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# LESLIE MATRIX CONSTRUCTION AND ITS PROPERTIES IN PREDICTING THE SIZE OF THE FEMALE POPULATION

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*ABSTRACT*

*The growth of the female population needs to be considered because if the female population is not controlled, it is feared that it could trigger a population explosion. Using the Leslie matrix, this study aimed to predict the number and growth rate of the female population in Batugajah Village, Mdona Hyera Subdistrict, Southwest Maluku Regency. Based on the results of the study, the female population in Batugajah Village, Mdona Hyera Subdistrict, Southwest Maluku Regency, is predicted to be 157 people in 2021, 150 people in 2022, and 142 people in 2023. Furthermore, the most significant positive eigenvalue is 0.6521, which indicates that the population growth rate will tend to decrease.*



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## 1. INTRODUCTION

In everyday life, mathematics is the basis of other sciences, for example, in the fields of biology and demography, which uses the concept of matrices to investigate the genotyping of offspring of a population and the projection of the number of female populations in a region [1]. Matrices are often tables of numerical data that arise from physical observations, but matrices are also constructed in various mathematical contexts [2]. The Leslie matrix model is a population growth model used by demographers [3] and named after its inventor, P. H Leslie, in 1945 [4]. The Leslie matrix is used to determine the prediction of the number and prediction of the growth rate of a population for the following year [5]. In the Leslie model, the birth and death process depend on age and is an integral part of population growth [6]. In addition, the Leslie matrix is only used in the female population [7].

Factors influencing population growth are fertility rate, survival rate, and age range [8]. These factors can determine the trend of population growth in an area, namely whether the population will increase, decrease, or stabilize in the following year [9]. Eigenvalues and eigenvectors are essential in determining the long-term population dynamics and whether the population is increasing, decreasing, or constant. The eigenvalues are used to define the population growth rate or provide valuable information about the state of the population, while the eigenvectors indicate the stability of the age distribution [10]. Furthermore, the population is said to increase if the positive eigenvalue is more significant than one; the population will decrease if the positive eigenvalue is less than one. The population will be stable if the positive eigenvalue equals one [11].

Several studies related to the Leslie matrix, namely research by Sanusi et al. [12], applied the Leslie matrix to predict the rate and population growth in Makassar City in 2017. Furthermore, in 2021, Maryati et al. [13] conducted research related to the application of the Leslie matrix and obtained the results that the number and growth rate of the female population in West Java Province in 2021 tended to increase. In the same year, Prayanti et al [14] made the Leslie matrix easier to work with by presenting it as *Perron root*. In 2022, Marizka et al. [15] constructed the Leslie matrix and studied its properties to predict the number and growth rate of the female population of the Yogyakarta Special Region in 2025.

From this description, this study determined the number of female populations in one of the villages in Southwest Maluku Regency, Maluku Province, namely Batugajah Village, based on birth rate and life expectancy using the eigenvalue and eigenvector of Leslie matrix.

## 2. RESEARCH METHODS

The research was conducted using a literature study. The data source used in this research is secondary data from population data taken in Batugajah Village, Mdona Hyera District, Southwest Maluku Regency. The stages or procedures of this research are as follows:

1. Understanding and studying the Leslie matrix model and its properties
2. Collecting data and determine the age classes
3. Finding the fertility value $(a_i)$ and survival value $(b_i)$
4. Constructing the Leslie matrix
5. Finding the eigenvalues and choosing the positive eigenvalue in the matrix to predict population growth rate in the following year.

The calculation of population prediction and determining the eigenvalue to determine the population growth rate was done using Maple13 software.

## 3. RESULTS AND DISCUSSION

In this study, researchers assumed that population changes were only influenced by births and deaths. Meanwhile, population movements in and out of the area under study are ignored or considered the same.

### 3.1 Leslie Matrix Construction

Fertility rate $a_i$ is the average number of daughters born to each cohort $i$, and resilience rate $b_i$ is the ratio of women who survive to enter the cohort to the number of women in cohort $i + 1$ group to the number of women in group $i$. Suppose $x_i^k$ is the number of women in the group $i$ in the observation $t_k$ for $i = 1,2,\cdots,n$. Then, the Leslie Model can be written with the equation

$$x^k = Lx^{k-1}, k = 1,2,\cdots,n$$

and

$$L = \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ b_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{n-1} & 0 \end{bmatrix}$$

is called the Leslie matrix.

For example, suppose the maximum age reached by any woman in a population is $T$ years (or any other unit of time). In that case, the population is divided into $n$ age classes. Thus, each class has a duration of $\frac{T}{n}$ years. The determination of these age classes can be seen in **Table 1.** below.

