following probability density function **[15]** :

$$f(t;z,a,b) = \frac{az\left(\frac{t}{b}\right)^{az-1}}{b\left(1+\left(\frac{t}{b}\right)^{a}\right)^{z+1}} \tag{1}$$

Where $t < \infty, z > 0, a > 0, b > 0$ with z, a are the shape parameters and b is the scale parameter. t is a response variable that has a distribution with Dagum 3 parameters distribution **[20] [21]**. Meanwhile, the Cumulative distribution function is as follows **[23]** :

$$\begin{aligned} F(t) &= p(T \le t) \\ &= \int_0^t \frac{az\left(t/b\right)^{az-1}}{b\left(1+\left(t/b\right)^{a}\right)^{z+1}}dt \end{aligned} \tag{2}$$

Example 1 Integral:

$$\begin{aligned} x &= \frac{t}{b} \\ dx &= \frac{dt}{b} \\ F(t) &= p(T \le t) \\ &= \int_0^x \frac{az(x)^{z-1}}{(1+x^a)^{z+1}}dx \end{aligned}$$

Example 2 Integral $x = y = y^{-a}$:

$$\begin{aligned} Y &= y^{-a} \\ \frac{1}{Y} &= y^{a} \\ dY &= -a * y^{-a-1}dY \\ &= -\frac{1}{a} * y^{-\frac{1}{a}} * y^{-1} \\ F(t) &= p(T \le t) \\ &= \int_0^Y \frac{az(y)^{z-1}}{(1+y^{\alpha})^{z+1}}dY \end{aligned}$$

So, the cumulative function is as follows:

$$\begin{aligned} F(t) &= p(T \le t) \\ &= \int_0^Y \frac{az(y)^{z-1}}{\left(1+y^a\right)^{z+1}} dY \\ &= \int_0^Y \frac{az\left(1/y\right)^z y^{1/a}}{\left(1+1/y\right)^{z+1}} - \frac{1}{a} * y^{-1/a} * y^{-1} dY \\ &= \int_0^Y \frac{z\left(1/y\right)^z}{\left(1+1/y\right)^{z+1}} - 1 * \frac{1}{y} dY = \int_0^Y \frac{-z\left(1/y\right)^{z+1}}{\left(1+1/y\right)^{z+1}} dY = \int_0^Y -z\left(\frac{1}{(1+y)}\right) dY \end{aligned} \tag{3}$$

Example 3 Integral:

$$\begin{aligned} r &= 1+y \\ dr &= dy \\ F(t) &= p(T \le t) \\ &= \int_0^y -z\left(\frac{1}{r}\right)^{z+1} dy \\ &= \int_0^r -z * r^{-z-1} dr \\ &= \frac{-z}{-z} * r^{-z} \\ &= r^{-z} = (1+y)^{-z} = (1+y^{-a})^{-z} = \left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z} \end{aligned}$$

$$F(t) = \left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z} \tag{4}$$

Based on the Survival function in **Equation (4)** it can be determined that the Survival function $S(t)$ of the Dagum 3 Parameter distribution is as follows **[22] [24]** :

$$\begin{aligned} S(t) &= 1 - F(t) \\ &= 1 - \left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z} \end{aligned} \tag{5}$$

Then the Hazard function $h(t)$ is as follows:

$$h(t) = \frac{f(t)}{S(t)} = \frac{\dfrac{az\left(\frac{t}{b}\right)^{az-1}}{b\left(1+\left(\frac{t}{b}\right)^{a}\right)^{z+1}}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}} = \frac{\frac{az}{b}\left(\frac{t}{b}\right)^{az-1}\left(1+\left(\frac{t}{b}\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}} \tag{6}$$

General *Cox* Survival Models on the **Equation (6)** can form a Dagum 3 parameter model as follows **[25]**:

$$\begin{aligned} h(t, X) &= h_0(t)\exp(b_1X_1 + b_2X_2 + \dots + b_pX_p) \\ &= az/b \, \frac{\left(t/b\right)^{az-1}\left(1+\left(t/b\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}} \end{aligned} \tag{7}$$

