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PROFITABILITY CALCULATION AND ANALYSIS FOR INTEREST RATE SWAP USING THE HULL WHITE MODEL

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ABSTRACT

The London Inter-Bank Offered Rate (LIBOR) volatility had resulted in higher interest rate risks faced by many big companies and financial institutions whose assets depend on the interest rate. Eventually, there was an appearance of the new financial product development that can be used for hedging, such as interest rate swap, one of the most popular methods, utilized by most financial institutions and big companies which use LIBOR as their benchmark. In fact, it is not uncommon for numerous companies to gain negative return from this swap transaction. Therefore, in this paper, we used the Hull-White model to predict the LIBOR rate, since this model has a decent level of accuracy, calculated using Root Mean Squared Error (RMSE) method. Furthermore, the estimation of LIBOR rate was used to calculate the net value of interest rate swap transaction, in three scenarios, using, respectively, the minimum value, the average value, and the maximum value of the LIBOR's estimation results to provide an analysis of potential P/L (Profit & Loss) exposure due to the realization of interest rate swaps.

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1. INTRODUCTION

The significant increase, in both interest rate levels and volatility, since the 1970s has resulted in substantially high interest rate risk faced by market participants such as financial institutions and large corporations [1]. The consequences that tend to be serious for market participants is the asset-liability mismatch, in which the duration of assets does not match the duration of the liabilities. As an example, most market participants utilizing the short-term floating-rate liabilities for financing the long-term fixed-rate assets, in result, when the relentless upward spike in the interest rate occur, they will lose. This is in line with the given fixed-rate of return on their assets, the short-term interest costs that they must pay rise with the market interest rates. Otherwise, market participants will gain the positive return when the short-term market interest rate has fallen sharply. In the past few years, many theories have been made to determine the new financial products in order to reduce the up-spike in the rate that tightened company's financial conditions – such as interest rate future [2], interest rate option, and interest rate swap [3]. In particular, interest rate swap is superior compared to 10-year treasury rate to determine mortgage interest rates, as shown in [4].

An interest rate swap is a contractual arrangement between two parties, often referred to as "counterparties". In an interest rate swap, one company agrees to pay to another company cash flows equal to interest at a predetermined fixed-rate on a notional amount for a predetermined number of years. In return, it receives interest at a floating-rate on the same notional amount for the same period of time from the other company [5]. Furthermore, the one who pay the fixed-rate is called a payer and the one who pay the floating-rate is called a receiver. The floating-rate in most interest rate swap agreements is the London Interbank Offered (LIBOR). Since 1980s, interest rate swap has become one of the most popular financial product conducted by many market participants to hedge against interest rate risk [6], [7]. One of the growing popularity reasons of an interest rate swap are they are easy to use. Unfortunately, during its implementation, interest rate swap not only used as a hedging method but also as a speculative motivation. When the average interest rate that incurred during the swap transactions corresponds to the company estimation, then the swap transaction can be said to be a hedging tool. Conversely, when the interest rate level that incurred during the swap transaction diverges from company's estimation, then the average diverge must be on the same level as the company's speculation.

Therefore, this paper will describe the P/L of an interest rate swap transaction. In addition, a model to predict the interest rate volatility must be created. Nowadays, there are lot of models that can be used for predicting the interest rate volatility. A pioneering paper about a short-rate model was the Vasicek model in 1977, which was the first paper that use a mean reversion characteristic [8]. A disadvantage of the model is that it is not able to reproduce the initial interest rate curve, caused by the fact that the model only uses constant parameters. The Hull-White model introduced in 1990 as an extension of the Vasicek model that allow to choose the parameters in such a way that the initial interest rate curve is reproduced [9], [10]. Before the 2007-2008 financial crisis [11], negative interest rates were not desirable. Therefore, the Cox-Ingersol-Ross (CIR) model was introduced as an extension of the Vasicek model, a model that does not allow negative interest rates [8]. However, after the financial crisis, many countries have lowered the interest rate as market stimulate to increase the economic growth and inflation. Until at the end of 2009, Central Bank of Sweden implement the negative interest rate for the first time [11]. Henceforth, the disadvantage of Hull-White interest rate models becomes their advantage; the possibility of negative rates.

For this reason, this paper used the Hull-White model to predict the interest rate volatility [12], instead of the classical martingale modelling approach. On its implementation, the Hull-White model required two parameters such as volatility level and mean reversion by using the Maximum Likelihood Estimation (MLE). Afterwards, we applied it to the LIBOR (London Interbank Offered Rate) historical data as our reference interest rate [13] to get the parameters, as LIBOR was actually risk-laden [14]. To evaluate our Hull-White model, we used Root Mean Squared Error (RMSE) method for calculating the error value, by comparing our LIBOR estimation using the Hull-White model with LIBOR's original rate. Therefore, we used the one-year LIBOR data, starting from Jan 2011 to Jan 2012 to predict the three-years LIBOR rate, starting from Jan 2013 to Jan 2016. Then, we also used the two-years LIBOR data, starting from Jan 2011 to Jan 2013 to predict the six-years LIBOR rate, starting from Jan 2014 to Jan 2020. After our Hull-White model is confirmed to have a highest level of accuracy to produce the LIBOR interest rate, the net value of swap transaction can therefore be calculated. Hence, readers can compare how much the profit or loss which they can get after using this swap transaction. The period of the swap transaction used in this paper is thirty years, because swap transactions usually have a quite a long period.