**Table 1. Determination of Age Classes**

| Age Class | Age Interval |
|---|---|
| 1 | $\left[0, \frac{T}{n}\right]$ |
| 2 | $\left[\frac{T}{n}, \frac{2T}{n}\right]$ |
| 3 | $\left[\frac{2T}{n}, \frac{3T}{n}\right]$ |
| $\vdots$ | $\vdots$ |
| $n-1$ | $\left[\frac{(n-2)T}{n}, \frac{(n-1)T}{n}\right]$ |
| $n$ | $\left[\frac{(n-1)T}{n}, T\right]$ |

For example, if the number of women is known in each class at time, $t = 0$. In particular, suppose there are $x_1^{(0)}$ women in the first class, $x_2^{(0)}$ women in the second class, and so on. With these $n$ numbers, a column vector is formed

$$x^{(0)} = \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \\ \vdots \\ x_n^{(0)} \end{bmatrix}$$

The vector $x^{(0)}$ is referred to as the initial age distribution vector. Furthermore, the parameter $a_i$ (fertility rate) is the average number of daughters born to each mother when the mother is in age class 1. $i$ where $a_i \geq 0$. At the same time, the parameter $x_i$ (initial population) is the number of women in the $i$-th age class. The following **Equation (1)** can be used to obtain the value of $a_i$.

$$a_i = \frac{A_i}{x_i} \qquad (1)$$

Parameter $A_i$ is the number of female births in the $i$-th age group, $(i = 1,2,\cdots,n)$. It is known that $a_i > 0$, because if $a_i = 0$, then no births occur in that class. Each age class with a value of $a_i > 0$ is called *fertile*. Next, the age distribution vector is defined by $x_k$ at time $t_k$ with

$$x^{(k)} = \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix}$$

where $x_1^{(k)}$ is the number of women in $i$-th age class at time $t_k$. Furthermore, at time $t_k$, women in the first age class are females born between time $t_{k-1}$ and $t_k$.

Furthermore, $b_i$ is defined as the survival rate of women in the $i$-th age class, i.e. the probability that an individual aged $i$ at time $t$ will stay alive until time $t+1$ and the individual reaches the age of $(i+1)$. The value of $b_i$ can be calculated using **Equation (2)** below.

$$b_i = \frac{x_{i+1}(t+1)}{x_i(t)} \tag{2}$$

with

$b_i$ : women's resilience level
$x_{i+1}$ : number of women in age class $1(i+1)$ at time zero$(t+1)$
$x_i(t)$ : number of females in $i$-th class at time $(t)$.

After the fertility rate $a_i$ and survival rate $b_i$ are known, then the Leslie matrix is constructed as follows.

$$L = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & a_n \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

This study assumes that population changes are only influenced by births and deaths, while population movements in and out are ignored or considered the same.

### 3.2 Properties of Leslie Matrix

The **Theorem 1 [16]** below is about the existence of a positive eigenvalue of the Leslie matrix.

**Theorem 1**. *A Leslie matrix L has a unique positive eigenvalue $\lambda_1$. This eigenvalue has multiplicity 1 and an eigenvector $x_1$ whose entries are all positive.*

The **Theorem 2 [16]** below is about the eigenvalue of the Leslie matrix.

**Theorem 2**. *If $\lambda_1$ is the unique positive eigenvalue of a Leslie matrix L, and $\lambda_k$ is any other real or complex eigenvalue of L, then $|\lambda_k| \leq \lambda_1$.*

The **Theorem 3 [16]** below is about the dominant eigenvalue of the Leslie matrix.

**Theorem 3**. *If two successive entries $a_i$ and $a_{1+1}$ in the first row of a Leslie matrix L are nonzero, then the positive eigenvalue of L is dominant.*

### 3.3 Application of Leslie Matrix in Predicting Female Population Growth

This subchapter will discuss the application of the Leslie matrix to predict the female population growth rate in Batugajah village, Mdona Hyera sub-district, Southwest Maluku Regency in 2020. The data used was the female population from January to December. From the research results, the maximum age of the female population in Batugajah Village is 73 years. Furthermore, the data obtained were data on the female population by age, data on the birth of females according to the age of the mother at childbirth, and data on the death of the female population. The data used can be seen in **Table 2** as follows.