Furthermore $h_0(t)$ is a function whose value depends on the value of t whereas $exp(b_1X_1 + b_2X_2 + \dots + b_pX_p)$ free from the t value so μ can be stated as follows **[15]** :

$$\mu = \frac{az}{b} = exp(b_1X_1 + b_2X_2 + \dots + b_pX_p) \tag{9}$$

And baseline hazard $h_0(t)$d is stated as follows:

$$h_0(t) = \frac{\left(\frac{t}{b}\right)^{az-1}\left(1+\left(\frac{t}{b}\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}} \quad (10)$$

So the hazard function is stated as follows [15]:

$$\begin{aligned} h(t) &= \frac{\left(t/b\right)^{az-1}\left(1+\left(t/b\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}} az/b \\ &= \frac{\left(t/b\right)^{az-1}\left(1+\left(t/b\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}} \exp(b_0+b_1X_1+b_2X_2+\ldots+b_pX_p) \end{aligned} \quad (11)$$

The Bayesian algorithm is a method for estimation parameters modelling [26]. The Bayesian perspective is an observation result of a parameter probability distribution that is not known with certainty [27]. Therefore it is necessary to determine a distribution of parameters, namely the prior distribution [28]. The combination of priors with sample data information (likelihood) produces a posterior distribution of parameters, the process is called Bayesian analysis with the Markov Chains Monte Carlo (MCMC) process [29]. MCMC is a numerical approach to obtain the posterior distribution [30]. The Posterior distribution as follows [31] :

$$f(\theta|x) = f(x|\theta)f(\theta)$$

$$p(b,a,z|t,x,\delta) = L(b,a,z;t,x,\delta)p(b)p(a)p(z) \quad (12)$$

The estimation for each parameter is obtained through the form of the full conditional distribution of each parameter a, z, and b, and by determining the prior distribution [32]. The prior parameter distribution of Equation (12) used is a combination of priors *conjugate* and *Informative* is as follows [19] [33] :

$a\sim$ Gamma (i,j)
$z\sim$ Gamma (i,j)
$b_i\sim$ Normal (u,v)

In Equation (12) the first form on the right side is *the probability* for hazard Dagum 3 parameters. The following Function *probability* can be explained as follows [23] :

$$L(\beta,a,z;t,x,\delta) = \prod_{i=1}^{I}\prod_{j=1}^{n_i} f_{ij}(t_{ij})^{\delta_{ij}}S_{ij}(t_{ij})^{1-\delta_{ij}} \quad (13)$$

Based on Equation (13) so that the equation is obtained described as follows [33] :

$$\begin{aligned} L(\beta,a,z;\boldsymbol{t},\boldsymbol{x},\boldsymbol{\delta}) &= \prod_{i=1}^{I}\prod_{j=1}^{n_i} f_{ij}(t_{ij})^{\delta_{ij}}S_{ij}(t_{ij})^{1-\delta_{ij}} \\ &= \prod_{i=1}^{I}\prod_{j=1}^{n_i}\left\{h_{ij}(t_{ij})S_{ij}(t_{ij})\right\}^{\delta_{ij}}S_{ij}(t_{ij})^{1-\delta_{ij}} \\ &= \prod_{i=1}^{I}\prod_{j=1}^{n_i} h_{ij}(t_{ij})^{\delta_{ij}}S_{ij}(t_{ij})^{\delta_{ij}}S_{ij}(t_{ij})^{1-\delta_{ij}} \\ &= \prod_{i=1}^{I}\prod_{j=1}^{n_i} h_{ij}(t_{ij})^{\delta_{ij}}S_{ij}(t_{ij}) \end{aligned} \quad (14)$$

