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IMPLEMENTATION OF MEWMA CHART USING TIME SERIES MODEL FOR MONITORING THE WHITE CRYSTAL SUGAR QUALITY

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ABSTRACT

The basic assumption of implementing a control chart is that the observed process values should be normally distributed and independent. However, the production process at the factory is carried out repeatedly with the same machine, so there is a possibility that the observation data is not independent and the resulting control chart becomes inaccurate. Therefore, a time series model approach is needed to create an accurate control chart. This study aims to conduct statistical quality control of white crystal sugar (WCS) production at Madukismo Sugar Factory (MSF) Yogyakarta to maintain product stability and quality according to the standards. MSF performs quality control on WCS products with several quality characteristic variables, including drying shrinkage (%), grain size (mm), polarization, and color of sugar solution (ICUMSA). This study is only limited to the four quality characteristics with research data in the form of secondary data from the MSF QC Laboratory. The data used were WCS quality characteristics from May 9 - July 16, 2022. The four characteristics influence each other, so a MEWMA control chart is used. This study found autocorrelation between observations so that time series modeling was carried out and resulted in VARIMA (1,1,2) as the best time series model. While the implementation of the MEWMA chart shows uncontrollable results with an optimal weighting value λ of 0.8. For process capability, the results show that the process is capable.

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1. INTRODUCTION

Quality is the fitness for use to meet consumer needs and satisfaction [1]. Quality is an important factor in the use of a product or service, so quality control is needed to maintain the stability of the manufactured product [2]. Statistical process control and control charts are commonly used techniques in industry to maintain quality.

The control chart is a chart used to observe whether a production process is under control or not. The control chart was first proposed by Walter A. Shewhart in 1924 so it is often referred to as the Shewhart control chart [1]. Shewhart charts are less sensitive in detecting relatively small process mean shifts ($\leq 1.5\sigma$). As a more effective alternative, the exponentially weighted moving average (EWMA) chart was developed by Roberts in 1959. This chart is able to detect relatively small average shifts in the production process [3].

There are two types of control charts, univariate control charts and multivariate control charts. The EWMA chart is one of the charts used in the univariate case. This chart becomes less effective in controlling one by one the quality characteristics of a product that has several interconnected quality characteristics. Therefore, the EWMA chart is developed in the multivariate case to control the average process with many interconnected quality characteristics. The development of the EWMA chart in the multivariate case was carried out by Lowry et al. in 1992 and is called the multivariate exponentially weighted moving average (MEWMA) chart [3].

The basic assumption in a control chart is that the process values should be normally distributed and independent. However, in industrial production processes, data is often dynamic so this assumption is not always met [4]. Repeated use of the same machine in industry leads to the possibility of autocorrelation in the observation data. This autocorrelation results in parameter bias in the process standard deviation, which results in tighter control limits on the control chart. This can decrease the average run length (ARL) value and increase the number of false alarms. To overcome this problem, the time series model approach is used to obtain accurate and effective control charts. In controlling the production process, the combination of the MEWMA chart and the time series model is an appropriate step if the basic assumptions are not met [5].

Previously, the quality control of WCS with time series-based multivariate charts was carried out by Taufiqi [6] in 2018 at the Tjoekir Jombang Sugar Factory. However, the quality characteristics used in this study were only 2 variables, including color of sugar solution (ICUMSA) and grain size. In this study, quality monitoring was carried out with the T^2 Hotelling control chart to monitor the process average and the M control chart to monitor the process variability based on the VAR(3) model. The results show that the process has not been statistically controlled, possibly due to machine damage, improper machine settings, and other factors.

Quality control with the Multivariate Exponentially Weighted Moving Average (MEWMA) control chart has been carried out by Wahyuni, et al. [7] in 2019 on the clean water production process at PDAM Tirta Khatulistiwa. In the control, five quality characteristics were observed, including color, turbidity, temperature, Electrical Conductivity, and pH. This study used an Upper Control Limit (UCL) of 12.91. The results showed that during the clean water production process, all quality characteristics remained below the UCL, indicating that the process was running in a controlled and stable manner.

Quality control with MEWMA graphs has then been carried out by Wibawati and Fadhila [8] in 2023 on the production of hot glass, specifically on the characteristics of the left and right CD edge distortion which affects the presence or absence of distortion in the visual check results. The use of weight $\lambda = 0.4$ in phase 1 resulted in adequate statistical control. However, in phase 2, the MEWMA control diagram showed that the hot glass production process was still not in statistical control. Process capability analysis indicated a low level of precision and accuracy of the hot glass products, so it was concluded that the production process had not reached the desired level of capability.

