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ON THE LOCATING CHROMATIC NUMBER OF DISJOINT UNION OF BUCKMINSTERFULLERENE GRAPHS

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ABSTRACT

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Let $G = (V, E)$ be a connected non-trivial graph. Let c be a proper vertex-coloring using k colors, namely $1, 2, \cdots, k$. Let $\Pi = \{S_1, S_2, \cdots, S_k\}$ be a partition of $V(G)$ induced by c, where S_i is the color class that receives the color i. The color code, denoted by $c_\Pi(v)$, is defined as $c_\Pi(v) = (d(v, S_1), d(v, S_2), \cdots, d(v, S_k))$, where $d(v, S_i) = min\{d(v, x) | x \in S_i\}$ for $1 \leq i \leq k$, and $d(v, x)$ is the distance between two vertices v and x in G. If all vertices in $V(G)$ have different color codes, then c is called as the locating-chromatic k-coloring of G. The locating-chromatic number of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring. Let B_{60} be the Buckminsterfullerene graph on 60 vertices. Buckminsterfullerene graph is a 3-connected planar graph and a member of the fullerene graphs, representing fullerene molecules in chemistry. In this paper, we determine the locating chromatic number of the disjoint union of Buckminsterfullerene graphs, denoted by $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$.

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1. INTRODUCTION

Many papers have discussed the locating-chromatic number since Chartrand *et al.* [1] introduced the concept in 2002. The locating-chromatic number of a graph is a newly interesting topic to study because there is no general theorem for determining the locating-chromatic number of any graph. All graphs considered in this paper are finite, undirected, and simple. Let $G = (V, E)$ be a connected graph. Let c be a proper coloring using k colors, namely $1,2,\cdots,k$. Let $\Pi = \{S_1, S_2, \cdots, S_k\}$ be a partition of $V(G)$ induced by c, where S_i is the color class that receives the color i. The color code, denoted by $c_\Pi(v)$, is defined as

$$c_\Pi(v) = (d(v, S_1), d(v, S_2), \dots, d(v, S_k)),$$

where $d(v, S_i) = \min\{d(v, x) | x \in S_i\}$ for $1 \leq i \leq k$, and $d(v, x)$ is the distance between two vertices v and x in G. If all vertices in $V(G)$ have different color codes, then c is called as the locating-chromatic k-coloring of G. The locating-chromatic number of G, denoted by $\chi_L(G)$, is the minimum k such that G has a locating coloring. The dominant vertex is the vertex v such that $d(v, S_i) = 0$ if $v \in S_i$, and $d(v, S_i) = 1$ if $v \notin S_i$. Other notations and definitions in graph theory are taken from Diestel [2].

Some interesting results are obtained to determine the locating-chromatic number of some connected graphs. The results are focused on certain families of graphs. Chartrand *et al.* [1] determined all graphs of order n with locating-chromatic number n, namely, a complete multipartite graph of n vertices. Moreover, Chartrand *et al.* [3] constructed trees on n vertices, $n \geq 5$, with locating-chromatic numbers varying from 3 to n, except for $n - 1$. In [4], Baskoro and Asmiati characterized all trees with locating-chromatic number 3. Next, Wellyyanti *et al.* [5] determined the locating-chromatic number for complete n-ary trees. Moreover, Syofyan *et al.* [6] characterized all trees of order with locating-chromatic number $n - 1$, for any integers n and t, where $n > t + 3$ and $2 \leq t \leq \frac{n}{2}$. In [7], Asmiati *et al.* determined the locating-chromatic number of the amalgamation of stars. In [8], Asmiati *et al.* obtained the locating-chromatic number for the amalgamation of stars linked by one path. Asmiati *et al.* [9] determined the characterization of some Petersen graphs having locating-chromatic numbers four or five. Next, Asmiati *et al.* [10] obtained the locating-chromatic number for certain barbell graphs.