2. RESEARCH METHODS

The data used in this paper is LIBOR daily historical data from January 2011 to January 2024 which calculated by Ice Benchmark Administration (IBA) [15]. Then, we started to find the Hull-White's parameters using the MLE method. After the formula of these parameters are successfully constructed, we do find the value of each parameters using the historical data of LIBOR and calculated using Excel. Furthermore, if we already have the value of Hull-White parameters, we could create the estimation of LIBOR rate in thirty years later using the Hull-White model and simulated using Python software. In addition, before we use our Hull-White model to predict the LIBOR rate, we also examine that model by comparing our estimation result with the original value of the LIBOR rate, using the RMSE method. After our Hull-White model is confirmed to have a highest level of accuracy to produce the LIBOR interest rate, the net value of swap transaction can therefore be calculated. Thus, this paper will present the net value into three cases: minimum, average, and maximum value of the LIBOR rate estimation, so we can conclude the P/L of the swap transactions.

2.1. Interest Rate Products

This section describes the different interest rate products that are used in this paper.

2.1.1 Short Rate

Short-rate $r(t)$ is the interest rate applying for a very short period of time [5]. Short-rate has a movement that is not completely random, but towards a certain level. When rates are high, the economy tends to slow down and there is low demand for funds from borrowers. As a result, rates decline. When rates are low, there tends to be a high demand for funds on the part of borrowers and rates tend to rise. This phenomenon is referred to as mean reversion, as shown in **Figure 1**, i.e., the tendency of a market variable (such as an interest rate) to revert to some long-run average level.

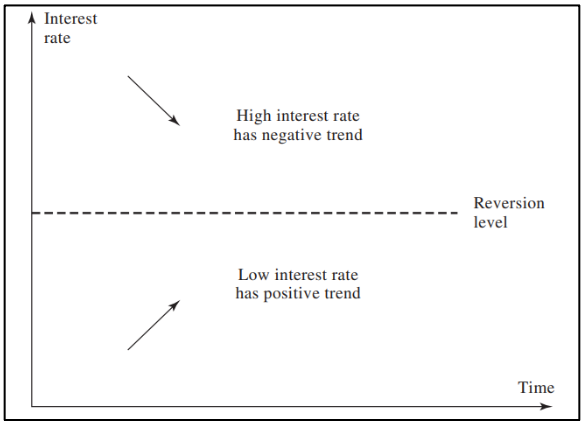

Figure 1. Mean Reversion [5]

We will discuss about short-rate on section 2.3.

2.1.2 Interest Rate Swap

An interest rate swap is a contractual arrangement between two parties, often referred to as "counterparties". In an interest rate swap, one company agrees to pay to another company cash flows equal to interest at a predetermined fixed-rate on a notional amount as the baseline value and is fixed for a predetermined number of years. In return, it receives interest at a floating-rate on the same notional amount for the same period from the other company. Furthermore, the one who pay the fixed-rate is called a payer and the one who pay the floating-rate is called a receiver. The floating-rate in most interest rate swap agreements is the LIBOR. Usually, the swap contract has a quite long period. Furthermore, in an interest rate swap, as in all economic transactions, it is presumed that both parties obtain economic benefits. The economic benefits in an interest rate swap are a result of the principle of comparative advantage, in line with the zero-sum concept of an interest rate swap where the benefits obtained by one party is come from losses suffered by another party, and vice versa. As we mentioned before, during its implementation, interest rate swap not only used as a hedging method but also as a speculative motivation. When the average interest rate that incurred during the swap transactions corresponds to the company estimation, then the swap transaction can be said to be a hedging tool. Conversely, when the interest rate level that incurred during the swap transaction diverges from company's estimation, then the average diverge must be on the same level as company's speculation. We will provide more details of the interest rate swap concept below.

Before Entering the Swap Transactions

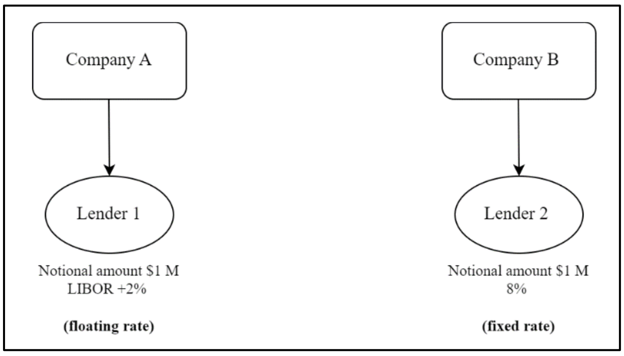

Figure 2. Cashflow for Company A and Company B before the swap transactions

From Figure 2, Companies A and B use the same notional amount, $1 million, but Company A transacts with floating-rate which follows LIBOR, while company B uses fixed-rate, 8%. Assume these two companies make two payments at the LIBOR rate on the first and second payments at 5% and 4%.