The aim of this study is to conduct statistical quality control of WCS production at MSF Yogyakarta in order to maintain product stability and quality according to the standards. Quality control is carried out on four quality characteristics, including drying shrinkage (%), grain size (mm), polarization, and color of sugar solution (ICUMSA). The remainder of this paper is organized as follows: Section 2, describes the variable data and the steps used in the analysis. In this section also briefly reviews the correlation test, VARIMA model, MEWMA chart to monitor the process mean shift, and process capability analysis of the controlled MEWMA chart. Results and discussions are clearly outlined in Section 3. Finally, conclusions are drawn in the last section.

2. RESEARCH METHODS

2.1 Data

This research uses secondary data in the form of WCS quality characteristics of the 2022 milling period of MSF Yogyakarta from May 9 - July 16, 2022 with a total data of 182 individual observations or only 1 subgroup ($n = 1$). The WCS quality variables used in this study were taken from the Indonesian National Standard (Standard Nasional Indonesia, SNI) for WCS issued by the government through the National Standardization Agency [9].

Table 1. Research Variables

Variable	Variable Description	Variable Unit	SNI Spec.
X_1	Drying Shrinkage	%	Maximum 0.1
X_2	Grain Size	mm	0.8 – 1.2
X_3	Polarization	"Z"	Minimum 99.6
X_4	Color of Sugar Solution (ICUMSA)	IU	81 – 200

2.2 Stages of Analysis

The analysis steps used in this study are as follows.

Step 1. Description of WCS quality data using descriptive statistics

Step 2. Checking the correlation between variables with Bartlett's test.

In the multivariate case, Bartlett's test is used to test for correlation with H_0: $\mathbf{R} = \mathbf{I}$ vs H_1: $\mathbf{R} \neq \mathbf{I}$ (there is a correlation between variables). The test statistic χ^2 in **Equation (1)** is calculated based on the sample size of residual observations (n), the number of variables or quality characteristics (m), and the correlation matrix between variables $(\mathbf{R})$ [10]. Reject H_0 if the $\chi^2 \geq \chi^2_{\left(\alpha, \frac{1}{2}m\,(m-1)\right)}$ or if the p-value $\leq \alpha$ (0.05 significance).

$$\chi^2 = -\left(n - 1 - \frac{(2m+5)}{6}\right)\ln|\mathbf{R}| \tag{1}$$

Step 3. Establishment of VARIMA(p,d,q) time series model.

The general form of the VARIMA(p,d,q) model is expressed in **Equation (2)**

$$\begin{aligned}\boldsymbol{Y}_t &= \boldsymbol{\delta} + \boldsymbol{\Phi}_1\boldsymbol{Y}_{t-1} + \cdots + \boldsymbol{\Phi}_p\boldsymbol{Y}_{t-p} + \boldsymbol{\varepsilon}_t - \boldsymbol{\Theta}_1\boldsymbol{\varepsilon}_{t-1} - \cdots - \boldsymbol{\Theta}_q\boldsymbol{\varepsilon}_{t-q} \\ &= \boldsymbol{\delta} + \sum_{i=1}^{p}\boldsymbol{\Phi}_i\boldsymbol{Y}_{t-i} + \boldsymbol{\varepsilon}_t - \sum_{i=1}^{q}\boldsymbol{\Theta}_i\boldsymbol{\varepsilon}_{t-i}\end{aligned} \tag{2}$$

with $\boldsymbol{Y}_t$ is an $m \times 1$ vector of variable values at time t; $\boldsymbol{\delta}$ is an $m \times 1$ vector of constant coefficients; $\boldsymbol{\Phi}_p$ is an $m \times m$ matrix of p-th VAR parameters, $p = 1,2,\ldots$; $\boldsymbol{\Theta}_q$ is an $m \times m$ matrix of q-th VMA parameters, $q = 1,2,\ldots$; $\boldsymbol{\varepsilon}_t$ is an $m \times 1$ vector of error values at time t [11]. Common stages in time series analysis to find the best model include stationarity tests, optimal lag determination, diagnostic checking, and model validation. At this stage, the SAS program is used for the analysis process.

a. Stationarity test using Augmented Dickey-Fuller (ADF) test.

Stationarity in time series data indicates statistical balance with H_0: $\phi = 0$ vs H_1: $\phi \neq 0$ (data is stationary) [12]. Test statistics

$$t_\phi = \frac{\hat{\phi} - 1}{SE(\hat{\phi})} \tag{3}$$

is used, with MacKinnon's critical value to determine stationarity. the data is considered stationary, if the $t_\phi <$ MacKinnon's critical value or the p-value ≤ 0.05.

b. Differencing if the data is not stationary.