Welyyanti *et al.* [11] generalized the concept of the locating-chromatic number such that it can be applied to disconnected graphs. The locating-chromatic number of a disconnected graph H, denoted by $\chi_L{}'(H)$, is the smallest k such that H admits a locating coloring with k colors. If no integer satisfies the above conditions, then the locating-chromatic number of H is defined as $\chi_L'(H) = \infty$. Some recent results are the locating-chromatic number of a graph with two components [12], two homogeneous components [13], and disconnected graphs with path, cycle, and star or double stars as its components [14]. Recently, Zikra et al. [15] determined the locating-chromatic number of disjoint unions of fan graphs.

In [16], Andova *et al.* discussed the fullerene graphs, the graph representation of fullerene molecules, where fullerenes are polyhedral molecules in chemistry made entirely of carbon atoms. They also determined that the fullerene graphs are 3-connected planar graphs with precisely 12 pentagonal faces, while all other faces are hexagons. Akhter *et al.* [17] obtained the metric dimension of some $(3,6)-$fullerene and $(4,6)-$fullerene graphs, where $(k, 6)-$fullerene graph is fullerene graph with all faces on k and six vertices, for $k = 3,4,5$. Next, Mehreen *et al.* [18] obtained the partition dimension of $(3,6)-$fullerene graphs and gave some conjecture related to the structure of $(5,6)-$fullerene graphs.

The most symmetric fullerene graph is the Buckminsterfullerene, B_{60}, discovered by Kroto *et al.* [19]. This $(5,6)-$fullerene graph comprises 60 carbon atoms and is named after Buckminster Fuller, whose geodetic dome resembled the structure of the molecules. Putri *et al.* [20] determined the metric dimension of Buckminsterfullerene B_{60}. Moreover, in [21], Mardimar *et al.* obtained the locating chromatic number of the Buckminsterfullerene graph B_{60}. The recent result is the locating chromatic number of the Buckminsterfullerene-type graph $B_{60}(1, t)$ for $1 \leq t \leq 5$ by Putri *et al.* [22]. Motivated by the result of [21] and of [22], we construct a new graph, namely the disjoint union of Buckminsterfullerene graphs, denoted by $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$, and determine its locating-chromatic number.

2. RESEARCH METHODS

Let G be a non-trivial connected graph. The upper bound of the locating chromatic number of G is obtained by constructing a partition $\Pi = \{S_1, S_2, \cdots, S_k\}$ for some positive integer k such that every two vertices in G have different color codes, denoted by $\chi_L(G) \leq k$. The lower bound of $\chi_L(G)$ is determined by showing that for every possibility of Π^* with $|\Pi^*| < k$, there are at least two vertices with the same color codes, denoted by $\chi_L(G) \geq k$. The proof for disconnected graph H is similar. If no integer k satisfies the above conditions, then the locating-chromatic number of H is defined as $\chi'_L(H) = \infty$.

Chartrand et al. [1] gave some characterization for the locating-chromatic coloring of a graph in Theorem 1. This theorem showed some restrictions in the construction of vertex coloring of a graph.

Theorem 1. [1] *Let G be a connected graph, and c be a locating-chromatic coloring of G. If $v, w \in V(G)$ and $v \neq w$ such that $d(v,x) = d(w,x)$ for all $x \in V(G) \setminus \{v,w\}$, then $c(v) \neq c(w)$. In particular, if v and w are two non-adjacent vertices of G such that the neighborhood of v is equal to that of w, then $c(v) \neq c(w)$.*

Welyyanti *et al.* [11] gave the upper and lower bounds of the locating-chromatic number for the disjoint union of arbitrarily connected graphs.