Table 1. Profitability Comparison Before the Swap Transactions

	Company A	Company B
First Payment	Payment to Lender 1: - LIBOR interest rate 5% + 2% = 7%, - Notional amount $1.000,000, - Total $70,000.	Payment to Lender 2: - Fixed-rate 8%, - Notional amount $1.000,000, - Total $80,000.
Second Payment	Payment to Lender 1: - LIBOR interest rate 4% + 2% = 6%, - Notional amount $1.000,000, Total $60,000.	Payment to Lender 2: - Fixed-rates 8%, - Notional amount $1.000,000, Total $80,000.
Total Payment	$70,000 + $60,000 = $130,000.	$80,000 + $80,000 = $160,000.

In Table 1, Companies A and B pay their loan with a different amount. Since Company A is worried that LIBOR rate may rise, it finds Company B that agrees to pay Company A the LIBOR annual rate. From the Company B viewpoint, even though the LIBOR rate is quite volatile, they might have a chance to pay the loan with lower amount as company A has a lower total payment compared with company B's. Therefore, these company agreed to perform the swap contract.

Swap Transaction: Declining LIBOR rate

Assume that the two parties enter into an interest swap agreement in which Company B will make yearly payments to Company A of LIBOR +1% on the notional amount of $1 million for 2 years. At the same time, Company A will make yearly payments to Company B of 7% on the notional amount of $1.000,000 for 2 years. This is standard interest rate swap where the notional amount of $1 million remains the same. Assume that in the following years, the LIBOR rate drop to 4%. In this case, Company B will receive a fixed payment of 7%. However, Company A will receive the new LIBOR +1%, i.e., 5% on the notional amount.

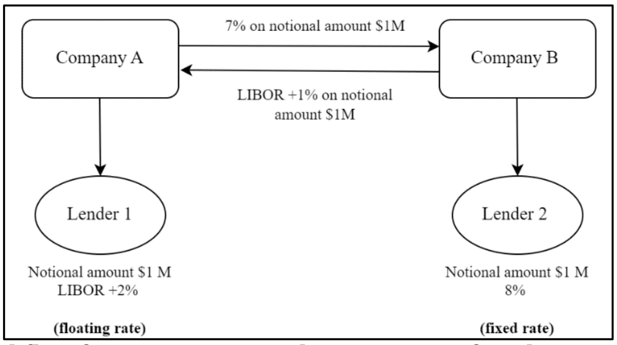

Figure 3. Cashflow for Company A and Company B after the swap transactions

Table 2. Profitability Comparison After the Swap Transactions in Declining LIBOR Rate Scenario

	Company A	Company B
First Payment	- Payment to Lender 1 $70,000, - Payment to Company B using the fixed-rates 7%, totaling $70,000, - Receive Company B's payment using the LIBOR rate 5% + 1% = 6%, totaling $60,000, - Total payment Company A: $70,000 + $70,000 - $60,000 = $80,000.	- Payment to Lender 2 $80,000, - Payment to Company A using the LIBOR rate 5% + 1% = 6%, totaling $60,000, - Receive Company A's payment using the fixed-rate 7%, totaling $70,000, - Total payment Company B: $80,000 + $60,000 - $70,000 = $70,000.
Second Payment	- Payment to Lender 1 $60,000, - Payment to Company B using the fixed-rates 7%, totaling $70,000, - Receive Company B's payment using the LIBOR rate 4% + 1% = 5%, totaling $50,000, - Total payment Company A: $60,000 + $70,000 - $50,000 = $80,000.	- Payment to Lender 2 $80,000, - Payment to Company A using the LIBOR rate 4% + 1% = 5%, totaling $50,000, - Receive Company A's payment using the fixed-rate 7%, totaling $70,000, - Total payment Company B: $80,000 + $50,000 - $70,000 = $60,000.
Total Payment	$80,000 + $80,000 = $160,000.	$70,000 + $60,000 = $130,000.

This way, both companies could achieve their goals; Company A has a stable amount in each yearly payment, i.e., $80,000 while Company B has a lower payment compared to Company A. Thus, the question is, if the LIBOR rate increasing during the second year, is Company B's total payment still lower than Company A's?

Swap Transaction: Increasing LIBOR rate

Assume that in the following years, the LIBOR rate rise to 6%. In this case, Company B will receive a fixed payment of 7%. However, Company A will receive the new LIBOR +1%, i.e., 7% on the notional amount.

Table 3. Profitability Comparison After the Swap Transactions in Increasing LIBOR Rate Scenario

	Company A	Company B
First Payment	- Payment to Lender 1 $70,000, - Payment to Company B using the fixed-rates 7%, totaling $70,000, - Receive Company B's payment using the LIBOR rate 5% + 1% = 6%, totaling $60,000, - Total payment Company A: $70,000 + $70,000 - $60,000 = $80,000.	- Payment to Lender 2 $80,000, - Payment to Company A using the LIBOR rate 5% + 1% = 6%, totaling $60,000, - Receive Company A's payment using the fixed-rate 7%, totaling $70,000, - Total payment Company B: $80,000 + $60,000 - $70,000 = $70,000.
Second Payment	- Payment to Lender 1 $80,000, - Payment to Company B using the fixed-rates 7%, totaling $70,000, - Receive Company B's payment using the LIBOR rate 6% + 1% = 7%, totaling $70,000, - Total payment Company A: $80,000 + $70,000 - $70,000 = $80,000.	- Payment to Lender 2 $80,000, - Payment to Company A using the LIBOR rate 6% + 1% = 7%, totaling $70,000, - Receive Company A's payment using the fixed-rate 7%, totaling $70,000, - Total payment Company B: $80,000 + $70,000 - $70,000 = $80,000.
Total Payment	$80,000 + $80,000 = $160,000.	$70,000 + $80,000 = $150,000.