If not stationary, differencing can be performed to stationarize the time series data by calculating the difference between the observed values. Differencing can be repeated until the data reaches stationarity. In general, the d-th order differencing is defined as $\nabla_d y_t = y_t - y_{t-d}$ [11].

c. Identify the time series model and determine the optimum lag with the smallest AICc value.

The criteria for selecting the best VARIMA model is to see the smallest value of Akaike Information Criterion corrected (AICc). The AICc calculation formula is

$$\mathrm{AIC_c} = n \log|\hat{\boldsymbol{\Sigma}}_{p+q}| + \frac{n(nm+(p+q)m^2}{n-(p+q)m-m-1} \tag{4}$$

with n is the sample size of observations, m is the number of variables, p is the number of VAR orders, and q is the number of VMA orders [13].

d. Diagnostic checking with Portmanteau test and multivariate normal distribution assumption.

Diagnostic checking is performed on two assumptions on the residuals of the time series model. First, the white noise assumption test aims to ensure that the residuals between observations are not correlated. The test uses the multivariate Portmanteau test with Ljung-Box statistics in **Equation (5)** with $H_0\colon \boldsymbol{\rho}_1 = \cdots = \boldsymbol{\rho}_k = 0$ while $H_1\colon \boldsymbol{\rho}_i \neq 0$; $i \in \{1,\dots,k\}$ (white noise is not met) [14].

$$Q_m(k) = n^2 \sum_{j=1}^{k} \frac{1}{n-j} tr(\hat{\boldsymbol{\Gamma}}_j' \hat{\boldsymbol{\Gamma}}_0^{-1} \hat{\boldsymbol{\Gamma}}_j \hat{\boldsymbol{\Gamma}}_0^{-1}) \tag{5}$$

where n is the sample size of the residuals, m is the dimension of $\boldsymbol{Y}_t$, k is the number of lags used in the Portmanteau test, which also determines the degrees of freedom of the Chi-Square distribution, $tr(\mathbf{A})$ is the trace of matrix $\mathbf{A}$, which is the sum of the diagonal elements of $\mathbf{A}$, and $\hat{\boldsymbol{\Gamma}}_j$ is the estimated covariance matrix of the residuals at the j-th lag, while $\hat{\boldsymbol{\Gamma}}_0$ is the estimated covariance matrix of the residuals at lag 0 or the initial period. $Q_m(k)$ follows the Chi-Square distribution with $m^2(k-p-q)$ degrees of freedom. If $\chi^2 \leq \chi^2_{(\alpha;m^2(k-p-q))}$ or the p-value > 0.05), the white noise assumption is met.

Next, a multivariate normal distribution assumption test is performed to verify that the residuals are normally distributed. This test uses the Mahalanobis distance in **Equation (6)** and the Chi-Square value, and the hypothesis tested is the existence of a multivariate normal distribution in the residuals.

$$d_t^2 = (\boldsymbol{Y}_t - \overline{\boldsymbol{Y}})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{Y}_t - \overline{\boldsymbol{Y}}) \tag{6}$$

with $t = 1,2,\dots,n$, where $\boldsymbol{Y}_t$ is the vector of observations at time t, $\overline{\boldsymbol{Y}}$ is the mean vector, and $\boldsymbol{\Sigma}^{-1}$ is the inverse covariance matrix of $\boldsymbol{Y}$ [15]. H_0: residual data is multivariate normally distributed vs H_1: residual data is not multivariate normally distributed. Don't reject H_0 if $d_t^2 \leq \chi^2_{(\alpha,m)}$ is half (50%).

e. Validating the time series model using MAPE.

Time series models are validated using the Mean Absolute Percentage Error (MAPE). The MAPE equation (in **Equation (7)**) calculates the absolute percentage difference between the actual value y_t and the forecast value $\hat{y}_t$ for one quality characteristic variable, and then calculates the average percentage over all observations n. The results are then put into percentage form.

$$\mathrm{MAPE} = \frac{\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right|}{n} \times 100\% \tag{7}$$

The model that is considered the best and most suitable is the one with the smallest MAPE value and no more than 10%. The lower the MAPE value, the better the prediction quality of the model [16].

Step 4. Defining the residuals of the time series model as new observation data.

Step 5. Developing a MEWMA chart with the following steps (at this stage, a program with Python is used to simplify the calculations):

a. State the weight value λ for each quality characteristic.

In this study, there is no difference between each weight, so $\lambda_1 = \lambda_2 = \cdots = \lambda_m = \lambda$.

b. Calculating the MEWMA statistic $(\boldsymbol{Z_t})$ for each observation.

