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COMMISSIONERS METHOD FOR LAST SURVIVOR WHOLE LIFE INSURANCE RESERVES WITH GOMPERTZ LAW

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ABSTRACT

Life insurance plays an important role in protecting one's life in terms of financial risks. The right required reserve is one aspect that should be secured to obtain a good company's financial health. A study reported that insufficient reserve is one of the primary causes of insolvency issues among bankrupted insurance companies. Therefore, this study aims to use the Commissioners method in calculating the reserve of the last-survivor whole life insurance policy The Gompertz mortality law is used to estimate Indonesian death probability of age x. Gompertz law is employed due to its practical convenience in dealing with continuous case calculation. Linear Least Square Method will be utilized to estimate the Gompertz parameters B and c. This study uses secondary data from Indonesian Mortality Table IV published by OJK. The result of this study is that the reserve of multiple life status policies, such as last-survivor insurance, depends on the states of individuals of each pair. In this study, the reserve has the highest amount in cases where only the male person is alive. Average Relative Error values of the estimated Gompertz p_x to the TMI IV show that the overall accuracy exhibits a deviation of up to 1.67%. MAPE values of Gompertz p_x show that the estimated model best fits the TMI IV for the interval age of 0-60.

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1. INTRODUCTION

In business practice, insurance companies must obey some provisions regarding the company's financial health, one of which is in managing premium funds so that the company can fulfill claims filed by the policyholder, or any other obligations arising from the insurance policy. In Indonesia, technical reserve, one of which is premium reserve, is one of the measurements of an insurance company's financial health as regulated in [1]. Inadequate reserves can contribute to the insolvency of insurance companies. Insurance insolvency occurs when the company faces financial distress and the regulator asses that they won't be able to fulfill their liabilities [2]. A 1999 study published by AM Best reported that out of 426 bankrupted insurance companies, 145 were identified as failing due to insufficient reserves [3]. Therefore, appropriate reserve calculations are needed to ensure the continuity of the insurance businesses and the policyholder's protection.

The Gompertz mortality law is a model frequently used in the analysis of survivorship. The usage of Gompertz mortality functions brings at least two advantages: it can describe the human survivorship phenomenon by a simple formula and it is practically easier to deal with continuous cases using fewer parameters of mortality function [4]. In a study comparing the Gompertz and Makeham mortality laws, an individual's life probability according to the Indonesian Mortality Table (TMI) was more accurate when modeled using the Gompertz law, as indicated by the Average Relative Error [5]. One of the methods that can be employed to estimate Gompertz parameters based on the TMI is the linear least square method [6].

Life insurance, based on the number of participants, can be divided into two categories: individual life insurance and multiple life insurance for more than one individual. Multiple life insurance can be categorized into last-survivor and joint life insurance. Joint life insurance paid benefits to survivors at the earliest death of the participants. The last-survivor policy, or second-to-die insurance, paid the benefits to the heirs at the last death of the participants [7]. Compared to an individual policy, a last-survivor status is typically cheaper with the same amount of benefit [8]. In a last-survivor policy, since the insurers might not be aware of the current state of the pairs, an appropriate solution would be to hold the weighted average of the different reserves [9].

The general principle used in premium calculation is the equivalence principle [10]. By employing this principle, the net level premium can be determined, which is constant throughout the year. In practice, an insurance policy requires higher expenses in the first year compared to the subsequent years [11]. Therefore, a modification in calculating reserves is needed besides the usage of the common prospective valuation method. In [12], where Commissioners, Illinois, and Canadian were evaluated to calculate reserve for single-life endowment policy, the Commissioner's reserve in the first year was less than the Illinois reserve but larger than the negative-value Canadian reserve. In [13], Commissioner and Illinois methods were explored to calculate reserves for single-life endowment life insurance, where the constants of the rate of mortality function influence the survival probability. Besides, under the Illinois method, the modified net premium matches the Commissioner method when there are up to 20 payments but exceeds it for more than 20 payments. Furthermore, in a study by [14], the Commissioners method gives the lowest amount of reserve compared to Canadian and Premium Sufficiency methods.