This way, both companies still achieve their goals; Company A has a stable amount in each yearly payment, i.e., $80,000 while Company B still has a lower payment compared to Company A's, even the LIBOR rate is increasing in the second year. This indicates that payment with LIBOR rate that fluctuated according to their volatility, not necessarily causing losses for its party. Therefore, on the next section, we explain how LIBOR impact their counterparties on the swap contract, including their P/L.

2.2. Net Present Value (NPV)

In its application, NPV of the cash flow of each party that agreed to a swap transaction should be the same. Assume that $\boldsymbol{R}$ is defined as a fixed-rate, $\boldsymbol{f}_{[t_{k-1},t_k]}$ is defined as the forward-rate, an annual interest rate charged at money borrowed (at $\boldsymbol{t_{k-1}}$) and paid at $\boldsymbol{t_k}$, with $\boldsymbol{t_k}$ is defined as a positive integer and $\boldsymbol{Q_k}$ is defined as the notional amount paid at time $\boldsymbol{k}$, where $\boldsymbol{k}$ represents a positive integer [16], [17]. Thus, the value paid by payer at $\boldsymbol{t_k}$ is

$$\sum_{k=1}^{n} Q_k \cdot R \cdot P_{t_k}, \tag{1}$$

whereas $\boldsymbol{P_t}$ is defined as present value of a payment at time $\boldsymbol{t}$ while the value paid by receiver is

$$\sum_{k=1}^{n} Q_k \cdot f_{[t_{k-1},t_k]} \cdot P_{t_k}. \tag{2}$$

Since Equation (1) and Equation (2) are equal or should be in the same value, then the payer net value can be found by subtracting Equation (2) with Equation (1). Conversely, the receiver net value can be found by subtracting Equation (1) with Equation (2).

$$\sum_{k=1}^{n} Q_k \cdot R \cdot P_{tk} = \sum_{k=1}^{n} Q_k \cdot f_{[t_{k-1},t_k]} \cdot P_{t_k},$$

$$R = \frac{\sum_{k=1}^{n} Q_k \cdot f_{[t_{k-1},t_k]} \cdot P_{t_k}}{\sum_{k=1}^{n} Q_k \cdot P_{t_k}},$$

$$= \frac{\sum_{k=1}^{n} Q_k \left(P_{t_{k-1}} - P_{t_k}\right)}{\sum_{k=1}^{n} Q_k \cdot P_{t_k}}. \tag{3}$$

As we mentioned before, the notional amount is always on the level over the swap contract period. Thus, we can assume that $\boldsymbol{Q_k} = \boldsymbol{Q}$ for every positive integer $\boldsymbol{k}$. Therefore, we can write Equation (3) as below

$$R = \frac{\sum_{k=1}^{n} Q \left(P_{t_{k-1}} - P_{t_k}\right)}{\sum_{k=1}^{n} Q \cdot P_{t_k}},$$

$$= \frac{P_{t_0} - P_{t_n}}{\sum_{k=1}^{n} P_{t_k}}. \tag{4}$$

2.3. Hull-White Model

The one-factor Hull-White model is a short-rate model, which is driven by a mean-reverting process. The dynamics of the Hull-white model are given by [3]

$$dr(t) = [\theta(t) - a \cdot r(t)]dt + \sigma \cdot dW(t). \tag{5}$$

with $\theta(t)$ is defined as the long-run mean to which the interest rate, $r(t)$, reverts, a as constant mean reversion parameter and $a > 0$, σ as the volatility of $r(t)$, and $W(t)$ as Wiener process. The drift factor of the Hull-White model is $\theta(t) - a \cdot r(t)$. Therefore, when the value of $\theta(t)$ is higher than $a \cdot r(t)$, then, the drift factor becomes positive and interest rates tend to move upwards towards the equilibrium point. Conversely, when the value of $\theta(t)$ is lower than $a \cdot r(t)$, then, the drift factor becomes negative and interest rates tend to move downwards towards the equilibrium point. Below is the solution of the Hull-White model

$$r(t) = r(0)e^{-at} + e^{-at}\int_0^t \theta(t)e^{au}du + \sigma e^{-at}\int_0^t e^{au}\sigma dW(u). \tag{6}$$

Since the increments of a Brownian motion have a normal distribution, we find that the short-rate, $r(t)$, has a normal distribution with mean

$$E[r_{t+1}|r_t] = r_t e^{-a\Delta t} + \frac{\theta}{a}\left(1 - e^{-a\Delta t}\right). \tag{7}$$