Multivariate Exponentially Weighted Moving Average (MEWMA) chart is used to monitor changes in the average production process with multivariate data. The MEWMA chart is developed from the EWMA statistic, is defined as

$$\boldsymbol{Z_t} = \boldsymbol{\Lambda X_t} + (\mathbf{I} - \boldsymbol{\Lambda})\boldsymbol{Z_{t-1}} \tag{8}$$

where $t = 1,2,\ldots,n$ is the number of observations, with $\boldsymbol{Z_0} = 0$, while $\boldsymbol{\lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_m)$ is a weight with a value of $0 < \lambda_i \leq 1, i = 1,2,\ldots,m$, and $\boldsymbol{X_t}$ is an $m \times n$ vector of quality characteristics observed in the t-th sample [1]. In this study, $\lambda_1 = \lambda_2 = \cdots = \lambda_m = \lambda$, so the Equation (8) becomes

$$\boldsymbol{Z_t} = \lambda \boldsymbol{X_t} + (1 - \lambda)\boldsymbol{Z_{t-1}}.$$

c. Calculate the covariance matrix and its inverse.

d. Draw the MEWMA chart with the observation sample point values (T_t^2) for each observation.

The MEWMA chart gives an out-of-control signal if the value of

$$T_t^2 = \boldsymbol{Z}_t' \boldsymbol{\Sigma}_{\boldsymbol{Z}_t}^{-1} \boldsymbol{Z}_t > h_4 \tag{9}$$

exceeds the upper control limit (h_4). The calculation of $\boldsymbol{Z}_t$, $\boldsymbol{\Sigma}_{\boldsymbol{Z}_t}$, and $\boldsymbol{T}_t^2$ is repeated for each observation data. The values of λ and h_4 with 4 quality characteristics were obtained from simulations conducted by Prabhu and Runger [1] and are presented in Table 2. The MEWMA chart is used to monitor the average production process by plotting the T_t^2 values on the chart.

Table 2. **Values of $\boldsymbol{\lambda}$ and $\boldsymbol{h_4}$ for $\boldsymbol{m} = 4$ MEWMA Chart**

m	λ	h_4
4	0.05	11.22
	0.10	12.73
	0.20	13.87
	0.30	14.34
	0.40	14.58
	0.50	14.71
	0.60	14.78
	0.80	14.85

e. Identify out-of-control sample observations and remove them.
f. Repeat steps (b-e) to get the MEWMA chart under control.

Step 6. Calculate the production process capability index value $(\hat{C}_p)$.

Process capability analysis is necessary to assess whether a process has successfully adhered to specifications or not. In process capability analysis, the terms upper specification limit (USL) and lower specification limit (LSL) are known. These limits can be seen in Table 1.

One method for assessing process capability involves calculating the process capability index (C_p), and a process is said to be capable if the value of $C_p > 1$. For the multivariate case, the estimated process capability index (C_p) can be calculated using the Equation (10),

$$\hat{C}_p = \frac{K}{\chi^2_{0.05;m}} \left(\frac{(n-1)m}{S} \right)^{\frac{1}{2}} \tag{10}$$

with $\chi^2_{0.05;m}$ is the critical value of the chi-square table with $\alpha = 0.05$ and m degrees of freedom due to the number of quality characteristic variables.

In the calculation of $\hat{C}_p$, K is the true process region calculated based on Equation (11)

$$K^2 = (\bar{\boldsymbol{X}} - \boldsymbol{\xi})'\boldsymbol{\Sigma}^{-1}(\bar{\boldsymbol{X}} - \boldsymbol{\xi}) \tag{11}$$

where $\boldsymbol{\Sigma}^{-1}$ as the inverse of the covariance matrix of the m quality characteristic variables, $\boldsymbol{\xi} = (\xi_1, \dots, \xi_m)$, and $\xi_j, j = 1, \dots, m$ is the target value for each variable based on **Equation (12)**.

$$\xi_j = \frac{1}{2}(USL_j + LSL_j) \tag{12}$$

Meanwhile, S is calculated using **Equation (13)**

$$S = \sum_{t=1}^{n}(\boldsymbol{X}_t - \bar{\boldsymbol{X}})'\mathbf{A}^{-1}(\boldsymbol{X}_t - \bar{\boldsymbol{X}}) \tag{13}$$

with matrix $\mathbf{A} = \mathbf{X}'\mathbf{X}$ [17].

