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COMPARISON OF CONTROL CHART $\bar{X}$ BASED ON MEDIAN ABSOLUTE DEVIATION WITH S

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ABSTRACT

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A stable and controlled process will produce products of good quality following predetermined specifications. A control chart is one of the statistical tools that can be used to measure the stability of a product process in a controlled state. Control charts commonly used to evaluate the statistical control process are Shewhart control charts ($\bar{X}$ and S). The control chart is used to control the process, as seen from the average and variability of the process. If the data used is not normally distributed or there are outliers, then an alternative control chart, namely the Median Absolute Deviation (MAD), can be used. MAD is used to monitor the process mean and process standard deviation because it has properties that are robust or resistant to deviations. This research aims to form a control chart $\bar{X}$ based on MAD, apply it to data on fat content in animal feed products, and compare control charts $\bar{X}$ based on S with the control chart based on $\bar{X}$ control chart based on MAD. The limitations in this study are the quality characteristics used consist of only one variable and the data is not normally distributed, only limited to the mean process, and the data used in this study are observation data on the fat content contained in animal feed products at PT Japfa Comfeed Indonesia Tbk Makassar Unit from December 2021 to January 2022. The results of this study show that the control chart $\bar{X}$ based on MAD detects more out-of-control points than the control $\bar{X}$ based on S. The performance of the control chart $\bar{X}$ based on MAD is better at detecting changes in the process than the control chart $\bar{X}$ based on S because it has a relatively smaller ARL value.

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1. INTRODUCTION

Quality is the overall characteristics of a product or service that can satisfy needs, both expressly stated and hidden [1]. Quality is something that consumers consider when choosing a product or service [2]. The good quality of a product depends on controlling the production process. A stable and controlled process produces products that follow predetermined specifications [3]. A control chart is one of the statistical tools that can be used to measure the stability of a product process in a controlled state [4]. Control charts are a basic tool commonly used to monitor changes in the production process [5].

Control chart is often used in the statistical quality control process, in the form of observations on a graph to evaluate the production process over time [6]. This control chart plots the average quality characteristics measured from the sample against the sample sequence or sampling time. Based on the plot of the average measurement of these quality characteristics, the variability of production quality can be controlled [7].

The control chart has three horizontal lines, namely the *center line* (CL), *upper control limit (*UCL), and *lower control limit* (LCL) [8] [9]. The average value of the process quality characteristics is around the CL if there are no unusual sources of variability. Control charts are very useful for monitoring process *means* because they can directly detect unusual sources of variability characterized by the average value of process quality characteristics that are *out of control*. *Control charts* that have been commonly used are *Shewhart* control charts [10].

Shewhart control chart (S), which is used to evaluate statistical process control, was first introduced by Walter Andrew Shewhart in 1924 [11] [12]. This control chart eliminates abnormal variation by separating *special-cause variation* from *common-cause variation* [13]. Shewhart control charts in the frequently used variable control charts are control charts $\bar{X}$ and S. The control chart is used to control the process seen from the average and variability of the process. If the data used is not normally distributed or there are *outliers*, an alternative control chart, namely the *Median Absolute Deviation* (MAD), can be used [14].

The MAD control chart is an estimator that can be used as an alternative in dealing with deviations from observation data. MAD can estimate the standard deviation of the process when there are deviations. Deviations that occur cause data not to meet the assumption of normal distribution. MAD is used to monitor the process *mean* and process standard deviation because it is *robust* or resistant to deviations [15].

Several studies that have used MAD control charts include reducing the control limits of standard deviation control charts using MAD. This research discusses how to adjust standard deviation control limits based on MAD as an alternative to the traditional approach based on standard deviation [16], modifying simple control limits and robust based on MAD, this research shows how MAD can be used to modify stronger control limits in combining process variability [17], using control charts based on median and MAD, This research discusses the use of a combination of median and MAD to integrate processes [7], using MAD control charts on abnormal data, this research focuses on how MAD can be applied when data does not follow a normal distribution [14], using MAD control charts on cayenne pepper price data in Central Java, research related to the use of MAD in special situations, such as commodity price data [15].

In this study, there are several new things that are significant differences with previous studies. First, the use of the latest data on the animal feed industry. This recent data provides a more accurate understanding of the actual conditions of the industry. Secondly, this study also includes the use of simulation data. The simulation approach provides the flexibility to test various scenarios and see how the control chart behaves under various possible conditions. In addition, this study introduces a performance comparison between two types of $\bar{X}$ control charts, namely based on standard deviation (S) and based on Median Absolute Deviation (MAD). The aim is to find out which control chart is superior in providing more accurate and efficient information in controlling the production process.

This study aims to create an $\bar{X}$ control chart based on Mean Absolute Deviation (MAD) as an alternative method of controlling fat content in animal feed products. By utilizing MAD, this research aims to provide a framework that is more sensitive and adaptive to fluctuations in production data. In addition, this study will apply the $\bar{X}$ control chart based on MAD to actual data to evaluate its effectiveness in detecting significant changes in the quality of feed products. In addition to introducing a new control chart, this study will also conduct a comparison with the commonly used $\bar{X}$ control chart based on standard deviation (S). It is expected that the results of this study will not only provide a new, more efficient tool for controlling the quality of animal feed products, but also provide a deeper understanding of the differences and advantages between the MAD approach and the S approach in this context.

2. RESEARCH METHODS

2.1 Data Source

This study used secondary data obtained from thesis data with the title "Nonparametric Control Chart Performance *Exponentially Weighted Moving Average Sign* on Animal Feed Production," namely animal feed production inspection data at PT Japfa Comfeed Indonesia Tbk Makassar Unit from December 2021 to January 2022 **[18]**. In addition, this study also uses simulated data derived from the Standard Normal distribution $\left(N(0.1)\right)$.

