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APPLICATION OF QUADRATIC PROGRAMMING ON PORTFOLIO OPTIMIZATION USING WOLFE'S METHOD AND PARTICLE SWARM OPTIMIZATION ALGORITHM

Syaripuddin[1], Fidia Deny Tisna Amijaya[2*], Wasono[3], Shanaz Tulzahrah[4], Rara Suciati[5]

[1,2,3,4,5] *Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Mulawarman*
Jln. Barong Tongkok No.04, Gunung Kelua, Samarinda, 75123, Indonesia

*Corresponding author's e-mail: * **fidiadta@fmipa.unmul.ac.id***

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ABSTRACT

Stock portfolios can be modeled into quadratic programming problems using the Markowitz mean-variance model. Quadratic programming problems can be solved using two methods, namely classical and heuristic methods. In this research, the classical method uses Wolfe's method, while the heuristic method uses the particle swarm optimization (PSO) algorithm. This research aims to determine optimal results in portfolio problems using two methods, namely Wolfe's method and the PSO algorithm. The data used in this research is data from 10 stock companies that distribute the highest dividends in the IDX High Dividend 20 category for the 2022 period. The research results discuss the portfolios of PT Astra International Tbk and PT. Indo Tambangraya Megah Tbk. Based on the result, using Wolfe's method, the ASII and ITMG stock portfolios are obtained, namely the optimal proportion of ASII shares = 0.76401 or 76.401% and ITMG shares = 0.23598 or 23.598%, while the PSO algorithm obtains a portfolio of ASII and ITMG shares, namely ASII shares = 75.02% and ITMG shares = 24.98%. Compared to Wolfe's method, the PSO algorithm has a smaller Z value 5.7.

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1. INTRODUCTION

Quadratic programming is an optimization problem that has the characteristics of a quadratic objective function with a linear constraint function [1]. In quadratic programming, we want to determine the variable values that minimize or maximize the quadratic objective function that satisfies several linear constraints [2]. There are two ways to solve quadratic programming problems: the classical method [3] and the heuristic method [4]. This research uses two methods, namely the classical method using Wolfe's method, while the heuristic method uses the particle swarm optimization (PSO) algorithm. Wolfe's optimization algorithm is developed based on the Karush-Kuhn-Tucker (KKT) conditions [5]. This method combines the simplex table method with the KKT conditions. Meanwhile, the PSO algorithm is inspired by the behavior of groups of insects, such as flocks of birds or fish [6]. PSO algorithm operates on a population of particles moving within a given solution space. Each particle in the population has a position and speed moving in the solution space. The particle position represents a possible solution, while the particle velocity indicates the direction and rate of change in the particle's position [7].

The portfolio optimization problem is motivated by investors' desire to invest in risky assets (shares) with the hope of still obtaining optimal returns by the risks they are willing to bear [8]. Therefore, investors can diversify by combining several securities to get returns and minimize risk, such as forming a portfolio. A stock portfolio combines several shares owned by an investor or a certain entity. A stock portfolio can be an alternative for potential investors when making investment decisions. Stock portfolios can be modeled into a quadratic programming model using the Markowitz mean-variance method [9].

Portfolio optimization problems have become one of the important research areas in modern risk management. Portfolio problems are the problem of how to allocate funds between several assets. Generally, an investor always wants the return from his portfolio to be as large as possible and the risk as small as possible. Markowitz proposed a mean-variance model, a quadratic programming problem [10].

Some previous research were solving quadratic problem using classical method and the other one using heuristic method [3]. Putri Z presented particle swarm optimization method for optimization portfolio [11]. Zhifeng Dai looked closer at the minimum variance portfolio optimization problem [12]. Muthohiroh presented the Markowitz method approach to optimizing portfolios with expected shortfall (ES) risk in stocksharia equipped with Matlab GUI [13]. Hartono presented determining the optimal portfolio using the Markowitz Model [14]. Rusmiati compared the Markowitz model optimal portfolio formation and the single index model on index stock IDX30 [15]. Assof presented portfolio management using methods Markowitz to make the weighting decision optimum investment [16]. Anugrahayu presented stock portfolio optimization using mean-variance and mean absolute deviation models based on k-medoid clustering by dynamic time warping [17]. Based on previous research, this research discussed solving quadratic programming models for optimal portfolio selection models using Wolfe’s method and particle swarm optimization (PSO).

2. RESEARCH METHODS

This research applies classical methods and heuristic methods to quadratic programming problems. The classical method is Wolfe’s, and the heuristic method is particle swarm optimization. The quadratic programming problem that will be solved is a portfolio problem. The data used in this research is from 10 stock companies that distribute the highest dividends in the IDX High Dividend 20 category for the 2022 period, which can be accessed at https://www.idx.co.id/id. In the final stage, the optimal portfolio results obtained by Wolfe’s method (classical method) and the PSO algorithm (heuristic method) will be seen. The stages of solving portfolio problems using Wolfe’s method and the PSO algorithm can be seen in **Figure 1**.