3. RESULTS AND DISCUSSION

3.1 Descriptive Statistics

Descriptive analysis of the four variables of white crystal sugar quality characteristics aims to determine the general description of data characteristics that can be seen in **Table 3**. In addition to looking at the descriptive statistics of each variable of WCS quality characteristics, the influence of external factors, namely rainfall, will also be considered, which can be seen in **Figure 1**.

Table 3. Descriptive Statistics of WCS Quality Characteristics

Quality Characteristics	Ave.	Var.	Min.	Max.	SNI Spec.
Drying Shrinkage (%)	0.02245	0.000051	0.016	0.043	Maks. 0.1
Grain Size (mm)	0.80786	0.000092	0.79	0.84	0.8 – 1.2
Polarization (“Z”)	99.646	0.00283	99.6	99.8	Min. 99.6
ICUMSA (IU)	174.68	103.68	158	200	81 – 200

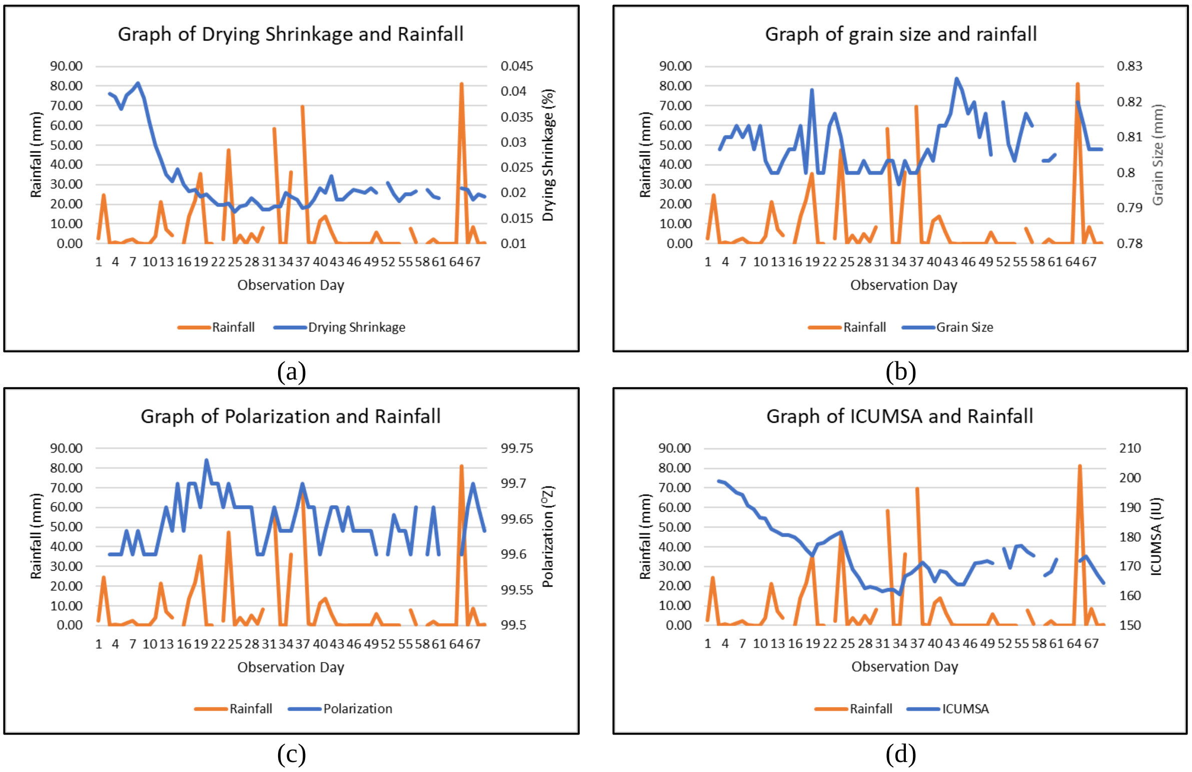

Figure 1. Graphs of Rainfall with (a) Drying Shrinkage, (b) grain size, (c) Polarization, and (d) ICUMSA

Low rainfall in Yogyakarta from May 9 to July 16, 2022 affected the dry season and WCS production. Average rainfall was only 8.8 mm, improving cane yield and sugar quality. Drying losses or moisture content remained constant at 0.02%, despite some high preliminary data. This could be due to the previous high rainfall or cane from outside Yogyakarta. The lower the rainfall, the lower the moisture content, benefiting sugar storage in warehouses.