Based on the presented background, reserve calculation is an essential element in the insurance business. This study aims to discuss reserve calculation of last-survivor whole life insurance policy with Gompertz mortality assumption employing linear least square estimation method towards TMI IV. Recognizing the need for a method modification, the Commissioner method will be applied in the reserve calculation.

2. RESEARCH METHODS

This study is conducted quantitatively, involving the collection and utilization of numerical data, which is then processed using the quantitative method. This study was conducted using the research and design approach, which involved studying previous findings related to the field being researched and then developing and/or revising the identified drawbacks [15]. In this study, the writer conducted a literature study to gather information from previous studies that had been previously conducted in reserve calculations and then applied them to calculate the whole life last-survivor insurance using the Commissioners method.

2.1 Commissioners Method

In this study, the Commissioner's method will be used to calculate reserves for a last-survivor whole life insurance policy. The method of Commissioners is a modified calculation system of net level premium assumptions, where the initial premium in the first year is set to be lower than the net level premium $\boldsymbol{P}$, and the renewal premium for the subsequent years is set to be higher, consequently. This method involves the general steps of calculating the actuarial present value of benefits and future premiums, calculating the net level premium, then calculating the initial and renewal premiums, calculating the reserves of each possible case, and lastly, calculating the weighted average reserve. Let $\boldsymbol{P}$ is the net level premium, then the steps of obtaining the Commissioner's reserves can be described as follows.

Table 1. The Commissioners Method of Last-Survivor Whole Life Insurance Reserve Steps

No.	Steps	Formula
1.	Calculation of renewal premium $\boldsymbol{\beta}$	$\boldsymbol{\beta} = \boldsymbol{P} + \boldsymbol{B}\dfrac{{}_{19}\boldsymbol{P}_{\overline{xy}+1} - \boldsymbol{c}_{\overline{xy}}}{\ddot{\boldsymbol{a}}_{\overline{xy}:\overline{n}\vert}}$
2.	Calculation of renewal premium $\boldsymbol{\alpha}$	$\boldsymbol{\alpha} = \boldsymbol{\beta} - \boldsymbol{B}\left({}_{19}\boldsymbol{P}_{\overline{xy}} + \boldsymbol{c}_{\overline{xy}}\right)$
3.	Calculation of Commissioners reserves if both male and female alive at $\boldsymbol{t}$	${}_{t}\boldsymbol{V}^{(0)} = \bar{\boldsymbol{A}}_{\overline{xy}+t:\overline{\omega-t}\vert} - \boldsymbol{\beta}\ddot{\boldsymbol{a}}_{\overline{xy}+t:\overline{\omega-t}\vert}$
4.	Calculation of Commissioners reserves if only male alive at $\boldsymbol{t}$	${}_{t}\boldsymbol{V}^{(1)} = \bar{\boldsymbol{A}}_{x+t:\overline{\omega-t}\vert} - \boldsymbol{\beta}\ddot{\boldsymbol{a}}_{x+t:\overline{n-t}\vert}$
5.	Calculation of Commissioners reserves if only female alive at $\boldsymbol{t}$	${}_{t}\boldsymbol{V}^{(2)} = \bar{\boldsymbol{A}}_{y+t:\overline{\omega-t}\vert} - \boldsymbol{\beta}\ddot{\boldsymbol{a}}_{y+t:\overline{n-t}\vert}$
6.	Calculation of weighted average Commissioners reserves at $\boldsymbol{t}$	${}_{t}\boldsymbol{V}^{w} = \dfrac{{}_{t}\boldsymbol{p}_{\overline{xy}}\,{}_{t}\boldsymbol{V}^{(0)} + \left({}_{t}\boldsymbol{p}_{x} - {}_{t}\boldsymbol{p}_{\overline{xy}}\right){}_{t}\boldsymbol{V}^{(1)} + \left({}_{t}\boldsymbol{p}_{x} - {}_{t}\boldsymbol{p}_{\overline{xy}}\right){}_{t}\boldsymbol{V}^{(2)}}{{}_{t}\boldsymbol{p}_{\overline{xy}}}$