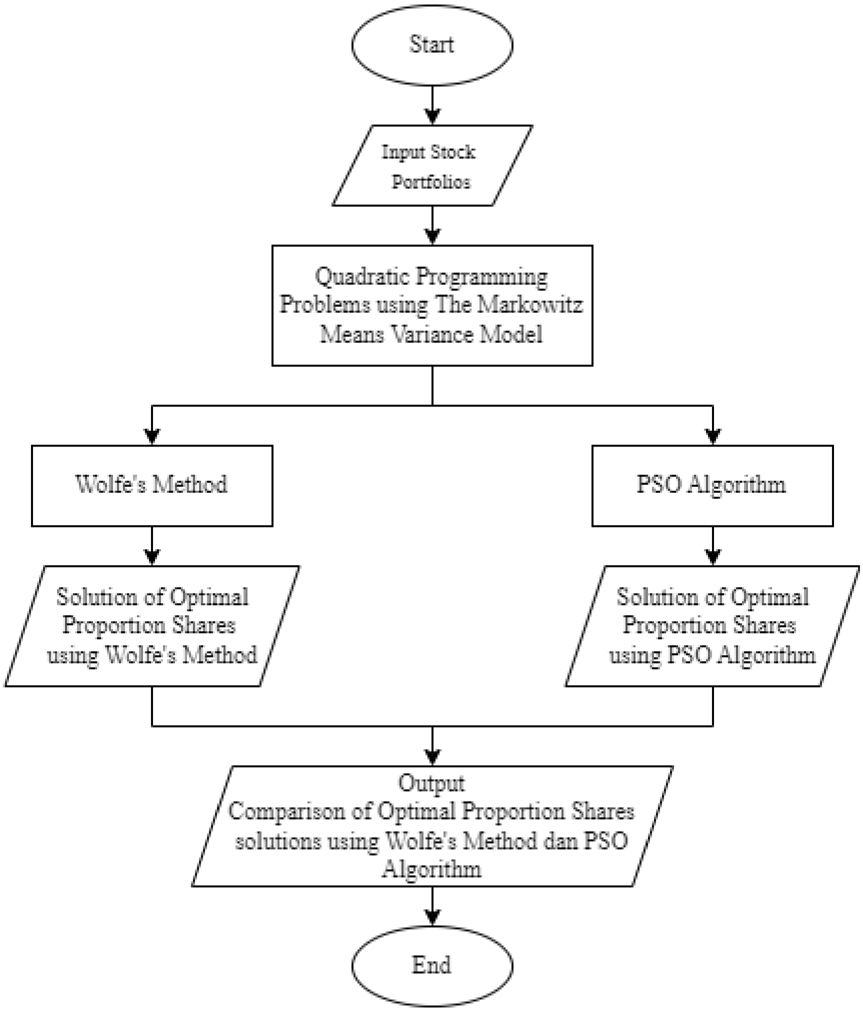

Figure 1. The Stages of Solving Portfolio Problems Using the Wolfe's Method and the PSO Algorithm

2.1 Quadratic Programming

Quadratic programming problems have a general form as shown in **Equation (1) [9]**.

$$\text{Maximize } Z = f(x) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} q_{jk} x_j x_k \tag{1}$$

with constraints

$$\sum_{j=1}^{n} c_j x_j \le b_i \quad , i = 1,2,\cdots,m$$

$$x_i \ge 0 \quad , j = 1,2,\cdots,n$$

$q_{jk} = q_{kj}$ for all $j = 1,2,\cdots,n$, k= $1,2,\cdots,n$ and $b_i \ge 0$ for all $i = 1,2,\cdots,m$, as well as quadratic form $\sum_{j=1}^{n}\sum_{k=1}^{n} q_{jk} x_j x_k$ is negative semidefinite.

2.2 Wolfe's Method

Wolfe's method was developed by Wolfe in 1959. This method is used to solve quadratic programming problems. The stages of Wolfe's method for quadratic programming problems involve the Lagrange method and the Karush-Kuhn-Tucker (KKT) condition **[12]**. The steps in Wolfe's method are presented as follows:

1. Determine the objective function in the form of a nonlinear equation and constraints in the form of linear inequalities.
2. Form the Karush-Kuhn-Tucker condition according to the objective function and constraints obtained.
3. Determine the condition of complementary slackness.
4. Adding artificial variables for the Karush Kuhn Tucker condition, which does not yet have a base variable
5. Determine the new objective function.
6. Carry out two-stage simplex iteration using Wolfe's method.
7. Substitute the results obtained from the simplex iteration process into the initial objective function so that optimal results are obtained.