Theorem 2. [11] *For every i, let G_i be the connected graph, and let $H = \bigcup_{i=1}^{m} G_i$. If $\chi'_L(G) < \infty$ then*

$$max\{\chi_L(G_i)|1 \leq i \leq m\} \leq \chi'_L(H) \leq min\{|V(G_i)|\,|\, 1 \leq i \leq m\}.$$

3. RESULTS AND DISCUSSION

In this section, we will discuss about the locating-chromatic number of the disjoint union of Buckminsterfullerene graphs, denoted by $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$. The vertex-set and edge-set of the Buckminsterfullerene graph B_{60} are given by Putri *et al.* [20] as follows.

$$V(B_{60}) = \{v_i, z_i | 1 \leq i \leq 5\} \cup \{w_j, y_j | 1 \leq j \leq 15\} \cup \{x_k | 1 \leq k \leq 20\}, \quad (1)$$

$$\begin{aligned} E(B_{60}) = &\{v_l v_{l+1}, z_l z_{l+1} | 1 \leq l \leq 4\} \cup \{w_m w_{m+1}, y_m y_{m+1} | 1 \leq m \leq 14\} \\ &\cup \{x_n x_{n+1} | 1 \leq n \leq 19\} \cup \{w_{3l} x_{4l+1}, x_{4l} y_{3l+1}, y_{3l} z_{l+1} | 1 \leq l \leq 4\} \\ &\cup \{v_i w_{3i-2}, w_{3i-1} x_{4i-2}, x_{4i-1} y_{3i-1} | 1 \leq i \leq 5\} \\ &\cup \{w_{15} x_1, x_{20} y_1, y_{15} z_1, v_1 v_5, z_1 z_5, w_1 w_{15}, y_1 y_{15}, x_1 x_{20}\}. \quad (2) \end{aligned}$$

Mardimar *et al.*[21] determined the locating-chromatic number for Buckminsterfullerene graph B_{60} as follows.

Theorem 3. [21] *Let B_{60} be the Buckminsterfullerene graph. Then $\chi_L(B_{60}) = 5$.*

Proof. The sketch of proof is as follows. Because B_{60} is a 3-regular graph, then it is clear that $\chi_L(B_{60}) \geq 4$. Next, assume that $\chi_L(B_{60}) = 4$. Because every vertex has degree 3, then every vertex in B_{60} is the dominant vertex. Therefore, we have four ways to color each vertex and its neighbors. Because $|V(B_{60})| = 60$, then at least two dominant vertices are in the same color class. Thus, $\chi_L(B_{60}) \geq 5$.

Next, by defining $c: V(B_{60}) \to \{1,2,3,4,5\}$ as follows:

$$c(v) = \begin{cases} 1, v = z_1, y_1, y_3, y_5, y_9, y_{12}, y_{14}, x_1, x_4, x_6, x_8, x_{12}, x_{16}, x_{18}, w_6, w_8, w_{12}, \\ 2, v = z_3, z_5, y_2, y_4, y_7, y_{10}, y_{13}, y_{15}, x_2, x_{11}, x_{15}, x_{17}, x_{19}, w_4, w_7, w_9, w_{11}, w_{13}, w_{15}, v_4, \\ 3, v = x_9, x_{14}, w_3, w_{10}, v_1, \\ 4, v = w_1, v_3, \\ 5, v = z_2, z_4, y_6, y_8, y_{11}, x_3, x_5, x_7, x_{10}, x_{13}, x_{20}, w_2, w_5, w_{14}, v_2, v_5, \end{cases} \quad (3)$$

it can be easily shown that for $\Pi = \{S_1, S_2, S_3, S_4, S_5\}$, where

$$\begin{aligned} S_1 &= \{z_1, y_1, y_3, y_5, y_9, y_{12}, y_{14}, x_1, x_4, x_6, x_8, x_{12}, x_{16}, x_{18}, w_6, w_8, w_{12}\}, \\ S_2 &= \{z_3, z_5, y_2, y_4, y_7, y_{10}, y_{13}, y_{15}, x_2, x_{11}, x_{15}, x_{17}, x_{19}, , w_4, w_7, w_9, w_{11}, w_{13}, w_{15}, v_4\}, \\ S_3 &= \{x_9, x_{14}, w_3, w_{10}, v_1\}, \\ S_4 &= \{w_1, v_3\}, \\ S_5 &= \{z_2, z_4, y_6, y_8, y_{11}, x_3, x_5, x_7, x_{10}, x_{13}, , x_{20}, w_2, w_5, w_{14}, v_2, v_5\}, \end{aligned}$$

every two vertices in B_{60} have different color codes. Therefore, $\chi_L(B_{60}) \leq 5$. ■