$\boldsymbol{x}$	:	the age of male person
$\boldsymbol{y}$	:	the age of female person
$\boldsymbol{B}$	:	policy benefit
$\boldsymbol{c}_{\overline{xy}}$	:	premium of one-year term policy issued at age for last-survivor status
${}_{19}\boldsymbol{P}_{\overline{xy}+1}$	:	premium of 19 payments whole life insurance for last-survivor status
$\ddot{\boldsymbol{a}}_{\overline{xy}:\overline{n}\vert}$	:	actuarial present value of $\boldsymbol{n}$-year temporary life annuity
$\boldsymbol{\omega}$	:	maximum age of of the policy

2.2 Data and Tools

This study uses secondary data, such as Indonesian Mortality Table IV, which was published by AAJI, OJK, PAI, PT Reasuransi Indonesia Utama, and Readi Project [16]. TMI IV was a study of Indonesian mortality from 2013 to 2017. This table contains information on the probability of death of Indonesian people aged 0-111 for men and women. To calculate the numerical data using quantitative methods, Microsoft Excel spreadsheet software will be mainly used due to its effectiveness in processing mortality data in the form of tables. Python will also be utilized to calculate continuous actuarial present value (APV), which involves numerous integrations.

2.3 Case Illustration

A married couple, aged 35 and 32 years old, purchased a last-survivor whole life insurance policy that provides a death benefit to the beneficiaries when both of them have passed away during the insurance period or if at least one of the insured remains alive until the age of 100. The death benefit is Rp1.000.000.000, and the premium payments are made for a duration of 15 years, with payments made at the beginning of each year. The interest rate is assumed to follow the BI rate in August 2023 of 5.75%.

Note that the age limit at policy termination is an assumption. In its calculation, this case is similar to a term life insurance policy. However, in this case, there is a difference in the insured's age at the end of the policy contract, which is predetermined at the inception of the policy, that is, when the insured reaches 100 years of age. Meanwhile, in a term life insurance policy, the insured's age at the end of the policy contract depends on the age at which the insured is registered on the policy. However, this research does not alter the initial definition of a whole life insurance policy, which stipulates that the benefit will be paid out whenever the insured passes away.

The process of calculating reserves in this study is given by the following steps:

1. Estimate Gompertz parameter $\boldsymbol{B}$ and $\boldsymbol{c}$.
2. Create Gompertz mortality table.
3. Calculate APV of benefit.
4. Calculate APV of life annuity.
5. Find net level premium $\boldsymbol{P}$ with equivalence principle.
6. Find modified Commissioners premium.
7. Calculate modified Commissioners reserve.

3. RESULTS AND DISCUSSION

3.1 Estimate Gompertz Parameter $\boldsymbol{B}$ and $\boldsymbol{c}$

Gompertz parameters B and c will be estimated using the method of Least Square. The Gompertz rate of mortality function is given by:

$$\mu_x = Bc^x, B > 0, c > 1 \tag{1}$$

x : age of a person.

The parameter B represents the initial mortality rate, while c represent the rate at which the mortality increases with age. From **Equation (1)**, the following linear estimation function can be defined:

$$\ln\left[\ln\left(\frac{1}{1-q_x}\right)\right] = \ln c \cdot x + \ln\left(\frac{B(c-1)}{\ln c}\right) \tag{2}$$

q_x: the probability of a person aged x dies between $[x, x+1)$.

Equation (2) is a linear form with following definitions:

$$y = \ln\left[\ln\left(\frac{1}{1-q_x}\right)\right], b = \ln c\,, a = \ln\left(\frac{B(c-1)}{\ln c}\right).$$

3.1.1 Linear Least Square Method

Given the data $\{(x_1, y_1), \dots, (x_N, y_N)\}$, a linear estimation function:

$$\hat{y} = bx + a \tag{3}$$

The method of linear least square is to find the numerical values of a and b such that to minimize the sum of squares error given by:

$$E = \sum_{n=1}^{N}(y_n - \hat{y}_n)^2 = \sum_{n=1}^{N}(y_n - a - bx_n)^2$$

This requires partial differentiation:

$$\frac{\partial E}{\partial a} = 0, \frac{\partial E}{\partial b} = 0.$$

The values of a and b are then obtained as:

$$b = \frac{\sum_{n=1}^{N} x_n y_n - \bar{y}\sum_{n=1}^{N} x_n}{\sum_{n=1}^{N}(x_n - \bar{x})x_n} \quad (4)$$

$$a = \frac{\sum_{n=1}^{N} y_n}{N} - b\frac{\sum_{n=1}^{N} x_n}{N} = \bar{y} - b\bar{x} \quad (5)$$

x_n: age, from 0 to 11

$y_n = \ln\left[\ln\left(\frac{1}{1-q_{x_n}}\right)\right]$.