Table 1. Observation Data on Fat Content (%) in Animal Feed Products from December 2021 to January 2022

Product Samples	Observation of Fat Content (%) in Each Animal Feed Product Sample (j)									
Animal Feed (i)	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
1	5,8	5,7	5,9	5,3	6	5,9	5,5	5,8	5,2	5
2	5,1	4,7	5,2	5	5,8	5,7	4,9	5	5,2	5,5
3	5,7	5,5	5,9	6,1	5,8	5,5	5,2	4,9	5,3	5,8
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
30	5,5	5,2	5,3	5,1	5,3	5,5	5,4	5,9	5,4	5,8

2.2 Variable Identification

The research variable used is the percentage of fat content contained in animal feed products. The data is 300 observations of $m = 30$ subgroups of size $n = 10$. The simulation data used consists of $m = 100$ and $n = 5. 10. 20.$ and 30.

2.3 Analysis Method

The analysis steps used in this study were:

1. Obtaining the control limits of the control chart $\bar{X}$ based on MAD.
2. Testing the normal distribution using the *Kolmogrov-Smirnov* test on the observation data of fat content contained in animal feed products.
3. Forming a control chart $\bar{X}$ based on S as follows:
   a. Determine the *mean* and standard deviation of each subgroup.
   b. Determine the *Upper Control Limit* (UCL), *Control Limit* (CL), and *Lower Control Limit* (LCL).
   c. Plot the *mean of* each subgroup with the obtained control limits to form a control chart $\bar{X}$ based on S.
   d. Interpret the results of control chart plots $\bar{X}$ based on S obtained.
4. Forming a control chart $\bar{X}$ based on MAD as follows:
   a. Control Chart $MD\text{-}MAD_R$
      1) Determine the *median of* each subgroup.
      2) Calculate the MAD of each subgroup and the average MAD.
      3) Determine the UCL, CL, and LCL.
      4) Plot the *median of* each subgroup with the obtained control limits to form a control chart $MD\text{-}MAD_R$.
      5) Interpret the results of control chart plots $MD\text{-}MAD_R$ obtained.
   b. Control Map $MD\text{-}MAD_M$
      1) Determine the *median of* each subgroup.
      2) Calculate the MAD of each subgroup and the average MAD.

3) Determine the UCL, CL, and LCL.

5. Plot the *median of* each subgroup with the obtained control limits to form a control chart $MD\text{-}MAD_M$.
    1) Interpret the results of control chart plots $MD\text{-}MAD_M$ obtained.

b. Control chart $\bar{X}\text{-}MAD_R$
    1) Determine the *mean of* each subgroup.
    2) Determine the *median* and MAD of each subgroup and the average MAD.
    3) Determine the UCL, CL, and LCL.
    4) Plot the *mean of* each subgroup with the obtained control limits to form a control chart $\bar{X}\text{-}MAD_R$.
    5) Interpret the results of control chart plots $\bar{X}\text{-}MAD_R$ obtained.

6. Determining the *Average Run Length* (ARL) of the control chart $\bar{X}$ based on S and control chart based on $\bar{X}$ based on MAD using simulated data.
7. Comparing control charts $\bar{X}$ based on S with the control chart based on $\bar{X}$ based on MAD by using the ARL value, then conclude.

3. RESULTS AND DISCUSSION

3.1 Control Chart $\bar{X}$ Based on MAD

Median absolute deviation (MAD) is an estimator to monitor data variability on a control chart [19]. MAD control charts are used to monitor the process *mean* [15]. Suppose $x_{i1}, x_{i2}, \ldots, x_{in}$ with $i = 1.2.\ldots, m$ is a sample that comes from m subgroups of size n, by sorting the sample data from smallest to largest first, then the median of the *i-th* subgroup is defined as follows (MD_i) is defined as follows:

$$MD_i = \begin{cases} X_{i\left(\frac{n+1}{2}\right)} & \text{, if n is odd} \\ \dfrac{X_{i\left(\frac{n}{2}\right)} + X_{i\left(\frac{n}{2}+1\right)}}{2} & \text{, if n is even} \end{cases} \tag{1}$$

MAD_R is defined as follows [20]:

$$MAD_i = b\ med|x_{i_{(i,j)}} - MD_i| \tag{2}$$

with $i = 1.2.\ldots, m$, $j = 1.2.\ldots, n$, and $b = 1.4826$ are constants.

MAD_M is defined as follows [7]:

$$MAD_i = med\left|x_{i_{(i,j)}} - MD_i\right| \tag{3}$$

with $i = 1.2.\ldots, m$, $j = 1.2.\ldots, n$.

The average of *the* m subgroup and *median* subgroup is given as follows:

$$\overline{MD} = \frac{MD_1 + MD_2 + \cdots + MD_m}{m} = \frac{1}{m}\sum_{i=1}^{m} MD_i \tag{4}$$

By using the *Moment* method to $k = 1$. It is obtained as follows:

$$M_1 = E(MD) = \hat{\mu}$$

$$M_1 = \sum_{i=1}^{m} \frac{MD_i}{m} = \overline{MD}$$

Thus, it is obtained $\hat{\mu} = \overline{MD}$.

The process *mean* is estimated by averaging over m *median* subgroup as follows:

$$\hat{\mu} = E(MD) = \frac{1}{m}\sum_{i=1}^{m} MD_i = \overline{MD} \tag{5}$$

The average of the m MAD subgroup is as follows:

$$\overline{MAD} = \frac{MAD_1 + MAD_2 + \cdots + MAD_m}{m} = \frac{1}{m}\sum_{i=1}^{m} MAD_i \tag{6}$$

The standard deviation of the process is estimated by averaging the following m the following MAD subgroups:

$$\hat{\sigma} = b_n \frac{1}{m}\sum_{i=1}^{m} MAD_i = b_n \overline{MAD} \quad (7)$$

with $b_n = \frac{n}{n-0.8}$ is the correction factor and n is the number of observations of the sample. The next is determining the control limits for the process *mean*.