Denote $B_{60}^{(s)}$ as the s^{th} Buckminsterfullerene B_{60}, for $1 \leq s \leq m$. We construct the disjoint union of five B_{60} denoted by $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$. The vertex set and edge set of H's are defined similarly as in **Equation (1)** and **Equation (2)** as follows.

$$V(H) = \{v_{s,i}, z_{s,i} | 1 \leq i \leq 5\} \cup \{w_{s,j}, y_{s,j} | 1 \leq j \leq 15\} \cup \{x_{s,k} | 1 \leq k \leq 20\}, \quad (4)$$

$$E(H) = \{v_{s,l}v_{s,l+1}, z_{s,l}z_{s,l+1} | 1 \leq l \leq 4\} \cup \{w_{s,m}w_{s,m+1}, y_{s,m}y_{s,m+1} | 1 \leq m \leq 14\} \quad (5)$$

$$\cup \{x_{s,n}x_{s,n+1} | 1 \leq n \leq 19\} \cup \{w_{s,3l}x_{s,4l+1}, x_{s,4l}y_{s,3l+1}, y_{s,3l}z_{s,l+1} | 1 \leq l \leq 4\}$$

$$\cup \{v_{s,i}w_{s,3i-2}, w_{s,3i-1}x_{s,4i-2}, x_{s,4i-1}y_{s,3i-1} | 1 \leq i \leq 5\}$$

$$\cup \{w_{s,15}x_{s,20}, x_{s,20}y_{s,1}, y_{s,15}z_{s,1}, v_{s,1}v_{s,5}, z_{s,1}z_{s,5}, w_{s,1}w_{s,15}, y_{s,1}y_{s,15}, x_{s,1}x_{s,20}\}.$$

In **Theorem 4**, we determine the chromatic location number of the disjoint union of Buckminsterfullerene graphs $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$.

Theorem 4. *Let $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$ as the disjoint union of Buckminsterfullerene graphs. Then $\chi_L'(H) = 5$.*

Proof. Let $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$ be the disjoint union of Buckminsterfullerene graphs. Since $\chi_L(B_{60}) = 5$ (**Theorem 3**), then we have that $\chi_L'(H) \geq max\{\chi_L(G_i) | 1 \leq i \leq m\} = 5$, from **Theorem 2**.

Let $c_1(a_{1,i}) = q$ for $a_{1,i} \in V(B_{60}^{(1)})$ for $1 \leq q \leq 5$ and $1 \leq i \leq 60$, following the 5-coloring in eq. (3). For the corresponding vertices $a_{p,i}$ in $B_{60}^{(p)}$, $2 \leq p \leq 5$, define the 5-coloring as follows. In constructing this coloring, we use **Theorem 1** as the restriction.

$$\begin{aligned} c_1(a_{2,i}) &= (q+1) mod\ 5, for\ a_{2,i} \in B_{60}^{(2)}, \\ c_1(a_{3,i}) &= (q+2) mod\ 5, for\ a_{3,i} \in B_{60}^{(3)}, \\ c_1(a_{4,i}) &= (q+3) mod\ 5, for\ a_{4,i} \in B_{60}^{(4)}, \\ c_1(a_{5,i}) &= (q+4) mod\ 5, for\ a_{5,i} \in B_{60}^{(5)}. \end{aligned}$$

By this coloring, we define $\Pi = \{S_1, S_2, S_3, S_4, S_5\}$, where:

$S_1 = \{z_{1,1}, y_{1,1}, y_{1,3}, y_{1,5}, y_{1,9}, y_{1,12}, y_{1,14}, x_{1,1}, x_{1,4}, x_{1,6}, x_{1,8}, x_{1,12}, x_{1,16}, x_{1,18}, w_{1,6}, w_{1,8}, w_{1,12}\} \cup$
$\{z_{2,2}, z_{2,4}, y_{2,6}, y_{2,8}, y_{2,11}, x_{2,3}, x_{2,5}, x_{2,7}, x_{2,10}, x_{2,13}, x_{2,20}, w_{2,2}, w_{2,5}, w_{2,14}, v_{2,2}, v_{2,5}\} \cup \{w_{3,1}, v_{3,3}\} \cup$
$\{x_{4,9}, x_{4,14}, w_{4,3}, w_{4,10}, v_{4,1}\} \cup \{z_{5,3}, z_{5,5}, y_{5,2}, y_{5,4}, y_{5,7}, y_{5,10}, y_{5,13}, y_{5,15}, x_{5,2}, x_{5,11}, x_{5,15}, x_{5,17}, x_{5,19}\} \cup$
$\{w_{5,4}, w_{5,7}, w_{5,9}, w_{5,11}, w_{5,13}, w_{5,15}, v_{5,4}\}$

$S_2 = \{z_{1,3}, z_{1,5}, y_{1,2}, y_{1,4}, y_{1,7}, y_{1,10}, y_{1,13}, y_{1,15}, x_{1,2}, x_{1,11}, x_{1,15}, x_{1,17}, x_{1,19}\} \cup$
$\{w_{1,4}, w_{1,7}, w_{1,9}, w_{1,11}, w_{1,13}, w_{1,15}, v_{1,4}\} \cup$
$\{z_{2,1}, y_{2,1}, y_{2,3}, y_{2,5}, y_{2,9}, y_{2,12}, y_{2,14}, x_{2,1}, x_{2,4}, x_{2,6}, x_{2,8}, x_{2,12}, x_{2,16}, x_{2,18}, w_{2,6}, w_{2,8}, w_{2,12}\} \cup$
$\{z_{3,2}, z_{3,4}, y_{3,6}, y_{3,8}, y_{3,11}, x_{3,3}, x_{3,5}, x_{3,7}, x_{3,10}, x_{3,13}, x_{3,20}, w_{3,2}, w_{3,5}, w_{3,14}, v_{3,2}, v_{3,5}\} \cup \{w_{4,1}, v_{4,3}\} \cup$
$\{x_{5,9}, x_{5,14}, w_{5,3}, w_{5,10}, v_{5,1}\}$,

$S_3 = \{x_{1,9}, x_{1,14}, w_{1,3}, w_{1,10}, v_{1,1}\} \cup$
$\{z_{2,3}, z_{2,5}, y_{2,2}, y_{2,4}, y_{2,7}, y_{2,10}, y_{2,13}, y_{2,15}, x_{2,2}, x_{2,11}, x_{2,15}, x_{2,17}, x_{2,19}\} \cup$
$\{w_{2,4}, w_{2,7}, w_{2,9}, w_{2,11}, w_{2,13}, w_{2,15}, v_{2,4}\} \cup$
$\{z_{3,1}, y_{3,1}, y_{3,3}, y_{3,5}, y_{3,9}, y_{3,12}, y_{3,14}, x_{3,1}, x_{3,4}, x_{3,6}, x_{3,8}, x_{3,12}, x_{3,16}, x_{3,18}, w_{3,6}, w_{3,8}, w_{3,12}\} \cup$
$\{z_{4,2}, z_{4,4}, y_{4,6}, y_{4,8}, y_{4,11}, x_{4,3}, x_{4,5}, x_{4,7}, x_{4,10}, x_{4,13}, x_{4,20}, w_{4,2}, w_{4,5}, w_{4,14}, v_{4,2}, v_{4,5}\} \cup \{w_{5,1}, v_{5,3}\}$,

$S_4 = \{w_{1,1}, v_{1,3}\} \cup \{x_{2,9}, x_{2,14}, w_{2,3}, w_{2,10}, v_{2,1}\} \cup$
$\{z_{3,3}, z_{3,5}, y_{3,2}, y_{3,4}, y_{3,7}, y_{3,10}, y_{3,13}, y_{3,15}, x_{3,2}, x_{3,11}, x_{3,15}, x_{3,17}, x_{3,19}\} \cup$