Furthermore, based on **Equation (13)** and **Equation (14)** it can be described as follows:

$$L(\beta, a, z, \boldsymbol{t}, \boldsymbol{x}, \boldsymbol{\delta}) = \prod_{i=1}^{I}\prod_{j=1}^{n_i} h_{ij}(t_{ij})^{\delta_{ij}} S_{ij}(t_{ij})$$

$$= \prod_{i=1}^{I}\prod_{j=1}^{n_i}\left\{\frac{\left(t/b\right)^{az-1}\left(1+\left(t/b\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}}\exp b^T \boldsymbol{x_{ij}}\right\}^{\delta_{ij}}\left(1-\left(\left(\frac{x}{b}\right)^{-a}\right)^{-z}\right)$$

$$= \prod_{i=1}^{I}\prod_{j=1}^{n_i}\left\{\frac{\left(t/b\right)^{az-1}\left(1+\left(t/b\right)^{a}\right)^{-z-1}}{1-\left(1+\left(\frac{t}{b}\right)^{-a}\right)^{-z}}\exp(b_0)\exp b^T \boldsymbol{x_{ij}}\right\}^{\delta_{ij}\,\delta_{ij}}\left(1-\left(\left(\frac{x}{b}\right)^{-a}\right)^{-z}\right) \quad (15)$$

The posterior distribution for each of the parameters z, a, and $b_1 - b_p$ is done by integrating out the relevant parameters and can be explained as follows **[34]** :

$$\begin{aligned}
p(a\,|\,z, b_{1+i}) &\cong \int_z\int_{b_1}\ldots\ldots\int_{b_{1+p}} I(t\,|\,z, b_1, \ldots b_p)\,p(z)\,p(b_1)\ldots.p(b_p)\,dz\,db_1\ldots.db_p \\
p(z\,|\,a, b_{1+i}) &\cong \int_a\int_{b_1}\ldots\ldots\int_{b_{1+p}} I(t\,|\,a, b_1, \ldots b_p)\,p(a)\,p(b_1)\ldots.p(b_p)\,da\,db_1\ldots.db_p \\
p(b_0\,|\,a, z, b_i) &\cong \int_a\int_z\int_{b_1}\ldots\ldots\int_{b_p} I(t\,|\,a, z, b_1, \ldots b_p)\,p(a)\,p(z)\,p(b_1)\ldots.p(b_p)\,da\,dz\,db_1\ldots.db_p \\
p(b_1\,|\,a, z, b_{1+i} \neq 1) &\cong \int_a\int_z\int_{b_2}\ldots\ldots\int_{b_{1+p}} I(t\,|\,a, z, b_2, \ldots b_p)\,p(a)\,p(z)\,p(b_2)\ldots.p(b_{1+p})\,da\,dz\,db_2\ldots.db_{1+p} \\
&\vdots \\
p(b_p\,|\,a, z, b_{1+i} \neq p) &\cong \int_a\int_z\int_{b_1}\ldots\ldots\int_{b_{1+p}} I(t\,|\,a, z, b_1, \ldots b_{p+1})\,p(a)\,p(z)\,p(b_1)\ldots.p(b_{1+p})\,da\,dz\,db_1\ldots.db_{p+1}
\end{aligned} \quad (16)$$

The **Equation (16)** is solved using the Gibbs Sampling algorithm. Gibbs sampling is a technique for generating a random distribution from the posterior distribution having to calculate its density **[35]**. Technique on *Gibbs Sampling* based on the arrangement *Markov chain* which converges in a posterior distribution **[36].** Steps in the algorithm process *Gibbs Sampling* **[37]** :

1. Determine the initial value for each parameter.

$$(a^0, z^0, b_1^0, \ldots.\, b_p^0)$$

2. Next, a random sequence is obtained

$$a^1 \text{ from } p\left(a, z^0, b_1^0, \ldots.\, b_p^0\right)$$
$$z^1 \text{ from } p(z, a^0, b_1^0, \ldots.\, b_p^0)$$
$$b_1^1 \text{ from } p\left(b_1, z^0, a^0, b_2^0, \ldots., b_p^0\right)$$
$$\vdots$$
$$b_p^1 \text{ from } p(b_p^1, z^0, a^0, b_2^0, \ldots., b_{p-1}^0)$$