1. Control Chart $MD\text{-}MAD_R$

The control limits for the process *mean are* derived from the sample *median*, and the MAD estimator is used to obtain the control limits and centerline. Thus, the control chart $\bar{X}$ based on MAD for the process *mean is* obtained:

$$CL = \overline{MD} \quad (8)$$

$$UCL = \overline{MD} + \frac{3\hat{\sigma}}{\sqrt{n}} = \overline{MD} + \frac{3b_n}{\sqrt{n}}\overline{MAD} \quad (9)$$

$$LCL = \overline{MD} - \frac{3\hat{\sigma}}{\sqrt{n}} = \overline{MD} - \frac{3b_n}{\sqrt{n}}\overline{MAD} \quad (10)$$

Suppose, $A_6 = \frac{3b_n}{\sqrt{n}}$. Then, the control limits for the control chart $\bar{X}$ based on MAD can be written as follows:

$$CL = \overline{MD} \quad (11)$$

$$UCL = \overline{MD} + A_6\overline{MAD} \quad (12)$$

$$LCL = \overline{MD} - A_6\overline{MAD} \quad (13)$$

2. Control Chart $MD\text{-}MAD_M$

The control limits for the process *mean are* derived from the sample *median*, and the MAD estimator is used to obtain the control limits and centerline. Thus, the control chart $\bar{X}$ based on MAD for the process *mean is* obtained:

$$CL = \hat{\mu} = \overline{MD} \quad (14)$$

$$UCL = \hat{\mu} + Z_{\frac{\alpha}{2}}\hat{\sigma}_{MD} = \overline{MD} + 3\hat{\sigma}_{MD} = \overline{MD} + 3\frac{1.253\hat{\sigma}}{\sqrt{n}}$$

$$UCL = \overline{MD} + 3\frac{1.253 b_n \overline{MAD}}{\sqrt{n}} = \overline{MD} + \frac{3.759 b_n \overline{MAD}}{\sqrt{n}} \quad (15)$$

$$LCL = \hat{\mu} - Z_{\frac{\alpha}{2}}\hat{\sigma}_{MD} = \overline{MD} - 3\hat{\sigma}_{MD} = \overline{MD} - 3\frac{1.253\hat{\sigma}}{\sqrt{n}}$$

$$LCL = \overline{MD} - 3\frac{1.253 b_n \overline{MAD}}{\sqrt{n}} = \overline{MD} - \frac{3.759 b_n \overline{MAD}}{\sqrt{n}} \quad (16)$$

Suppose, $R_1 = \frac{3.759 b_n}{\sqrt{n}}$. Then, the control limits for the control chart $\bar{X}$ based on MAD can be written as follows:

$$CL = \overline{MD} \quad (17)$$

$$UCL = \overline{MD} + R_1\overline{MAD} \quad (18)$$

$$LCL = \overline{MD} - R_1\overline{MAD} \quad (19)$$

3. Control Chart $\bar{X}\text{-}MAD_R$

The control limits for the process *mean are* derived from the sample *median*, and the MAD estimator is used to obtain the control limits and centerline. Thus, the control chart $\bar{X}$ based on MAD for the process *mean is* obtained:

$$CL = \bar{X} \quad (20)$$

$$UCL = \bar{X} + \frac{3\hat{\sigma}}{\sqrt{n}} = \bar{X} + \frac{3b_n}{\sqrt{n}}\overline{MAD} \quad (21)$$

$$LCL = \bar{X} - \frac{3\hat{\sigma}}{\sqrt{n}} = \bar{X} - \frac{3b_n}{\sqrt{n}}\overline{MAD} \quad (22)$$

Suppose, $A_6 = 3b_n/\sqrt{n}$. Then, the control limits for the control chart $\bar{X}$ based on MAD can be written as follows:

$$CL = \bar{X} \quad (23)$$

$$UCL = \bar{X} + A_6\overline{MAD} \quad (24)$$

$$LCL = \bar{X} - A_6\overline{MAD} \tag{25}$$

3.2 Case Study

The first step taken in determining the control chart $\bar{X}$ based on MAD was to test the normality of the data. The purpose of the normality test in this study was to ensure that the data did not fulfill the normal distribution assumption so that it could form a control chart based on MAD. $\bar{X}$ based on MAD. The *Kolmogorov-Smirnov normality* test was the most widely used normality test [21]. The Kolmogorov-Smirnov test was conducted to assess the normality of the observation data of fat composition in animal feed. The hypotheses were set as follows: H_0: the observation data of fat composition in animal feed is normally distributed, while H_1: the observation data of fat composition in animal feed is not normally distributed. The test statistic Test statistics calculation results is $D_{count} = max|F_t(x) - F_s(x)|$, D_{count} was calculated as 0.123936. Using the Kolmogorov-Smirnov critical value table approach with a significance level (α) of 0.05 and sample size (n) of 300, the critical value $D_{\alpha;n}$ was determined to be $D_{\alpha;n} = D_{(0.05;300)} = 0.077942$. Comparing the test statistic to the critical value, it was found that $D_{count} = 0.123936 > D_{(0.05;300)} = 0.077942$. Therefore, there is not enough evidence to accept H_0, indicating that the observation data on fat composition in animal feed does not follow a normal distribution.

The next step was to form a control chart $\bar{X}$ based on S and a control chart $\bar{X}$ based on MAD.

1. Control Chart $\bar{X}$ based on S

The first step in forming a control chart $\bar{X}$ based on S is to determine the *mean* and standard deviation of each subgroup using the following formula:

$$\bar{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n}$$

$$S_i = \sqrt{\frac{\sum_{j=1}^{n}\left(x_{ij} - \bar{x}_i\right)^2}{n-1}}$$

The results of the calculation of the *mean* and standard deviation of each subgroup obtained $\bar{X} = 5.769333$ and $\bar{S} = 0.293562$. The next step was determining CL, UCL, and LCL to form a control chart $\bar{X}$ based on S. For *the* $n = 10$ value; it is obtained $A_3 = 0.975309$. Thus, the boundaries of the control chart $\bar{X}$ based on S are as follows:

$CL = \bar{X} = 5.769333$

$UCL = \bar{X} + A_3\bar{S}$

$= 5.769333 + 0.975309(0.293562) = 5.769333 + 0.286314 = 6.055647$

$LCL = \bar{X} - A_3\bar{S}$

$= 5.769333 - 0.975309(0.293562) = 5.769333 - 0.286314 = 5.483019$

Based on the results of these calculations, the values $CL = 5.769333$. $UCL = 6.055647$, and $LCL = 5.483019$. The next step is plotting the *mean of* each subgroup with the control limits obtained to form a control chart $\bar{X}$ based on S, as shown in **Figure 1** below.