$\{w_{3,4}, w_{3,7}, w_{3,9}, w_{3,11}, w_{3,13}, w_{3,15}, v_{3,4}\} \cup$
$\{z_{4,1}, y_{4,1}, y_{4,3}, y_{4,5}, y_{4,9}, y_{4,12}, y_{4,14}, x_{4,1}, x_{4,4}, x_{4,6}, x_{4,8}, x_{4,12}, x_{4,16}, x_{4,18}, w_{4,6}, w_{4,8}, w_{4,12}\} \cup$
$\{z_{5,2}, z_{5,4}, y_{5,6}, y_{5,8}, y_{5,11}, x_{5,3}, x_{5,5}, x_{5,7}, x_{5,10}, x_{5,13}, x_{5,20}, w_{5,2}, w_{5,5}, w_{5,14}, v_{5,2}, v_{5,5}\},$

$S_5 = \{z_{1,2}, z_{1,4}, y_{1,6}, y_{1,8}, y_{1,11}, x_{1,3}, x_{1,5}, x_{1,7}, x_{1,10}, x_{1,13}, x_{1,20}, w_{1,2}, w_{1,5}, w_{1,14}, v_{1,2}, v_{1,5}\} \cup \{w_{2,1}, v_{2,3}\} \cup$
$\{x_{3,9}, x_{3,14}, w_{3,3}, w_{3,10}, v_{3,1}\} \cup \{z_{4,3}, z_{4,5}, y_{4,2}, y_{4,4}, y_{4,7}, y_{4,10}, y_{4,13}, y_{4,15}, x_{4,2}, x_{4,11}, x_{4,15}, x_{4,17}, x_{4,19}\} \cup$
$\{w_{4,4}, w_{4,7}, w_{4,9}, w_{4,11}, w_{4,13}, w_{4,15}, v_{4,4}\} \cup$
$\{z_{5,1}, y_{5,1}, y_{5,3}, y_{5,5}, y_{5,9}, y_{512}, y_{5,14}, x_{5,1}, x_{5,4}, x_{5,6}, x_{5,8}, x_{5,12}, x_{5,16}, x_{5,18}, w_{5,6}, w_{5,8}, w_{5,12}\}.$

From **Theorem 3**, it is clear that every two vertices in $B_{60}^{(1)}$ have different color codes. Then we can conclude that every two vertices in $B_{60}^{(p)}$, for $2 \le p \le 5$ also have different color codes. Now, consider two vertices in different $a \in V(B_{60}^{(k)})$ and $b \in V(B_{60}^{(t)})$ for $1 \le k,t \le 5, k \ne t$, where $c_1(a) = c_1(b)$. We will show that $c_\Pi(a) \ne c_\Pi(b)$. Without loss of generalities, let $a \in V(B_{60}^{(1)})$ and $b \in V(B_{60}^{(2)})$, with $c(a) = c(b) = 1$. For every two vertices in $V(B_{60}^{(k)})$ and $V(B_{60}^{(t)})$ for $1 \le k,t \le 5$ and $k \ne t$, the proofs are similar. If $c(a) = 1$ then the corresponding vertices in $B_{60}^{(2)}$ is given color 2. For example, if $c(x_{1,12}) = 1$ then $c(x_{2,12}) = 2$. Similarly, if $c(b) = 1$ then the corresponding vertices in $B_{60}^{(1)}$ is given color 5. For example, if $c(x_{2,13}) = 1$ then $c(x_{1,13}) = 5$. Because the distance to the vertex of color 5 are different, $d(x_{1,12}, x_{1,13}) = 1$, and $d(x_{2,13}, v_{2,3}) = 4$

then $c_\Pi(a) \ne c_\Pi(b)$. Therefore, every two vertices in $H = \bigcup_{s=1}^{m} B_{60}^{(s)}$ have different color codes, and $\chi_L'(H) \le 5$. ■

In **Example 1** we give the 5-locating coloring of $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$.