Then, using the Itô's isometry, the variance for $r(t)$ is given by

$$\mathrm{Var}[r_{t+1}|r_t] = \frac{\sigma^2}{2a}\left(1 - e^{-2a\Delta t}\right). \tag{8}$$

2.4. Maximum Likelihood Estimator (MLE) of the Hull-White Model

To get the Hull-White parameters $\boldsymbol{a}$ and $\boldsymbol{\sigma^2}$, the MLE method is developed in this section. The MLE method is a general method for estimating the unknown parameters in any probability distributions. Assume we have a sample $\boldsymbol{x}$ with $\boldsymbol{n}$ independent and identically distributed (i.i.d.) random variables, then the MLE maximizes the probability of obtaining the same sample once again. By $\boldsymbol{x} = \{\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}\}$ we denote our sample of size $\boldsymbol{n}$, and by $\boldsymbol{\mu} = \{\boldsymbol{\mu_1}, \boldsymbol{\mu_2}, \dots, \boldsymbol{\mu_m}\}$ also $\boldsymbol{\sigma} = \{\boldsymbol{\sigma_1}, \boldsymbol{\sigma_2}, \dots, \boldsymbol{\sigma_m}\}$, we denote the $\boldsymbol{m}$ different parameters in the probability density function (pdf) [19], [20]:

$$f(x_i; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2},$$

under the assumption that the random variables are i.i.d., the joint probability distribution is expressed as

$$\begin{aligned} f(x_1, x_2, \dots, x_n; \mu, \sigma^2) &= \prod_{i=1}^{n} f(x_i; \mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2} \\ &= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}. \end{aligned} \quad (9)$$

The objective is then to find the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\sigma^2}$ that maximizes the likelihood function defined in (9). The first step is to simplify the algorithm by taking the logarithm of the likelihood function and thus obtain the log-likelihood function which we denote by $\boldsymbol{\mathcal{L}(\mu, \sigma^2)}$. Since the logarithm is a monotonic function, the values that maximizes $\boldsymbol{\mathcal{L}(\mu, \sigma^2)}$ also maximizes the likelihood function. Thus, we define $\boldsymbol{\mathcal{L}(\mu, \sigma^2)}$ as

$$\begin{aligned} \ln \mathcal{L}(\mu, \sigma^2) &= \ln[f(x_1, x_2, \dots, x_n; \mu, \sigma^2)] = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \\ &= \frac{-n}{2}\ln(\sigma^2) - \frac{n}{2}\ln(2\pi) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2. \end{aligned} \quad (10)$$

By substituting **Equation (7)** and **Equation (8)** into $\boldsymbol{\mu}$ and $\boldsymbol{\sigma^2}$ in (10) consecutively, we obtain the parameter estimates of the Hull-White formula:

$$\hat{\sigma}^2 = \frac{2\hat{a}}{n(1-e^{-2\hat{a}\Delta t})}\sum_{i=1}^{n}\left(r_{i+1} - r_i e^{-\hat{a}\Delta t}\right)^2. \quad (11)$$

$$\hat{a} = -\frac{1}{\Delta t}\ln\left(\frac{\sum_{i=1}^{n} r_i \cdot r_{i+1}}{\sum_{i=1}^{n} r_i^2}\right). \quad (12)$$

2.5. Root Mean Squared Error (RMSE)

To test on normal errors, we use RMSE to test the accuracy of the Hull-White model [21]. RMSE is the root of mean squared error that is usually used to calculate accuracy between observed data and estimation. The smaller the RMSE value is, the better the accuracy will be. The root mean squared error can be calculated using

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(s_i - o_i)^2}, \quad (13)$$

with RMSE is the root mean squared error value, n is the amount of data, s_i is the observed data, and o_i is the estimation for observed data.

The framework of this research is as follows:

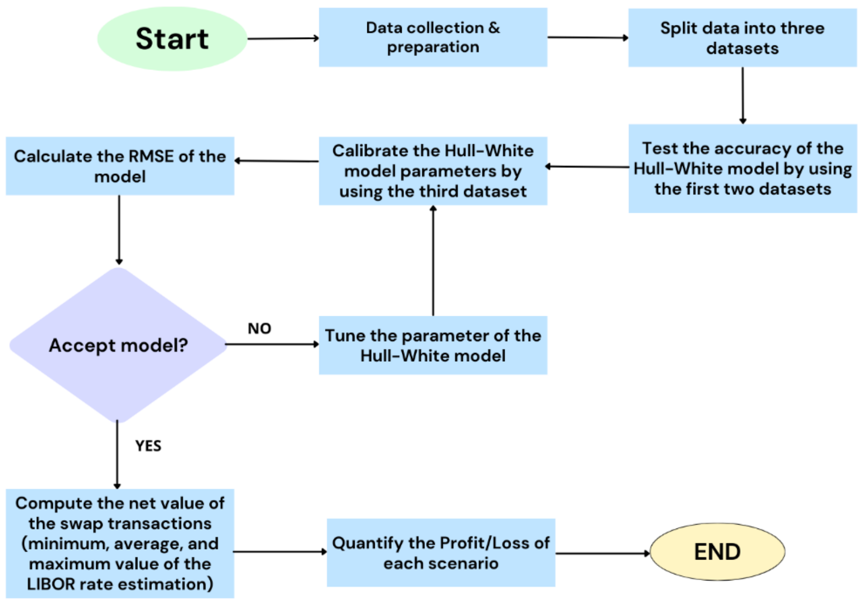

Figure 4. Flowchart of the LIBOR Calculation Using the Hull-White Model

3. RESULTS AND DISCUSSION

The following section aims to present our findings within the scope of this paper. This mainly includes accuracy of the model, result of the model estimation, and net value with three scenarios; using minimum, average, and maximum value of the model estimation.