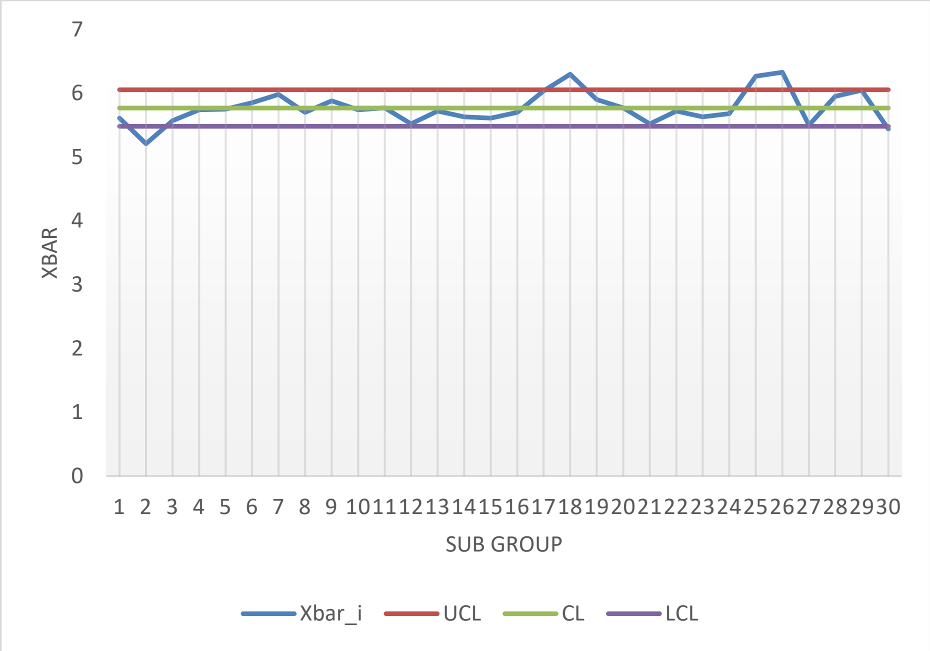

Figure 1. Control Chart $\bar{X}$ based on S

Figure 1. It can be seen that the process is uncontrolled. This is because five subgroup *mean* points are located outside the control limits *(out of control*), namely $\bar{x}_2, \bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}$, and $\bar{x}_{30}$. Thus, it can be concluded that the *mean of the* fat content composition process in animal feed production shows three points above the upper control limit (UCL), namely $\bar{x}_{18}, \bar{x}_{25}$, and $\bar{x}_{26}$. Two points are below the lower control limit (LCL), namely $\bar{x}_2$ and $\bar{x}_{30}$.

2. Control Chart $\bar{X}$ based on MAD

a. Control Chart $MD\text{-}MAD_R$

The first step in forming a control chart $MD\text{-}MAD_R$ was to determine the *median* and MAD of each subgroup using the formulas in **Equation (1)** and **Equation (2)**.

The results of the calculation of the *median* and MAD of each subgroup obtained $\overline{MD} = 5.778333$ and $\overline{MAD} = 0.27181$. The next step was determining CL, UCL, and LCL to form a control chart. $MD\text{-}MAD_R$ by using **Equation (11)**, **Equation (12)**, and **Equation (13)**. For $n = 10$, the value obtained is $A_6 = 1.031219$. Thus, the boundaries of the control chart $MD\text{-}MAD_R$ are as follows:

$$CL = \overline{MD} = 5.778333$$

$$UCL = \overline{MD} + A_6\overline{MAD} = 5.778333 + 1.031219(0.27181) = 5.778333 + 0.280296 = 6.058629$$

$$LCL = \overline{MD} - A_6\overline{MAD} = 5.778333 - 1.031219(0.27181) = 5.778333 - 0.280296 = 5.498037$$

Based on the results of these calculations, the values $CL = 5.778333. UCL = 6.058629$, and $LCL = 5.498037$. The next step is plotting the *median* of each subgroup with the control limits obtained to form a control chart $MD\text{-}MAD_R$, as shown in **Figure 2** below.

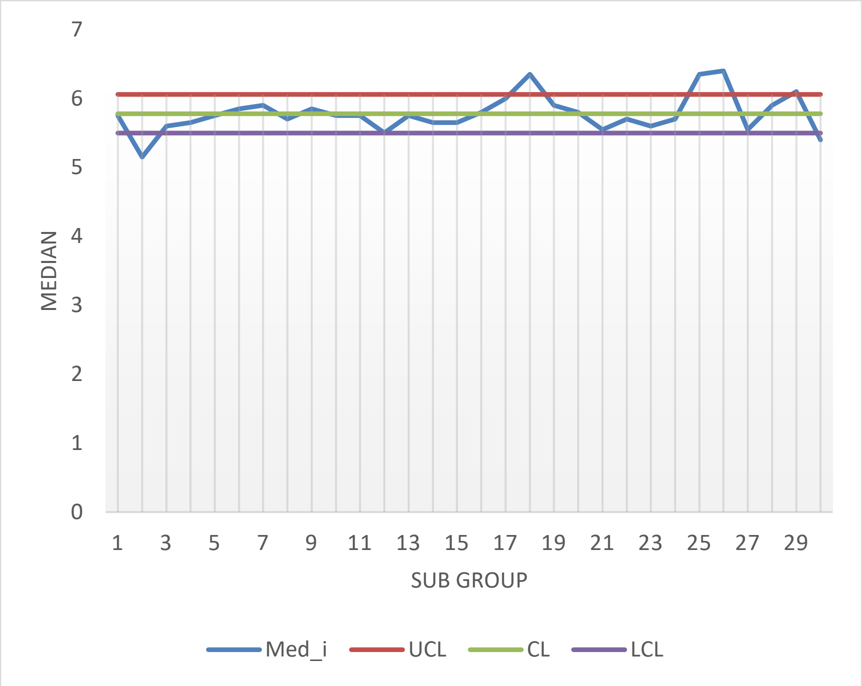

Figure 2. Control Chart $MD\text{-}MAD_R$

In **Figure 2,** It can be seen that the process is out of control. This is because six subgroup *median* points lie outside the control limits *(out of control*), namely $\bar{x}_2, \bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}, \bar{x}_{29}$ and $\bar{x}_{30}$. Thus, it can be concluded that the *mean* process of fat content composition in animal feed production shows four points above the upper control limit (UCL), namely $\bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}$ and $\bar{x}_{29}$. Two points are below the lower control limit (LCL), namely $\bar{x}_2$ and $\bar{x}_{30}$.

b. Control Chart $MD\text{-}MAD_M$

Initial steps to form a control chart $MD\text{-}MAD_M$ is first to determine the *median* and MAD of each subgroup, using the formulas in **Equations (1)** and **(3)**.