Therefore, Gompertz parameters can be expressed as [17]

$$c = e^b \quad (6)$$

$$B = \frac{e^a \ln c}{c-1} \quad (7)$$

After obtaining the Gompertz parameters for male and female, the estimated Gompertz parameters are summarized in Table 2. The variable x denotes the age of the male person, while y denotes the age of female person.

Table 2. Estimated Gompertz Parameters Using Linear Least Square

Gender	Gompertz Parameter	
	B	c
Male (x)	0.00009051	1.08441
Female (y)	0.00006608	1.08447

Based on Table 2, B=0.00006608 for female means that at the age of 0, the mortality rate is 0.00006608. As age increases, the mortality increases at a rate of 1.08447.

3.2 Create Gompertz Mortality Table

The establishment of the Gompertz mortality table involves the calculation of survival functions of man and woman aged 0-111. Based on the estimated parameters, the survival probability of a man aged x is given by:

$$p_x = \exp\left[\frac{-Bc^x}{\ln c}(c-1)\right] = \exp\left[\frac{-(0.00009051)1.08441^x}{\ln 1.08441}(0.08441)\right] \quad (8)$$

Similarly, the survival probability of a woman aged y is given by:

$$p_y = \exp\left[\frac{-Bc^x}{\ln c}(c-1)\right] = \exp\left[\frac{-(0.00009051)1.08441^x}{\ln 1.08441}(0.08441)\right] \quad (9)$$

Using the relationship $q_x = 1 - p_x$, the mortality table with Gompertz mortality law assumption is shown in Table 3.

Table 3. Gompertz mortality table

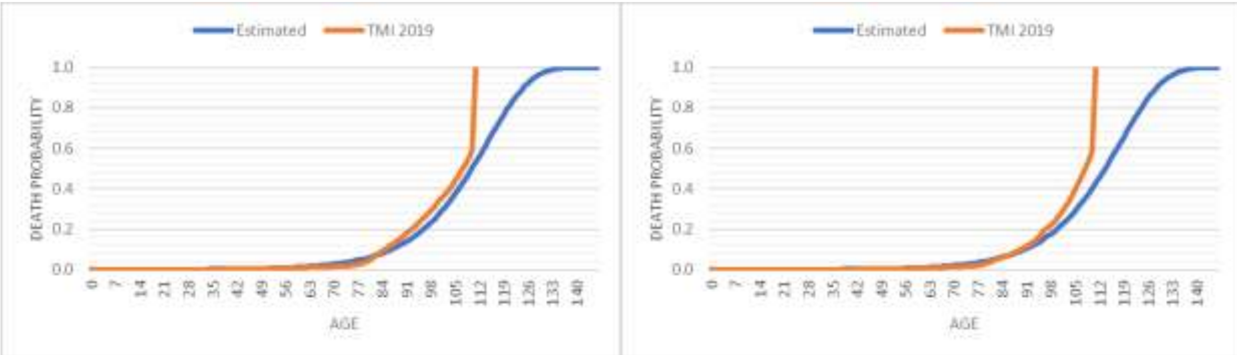
Age	q_x	q_y
0	0.00009	0.00007
1	0.00010	0.00007
2	0.00011	0.00008
3	0.00012	0.00009
4	0.00013	0.00010
5	0.00014	0.00010
6	0.00009	0.00007
⋮	⋮	⋮
108	0.44913	0.33214

Age	q_x	q_y
109	0.47617	0.35453
110	0.50400	0.37796
111	0.53250	0.40241

q_x: the probability of a man aged x dies between $[x, x+1)$.

q_y: the probability of a woman aged y dies between $[y, y+1)$.