Example 1. Using the 5-locating coloring defined in **Theorem 4**, we will show that every two vertices have different color codes. Consider the following cases.

Case 1. Vertices with color *1* in $B_{60}^{(s)}$ for $1 \le s \le 5$. Without loss of generality, let $c(w_{1,8}) = c(w_{2,5}) = c(w_{3,1}) = c(w_{4,3}) = c(w_{5,9}) = 1$. Then their color codes are $c_\Pi(w_{1,8}) = (0,1,2,2,1)$, $c_\Pi(w_{2,5}) = (0,1,1,2,3)$, $c_\Pi(w_{3,1}) = (0,1,2,1,1)$, $c_\Pi(w_{4,3}) = (0,2,1,2,1)$ and $c_\Pi(w_{5,9}) = (0,1,3,1,1)$.

Case 2. Vertices with color *2* in $B_{60}^{(s)}$ for $1 \le s \le 5$. Without loss of generality, let $c(w_{1,7}) = c(w_{2,8}) = c(w_{3,5}) = c(w_{4,1}) = c(w_{5,10}) = 2$. Then their color codes are $c_\Pi(w_{1,7}) = (1,0,2,1,2)$, $c_\Pi(w_{2,8}) = (1,0,1,2,2)$, $c_\Pi(w_{3,5}) = (3,0,1,2,2)$, $c_\Pi(w_{4,1}) = (1,0,1,2,1)$ and $c_\Pi(w_{5,10}) = (1,0,4,2,2)$.

Case 3. Vertices with color *3* in $B_{60}^{(s)}$ for $1 \le s \le 5$. Without loss of generality, let $c(w_{1,10}) = c(w_{2,9}) = c(w_{3,8}) = c(w_{4,5}) = c(w_{5,1}) = 3$. Then their color codes are $c_\Pi(w_{1,10}) = (2,1,0,4,2)$, $c_\Pi(w_{2,9}) = (1,1,0,1,3)$, $c_\Pi(w_{3,8}) = (2,1,0,1,2)$, $c_\Pi(w_{4,5}) = (2,3,0,1,2)$ and $c_\Pi(w_{5,1}) = (1,1,0,1,2)$.

Case 4. Vertices with color *4* in $B_{60}^{(s)}$ for $1 \le s \le 5$. Without loss of generality, let $c(w_{1,1}) = c(w_{2,3}) = c(w_{3,4}) = c(w_{4,6}) = c(w_{5,5}) = 4$. Then their color codes are $c_\Pi(w_{1,1}) = (2,1,1,0,1)$, $c_\Pi(w_{2,3}) = (1,2,1,0,2)$, $c_\Pi(w_{3,4}) = (3,1,2,0,1)$, $c_\Pi(w_{4,6}) = (1,2,2,0,1)$ and $c_\Pi(w_{5,5}) = (1,2,3,0,1)$.

Case 5. Vertices with color *5* in $B_{60}^{(s)}$ for $1 \le s \le 5$. Without loss of generality, let $c(w_{1,14}) = c(w_{2,1}) = c(w_{3,10}) = c(w_{4,9}) = c(w_{5,8}) = 1$. Then their color codes are $c_\Pi(w_{1,14}) = (1,1,3,2,0)$, $c_\Pi(w_{2,1}) = (1,2,1,1,0)$, $c_\Pi(w_{3,10}) = (4,2,2,1,0)$, $c_\Pi(w_{4,9}) = (1,3,1,1,0)$ and $c_\Pi(w_{5,8}) = (1,2,2,1,0)$.

Graph $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$ and its 5-locating coloring is given in **Figure 1**.