The results of the calculation of the *median* and MAD of each subgroup obtained $\overline{MD} = 5.778333$ and $\overline{MAD} = 0.183333$. The next step was to determine CL, UCL, and LCL using **Equations (17)**, **(18),** and **(19)**. For $n = 10$, the value of $R_1 = 1.29212$ was obtained. Thus, the boundaries of the control chart $MD\text{-}MAD_M$ are as follows:

$$CL = \overline{MD} = 5.778333$$
$$UCL = \overline{MD} + R_1\overline{MAD} = 5.778333 + 1.29212(0.183333) = 5.778333 + 0.236888 = 6.015221$$
$$LCL = \overline{MD} - R_1\overline{MAD} = 5.778333 - 1.29212(0.183333) = 5.778333 - 0.236888 = 5.541445$$

Based on the results of these calculations, the values $CL = 5.778333$, $UCL = 6.015221$, and $LCL = 5.541445$. The next step is plotting the *median* of each subgroup with the control limits obtained to form a control chart $MD\text{-}MAD_M$, shown in **Figure 3** below.

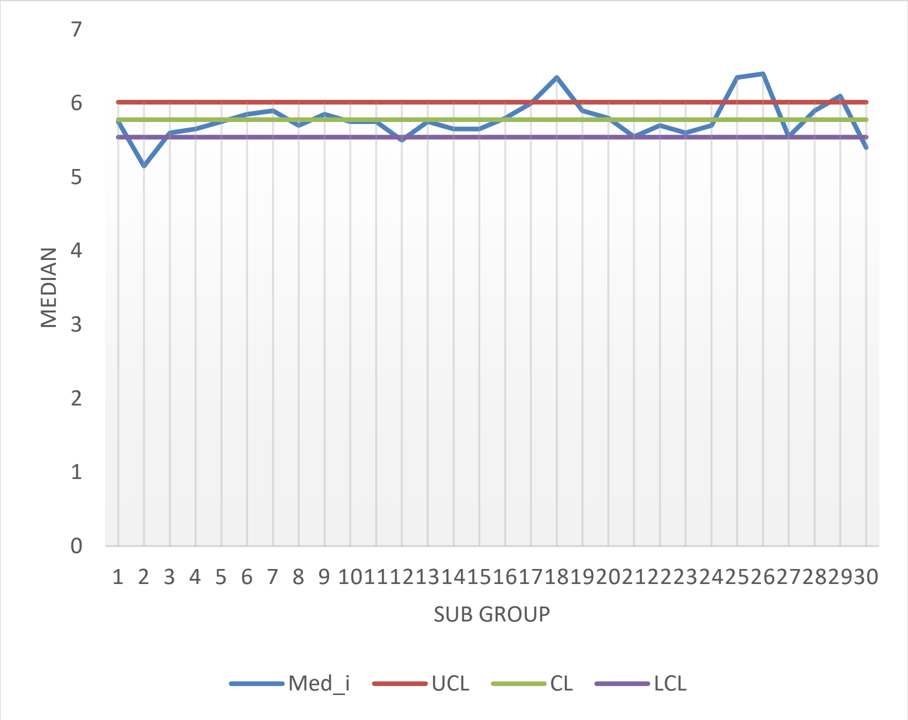

Figure 3. Control Chart $MD\text{-}MAD_M$

In **Figure 3.** The process is out of control. This is because seven subgroup *median* points lie outside the control limits *(out of control*), namely $\bar{x}_2, \bar{x}_{12}, \bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}, \bar{x}_{29}$, and $\bar{x}_{30}$. Thus, it can be concluded that the *mean* process of fat content composition in animal feed production shows four points above the upper control limit (UCL), namely $\bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}$ and $\bar{x}_{29}$. Three points are below the lower control limit (LCL), namely $\bar{x}_2, \bar{x}_{12}$ and $\bar{x}_{30}$.

c. Control Chart $\bar{X}\text{-}MAD_R$

The first step in forming a control chart $\bar{X}\text{-}MAD_R$ was to determine the *mean* and MAD of each subgroup using the following formula:

$$\bar{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n}$$
$$MAD_i = b\ med|x_{i_{(i,j)}} - MD_i|$$

The results of the calculation of the *mean* and MAD of each subgroup are obtained $\bar{X} = 5.769333$ and $\overline{MAD} = 0.27181$. The next step was determining CL, UCL, and LCL to form a control chart. $\bar{X}\text{-}MAD_R$ by using **Equation (23)**, **Equation (24)**, and **Equation (25)**. For $n = 10$, the value obtained is $A_6 = 1.031219$. Thus, the boundaries of the control chart $\bar{X}\text{-}MAD_R$ are as follows:

$$CL = \bar{X} = 5.769333$$
$$UCL = \bar{X} + A_6\overline{MAD} = 5.769333 + 1.031219(0.27181) = 5.769333 + 0.280296 = 6.049629$$
$$LCL = \bar{X} - A_6\overline{MAD} = 5.769333 - 1.031219(0.27181) = 5.769333 - 0.280296 = 5.489037$$

Based on the results of these calculations, the values $CL = 5.769333. UCL = 6.049629,$ and $LCL = 5.489037$. The next step is plotting the *mean of* each subgroup with the control limits obtained to form a control chart $\bar{X}\text{-}MAD_R$, shown in **Figure 4** below.