3.1 Accuracy of the Hull-White Model

Before we implement the Hull-White model to forecast LIBOR rate in thirty years later, we analyze the accuracy of the model first, to measure the error and determine its accuracy.

3.1.2 Using the One-Year Historical LIBOR rate to predict the Three-Years LIBOR rate

The one-year daily historical LIBOR rate data that we used to predict the three-years LIBOR rate is presented in **Figure 5**. Using the data in which we input it to **Equation (7)** and **Equation (8)**, we obtain that the value for σ and a are 0.022 and 0.001, respectively. Then, we can input these parameters to Python software to get the LIBOR rate estimation, starting from January 2013 to January 2016, using the Hull-White model. To get the analysis of the Hull-White model accuracy, we compare the original value of LIBOR rate with its estimation value, displayed on a line chart below, shown in **Figure 6**.

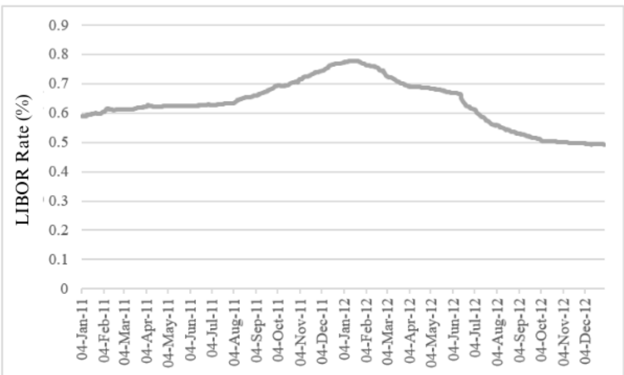

Figure 5. Historical LIBOR rate (January 2011 – January 2012)

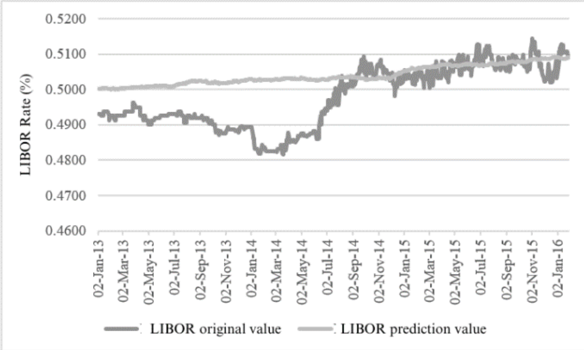

Figure 6. Accuracy of Three-Years LIBOR Rate using One-Year LIBOR Rate Data

Using this result, we can calculate the RMSE to understand how big the error of our model estimation is. Applying the **Equation (13)**, gives the error value 0.0116%. Thus, we can conclude that our model can be used for predicting the LIBOR rate.

3.1.2 Using the Three-Years Historical LIBOR rate to predict the Six-Years LIBOR rate

To ensure the accuracy of Hull-White model, we use the daily historical data with a longer period, which is three-years, starting from January 2011 to January 2013 as displayed in **Figure 7** below.

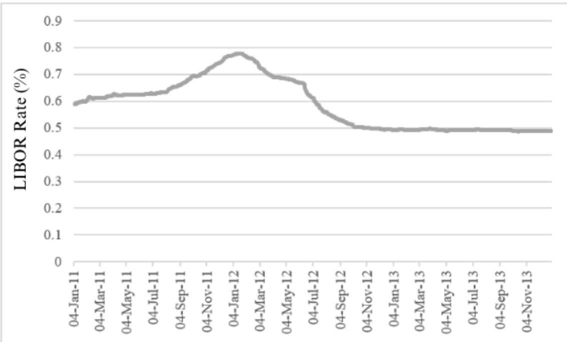

Figure 7. Historical LIBOR Rate (January 2011 – January 2013)

Using the data above which we input it to **Equation (7)** and **Equation (8)**, we can obtain that the value for σ and a are 0.018 and 0.001, respectively. Then, we can input these parameters to Python software to get the LIBOR rate estimation, starting from January 2014 to January 2020, using the Hull-White model. To get the analysis of the Hull-White model accuracy, we compare the original value of LIBOR rate with its estimation value, displayed on a line chart, as shown in **Figure 8**.

