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LOSS MODEL OF CLIMATE INSURANCE BASED ON EFFECT OF GROWING DEGREE DAYS INDEX

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ABSTRACT

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Climate change is a threat to agriculture, especially food crops such as rice. Climate index insurance is an alternative to cover the risk of agricultural losses due to crop failure due to climate change factors. The observed climate index is the effect of growing degree days which measures the impact of temperature on plant growth and development. The data used in this study is daily temperature data at Climatology Station Class 1 Darmaga, Bogor and Citeko Class 3 Meteorological Station, West Java, during the gadu (rice that is planted in the Gadu/Dry season) planting period. In determining the amount of loss, the average daily temperature on growing degree days is calculated using a combination of a time series model and a deterministic model. The deterministic model describes the trend and seasonality of the time series at each station. The parameters contained in the model will be estimated using least-square. To see the dependence of temperature at different stations using a normal bivariate distribution. The result show that the amount of loss based on the index of growing degree days per unit rupiah per degree Celsius (°C) for Citeko Class 3 Meteorological Station only occurs for certain percentages, namely 80%, 90%, and 95%, while for Climatology Station Class 1 Darmaga Bogor it can occur for each percentage. This indicates that the amount of losses obtained will depend on determining the strike level by using the mean and standard deviation of the growing degree days index distribution. Furthermore, these findings suggest that Climatology Station Class 1 Darmaga Bogor have higher risk of crop failure due to climate change than Citeko Class 3 Meteorological Station.



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1. INTRODUCTION

Almost parts of Indonesia have superior agricultural potential, especially rice plants which play an important role in the national economy. Farming activities are closely related to climate change, especially the increase in extreme air temperature which is very risky to have a bad impact on rice plants. The increase in air temperature will have an impact on increasing transpiration, increasing water consumption, accelerating fruit or seed ripening so that it has an impact on the development of yield quality and shifting patterns and types of plants. Each minimum temperature increase of 1°C will reduce rice yields by 10% [1]. The extreme climate is not only a risk for rice plants, but also has an impact on the environment and the sustainability of human life. There are four indicators that indicate climate change, namely the increase in air temperature, sea level rise, changes in rain patterns and an increase in the frequency of extreme climate events (dry and wet anomalies) [2], [3].

One of the protection mechanisms for the risk of loss that will occur is to transfer risk, which is generally done through insurance. In agriculture, risk can be caused by crop failure, disasters, plant pest attacks, extreme climate change, and others. Of these risks, the risk of crop failure is the most often the main subject of the risks that are covered. Harvest failure due to disaster risk requires high costs because it requires verification in the field to avoid fraudulent claims [4], [5], [6]. Climate index-based agricultural insurance is the main alternative that is fair for farmers and insurance management companies because the system is more practical and verification of claims is much cheaper. Climate index-based insurance provides compensation due to decreased harvest rates or crop failure due to extreme climatic conditions or climate change [7], [8]. The parameters commonly used in determining the insurance loss model based on the climate index are temperature, rainfall, humidity, wind speed, vegetation biomass and growing degree days [4], [9].

This study focuses on modeling losses by looking at the effect of growing degree days. The concept of using the growing degree days index is based on the theory of plant development that depends on the amount of heat during growth. The amount of heat needed by plants every day depends on the average air temperature which must exceed one degree above the basic temperature of certain plants [5]. Growing degree days (GDD) measures accumulated exposure to heat over the growing season and provides a useful approach for estimating the growth and development of plants during the growing season [10]. In this concept, what is covered is not rice but a climate index of growing degree days which is determined based on the relationship between crop failure and climate change using climatological and agronomic approaches. If the specified index occurs, the farmer will receive compensation, whether it is crop failure or not. On the other hand, if there is a crop failure but the specified index is not met, the farmer will not receive compensation for the risk. Several calculation methods related to determining crop failure based on the growing degree days (GDD) index refer to research conducted by Okhrin et al [4], and Zhou [10]. In this research, the average daily temperature which is related to determining the amount of loss will be calculated using a combination of the Time Series method and deterministic elements of trend and seasonality which depend on time where the parameters will be estimated using Least Squares. Modeling relationship of weather index and yield losses is a basis for developing weather-based index crop insurance [11].