The Gompertz death probability, as shown in **Table 3**, has a noticeable difference in which the death probability at the maximum age is not equal to 1. Calculations can be continued until the probability of death is equal to 1 to determine the maximum age under the Gompertz assumption. **Figure 1** shows the comparison between the estimated probability of death and the probability of death values in the TMI IV.



(a) **(b)**

Figure 1. The Comparison Graph between the Estimated Probability of Death and TMI IV, (a) Male, (b) Female

From **Figure 1**, it can be seen that the probability of death with the Gompertz assumption at the age of 111 is not equal to 1, as in the TMI IV. Based on the calculation, it can be inferred that the application of the Gompertz mortality law on the TMI IV, employing the linear least square parameter estimation method, leads to an extension of the maximum age up to the age of 146. Besides, there is a significant difference between the estimated model and TMI for ages 65 and above. This error can be measured in terms of Average Relative Error (ARE) that can be obtained by [18]:

$$ARE = \frac{\sum_{i=0}^{n} |e_i - o_i|}{\sum_{i=0}^{n} o_i} \qquad (10)$$

e_i: the estimated value
o_i: the actual value.

The ARE values with death probability values are too much larger, hence using one-year survival probability values yields more meaningful results. The values of survival probability must be found first based on the data in **Table 3**. Then, the Absolute Relative Error is given by:

Table 4. Gompertz ARE of p_x and p_y

Gender	ARE (%)
Male	1.67
Female	1.66

According to the data presented in **Table 4**, it is evident that the overall accuracy of the estimated p_x and p_y values derived from the Gompertz model in relation to the TMI IV exhibit a deviation of up to 1.67% for males and 1.66% for females.

By evaluating the error in intervals of ages, the error can be analyzed further using the Mean Absolute Percentage Error (MAPE) which can be expressed as [5]

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{o_i - e_i}{o_i} \right|}{n} 100\% \qquad (11)$$

Dividing the ages into 11 equal intervals, the MAPE values of p_x and p_y is obtained as shown in **Table 5**.

Table 5. MAPE Values of the Estimated p_x and p_y

Age	MAPE (%)	
	Male	Female
1-10	0.01	0.01
11-20	0.00	0.00
21-30	0.02	0.02
31-40	0.05	0.04
41-50	0.05	0.08
51-60	0.07	0.13
61-70	0.74	0.49
71-80	1.81	1.17
81-90	2.19	0.53
91-100	7.23	4.06
101-110	13.66	21.92

Table 5 shows the MAPE values of one-year survival probability of male (p_x) and female (p_y) of each 10-year interval. The values of p_x and p_y is obtained from the death probability for male and female in **Table 3**. These intervals are divided into 11 equal length intervals starting from age 1 to 110 to provide a general overview of the absolute error of the Gompertz model against the already created TMI IV in that range.

Based on these results, there are two things that can be inferred regarding the situation in Indonesia. First, the use of the Gompertz mortality law in the TMI IV is quite effective in describing the likelihood of death in the Indonesian population for ages 0 to approximately 60 years. Second, the use of the Gompertz mortality law to assess the likelihood of death in the Indonesian mortality table for ages 60 and above is not recommended due to a significant difference in probabilities and the presence of different maximum ages. In the Gompertz assumption, the maximum age is approximately 146 years, while the maximum age in the IV TMI is 111 years.

3.3 APV of Benefit

In calculating the APV of the benefit, several things to consider are: the benefit is paid at the moment of the second death; the benefit is paid if at least one of the insured remains alive until 100; the maximum insurance period is 68 years (100-32). The relationship between APV of benefit and life annuity will be utilized here. Since the formulation for $\bar{a}_{\overline{35,32}:\overline{68}|}$ is relatively simpler to compute, the APV of future benefit in this case can be obtained by

$$\begin{aligned}\bar{A}_{\overline{35,32}:\overline{68}|} &= 1 - \delta\bar{a}_{\overline{35,32}:\overline{68}|}\\ &= 1 - \ln(1.0575)\int_0^{68} e^{-0.05591t}\\ &\cdot\left(e^{-\frac{B_m c_m^{35}}{\ln c_m}(c_m^t-1)} + e^{-\frac{B_f c_f^{32}}{\ln c_f}(c_f^t-1)} - e^{-\frac{B_m c_m^{35}}{\ln c_m}(c_m^t-1)-\frac{B_f c_f^{32}}{\ln c_f}(c_f^t-1)}\right)dt\\ &= 0.05763\end{aligned}$$