3. Repeat the second step until it converges.

The following are the steps in completing a survival regression analysis with a Bayesian approach that has a Dagum 3-Parameter distribution:

1. Collect data on HIV/AIDS care sufferers at Soetomo Hospital in Surabaya.
2. Identify events, the presence of censored and uncensored data stated as follows :
   a. δ: 0 which is censored data, for example, if the sufferer does not experience failure (*failure*) event in this case followed ARV therapy until the patient was declared dead, stopped, and skipped follow-up/last visit.
   b. δ: 1 which is uncensored data, for example, sufferers of HIV/AIDS treatment experience *failure events* or declared referred out of ARVs or condition improved.
3. Determine the model and survival parameters with the Dagum 3 Parameter distribution (α , β and k) which can be explained as follows:
   a. Define *Likelihood* function

b. Determine the distribution of prior parameters of the Dagum survival model parameters based on information from the data. Parameter a follows the gamma distribution (1,1). Parameter b follows the gamma distribution (1,1) dan parameter z follows the gamma distribution (1,1).
c. Parameter initialization determination (a, b, z) use 1-Step MCMC which is initial for parameter and k initial is 1 whereas β initials 1.
d. Calculates function value *hazard* and the survival function on the distribution of Dagum 3 parameter based on the posterior *summaries* which have been obtained.

4. Generating T samples $\theta^1, \theta^2, \dots \dots \theta^T$ form the posterior distribution $p(\theta|x)$ done *update* T as much as needed n times with *thin* enough to process *Markov Chain* fulfilled.
5. Obtain summaries of the posterior distribution
6. Interpret the mode *of survival* the Dagum 3 Parameter distribution.
7. Determining the factors that affect the rate of recovery of people with HIV/AIDS

3. RESULTS AND DISCUSSION

Estimation of the distribution is done against *survival* (t) where in this study is the length of HIV/AIDS care in the districts/cities of East Java. The hypothesis to estimate the appropriate distribution using testing *Kolmogorov Smirnov* through the following *Easy-Fit* program:

H_0 : Selection of survival time by the Dagum 3 Parameter distribution

H_1 : Selection of survival time is not by the Dagum 3 Parameter distribution

Conditions reject H_1 if $D > \alpha_{n,1-\alpha}$ or $P\_value > \alpha = 0{,}05$ with $\alpha_{n,1-\alpha}$ is the table value of *Kolmogorov Smirnov*. The smaller the value *Kolmogorov Smirnov* the alleged distribution, the data is getting closer to the alleged distribution

Table 1. Survival Time Distribution Test

Distribution	Statistical test	significance	Rank	Decision
Dagum 3 Parameter	0.0316	0,75884	1	Reject H_1

Based on the results of testing the distribution of survival time data on care for HIV/AIDS sufferers in **Table 1** it shows that the distribution of the appropriate guess is the Dagum 3 Parameter distribution with a Significant. *Kolmogorov Smirnov* is greater than the critical value $\alpha = 0.05$ and has a ranking of 1. **Table 2** in Parameter z shows that the patient will recover with an improved condition after treatment at Soetomo Hospital in Surabaya because at an interval of 2,5% to 97,50% it does not contain a value 0. Survival function and hazard function in treating HIV/AIDS sufferers in Hospital Soetomo Surabaya based on the estimation results of the Dagum 3 parameter distribution using the Bayesian approach. Results Output parameter estimation as a whole through the WinBUGS program package. Following are the results of the estimation of the distribution of Dagum 3 parameters for calculating the survival function and hazard function.