2. RESEARCH METHODS

The calculation method related to determining the risk of crop failure based on the growing degree days (GDD) index used is a combination of the time series method and the deterministic element of trend and seasonal which depends on time by estimating parameters using least squares [12], [13]. This study modeled the average daily temperature associated in determining the amount of loss calculated using the combination but determining its time dependence with the copula [5], [14], [15]. This is done because there are two different locations which are assumed to have independent dependence on each other. The period of the growing season observed was in the *gadu* season from April to July. The amount of loss is calculated by the accumulated value of growing degree days which is less than the strike level.

Loss model based on growing degree days index (L_{GDD}) :

 $L_{GDD_i} = \max\left(0, K_i^{GDD} - A_{GDD_i}\right) \times V \tag{1}$

$$\begin{array}{l} A_{GDD_{i}} = \sum_{t=1}^{L} \max\left(0, T_{i,t} - T\right) \\ K_{i}^{GDD} = A_{GDD_{i}}^{K} + \% \, \sigma_{GDD_{i}}^{K} \end{array}$$
(2) (3)

where:

V = Conversion value per unit of rupiah $K_i^{GDD} = \text{GDD strike level location-i}$ $A_{GDD_i} = \text{GDD accumulation location-i}$ $T_{i,t} = \text{Daily average temperature in units of at location-i on day t, observation 122 days}$ $\hat{T} = \text{Base temperature/minimum temperature}$ $\% \sigma_{GDD_i}^{K} = \text{Large percentage of losses determined by farmers}$

The method used to estimate model parameters is the least squares method [16], [17] with the following parameters:

$$T_{i,t} = \Lambda_i(t) + \sum_{j=1}^{j_i} b_{i,j} \Psi_{i,t-j} + \sigma_i(t) \varepsilon_{i,t}$$

$$\tag{4}$$

$$\begin{aligned} \Lambda_i(t) &= Trend(t) + Seasonal(t) \end{aligned} (5) \\ \Psi_{i,t-j} &= T_{i,t} - \Lambda_i(t) \end{aligned} (6)$$

where:

 $\begin{array}{ll} \Psi_{i,t} &= \text{Adjusted temperature} \\ \Lambda_i(t) &= \text{Seasonal component and trend at location-}i \text{ at time } t \\ \varepsilon_{i,t} &= \text{Residual standard} \end{array}$



Figure 1. Location Map of Darmaga Bogor Climatology Station Class 1 and Class 3 Meteorological Stasiun Citeko

Observations were made for 122 days, which is the ideal time for rice harvesting, namely the *gardu* planting period from April 1 to July 31, 2023. Observations were made in two locations, namely Climatology Station Class 1 Darmaga Bogor and Citeko Class 3 Meteorological Station. The standard residual value for each location will be used to determine the distribution of dependence between temperatures at each location.

3. RESULTS AND DISCUSSION

Data analysis using R-studio software [18], [19]. The general description of the observation locations can be identified by descriptive statistics which are presented in the following Table 1

	— — — — — — — — — — — — — — — — — — — —	
	Darmaga Bogor Class 1 Climatology Station	Citeko Class 3 Meteorological Station
Ν	122	122
Mean	26.2	21.7
Median	26.1	21.7
Modus	25.9	21.7
Variansi	0.5	0.6
Skewness	-0.077	-0.021
Kurtosis	0.706	0.911
Range	4.3	4.8
Minimum	23.8	19.2
Maximum	28.1	24.0

Table 1	. Descriptive	Statistics (of Temperature	Data
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From **Table 1** it can be seen that the average temperature at the Climatology Station Class 1 Darmaga Bogor during the gadu planting period is 26.2 °C and at the Citeko Class 3 Meteorological Station it is 21.7 °C. While the medians are 26.1 °C at the Climatology Station Class 1 Darmaga Bogor and 21.7 °C at the Citeko Class 3 Meteorological Station. In general, the descriptive statistics of the two locations are not much different.