B_m : B value for male

B_f : B value for female

c_m : c value for male

c_f : c value for female

Note that this policy ends if at least one of the pair individuals reaches the age of 100. The upper limit for the integration calculation for actuarial present value of benefit is therefore 68. This might be similar to a term life policy where the calculation involves upper limit. But in this case, the contract termination is determined at the policy inception, aside from the ages of the insured. On the other hand, the age of the insured at the end of the term life policy contract depends on the age when the insured entered the policy.

3.4 APV of Life Annuity

Furthermore, since the premiums are paid annually, annuities are calculated discretely using the mortality table that has been computed based on the Gompertz assumption. When calculating last-survivor annuities, the symmetric relationship will frequently make the calculation easier [19]. The 15-year temporary life annuity for last-survivor status of 35 and 32 can be calculated as follows:

$$\begin{aligned}\ddot{a}_{\overline{35,32}:\overline{15}|} &= \ddot{a}_{35:\overline{15}|} + \ddot{a}_{32:\overline{15}|} - \ddot{a}_{35,32:\overline{15}|}\\ &= \left(\frac{N_{35} - N_{50}}{D_{35}}\right) + \left(\frac{N_{32} - N_{47}}{D_{32}}\right) - \left(\frac{N_{35,32} - N_{50,47}}{D_{35,32}}\right)\\ &= 10.29846 + 10.35865 - 10.21845\\ &= 10.43866.\end{aligned}$$

3.5 Net Level Premium *P*

The annual net premium of last-survivor whole life insurance policy for 35 and 32 years old with the insured benefit of Rp1.000.000.000 can be calculated as follows:

$$\begin{aligned}P &= B\,\frac{\bar{A}_{\overline{35,32}:\overline{68}|}}{\ddot{a}_{\overline{35,32}:\overline{15}|}}\\ &= Rp1.000.000.000\,\frac{0.05763}{10.43866}\\ &= Rp5.521.150,50\end{aligned}$$

3.6 Modified Premiums α and β

Then, the renewal premium of Commissioners method is calculated as follows:

$$\begin{aligned}\beta &= P + Rp1.000.000.000\left(\frac{{}_{19}P_{\overline{36,33}} - c_{\overline{35,32}}}{\ddot{a}_{\overline{35,32}:\overline{15}|}}\right)\\ &= P + Rp1.000.000.000\left(\frac{0.005032 - 0.000001426}{\ddot{a}_{\overline{35,32}:\overline{15}|}}\right)\\ &= Rp6.003.061,03\end{aligned}$$

and the renewal premium

$$\alpha = \beta - Rp1.000.000.000\left({}_{19}P_{\overline{36,33}} + c_{\overline{35,32}}\right) = \text{Rp}483.336,51.$$

3.7 Commissioners Reserves

In calculating the reserve of last-survivor status, three possible states of the status will be defined as follows:

- ${}_{t}V^{(0)} = \bar{A}_{\overline{35,32}+t:\overline{68-t}|} - \beta\ddot{a}_{\overline{35,32}+t:\overline{15-t}|}$ if both x and y alive at t
- ${}_{t}V^{(1)} = \bar{A}_{35+t:\overline{68-t}|} - \beta\ddot{a}_{35+t:\overline{15-t}|}$ if only x alive at t
- ${}_{t}V^{(2)} = \bar{A}_{32+t:\overline{68-t}|} - \beta\ddot{a}_{32+t:\overline{15-t}|}$ if only y alive at t

After calculate the reserves for each possible cases, then the weighted average of the different reserves can be calculated as follows [9]:

$$ {}_{t}V^{w} = \frac{ {}_{t}p_{35,32}\,{}_{t}V^{(0)} + \left({}_{t}p_{35} - {}_{t}p_{35,32}\right){}_{t}V^{(1)} + \left({}_{t}p_{32} - {}_{t}p_{35,32}\right){}_{t}V^{(2)} }{ {}_{t}p_{\overline{35,32}} } \quad (12) $$