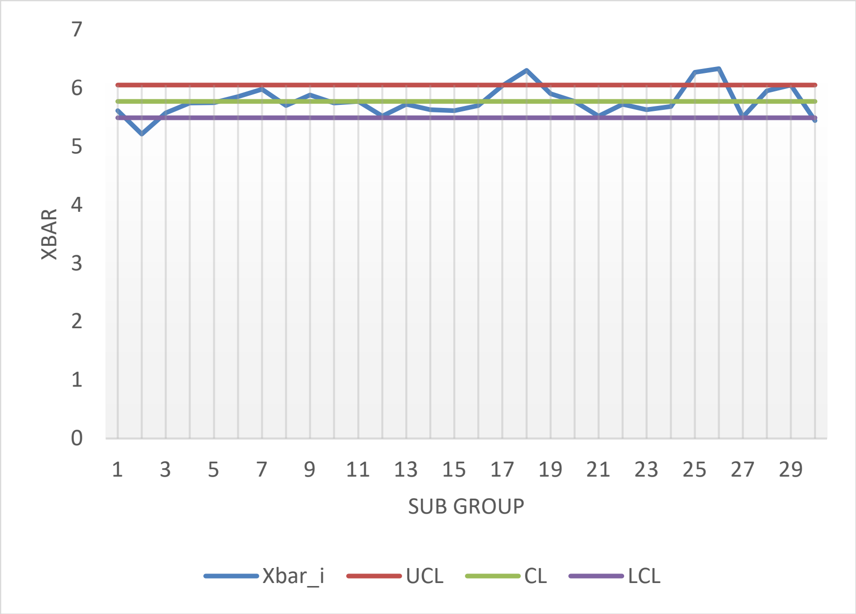

Figure 4. Control Chart $\bar{X}$-MAD_R

In **Figure 4,** the process is uncontrolled. This is because six subgroup *mean* points are located outside the control limits *(out of control*), namely $\bar{x}_2, \bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}, \bar{x}_{29}$ and $\bar{x}_{30}$. Thus, it can be concluded that the *mean* process of fat content composition in animal feed production shows four points above the upper control limit (UCL), namely $\bar{x}_{18}, \bar{x}_{25}, \bar{x}_{26}$ and $\bar{x}_{29}$. Two points are below the lower control limit (LCL), namely $\bar{x}_2$ and $\bar{x}_{30}$.

The point above the UCL is a warning that a large mixture of fat content exceeds the average standard that has been set. The cause of the case must be resolved immediately to avoid variations in the form of more quality differences because the excess fat content will cause diarrhea in livestock, and feed will easily go rancid. Meanwhile, the point below the LCL is a form of warning that there is a mixture of less fat content so that it does not meet the average standard that has been set. The cause of the case must be resolved immediately to avoid variations in the form of more and more quality differences because if the fat content is less, it will hinder the growth and development of livestock and feed will also easily go rancid.

3.3 Control Chart Comparison $\bar{X}$ Based on S with Control Chart $\bar{X}$ Based on MAD

Measuring the performance of a control chart usually uses *Average Run Length* (ARL). The smaller the ARL value obtained, the better the performance of the control chart in detecting process changes. Determination of the ARL value of the control chart $\bar{X}$ based on S and control chart $\bar{X}$ based on MAD in this study using simulated data normally distributed $X \sim N(0.1)$, as many as $m = 100$ with each $n = 5.10.20.$ and 30.

Value determination of β for the ARL of the control chart $\bar{X}$ based on S and control chart based on $\bar{X}$ based on MAD are as follows.

1. Value determination of β for the ARL of the control chart $\bar{X}$ based on S is as follows.

   Hypothesis:

   $H_0 : \mu = \mu_0$
   $H_1 : \mu \neq \mu_0$; $\mu = \mu_0 + k\sigma$

$$\beta = Pr(LCL < \bar{X} < UCL | \mu = \mu_0 + k\sigma)$$

$$\beta = Pr\left(\frac{LCL-\mu}{\sqrt{\sigma^2}} < \frac{\bar{X}-\mu}{\sqrt{\sigma^2}} < \frac{UCL-\mu}{\sqrt{\sigma^2}} \middle| \mu = \mu_0 + k\sigma\right)$$

$$\beta = Pr\left(\frac{LCL-\mu}{\sigma} < \frac{\bar{X}-\mu}{\sigma} < \frac{UCL-\mu}{\sigma} \middle| \mu = \mu_0 + k\sigma\right)$$

$$\beta = Pr\left(\frac{LCL-\mu}{\sigma} < Z < \frac{UCL-\mu}{\sigma}\middle| \mu = \mu_0 + k\sigma; Z\sim N(0.1)\right)$$
$$\beta = Pr\left(\frac{LCL-(\mu_0+k\sigma)}{\sigma} < Z < \frac{UCL-(\mu_0+k\sigma)}{\sigma}\middle| \mu_0 = 0.\sigma = 1\right)$$
$$\beta = Pr\ (LCL-k < Z < UCL-k)$$
$$\beta = \Phi(Z < UCL-k) - \Phi(Z < LCL-k)$$
$$\beta = \Phi(UCL-k) - \Phi(LCL-k)$$
$$\beta = \Phi\left(\left(\bar{X} + \frac{3}{c_4\sqrt{n}}\bar{S}\right) - k\right) - \Phi\left(\left(\bar{X} - \frac{3}{c_4\sqrt{n}}\bar{S}\right) - k\right) \tag{26}$$

2. Value determination of β for the ARL of the control chart $\bar{X}$ based on MAD

a. Value determination of β for the ARL of the control chart $MD\text{-}MAD_R$ is as follows.