3.1 Trend and Seasonal Parameter Estimation Results and Analysis $\Lambda_i(t)$

The results and analysis of trend and seasonal parameter estimates for Climatology Station Class 1 Darmaga Bogor can be seen in the following table:

Parameter	Parameter Estimation Results
$a_{1,\text{Darmaga}}$	26.3311
$a_{2,\text{Darmaga}}$	-0.0028
a _{3.Darmaga}	-0.8738
a _{4,Darmaga}	49.1878

Table 2. Parameter Estimation Results $\Lambda_i(t)$ Darmaga Bogor Class 1 Climatology Station

From Table 2, it is known that the estimated *trend* and *seasonal* parameters $\Lambda_i(t)$ for the period $W_1 = 18$ with 5000 iterations produce convergent parameter values. The selection of the W_1 period value is obtained by looking at the pattern of the data plot that is close to the original data. From the estimation results, the following parameter models are obtained:

$$\widehat{\Lambda}_1(\boldsymbol{t}) = 26.3311 - 0.0028 \, \boldsymbol{t} - 0.8738 \cos(2\pi \frac{\boldsymbol{t} - 49.1878}{18}) \tag{7}$$

From the estimation results of *trend* and *seasonal* parameters $\Lambda_i(t)$, a plot is formed that describes the closeness of the estimation results as follows:



Figure 2. Plot of Parameter Estimation $\Lambda_i(t)$ Dermaga Bogor Class 1 Climatology Station (in temperature celcius, and days)

The results and analysis of *trend* and *seasonal* parameter estimates for the Citeko Class 3 Meteorological Station can be seen in the following Table 3:

Parameter	Parameter Estimation Results
$a_{1,Citeko}$	22.007
$a_{2,Citeko}$	-0.0042
$a_{3.Citeko}$	-1.3010
$a_{4,Citeko}$	0.1782

Table 3.	Parameter	Estimation	Citeko	Class 3	6 Meteoro	ological	Station
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From Table 3 with the same treatment, it is known that the estimated *trend* and *seasonal* parameters $(\Lambda_i(t))$ for period $W_2 = 15$ with 5000 iterations produce convergent parameter values. The selection of the value of the period W_2 is obtained by looking at the pattern of the data plot that is close to the original data. From the estimation results, the following parameter models are obtained:

$$\widehat{\Lambda}_2(t) = 22.007 - 0.0042 t - 1.3010 \cos(2\pi \frac{t - 0.1782}{15})$$
(8)

From the estimation results of *trend* and *seasonal* parameters ($\Lambda_i(t)$), a plot is formed that describes the closeness of the estimation results as follows:



Figure 3. Plot of Parameter Estimation $\Lambda_i(t)$ Citeko Class 3 Meteorological Station

3.2 Autoregressive Parameter Estimation Results and Analysis

The results and analysis of the AR parameter estimation can be seen in the following Table 4:

Table 4. Parameter Estimation Results AR(1)				
Parameter	Parameter Estimation Results			
b _{1,Darmaga}	0.58552			
b _{1,Citeko}	0.60861			

. .

From Table 4, a model with AR(1) parameters is obtained for Darmaga Bogor Class 1 Climatology Station and Citeko Class 3 Meteorological Station as follows:

$$\Psi_{t,1} = 0.58552 \Psi_{t-1,1}$$

$$\Psi_{t,2} = 0.60861 \Psi_{t-1,2}$$
(10)

(9)

To see the closeness of the estimation results with the original data, a plot of the estimation results of the AR(1) parameter was carried out using the *least square* method with temperature data at each station as follows:



Figure 4. Plots of Parameter Estimation AR(1) (a) Darmaga Bogor Class 1 Climatology Station, and (b) Citeko Class 3 Meteorological Station

3.3 Result and Analysis of Variance Parameter Estimation σ^2

The results and analysis of trend and seasonal parameter estimates for Darmaga Bogor Class 1 Climatology Station can be seen in the following Table 5:

Fable 5. Result of	' Variance Parameter	• Estimation of	Darmaga Bogor	Class 1	Climatology Station
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Parameter	Parameter Estimation Results
$d_{1,\text{Darmaga}}$	0.4328
$d_{2,\text{Darmaga}}$	0.0037
d _{3.Darmaga}	0.2419
$d_{4,\mathrm{Darmaga}}$	-0.0952

From the estimation results obtained a model with the estimated parameters as follows:

$$\hat{\sigma}_1^2(t) = 0.4328 + 0.0037 t + 0.2419 \cos\left(\frac{2\pi t}{18}\right) - 0.0952 \sin\left(\frac{2\pi t}{18}\right) \tag{11}$$

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The results and analysis of the estimated variance parameters for the Citeko Class 3 Meteorological Station can be seen in the following Table 6:

Parameter	Parameter Estimation Results
d _{1,Citeko}	1.0989
$d_{2.Citeko}$	-0.0027
$d_{3.Citeko}$	0.4064
$d_{4 Citeko}$	0.2109

 Table 6. Result of Variance Parameter Estimation of Citeko Class 3 Meteorological Station

From the estimation results obtained a model with the estimated parameters as follows:

$$\hat{\sigma}_2^2(t) = 1.0989 - 0.0027 t + 0.4604 \cos\left(\frac{2\pi t}{15}\right) + 0.2109 \sin\left(\frac{2\pi t}{15}\right)$$
 (12)

3.4 Standard Residual Distribution Estimation Results and Analysis

The results and analysis of the estimated variance parameters for Darmaga Bogor Class 1 Climatology Station can be seen in the following Table 7:



Figure 5. Plots of Standard Residual Fit Distribution (a) Climatology Station Class 1 Darmaga Bogor, and (b) Citeko Class 3 Meteorological Station

From **Figure 5**, it can be seen that both models follow the Normal distribution. Thus, the normal conditions are met so that the standard residual model for each station meets the assumption of normality.

3.5 Results and Analysis of the Loss Model of the Growing Degree Days Index

From the standard residual model with a normal bivariate distribution, it can be calculated the value of the accumulated *growing degree days* index. The accumulated value of the GDD index for Darmaga Bogor Class 1 Climatology Station is as follows:

$$A_{GDD_1} = \sum_{t=1}^{122} \max(0, T_{1,t} - \hat{T})$$

= $\max(0, T_{1,1} - \hat{T}) + \max(0, T_{1,2} - \hat{T}) + \dots + \max(0, T_{1,122} - \hat{T})$

$$= \max(0,26.7 - 10) + \max(0,26.6 - 10) + \dots + \max(0,25.8 - 10)$$

= 1969.2

While the accumulated value of the GDD index for Citeko Class 3 Meteorological Station is as follows:

$$A_{GDD_2} = \sum_{t=1}^{122} \max(0, T_{2,t} - \hat{T})$$

= $\max(0, T_{2,1} - \hat{T}) + \max(0, T_{2,2} - \hat{T}) + \dots + \max(0, T_{2,122} - \hat{T})$
= $\max(0, 19.7 - 10) + \max(0, 21.4 - 10) + \dots + \max(0, 21.1 - 10)$
= 1376.6

After obtaining the value from the accumulated growing degree days index for each station, the amount of loss can be calculated based on the growing degree days for each station using **Equation** (1) and **Equation** (3). The values of $A_{GDD_i}^K$ and $\sigma_{GDD_i}^K$ converge at the 1000th generation with the strike level at a certain percentage as shown in the following **Table 8**:

Darmaga Bogor Class 1 Climatology Station			Citeko Class 3 Meteorological Station			Station	
$A_{GDD_1}^K$	$\sigma_{GDD_1}^{\scriptscriptstyle K}$	%	K_1^{GDD}	$A_{GDD_2}^K$	$\sigma_{GDD_2}^{K}$	%	K_2^{GDD}
		25	1970.0			25	1375.4
		50	1971.1			50	1376.0
1968.9	4.3819	75	1972.2	1374.8	2.3798	75	1376.6
		80	1972.4			80	1376.7
		90	1972.8			90	1376.9
		95	1973.1			95	1377.1