In **Figure 1** (b) and **Figure 1** (c), it can be seen that grain size and polarization are high when rainfall is close to zero. This means that when rainfall is low, the sugar content in sugarcane is high, resulting in high sucrose content and grain size. This condition is inversely proportional to the drying shrinkage in **Figure 1** (a), where drying shrinkage is low when rainfall is low. Meanwhile, **Figure 1** (d) is similar to **Figure 1** (a), in that there is an increase in ICUMSA at the beginning of the observation, then it appears constant on the following days. This indicates that the lower the rainfall, the lower the moisture content and ICUMSA, but the higher the sucrose content or polarization. This indicates that rainfall patterns can affect the quality of the WCS produced.

3.2 VARIMA Modelling

Before performing VARIMA modeling, the correlation test between variables is first tested using the Bartlett test. Based on the variable data, the correlation matrix $\mathbf{R}$ for the WCS quality characteristics was obtained and the determinant value of the correlation matrix, $|\mathbf{R}|$ was 0.2732. For significance testing, the value of $\chi^2_{(0.05;6)}$ in the chi-square table was obtained as 12.5916. Meanwhile, the value of χ^2 based on **Equation (1)** is 232.0365. Thus, the value of χ^2 is greater than $\chi^2_{(0.05;6)}$, it can be concluded that the WCS quality characteristic variables are correlated, so the null hypothesis (H_0: $\mathbf{R} = \mathbf{I}$) is rejected.

3.2.1 Data Stationarity

Data stationarity can be tested with the Augmented Dickey-Fuller (ADF) test using the original observation data and the results are shown in **Table 4**. If the ADF t-statistic $\left(t_\phi\right) <$ the MacKinnon critical value of 5% and the p-value < 0.05, then H_0 is rejected and the data is said to be stationary.

Table 4. ADF Test Results WCS Quality Characteristics Data

Quality Characteristics	t-statistic ADF	t-statistic MacKinnon	p-value
Drying Shrinkage (%)	-2.5166	-2.8775	0.1132
Grain Size (mm)	-3.3801	-2.8777	0.0130
Polarization ("Z")	-2.8569	-2.8779	0.0526
ICUMSA (IU)	-2.3799	-2.8776	0.1489

Based on **Table 4**, there are three WCS quality characteristics that do not reject the null hypothesis (H_0) or are not stationary based on the original data, so it is necessary to perform first differencing for all variables (subject to equal treatment). The results of this first differencing are shown in **Table 5**.

Table 5. ADF Test Results After First Differencing

Quality Characteristics	t-statistic ADF	t-statistic MacKinnon	p-value
Drying Shrinkage (%)	-13.8454	-2.8775	0.0000
Grain Size (mm)	-6.4293	-2.8785	0.0000
Polarization ("Z")	-10.4603	-2.8779	0.0000
ICUMSA (IU)	-8.0187	-2.8776	0.0000

Based on **Table 5**, the WCS quality characteristics after first differencing show that the four variables have an ADF t-statistic $\left(t_\phi\right) <$ the MacKinnon 5% t-statistic value and p-value < 0.05. Therefore, it can be concluded that the WCS quality characteristics data after differencing has become stationary.

3.2.2 VARIMA Model Identification

The cross-correlation matrix function is used to identify the VMA model components, while the partial autoregression matrix function is used to identify the VAR model components in the VARIMA model order.

Analysis of both functions helps in determining the optimal VARIMA model order for WCS quality characteristics data. To be clearer and easier about the most suitable VARIMA model order, we can see the smallest Akaike Information Criterion corrected (AICc) in **Equation (4)** value summarized in **Table 6**.

Based on **Table 6**, it can be seen that the model that has the smallest AICc value (bold) is the VARIMA (1,1,2) model, so the residuals from this model will be defined as new observation data and a MEWMA chart will be formed. But before that, the residual assumption test will be carried out first.

Table 6. AICc Values

Lag	MA 0	MA 1	MA 2	MA 3	MA 4	…	MA 11
AR 0	-24.9444	-25.5915	-25.5584	-25.6020	-25.5471	…	-24.9144
AR 1	-25.5759	-25.6555	**-25.8086**	-25.7794	-25.6621	…	-24.8198
AR 2	-25.6889	-25.6929	-25.7317	-25.6744	-25.5709	…	-24.6867
AR 3	-25.7688	-25.7239	-25.6428	-25.4997	-25.4256	…	-24.3514
AR 4	-25.7884	-25.7335	-25.6128	-25.5003	-25.3303	…	-24.0813
AR 5	-25.7119	-25.6484	-25.4998	-25.4033	-25.2113	…	-23.8711
AR 6	-25.5942	-25.5225	-25.4040	-25.2933	-25.0627	…	-23.5263
AR 7	-25.4290	-25.3699	-25.2551	-25.1438	-24.9188	…	-23.1634
AR 8	-25.3260	-25.3524	-25.2423	-25.1010	-24.8582	…	-22.6854
AR 9	-25.2032	-25.2793	-25.0942	-24.8796	-24.6628	…	-22.1906

3.2.3 Diagnostic Checking

At this stage, assumptions are tested on the residuals of the time series model formed. The white noise assumption test is used to see if the model residuals are autocorrelated. The white noise assumption is met if the value of $\chi^2 \leq \chi^2_{\left(\alpha;m^2(k-p-q)\right)}$ or the p-value > 0.05. The results of this test are shown in **Table 7**.