**Table 2. Female Population in Batugajah Village, Mdona Hyera Sub-district, Southwest Maluku Regency, January - December 2020.**

| Age Class $(i)$ | Age Interval (Year) | Initial Population $(x_i^0)$ | Birth $(A_i)$ | Death $(B_i)$ |
|---|---|---|---|---|
| 1 | 0-11 | 45 | 0 | 0 |
| 2 | 12-22 | 34 | 0 | 0 |
| 3 | 23-33 | 20 | 3 | 0 |
| 4 | 34-44 | 24 | 2 | 0 |
| 5 | 45-55 | 17 | 0 | 0 |
| 6 | 56-66 | 13 | 0 | 1 |
| 7 | 67-77 | 6 | 0 | 1 |
| **Total** | | 159 | 5 | 2 |

Based on **Table 2.** the initial population of female residents $(x_i^0)$ in Batugajah Village is as many as 159 people, the number of births of females $(A_i)$ is as many as 5 people and the number of deaths of the

female population $(B_i)$ is as many as 2 people. Furthermore, the value of the fertility rate $(a_i)$ and the level of women's survival $(b_i)$ can be seen in **Table 3**.

**Table 3. Fertility and survival rates of women in Batugajah Village, Mdona Hyera Sub-district, Southwest Maluku District, January - December 2020.**

| Age Class $(i)$ | Age Interval (Year) | Fertility Rate $(a_i)$ | Survival Rate $(b_i)$ |
|---|---|---|---|
| 1 | 0-11 | 0 | 1 |
| 2 | 12-22 | 0 | 1 |
| 3 | 23-33 | 0.15 | 1 |
| 4 | 34-44 | 0.083 | 1 |
| 5 | 45-55 | 0 | 1 |
| 6 | 56-66 | 0 | 0.923 |
| 7 | 67-77 | 0 | 0.834 |

Based on **Table 3.** It is a Leslie matrix model with a size of 7 x 7 whose elements consist of fertility rates $(a_i)$ and survival rate $(b_i)$ female population as follows.

$$L = \begin{bmatrix} 0 & 0 & 0.15 & 0.083 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.923 & 0 \end{bmatrix}$$

The number and growth rate of the population for the next 3 years will be predicted using **Equation (3)** below.

$$x^{(k)} = L^k x^{(0)} \tag{3}$$

where

$L$ : Leslie matrix model (Leslie Matrix)
$x^{(k)}$ : Total number of women in $i$-th age class
$x^{(0)}$ : Initial population size of women

$$x^{(1)} = Lx^{(0)} = \begin{bmatrix} 0 & 0 & 0.15 & 0.083 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.923 & 0 \end{bmatrix} \begin{bmatrix} 45 \\ 34 \\ 20 \\ 24 \\ 17 \\ 13 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 45 \\ 34 \\ 20 \\ 24 \\ 17 \\ 12 \end{bmatrix}$$

$$x^{(2)} = L^2x^{(0)} = \begin{bmatrix} 0 & 0 & 0.15 & 0.083 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.923 & 0 \end{bmatrix}^2 \begin{bmatrix} 45 \\ 34 \\ 20 \\ 24 \\ 17 \\ 13 \\ 6 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 45 \\ 34 \\ 20 \\ 24 \\ 16 \end{bmatrix}$$

$$x^{(3)} = L^3x^{(0)} = \begin{bmatrix} 0 & 0 & 0.15 & 0.083 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.923 & 0 \end{bmatrix}^3 \begin{bmatrix} 45 \\ 34 \\ 20 \\ 24 \\ 17 \\ 13 \\ 6 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \\ 5 \\ 45 \\ 34 \\ 20 \\ 22 \end{bmatrix}$$

Thus, the number of female residents in Batugajah Village, Mdona Hyera Sub-district, Southwest Maluku Regency in 2021 is 157 residents. In 2022, it is 150 residents; in 2023, it is 142. Next, the growth

rate of the female population in Batugajah Village will be predicted using the eigenvalue of the Leslie matrix. Using the Maple13 application, the largest eigenvalue of the Leslie matrix is obtained in **Table 4.** as follows.

**Table 4. Eigenvalues of Leslie Matrix**

| | |
|---|---|
| $\lambda_1$ | 0 |
| $\lambda_2$ | 0 |
| $\lambda_3$ | 0 |
| $\lambda_4$ | 0.6521 |
| $\lambda_5$ | 0.1293 + 0.5538i |
| $\lambda_6$ | -0.1293 - 0.5538i |
| $\lambda_7$ | -0.3935 |

The actual positive eigenvalue is 0.6521. Thus, the growth rate of the female population in Batugajah Village, Mdona Hyera Sub-district, Southwest Maluku Regency will decrease.

## 4. CONCLUSIONS

Based on the results and discussion, it can be concluded that the prediction of the total female population in Batugajah Village, Mdona Hyera Subdistrict, Southwest Maluku Regency in 2021 is 157 people, 2022 is 150 people, and in 2023 is 142 people, with the most significant positive eigenvalue is 0.6521 which indicates that the population will decrease. In this study, only one village data was used by researcher. In the next study, it is recommended to use wider data at the sub-district or district/city level and even the provincial level.

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