Table 8. Strike Level Calculation

From the acquisition of strike level values in **Table 8**, it can then be determined the amount of loss based on growing degree days for Darmaga Bogor Class 1 Climatology Station and Citeko Class 3 Meteorological Station as follows:

Darmaga Bogor Class 1 Climatology Station			Citeko Class 3 Meteorological Station				
A_{GDD_1}	K_1^{GDD}	$K_1^{GDD} - A_{GDD_1}$	L _{GDD1}	A _{GDD2}	K_2^{GDD}	$K_2^{GDD} - A_{GDD_2}$	L_{GDD_2}
	1970.0	0.80	0.80		1375.4	-1.21	0.00
	1971.1	1.89	1.89		1376.0	-0.61	0.00
1969.2	1972.2	2.99	2.99	1376.6	1376.6	-0.02	0.00
	1972.4	3.21	3.21		1376.7	0.10	0.10
	1972.8	3.64	3.64		1376.9	0.34	0.34
	1973.1	3.86	3.86		1377.1	0.46	0.46

Table 9. Loss Value for Each Station

The amount of loss shown in **Table 9** is the amount of loss in rupiah per degree Celsius (°C). For Darmaga Bogor Class 1 Climatology Station, the loss, based on the growing degree days index, occurs every specified percentages. In contrast, Citeko Class 3 Meteorological Station indicates losses occurring at percentages of $\geq 80\%$. This indicates that the level of station losses is highly dependent on determining the magnitude of the strike level using the mean and standard deviation of the growing degree days index distribution. Additionally, these findings suggest that Darmaga Bogor Class 1 Climatology Station have higher risk of crop failure due to climate change compared to Citeko Class 3 Meteorological Station. As a result, insurance premiums for farmers in Darmaga Bogor Class 1 Climatology Station are higher than for farmers in Citeko Class 3 Meteorological Station.

For simple calculation, if farmers' annual expenses in both regions total IDR 100 million, the estimated annual loss for Darmaga Bogor Class 1 Climatology Station, with a minimum loss percentage of 25%,

amounts to IDR 80 million per year, $L_{GDD_1} = 0.80 \times IDR 100$ million = IDR 80 million. Meanwhile, for Citeko Class 3 Meteorological Station, with a minimum loss percentage of 80%, the estimated loss is IDR 10 million per year, $L_{GDD_2} = 0.10 \times IDR 100$ million = IDR 10 million. If the insurance company wants to cover crop failure losses due to climate change with a minimum risk percentage, the premium for Darmaga Bogor Class 1 Climatology Station farmers is IDR 20 million per year, $Premium_{GDD_1} = 25\% \times$ IDR 80 million = IDR 20 million, whereas for Citeko Class 3 Meteorological Station, it's IDR 8 million per year, $Premium_{GDD_2} = 80\% \times IDR 10$ million = IDR 8 million. The determination in the selection of the percentage is determined by the farmer as the policyholder.

4. CONCLUSIONS

From the results and analysis, it can be concluded that the temperature dependence at different locations, namely Darmaga Bogor Class 1 Climatology Station and Citeko Class 3 Meteorological Station follows a normal bivariate distribution. The amount of loss based on the index of growing degree days per unit rupiah per degree Celsius (°C) for Citeko Class 3 Meteorological Station only occurs for certain percentages, namely 80%, 90%, and 95%, while for Darmaga Bogor Class 1 Climatology Station it can occur for each percentage. The results show that the level of station losses is highly dependent on determining the magnitude of the strike level using the mean and standard deviation of the growing degree days index distribution. Furthermore, these findings suggest that Darmaga Bogor Class 1 Climatology Station. Consequently, insurance premiums for farmers in Darmaga Bogor Class 1 Climatology Station are higher than for farmers in Citeko Class 3 Meteorological Station.

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