Hypothesis:
$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0\ ;\ \mu = \mu_0 + k\sigma$

$$\beta = Pr(LCL < MD_i < UCL)$$
$$\beta = Pr\left(\frac{LCL - E(MD_i)}{\sqrt{var(MD_i)}} < \frac{MD_i - E(MD_i)}{\sqrt{var(MD_i)}} < \frac{UCL - E(MD_i)}{\sqrt{var(MD_i)}}\right)$$
$$\beta = Pr\left(\frac{LCL-\mu}{\sqrt{(MAD_i)^2}} < Z < \frac{UCL-\mu}{\sqrt{(MAD_i)^2}}\middle| \mu = \mu_0 + k\sigma\right)$$
$$\beta = Pr\left(\frac{LCL-\mu}{MAD_i} < Z < \frac{UCL-\mu}{MAD_i}\right)\middle| \mu = \mu_0 + k\sigma$$
$$\beta = Pr\left(\frac{LCL-\mu}{\sigma} < Z < \frac{UCL-\mu}{\sigma}\right)\middle| \mu = \mu_0 + k\sigma; Z\sim N(0.1)$$
$$\beta = Pr\left(\frac{LCL-(\mu_0+k\sigma)}{\sigma} < Z < \frac{UCL-(\mu_0+k\sigma)}{\sigma}\middle| \mu_0 = 0.\sigma = 1\right)$$
$$\beta = \ Pr\ (LCL-k < Z < UCL-k)$$
$$\beta = \Phi(Z < UCL-k) - \Phi(Z < LCL-k)$$
$$\beta = \Phi(UCL-k) - \Phi(LCL-k)$$
$$\beta = \Phi\left((\overline{MD} + A_6\overline{MAD}) - k\right) - \Phi\left((\overline{MD} - A_6\overline{MAD}) - k\right) \tag{27}$$

b. Value determination of β for the ARL of the control chart $MD\text{-}MAD_M$ is as follows.
Hypothesis:
$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0\ ;\ \mu = \mu_0 + k\sigma$

$$\beta = Pr(LCL < MD_i < UCL)$$
$$\beta = Pr\left(\frac{LCL - E(MD_i)}{\sqrt{var(MD_i)}} < \frac{MD_i - E(MD_i)}{\sqrt{var(MD_i)}} < \frac{UCL - E(MD_i)}{\sqrt{var(MD_i)}}\right)$$
$$\beta = Pr\left(\frac{LCL-\mu}{\sqrt{(MAD_i)^2}} < Z < \frac{UCL-\mu}{\sqrt{(MAD_i)^2}}\middle| \mu = \mu_0 + k\sigma\right)$$
$$\beta = Pr\left(\frac{LCL-\mu}{MAD_i} < Z < \frac{UCL-\mu}{MAD_i}\middle| \mu = \mu_0 + k\sigma\right)$$
$$\beta = Pr\left(\frac{LCL-\mu}{\sigma} < Z < \frac{UCL-\mu}{\sigma}\middle| \mu = \mu_0 + k\sigma; Z\sim N(0.1)\right)$$
$$\beta = Pr\left(\frac{LCL-(\mu_0+k\sigma)}{\sigma} < Z < \frac{UCL-(\mu_0+k\sigma)}{\sigma}\middle| \mu_0 = 0.\sigma = 1\right)$$
$$\beta = \ Pr\ (LCL-k < Z < UCL-k)$$
$$\beta = \Phi(Z < UCL-k) - \Phi(Z < LCL-k)$$
$$\beta = \Phi(UCL-k) - \Phi(LCL-k)$$
$$\beta = \Phi\left((\overline{MD} + R_1\overline{MAD}) - k\right) - \Phi\left((\overline{MD} - R_1\overline{MAD}) - k\right) \tag{28}$$

c. Value determination of β for the ARL of the control chart $\bar{X}\text{-}MAD_R$ is as follows.
Hypothesis:

$H_0 : \mu = \mu_0$
$H_1 : \mu \neq \mu_0\ ;\ \mu = \mu_0 + k\sigma$

$$\beta = Pr(LCL < \bar{X} < UCL | \mu = \mu_0 + k\sigma)$$
$$\beta = Pr\left(\frac{LCL - \mu}{\sqrt{\sigma^2}} < \frac{\bar{X} - \mu}{\sqrt{\sigma^2}} < \frac{UCL - \mu}{\sqrt{\sigma^2}} \middle| \mu = \mu_0 + k\sigma\right)$$
$$\beta = Pr\left(\frac{LCL - \mu}{\sigma} < \frac{\bar{X} - \mu}{\sigma} < \frac{UCL - \mu}{\sigma} \middle| \mu = \mu_0 + k\sigma\right)$$
$$\beta = Pr\left(\frac{LCL - \mu}{\sigma} < Z < \frac{UCL - \mu}{\sigma} \middle| \mu = \mu_0 + k\sigma; Z \sim N(0.1)\right)$$
$$\beta = Pr\left(\frac{LCL - (\mu_0 + k\sigma)}{\sigma} < Z < \frac{UCL - (\mu_0 + k\sigma)}{\sigma} \middle| \mu_0 = 0. \sigma = 1\right)$$
$$\beta = Pr\ (LCL - k < Z < UCL - k)$$
$$\beta = \Phi(Z < UCL - k) - \Phi(Z < LCL - k)$$
$$\beta = \Phi(UCL - k) - \Phi(LCL - k)$$
$$\beta = \Phi\big((\bar{X} + A_6\overline{MAD}) - k\big) - \Phi\big((\bar{X} - A_6\overline{MAD}) - k\big) \quad (29)$$

With Φ is standard normal probability cumulative distribution function, UCL is upper control limit, LCL is lower control limit, μ_0 is average of the data, k is the amount of shift/change, and σ is standard deviation of the data.

Determination of the ARL of the control chart $\bar{X}$ based on S and control chart $\bar{X}$ based on MAD are as follows: The initial step in determining the ARL of the control chart was to generate normally distributed data $X \sim N(0.1)$, as many as $m = 100$ with each $n = 5, n = 10, n = 20$, and $n = 30$. Then, each value of the control limits, namely UCL, LCL and CL was determined. The next step was to determine each value β from the control chart, then determine the ARL value to compare the performance of the control chart $\bar{X}$ based on S with the MAD control chart . The ARL value of the control chart $\bar{X}$ based on S, and the control chart based on $\bar{X}$ and MAD, as shown in **Table 2** as follows.