Table 7. Multivariate Portmanteau Test Results VARIMA(1,1,2) Model

Up To Lag	df	χ^2	$\chi^2_{(0,05;m^2(k-p-q))}$	p-value
4	16	23.52	26.2962	0.1006
5	32	41.24	46.1943	0.1270
6	48	55.06	65.1708	0.2249
7	64	78.78	83.6753	0.1011
8	80	101.36	101.8795	0.0537
9	96	122.03	119.8709	0.0376
10	112	131.31	137.7015	0.1027
11	128	143.56	155.4047	0.1644
12	144	151.48	173.0041	0.3183

Based on **Table 7**, the white noise assumption is fulfilled because most of the $\chi^2 < \chi^2_{\left(0.05;m^2(k-p-q)\right)}$ and the p-value > 0.05 for most lags. The next assumption is multinormality, seen by calculating the value of the d_t^2 statistic in **Equation (6)** for $t = 1,2,\ldots,180$. Based on the Chi-Square table, the value of $\chi^2_{(0.05;4)}$ is 9.4877. Next, it is calculated how many d_t^2 values are less than or greater than this value. The calculation results show that there are 166 d_t^2 data that are less than 9.4877 and 14 d_t^2 data that are greater than that value. Thus, more than 50% of the d_t^2 values are less than $\chi^2_{(0.05;4)}$, so it can be concluded that we do not reject H_0 or the data is multivariate normally distributed.

3.2.4 VARIMA Model Validation

The VARIMA(1,1,2) model was validated using Mean Absolute Percentage Error (MAPE) to evaluate the accuracy of the prediction. The MAPE calculation results showed that all WCS quality characteristic variables had error values below 10%, indicating highly accurate predictions (see **Table 8**). Thus, the VARIMA(1,1,2) model can be considered as the best model for WCS quality control. The residuals of the model will be considered as new observation data and used in the process of forming the MEWMA controller chart.

Table 8. MAPE Values

Quality Characteristics	MAPE Value (%)
Drying Shrinkage (%)	6.2841
Grain Size (mm)	0.8071
Polarization (“Z”)	0.0391
ICUMSA (IU)	1.1071

3.3 MEWMA Chart

Before creating the MEWMA chart, there are several assumptions that need to be met. First, the assumptions of independence between observations and multivariate normal distribution have been verified at the diagnostic checking stage of VARIMA time series modelling. The results show that the data has met both assumptions. Furthermore, the requirement for constructing multivariate control charts is the presence of correlation between variables, which has also been confirmed in the previous section.

The first step in constructing the MEWMA chart is to determine the weight value (λ) for each quality characteristic. In this study, there is no different weight for each characteristic, so the value of $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda$. The weight values and upper control limit (h_4) for Average Run Length $(ARL_0) = 200$ have been tested through simulations conducted by Prabhu and Runger **[1].** In summary, the results of the MEWMA chart formation experiments with various weight values λ and control limit values h_4 are summarized in **Table 9**.

Table 9. Summary of MEWMA Chart Experiment Results

λ	h_4	$\max(T_t^2)$	**Criterion 1**	**Criterion 2**	**Criterion 3**
			Number of Out of Control	$\lvert\max(T_t^2) - h_4\rvert$	$\overline{\lvert T_t^2 - h_4\rvert}$
0.05	11.22	28.3827	2	17.1627	**7.8798**
0.10	12.73	28.3827	2	15.6527	8.9739
0.20	13.87	28.3827	1	14.5127	9.9511
0.30	14.34	28.3827	1	14.0427	10.4256
0.40	14.58	28.3827	1	13.8027	10.6743
0.50	14.71	28.3827	1	13.6727	10.8047
0.60	14.78	28.3827	2	13.6027	10.8820
0.80	14.85	28.3827	**4**	**13.5327**	10.9776

The selection of the optimum weight value (λ) for the MEWMA chart can be done with the following three criteria.