By applying the **Equation (13)**, the error value gave 0.1715%. Thus, we can conclude that our model can be used for predicting the LIBOR rate. If we compared the error value for **Figure 6** and **Figure 8**, we found that **Figure 8** gave the bigger error value. This is understandable as **Figure 8** use the longer period i.e., six-years. The Financial Conduct Authority (FCA), which regulated the administrator of LIBOR **[22]**, announced that they would launch the new regulation for participant which involved on the process of calculating the LIBOR rate, thus, impacting the declining of LIBOR rate at that time. Furthermore, from 2015 to 2018, the Federal Reserve raised its federal funds interest rate seven times **[23]**. As a result, LIBOR rate also upsurge to their high-level. If we look at the chart, our model has successfully predicted an increase, but not as high as its origin. Hence, we can conclude that our model is sufficient to forecast the LIBOR rate.

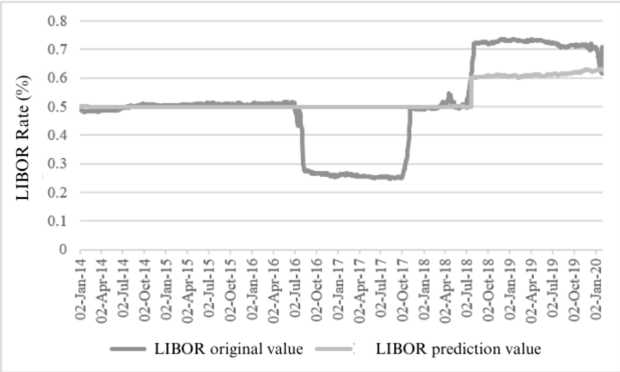

Figure 8. Accuracy of Six-Years LIBOR Rate using Three-Years LIBOR Rate Data

3.2 Hull-White Simulation

As we mentioned before, swap transactions usually have a quite a long period, up to fifteen years, thus, we use thirty years period. Below is the data that we used.

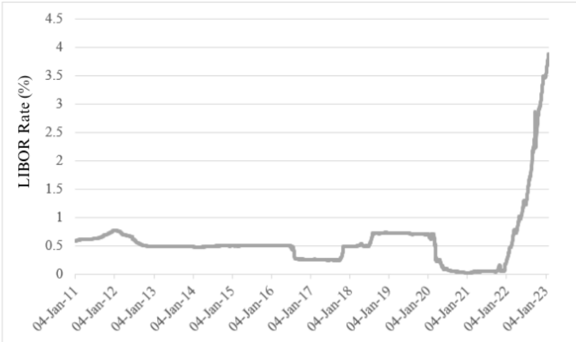

Figure 9. Historical LIBOR rate (January 2011 – January 2024)

From **Figure 9**, we can see that the LIBOR rate has an up-spike trend, especially on the beginning of the year 2022, because of Covid-19 **[18]** which stimulate the government to raise its interest rate benchmark to anticipate high inflation in the future. Using this data, we can conclude that the value of Hull-White parameters, σ and a are 0.07 and 0.004, respectively. Then, we can build the Hull-White model using the Python software to predict the LIBOR rate on the thirty years later, using 100 simulations. The result is presented below.

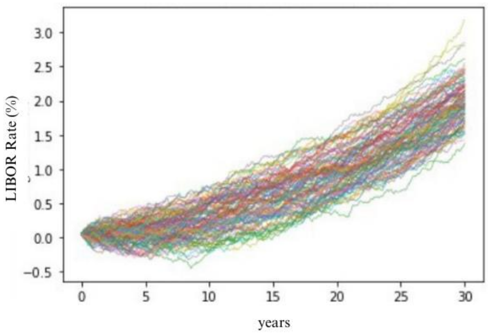

Figure 10. Hull-White Model Simulation

The interesting point from Hull-White simulation is the LIBOR rate increasing over the year, so there is a possibility that receiver will lose because they paid the higher rate. Therefore, in the Section 3.3 we will calculate whether the receiver will lose their money or not.

3.3 Net Value Calculation

As we mentioned before, the aim of this paper is determining whether the swap transaction using LIBOR rate as its reference will be beneficial for their counterparties. Therefore, using the thirty years LIBOR rate as we already predicted before, we will then calculate net value by using three scenarios: minimum, average, and maximum value of the LIBOR estimation. Assume that we will use the notional amount $10 million, hence, the result of each net value will be explained below.

3.3.1 Net Value Based on Minimum Predicted LIBOR Rate

With the Hull-White model, we have effectively projected the LIBOR interest rates for the next thirty years through 100 simulations. By selecting the minimum value from these 100 simulations, we can determine the profit and loss for each party by calculating the net value. To make it easier to do the analysis, the profit and loss of each counterparties is presented below.

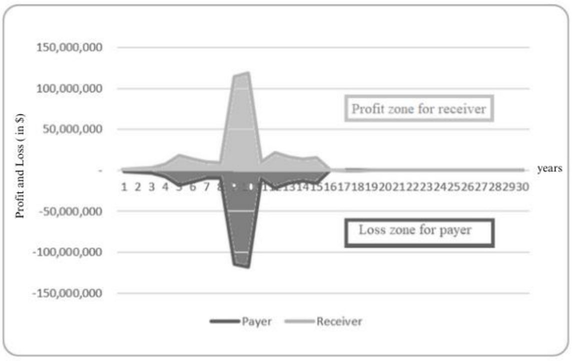

Figure 11. Net Value with Minimum LIBOR Rate

From **Figure 11**, we can infer that the recipient enjoys significantly greater advantages compared to the payer, with the benefits accruing to the recipient steadily increasing from the first year through the sixteenth year. Then, after the sixteenth year, the payer will earn the profit, but, if we compared with receiver's profit, payer gets much lower. Eventually, the net value converges towards zero. This suggests that the swap functions as a zero-sum instrument, as previously mentioned, where one party's gains stem from the losses incurred by the other party. Additionally, the recipient's higher profits can be attributed to the lower LIBOR rate compared to the fixed-rate, 0.0180% and 0.188%, respectively.