The commissioner's method is a modification of prospective calculations by replacing the net level premium P by renewal year premium β. This modification is made due to the fact that the policy expense in the first year is higher than in the subsequent years. This implies a modification of the initial premium to be lower than P and renewal year premium β to be higher than P. In the modification system, in this case, the Commissioner's method, the total premium paid remains unchanged (i.e., the same as the gross premium using net level premium). In other words, the modification is implemented to allocate a larger portion of the costs in the first year. Meanwhile, in the assumption of net level premium, the cost component in the gross premium remains the same every year.

Table 6. Commissioners Reserve of Last-Survivor Status for 35 and 32 Years Old (in Rp1,000,000)

t	${}_{t}V^{(0)}$	${}_{t}V^{(1)}$	${}_{t}V^{(2)}$	${}_{t}V^{(w)}$	t	${}_{t}V^{(0)}$	${}_{t}V^{(1)}$	${}_{t}V^{(2)}$	${}_{t}V^{(w)}$
1	0.90	76.15	39.39	1.17	35	320.80	493.23	401.02	363.18
2	7.15	85.23	47.03	7.59	36	334.18	507.50	414.49	379.29
3	13.74	94.72	55.02	14.37	37	347.89	521.85	428.16	395.73
4	20.69	104.62	63.40	21.54	38	361.92	536.25	442.02	412.48
5	28.01	114.96	72.18	29.12	39	376.25	550.67	456.05	429.48
6	35.74	125.74	81.37	37.13	40	390.88	565.10	470.25	446.72
7	43.87	136.99	90.99	45.59	41	405.78	579.52	484.60	464.13
8	52.45	148.73	101.06	54.52	42	420.95	593.89	499.07	481.68
9	61.48	160.97	111.61	63.96	43	436.37	608.21	513.67	499.32
10	70.99	173.73	122.64	73.92	44	452.01	622.45	528.36	517.01
11	81.00	187.03	134.19	84.44	45	467.87	636.59	543.15	534.70
12	91.54	200.89	146.27	95.55	46	483.93	650.60	558.00	552.34
13	102.63	215.35	158.91	107.27	47	500.17	664.47	572.92	569.88
14	114.30	230.41	172.14	119.63	48	516.58	678.18	587.88	587.30
15	126.57	246.10	185.97	132.68	49	533.14	691.72	602.88	604.55
16	133.12	256.08	194.07	140.08	50	549.84	705.05	617.92	621.59
17	139.97	266.34	202.45	147.86	51	566.69	718.19	632.98	638.42
18	147.12	276.88	211.11	156.04	52	583.68	731.10	648.07	655.00
19	154.59	287.70	220.04	164.62	53	600.82	743.80	663.20	671.34
20	162.37	298.79	229.27	173.62	54	618.13	756.27	678.38	687.44
21	170.48	310.16	238.77	183.05	55	635.65	768.52	693.66	703.33
22	178.92	321.79	248.57	192.91	56	653.44	780.58	709.06	719.04
23	187.70	333.69	258.65	203.23	57	671.59	792.47	724.66	734.66
24	196.83	345.84	269.01	214.01	58	690.20	804.24	740.55	750.29
25	206.31	358.23	279.67	225.26	59	709.47	815.97	756.85	766.06
26	216.14	370.86	290.60	236.97	60	729.63	827.78	773.75	782.19
27	226.33	383.72	301.82	249.16	61	751.03	839.86	791.51	798.96
28	236.88	396.80	313.31	261.83	62	774.14	852.48	810.47	816.77
29	247.79	410.08	325.08	274.96	63	799.63	866.10	831.14	836.16
30	259.07	423.55	337.11	288.57	64	828.38	881.46	854.23	857.91
31	270.70	437.20	349.40	302.63	65	861.57	899.72	880.76	883.14
32	282.70	451.01	361.95	317.14	66	900.58	922.86	912.23	913.45
33	295.05	464.96	374.74	332.08	67	946.73	954.29	950.86	951.21
34	307.75	479.04	387.77	347.43	68	1,000	1,000	1,000	1,000