1. Criterion 1 is by looking at the largest number of out of control observations on each MEWMA chart with different weights λ **[18]**.
2. Criterion 2 is by calculating the minimum absolute value difference between highest T_t^2 value and the control limit h_4 **[19]**.
3. Criterion 3 is to calculate the minimum average value of the distance between the T_t^2 value of each observation sample and the control limit h_4 for each weight λ **[20]**.

Based on **Table 9**, the weight λ of 0.8 fulfills two criteria, namely criteria 1 and 2. Meanwhile, the weight λ of 0.05 only fulfills one criterion, namely criterion 3. Therefore, the weight λ of 0.8 is chosen as the optimum weight value. The MEWMA chart with the weight λ is the most sensitive in detecting the average shift of WCS production process in MSF Yogyakarta. The MEWMA chart with $\lambda = 0{,}8$ is presented in **Figure 2**.

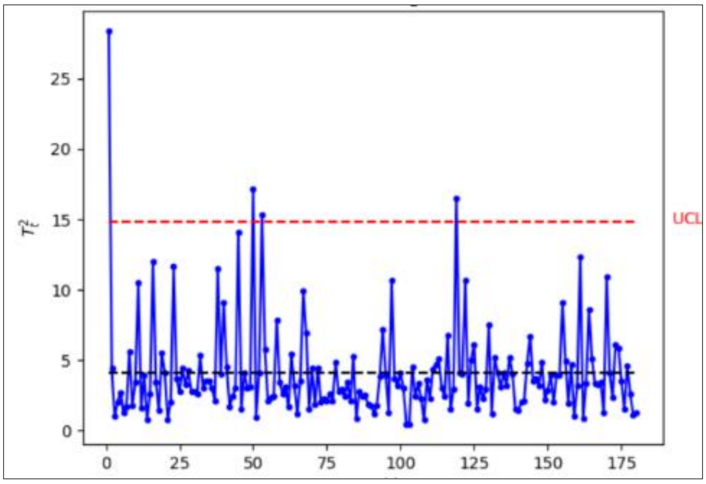

Figure 2. MEWMA Chart with $\lambda = 0.8$ and $h_4 = 14.85$

Based on **Figure 2**, there are four observation sample points that are out of control, which are the 1^{st}, 50^{th}, 53^{rd}, and 119^{th} samples. The existence of out of control, especially at the beginning of the milling season at MSF Yogyakarta can be caused by several factors. First, the use of machines after stopping operation for ±5 months can affect the performance and stability of the production process. In addition, climatic factors such as rainfall can also affect the quality of WCS produced.

In addition to machine and climate factors, the raw material factor of sugarcane also plays an important role in the quality of WCS. Sugarcane that is not clean and fresh can affect the production process and final product quality. In addition, the delay in milling carried out by MSF due to limited raw materials results in shrinkage of cane weight and an increase in reducing sugar content and can affect the quality of the resulting WCS. Transportation of sugarcane from outside Yogyakarta can also cause sugarcane to be less fresh. These factors cause the possibility of a shift in the average WCS production process and the occurrence of out of control at MSF Yogyakarta.

3.4 Capability Analysis

In addition to monitoring the production process, in quality control it is also necessary to analyze the process capability which can be measured by the process capability index value ($\hat{C}_p$) to assess the extent to which the process meets the specification limits. The data used is the original controlled data after eliminating uncontrolled data, with a total of 176 observations after eliminating 4 observations that are considered out of control. The specification limits used can be seen in **Table 1**. With **Equation (10)**, the estimated value of the process capability index ($\hat{C}_p$) is obtained as a result of 40.4005. This value is greater than 1 and meets the requirements for a process to be declared capable ($\hat{C}_p > 1$). Thus, it can be concluded that the WCS production process at MSF Yogyakarta meets the process capability and is declared a capable process.

4. CONCLUSIONS

The application of the MEWMA control chart with 8 weight values of λ showed statistically uncontrollable results in detecting the average shift of the WCS production process at Maduksimo Sugar Factory Yogyakarta in the milled period under study. Of all the λ weights tested the λ weight of 0.8 was chosen as the optimum weight because it fulfils two of the three criteria for determining the optimum weight. The MEWMA chart based on the original data with a λ weight of 0.8 shows that there are 4 observation data that are considered out of control on 11 May 2022, 27 May 2022, 28 May 2022, and 19 June 2022. However, in the process capability analysis using the controlled MEWMA chart after removing the out-of-control samples with a weighting λ of 0.8, the process capability index value $\hat{C}_p$ is 40.4005, which is greater than 1. Thus, the WCS production process at Maduksimo Sugar Factory Yogyakarta in the milling period studied can be declared capable or good.

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