Table 2. Estimation of Dagum 3 parameter Distribution

Parameter	mean	2,50%	Median	97,50%
a	0.743	0.0011	0.690	0.742
b	17.4	0.16	10.4	16.92
z	10.6	0.057	7.93	10.52

Based on **Table 2** shows the estimated values of the three Dagum parameters where the values of the shape parameters $\alpha = 0.7425$, and $k = 10.58$ which increases the value of the hazard function over time. Based on the results of the estimated survival parameters, the calculation uses the formula in **Equation (5) and (6)**. From these calculations, the survival function and hazard function are obtained in **Table 3**. If the value of the survival function decreases and the hazard function increases with survival time, it can be concluded that the longer the patient's treatment for HIV/AIDS, the chances of surviving HIV/AIDS sufferers will be lower and the patient's recovery rate will be high, as presented in **Table 3** below :

Table 3. Hazard and Survival Function for Dagum 3 parameter distribution

Day	$S(t)$	$h(t)$
1	1	0.00000
2	0.99	0.00000
3	0.99	0.00000
4	0.99	0.00000
5	0.99	0.00000
6	0.99	0.00000
7	0.99	0.00000
8	0.99	0.00000
9	0.99	0.00000
10	0.99	0.00000
11	0.99	0.00000
12	0.99	0.00000
13	0.99	0.00006
14	0.99	0.00007
15	0.99	0.00007
.	.	.
.	.	.
2775	0.21	0,001

Based on **Table 3** it shows that the longer the longer HIV/AIDS patients take care of, the patient's recovery rate will increase and chances of survival during time t will decrease so that HIV/AIDS patients the longer the treatment, the better it will. For example, a person's chance of surviving on day 13 is 0.99 meaning that the number of patients who will not recover on the 13th day was 99 percent, while based on the hazard function on the 13th day the patient's recovery rate was 0.00006, meaning that the number of patients who recovered on the 13th day was 6 per hundred thousand.

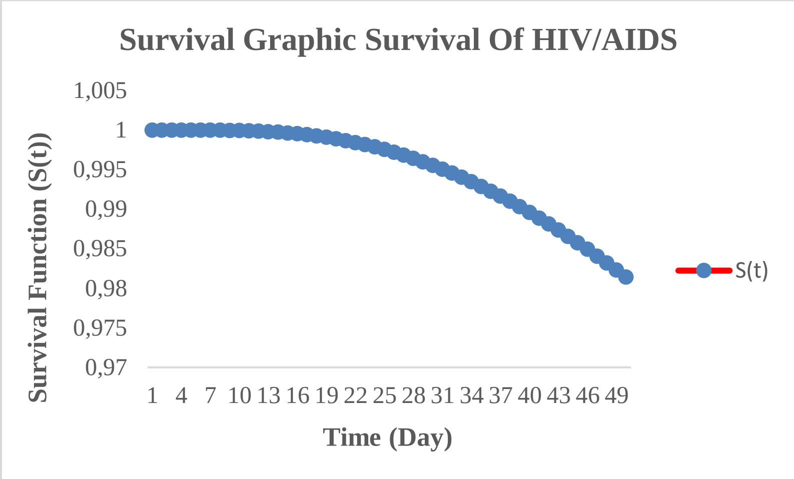

Figure 2. Plot of Survival Functions of HIV/AIDS Patients

Based on **Figure 2** shows the survival function over time. From this figure, it can be seen that the probability of surviving HIV/AIDS infection on a day by day basis is close to 0, meaning that treatment for HIV/AIDS sufferers is improving (all people with HIV/AIDS are cured). Based on **Figure 2**, it can be explained that HIV/AIDS patients in the ARV drug treatment program at Soetomo Hospital, Surabaya East Java experience an improvement every day, this can be seen from the chances of the survival curve decreasing day by day. Antiretroviral therapy in HIV/AIDS patients is effective for the patient's recovery rate.