3.3.2 Net Value Based on Average Predicted LIBOR Rate

When the scenario involves a situation where the LIBOR rate is the average of its predictions, the counterparties will both benefit, but the magnitude of this benefit depends on time. Such cases commonly occur in swap transactions. The time-dependent benefit arises because the LIBOR rate is lower in the first seven years compared to the fixed-rate, approximately 0.095% for LIBOR and 0.188% for the fixed-rate. However, as time progresses, the expanding LIBOR rate continues to rise until it becomes higher than the fixed-rate, reaching around 1.238% for LIBOR, while the fixed-rate remains constant over time, at 0.188%. Thus, the turning point for determining the level of profit or loss for the counterparties occurs in the seventh year, as displayed in **Figure 12**.

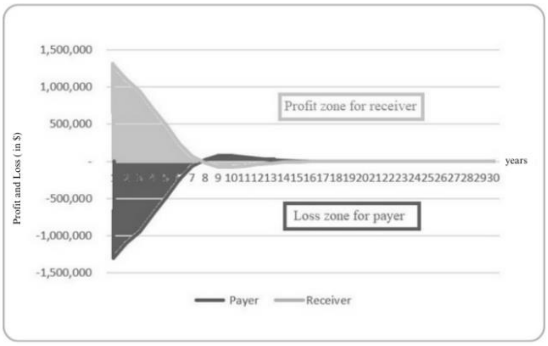

Figure 12. Net Value with Average LIBOR Rate

3.3.3 Net Value Based on Maximum Predicted LIBOR Rate

In this case, the LIBOR rate is significantly higher than the fixed-rate, at 1.395% for LIBOR and 0.188% for the fixed-rate. Therefore, the profit will be experienced by the payer. However, for ease of analysis, the net value calculation results with the maximum value of the LIBOR predictions are presented in Figure 13.

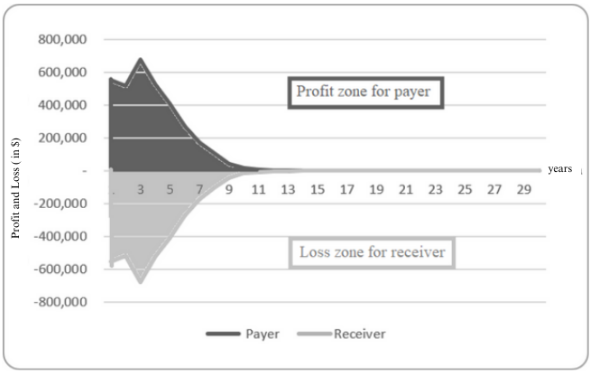

Figure 13. Net Value with Maximum LIBOR Rate

3.4 Analysis of Net Value

From the results obtained in the previous section, it can be concluded that both counterparties will experience time-dependent profit. There are times when the LIBOR rate will be significantly lower than the fixed-rate. However, the LIBOR rate does not always remain at a low level; there are times when it will rise to its highest level, becoming higher than the fixed-rate. When the LIBOR rate is lower than the fixed-rate, the receiver will profit, while the payer will incur losses. Conversely, when the fixed-rate is lower than the LIBOR rate, the payer will benefit.

When comparing the profit of both counterparties, the receiver is significantly more advantaged than the payer. This can be observed in the scenario results for net value using the minimum and average values of LIBOR predictions. Although the payer also gains profit, the amount they receive is much lower than what the receiver receives. Therefore, even though the receiver's payments depend on the fluctuations of the LIBOR interest rate, they could receive much greater profits than the payer. Conversely, for the payer, despite having fixed interest rate payments, they are not always the more advantaged party.

4. CONCLUSIONS

Based on the simulation results for predicting the LIBOR rate using the Hull-White model, the following conclusions can be drawn:

1. The Hull-White model can be used to construct the LIBOR interest rate for the next thirty years because it produces relatively low errors, ranging from 0.0116% to 0.1715%. The longer the forecasting duration of the Hull-White model, the larger the error.
2. To enhance the accountability of the Hull-White model, three scenarios of LIBOR rate constructions were used to calculate the net value of swap transactions: net value with the minimum, average, and maximum values of the LIBOR predictions. From these three scenarios, it is evident that neither the receiver nor the payer is always at a disadvantage. For the payer, the fact that they have a fixed-rate commitment until the end of the swap transaction does not necessarily mean they are more advantageous.
3. Swap transactions can be used by large companies and financial institutions for hedging purposes, but they are not immune to the possibility of speculative elements. Speculation can arise when the interest rates during the transaction deviate from what was anticipated by the company. Thus, the average deviations must align with the company's speculative intentions.

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