Table 2. Comparison of Control Chart ARL Values $\bar{X}$ Based on S with Control Chart $\bar{X}$ Based on MAD

$n = 5$				
k	**Control chart $\bar{X}$ based on S**	**Control chart $\bar{X}$ based on MAD**		
		$MD\text{-}MAD_R$	**$MD\text{-}MAD_M$**	**$\bar{X}\text{-}MAD_R$**
0	5.429077	5.929141	4.086871	5.920725
0.25	4.957160	5.453961	3.832653	5.378037
0.5	4.079719	4.474638	3.281619	4.381409
0.75	3.215989	3.494304	2.690246	3.415805
1	2.538298	2.726186	2.196404	2.668685
1.25	2.051236	2.177420	1.826102	2.137271
1.5	1.711520	1.796904	1.560760	1.769220
1.75	1.476527	1.534769	1.374364	1.515707
2	1.314482	1.354329	1.245007	1.341220
$n = 10$				
k	**Control chart $\bar{X}$ based on S**	**Control chart $\bar{X}$ based on MAD**		
		$MD\text{-}MAD_R$	**$MD\text{-}MAD_M$**	**$\bar{X}\text{-}MAD_R$**
0	2.823084	2.770134	2.272168	2.770676
0.25	2.665440	2.616340	2.172582	2.619002
0.5	2.363725	2.324329	1.977609	2.327877
0.75	2.033171	2.004289	1.754699	2.007674
1	1.744147	1.723882	1.551209	1.726652
1.25	1.517891	1.503916	1.386270	1.506012
1.5	1.350670	1.341101	1.261453	1.342621
1.75	1.231218	1.224712	1.171176	1.225784
2	1.148094	1.143727	1.108213	1.144462
$n = 20$				
k	**Control chart $\bar{X}$ based on S**	**Control chart $\bar{X}$ based on MAD**		
		$MD\text{-}MAD_R$	**$MD\text{-}MAD_M$**	**$\bar{X}\text{-}MAD_R$**
0	1.960180	1.961542	1.731657	1.961576
0.25	1.909026	1.915922	1.699550	1.910325
0.5	1.781072	1.791179	1.610361	1.782142
0.75	1.619680	1.630312	1.492266	1.620487

1	1.462265	1.471694	1.372377	1.462840
1.25	1.328601	1.336170	1.267261	1.328998
1.5	1.224229	1.229926	1.183244	1.224497
1.75	1.147240	1.151329	1.120346	1.147418
2	1.092969	1.095784	1.075688	1.093085
		$n = 30$		
k	**Control chart $\bar{X}$ based on S**	**Control chart $\bar{X}$ based on MAD**		
		$MD\text{-}MAD_R$	$MD\text{-}MAD_M$	$\bar{X}\text{-}MAD_R$
0	1.724592	1.717231	1.557467	1.717305
0.25	1.694135	1.688276	1.536385	1.687238
0.5	1.606822	1.602778	1.473395	1.600994
0.75	1.490273	1.487803	1.386920	1.485772
1	1.371413	1.370046	1.296232	1.368143
1.25	1.266880	1.266188	1.214489	1.264601
1.5	1.183154	1.182835	1.147765	1.181609
1.75	1.120378	1.120250	1.097111	1.119357
2	1.075757	1.075718	1.060888	1.075099

In **Table 1**, the control chart $\bar{X}$ based on MAD performs better in detecting process changes than the control chart based on MAD. $\bar{X}$ based on S. It is shown with the change value $0 \leq k \leq 2$. For $n = 5, 10, 20,$ and 30, ARL value of control chart $\bar{X}$ based on MAD ($MD\text{-}MAD_M$) has a relatively smaller value when compared to the ARL value of the control chart based on the $\bar{X}$ based on S. The smaller the ARL, the smaller the expected number of samples required until there is an out-of-control signal. This means that the smaller the ARL, the faster the graphics control detects a shift [22]. So, the smaller the ARL value, the better the type of control chart in question [23].

Based on the results and analysis in this study, it can be concluded that the use of $\bar{X}$ control chart based on Median Absolute Deviation (MAD) on observed data of fat content in animal feed products at PT Japfa Comfeed Indonesia Tbk Makassar Unit is significantly more sensitive in detecting out-of-control points compared to $\bar{X}$ control chart based on Standard Deviation (S). The $\bar{X}$ control chart based on MAD also excels at capturing changes in the production process, with a better ability to identify small deviations or shifts in the fat content of animal feed. This provides an early warning of potential problems or discrepancies in production, which in turn can improve quality control and overall production efficiency. Nonetheless, this study has limitations as it focused on one case study, namely on fat data in animal feed at PT Japfa Comfeed Indonesia Tbk Makassar Unit. Therefore, generalization of the findings to other industries or products may require additional research. Recommendations for future research include developing the application of MAD with more than one variable, comparing the performance of $\bar{X}$ control chart based on S and $\bar{X}$ control chart based on MAD based on ARL value can use various distributions other than Normal distribution, i.e. Exponential, Weibull, Chi-square, logistic, and Cauchy, and further study to understand the impact of applying $\bar{X}$ control chart based on MAD in the animal feed industry as a whole. With these steps, future research can expand the understanding of the effectiveness of MAD in the $\bar{X}$ control chart and its contribution to quality control and production efficiency in the animal feed industry.

4. CONCLUSIONS

Based on the research results in this paper, it can be concluded that the application of $\bar{X}$ control chart based on Mean Absolute Deviation (MAD) on the observation data of fat content in animal feed products at PT Japfa Comfeed Indonesia Tbk Makassar Unit is more efficient than $\bar{X}$ control chart based on standard deviation (S). This finding has significant implications in quality control of animal feed products. By using an $\bar{X}$ control chart based on MAD, companies can more quickly and more accurately detect significant changes in product quality, which in turn can reduce the risk of nonconforming products and improve the efficiency of the production process. This provides a strong justification for applying this new approach to quality control of animal feed products. Thus, this research not only makes an important contribution to the literature in the field of statistical process control, but also provides practical guidance for companies in improving their efficiency and product quality.

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