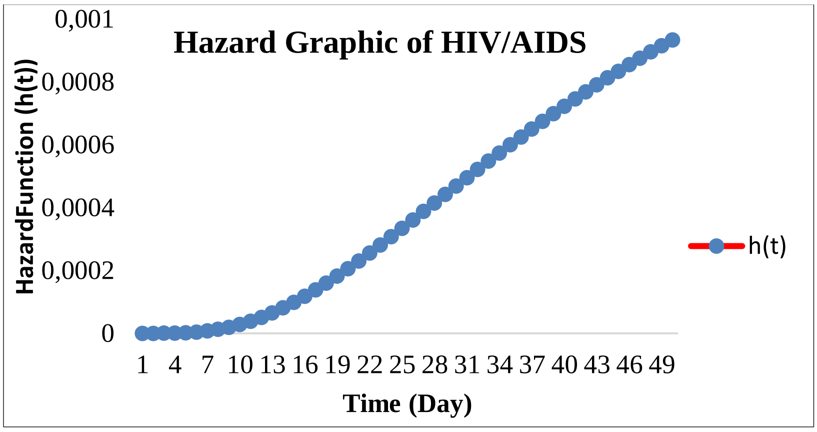

Figure 3 Plot of Hazard Functions of HIV/AIDS Patients

Based on **Figure 3**, it can be seen that the hazard value or rate of examination of HIV/AIDS sufferers has increased. The increase in value indicates that the longer the treatment for HIV/AIDS sufferers, the recovery rate will increase and the chances of survival will be greater. shows the slow recovery rate in the following days because the patient's condition is different and can be caused by other factors such as failure to take ARV medication. Furthermore, testing the factors that influence HIV/AIDS care in East Java. Factors that are thought to affect the hazard or rate of recovery from death in HIV/AIDS care are Sex (X_1), Age (X_2), Education (X_3), Employment Status (X_4), Marital Status (X_5), Weight (X_6), absolute CD4 levels (X_7), Suffering Stage (X_8), Functional State (X_9), Therapy adherence (X_{10}), Opportunistic infections (X_{11}), Status TB (Tuberculosis) (X_{12}), Transmission Risk Factors (X_{13}), and Companion taking medication (X_{14}) which is shown in **Table 4**. The following are the posterior results of constructing a survival regression model with Dagum 3 parameters to find out which predictor variables affect survival time until the patient is declared referred out for ART or died, moved, and missed follow-up.

Table 4. Parameter Estimation of Regression Survival Dagum 3 Parameter distributions

Variable	Parameter	mean	2.5%	median	97.5%	Exp(β)	Explanation
alpha	α	8.443	6.231	8.307	11.47		Significant
k	k	1.534	0.738	1.335	3.510		Significant

Variable	Parameter	mean	2.5%	median	97.5%	Exp(β)	Explanation
X_1	b_1	-0.023	-0.128	-0.025	0.086	0.977	Not Significant
X_2	b_2	0.083	-0.078	0.083	0.242	1.087	Not Significant
X_3	b_3	0.053	-0.009	0.053	0.115	1.054	Not Significant
X_4	b_4	0.026	-0.098	0.026	0.148	1.026	Not Significant
X_5	b_5	0.166	0.037	0.165	0.298	1.181	Significant
X_6	b_6	-0.053	-0.134	-0.058	0.028	0.948	Not Significant
X_7	b_7	0.189	0.020	0.189	0.361	1.208	Significant
X_8	b_8	0.076	0.019	0.076	0.131	1.079	Significant
X_9	b_9	0.149	0.029	0.146	0.283	1.161	Significant
X_{10}	b_{10}	-0.142	-0.267	-0.142	-0.020	0.868	Significant
X_{11}	b_{11}	0.081	-0.029	0.081	0.191	1.084	Not Significant
X_{12}	b_{12}	0.015	-0.026	0.014	0.056	1.015	Not Significant
X_{13}	b_{13}	0.054	-0.017	0.051	0.141	1.055	Not Significant
X_{14}	b_{14}	-0.084	-0.180	-0.083	0.012	0.919	Not Significant