Table 6 presents the weighted average of the Commissioner's reserves using **Equation (12)**. From this table, it can be seen that the last survivor status influences the amount of the reserves. In the case where only the male of the couple is still alive, the reserve size increases significantly. This occurs because when the female dies, their status is affected by a greater risk of failure, which now depends only on the survival of the

male, resulting in a larger required reserve. Then, as expected, at the end of the insurance term, the value is equal to the benefit, that is $Rp. 1.000.000.000$.

The approach of the weighted average is applicable when the insurers will not know the actual status of the couple immediately, whether the couples still alive or one of them has passed away. By this method, the actual. In this case, the appropriate solution would be to hold a weighted average of the different reserves [14].

3.8 Comparison with Prospective Reserves

To see the differences that occur, a prospective reserve calculation will be performed, and the results will be compared with the reserves using the Commissioners method. The results are stated in Table 7. The results are obtained using Equation (12) where weighted average reserve is calculated on net level premium and modified reserve. The process includes the calculation of actuarial present value of benefit and life annuities, calculation of premium P assuming net level premium assumption, then calculation of Commissioners premiums, β and α. Lastly, using premiums value with respect to each method, the weighted average reserves can be obtained. In other words, the difference of the two methods is based on the usage of net level premium P and renewal premium β in the calculation of the weighted average reserves.

Table 7. Comparison of Prospective and Commissioners Reserve

n	Commissioners	Prospective
1	1,168,506.31	5,978,520.00
2	7,587,686.64	12,164,684.21
3	14,372,501.31	18,703,103.58
4	21,543,065.84	25,613,126.41
5	29,120,483.36	32,915,044.15
6	37,126,875.20	40,630,119.23
7	45,585,410.01	48,780,611.10
8	54,520,331.13	57,389,800.25
9	63,956,981.92	66,482,009.90
10	73,921,828.83	76,082,624.97
11	84,442,482.23	86,218,108.31
12	95,547,714.75	96,916,014.04
13	107,267,477.68	108,204,998.05
14	119,632,915.62	120,114,826.16
15	132,676,380.45	132,676,380.45

From Table 7, it can be clearly seen that in the Commissioners method, the reserve size is much smaller compared to the reserve with the prospective method. This can occur due to the size of the renewal premium β that larger than the net level premium. By rechecking the reserve formula in section 3.5, the value of the renewal premium is inversely proportional to the reserve size. In other words, if the renewal premium is larger than the net level premium, then the reserve with the net level premium assumption will be larger than the reserve with the Commissioner’s method.

From this result, it can be understood that the use of the Commissioner's reserve calculation method can be applied to newly established insurance companies with limited cash amounts. With limited cash amounts, they are limited in number of policies issued during their first year. Hence, with the usage of the Commissioner's method, reserves in the first year can be reduced, allowing a larger portion to be allocated to policy costs in that year, allowing more new policies issued [20].

4. CONCLUSIONS

The application of the Gompertz mortality law with linear least square estimation method has its advantages and drawbacks. The use of the Gompertz mortality law in the TMI IV is effective in describing the likelihood of death in the Indonesian population for ages 0 to approximately 60 years. This is shown by the MAPE values of the estimated p_x, where the estimation process through linear least squares for 10-year intervals is close to the actual value in TMI until the age of 60. In other words, the model is best used to describe the younger population until before the pension age, or 60 years old. Furthermore, the usage of Gompertz mortality law enables us to calculate reserve where the benefit is paid at the moment of death, in this case, semi-continuous. However, the Gompertz model in this study deviates more after the age of 60, which can't be relied upon much when used to calculate reserves.

Besides, it is also concluded that for last-survivor status, the amount of reserve depends on the current situation of the survivorship of the couple. In this study, the case where only x alive gives the highest amount of reserve.

Lastly, it can also be concluded that the Commissioners reserves give a significantly lower amount of reserve compared to regular prospective reserves in the first year. This occurrence happens due to the modification of net level premium into initial and renewal premiums.

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