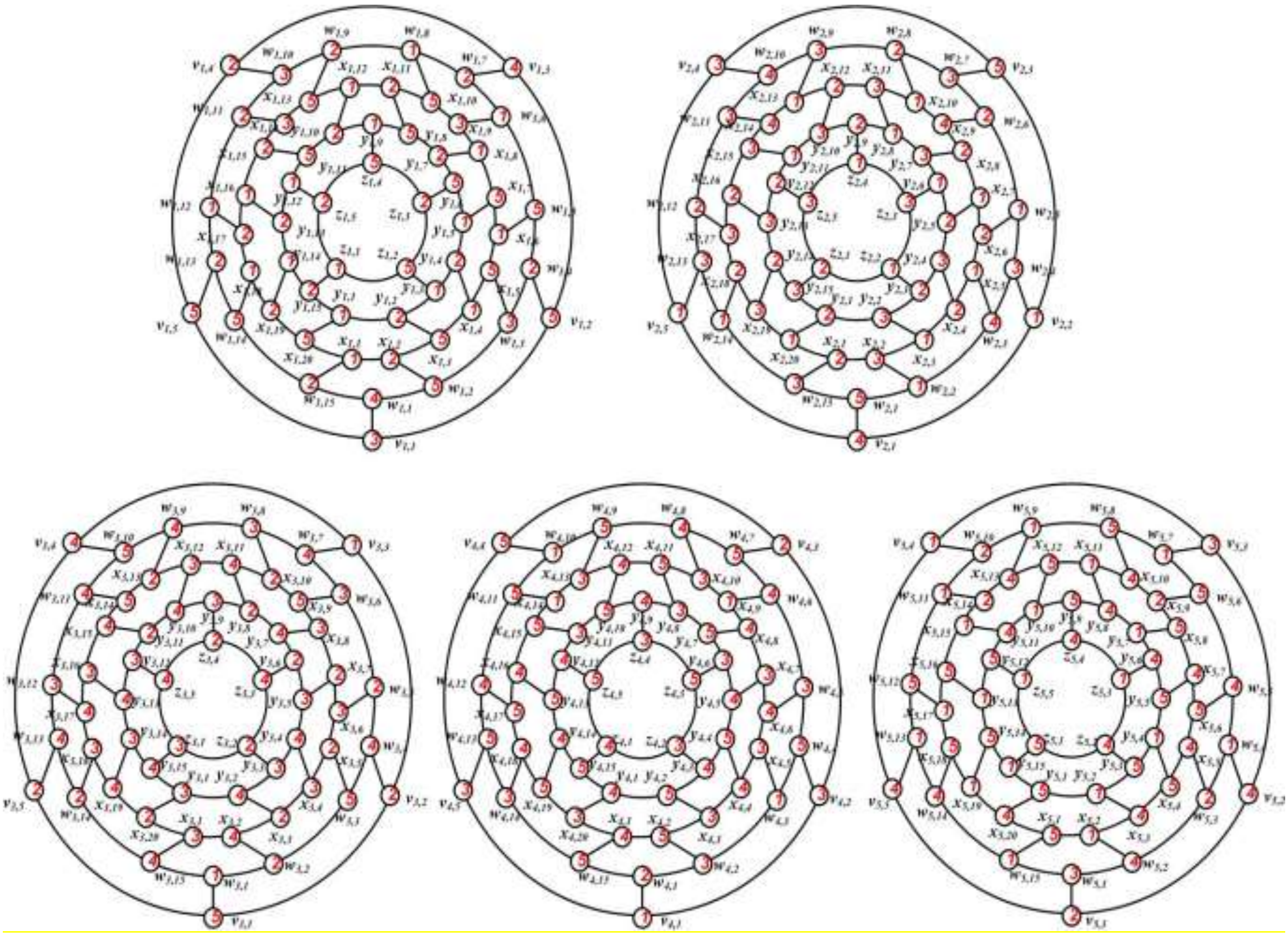

Figure 1. The 5-locating-chromatic coloring of $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$. ■

4. CONCLUSIONS

In this paper, we modified the locating chromatic 5-coloring of one Buckminsterfullerene to construct the locating chromatic 5-coloring of the disjoint union of the Buckminsterfullerene graphs, denoted by $H = \bigcup_{s=1}^{5} B_{60}^{(s)}$.

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