Based on the calculations by the WinBUGS Software shown in **Table 4**. the factors that are considered to influence the rate of recovery for HIV/AIDS sufferers in East Java if the confidence interval value of 2.5% to 97.5% does not contain a value of 0. **Table 4** shows the factors that influence Significant effects on the rate of recovery in care for HIV/AIDS sufferers in East Java Marital Status (X_5). absolute CD4 Levels (X_7). Suffering Stage (X_8), Functional State (X_9), Therapy adherence (X_{10}). **Table 4** The mean column is the size of the model parameter while the next three columns are the estimated values at the 97.5% confidence interval. Parameters alpha and k are significant for the effect of caring for people with HIV/AIDS because the range of 2.5% to 97.5% does not contain a value of 0 which indicates there is an unexplained dependency/error in the survival regression model. The following is a survival regression model with Dagum 3 parameters formed:

$$\begin{aligned} h(t) &= h_0(t)\exp(\beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p) \\ &= h_0(t)\exp(-0,023X_1 + 0,083X_2 + ... - 0,084X_{14}) \end{aligned} \tag{17}$$

Furthermore. to determine the level of risk of a particular factor is calculated with the value of the Hazard ratio. Hazard is a comparison of individual ratios in factor conditions in the expected category with factors in the comparison category. **Table 4** provides a summary of the posterior survival regressions using the 3-parameter link function. Hazard calculated from two individuals can be used as a comparison function based on categorical values and the Cox regression coefficient can be interpreted. The hazard ratio indicates the survival rate of death in individuals with a factor *hazard* bigger is as big Exp(β) times the individual survival rate on the factor *hazard* smaller one. The interpretation of the variable Gender (X_1) shows that female sufferers who take part in the ARV program affect the rate of HIV/AIDS recovery or survival from death by 0.977 times compared to male sufferers. This shows that overall the death of people with HIV/AIDS is more male than female because men have physical endurance (body) that is more susceptible to the virus. HIV/AIDS. The interpretation is the same for all observed variables.

The posterior summary in **Table 4** explains the factors that have a significant effect on the rate of recovery in care for HIV/AIDS sufferers in East Java the marital status (X_5). absolute CD4 Levels (X_7). Suffering Stage (X_8), Functional State (X_9), Therapy adheren*ce* (X_{10}) using a Bayesian approach. This method estimates survival regression parameters on the Dagum 3 Parameter link function distribution with the MCMC-Gibbs Sampling algorithm. Data distribution has an important role in the results of the research conducted. This is supported in research **[37]** Survival regression with a Weibull distribution produces factors that influence the healing of HIV/AIDS patients at Shashemene Referral Hospital. Ethiopia. Factors that influence the healing of HIV / AIDS are TB status. knowledge of ART. and condom-use. Research conducted **[4]** related to the treatment of HIV/AIDS patients in the East has a Lognormal distribution of 3 Parameters. The results of this study were education, marital status, patient stage, adherence to therapy, opportunistic infections, and transmission risk. Research is also being conducted **[1]** about the Analysis of Kaplan Meier's Survival Curve in HIV/AIDS Patients on Antiretroviral Therapy (ART) at RSUD Prof. Dr. Soekandar Mojokerto using the Log Rank test. Kaplan Meier's survival curve analysis explains that the chances of survival for HIV/AIDS patients having a severe stage are lower than patients who have a mild stage so the stage factor can influence the survival of HIV/AIDS patients on ART. Modeling on survival data has a variety of distributions that can represent research results well using the Bayesian method approach.

4. CONCLUSIONS

The results of Regression Survival modeling with 3 link function parameters show that the longer the treatment of HIV/AIDS sufferers in the hospital. the recovery rate of a person will increase and the chances of survival over time t will decrease so that the sufferer will be more length of stay will improve as a result of joining the ARV program. The effectiveness of the ARV program followed by HIV/AIDS sufferers significantly increases the patient's immune system. Then the results of the computational Bayesian survival regression with Dagum 3 parameters link function show that the factor that has a significant effect on the care of HIV/AIDS sufferers in East Java is Marital Status (X_5). absolute CD4 Levels (X_7). Suffering Stage (X_8), Functional State (X_9), Therapy adherence (X_{10}). The results of research related to Dagum Survival regression 3 Parameters link function in Bayesian computing can provide information regarding the factors that influence HIV/AIDS and the rate of recovery. Information related to HIV/AIDS care can be used by the local government as socialization materials related to subscribing to and reducing HIV/AIDS sufferers in East Java.

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