Based on the Wolfe's method algorithm, **Equation (1)** can be modeled into a linear programming model which can be seen in **Equation (2)**.

$$\text{Minimize } Z_v = v_1 + v_2 + \cdots + v_n \tag{2}$$

with constraints

$$-\sum_{k=1}^{n} q_{jk}x_k + \sum_{i=1}^{m} a_{ij}\lambda_i - \mu_j + v_j \leq c_i \quad , j = 1,2,\cdots,n$$

$$\sum_{k=1}^{n} a_{ij}x_j + s_i = b_i \quad , i = 1,2,\cdots,m$$

$$v_i, \lambda_i, \mu_i, x_i \geq 0 \quad , j = 1,2,\cdots,n, i = 1,2,\cdots,n$$

and fulfill complementary requirements

$$\sum_{j=1}^{n} \mu_j x_j + \sum_{i=1}^{m} \lambda_i s_i = 0$$

with

$$\lambda_i s_i = 0 \text{ and } \mu_j x_j = 0 \text{ , } (i = 1,2,\cdots,m \text{ and j} = 1,2,\cdots,n).$$

Additional rules for the Wolfe's method are for selecting the base variables as follows:

1. If x_i has been selected as a base variable, then its complement, namely μ_i, cannot be selected as a base variable.
2. If s_i has been selected as a base variable, then its complement, namely λ_i, cannot be selected as a base variable.

2.3 Particle Swarm Optimization (PSO)

The steps of the PSO algorithm are as follows **[18]**:

1. Initialize the location x_i randomly, the velocity v_i randomly, population size n, inertia weight w, acceleration constant c_1, c_2 (iteration 0).
2. Repeat the following steps:
   a. Calculate the fitness value of each particle $\{f(x_1), \ldots, f(x_n)\}$.
   b. Determine the best location for each particle $pbest$.
   c. Determine the best location of all particles $gbest$.
   d. For each particle and each dimension, repeat the following steps:

1) Calculate the velocity v_i using **Equation (3)**

$$v_i^{t+1} = wv_i^t + c_1.rand(0,1).\left[gbest - x_i^t\right] + c_2.rand(0,1).\left[pbest_i - x_i^t\right] \quad (3)$$

2) Calculate the location x_i using **Equation (4)**

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (4)$$

e. Update $t = t + 1$

Until the stopping criteria are met.

3. The final result is the best location of all the particles $gbest$

2.4 Portfolio Theory

A portfolio is an investment consisting of various shares so that efficient combinations can be made so that investors can obtain the desired level of return with minimal risk. The steps in determining the optimal portfolio using the Markowitz model are as follows [19]:

1. Calculate the return individual using **Equation (5)**

$$R_{it} = \frac{P_{it} - P_{i(t-1)} + \left(\frac{D}{12}\right)}{P_{i(t-1)}} \quad (5)$$

with

R_{it} : individual realized return

P_{it} : closing price of stock i in period t

$P_{i(t-1)}$: closing price of stock i in period $(t-1)$

D : share dividend value

2. Calculate the individual expected returns using **Equation (6)**

$$E(R_i) = \frac{\sum_{i=1}^{N} R_{it}}{N} \quad (6)$$

with

$E(R_i)$: expected return of asset i

N : the number of returns that occurred during the observation period

R_p : portfolio realized return

3. Calculate individual risk using **Equation (7)**

$$\sigma_i^2 = \frac{\sum_{j=1}^{N} (R_{it} - \left(E(R_i)\right)^2}{N-1} \quad (7)$$

with

σ_i^2 : variance of investment in stock i

$E(R_i)$: expected return of asset i

R_{it} : return on asset i in period t

N : the number of returns that occurred during the observation period

4. Calculate covariance using **Equation (8)**

$$cov\ R_iR_j = \frac{\sum_{i,j=1}^{N}\left[R_{it} - \sum_{i=1}^{N} E(R_i)\right]\left[R_{jt} - \sum_{j=1}^{N} E(R_j)\right]}{N-1} \tag{8}$$

with

σ_i^2 : variance of investment in stock i

σ_p^2 : portfolio risk

$E(R_i)$: expected return of asset i

R_{it} : return on asset i in period t

N : the number of returns that occurred during the observation period

$cov\ R_iR_j$: covariance of the return on the i and j assets

5. Determine the proportion of funds from each stock portfolio using a quadratic programming model based on the Markowitz mean-variance model, which can be seen in **Equation (9)**. The model chosen is to minimize risk with a certain level of return.

$$\text{Minimize } Z = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{9}$$

with constraints

$$\sum_{i=1}^{N} x_i = 1$$

$$\sum_{i=1}^{N} E(R_i)x_i \geq E(R_i)$$

$$x_i \geq 0, i = 1, \cdots, N$$

and

Z : objective function

$E(R_i)$: individual expected return

σ_i^2 : individual stock risk

σ_{ij} : portfolio stock risk

N : the number of returns that occurred during the observation period

Or you can use the following model as shown in **Equation (10)**.

$$\text{Minimize } Z = \sum_{i=1}^{N} -E(R_i)x_i + \sum_{i=1}^{N} x_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{10}$$

With constraints

$$\sum_{i=1}^{N} x_i \leq 1$$

$$x_i \geq 0, i = 1, \dots, N$$

and

Z : objective function

$E(R_i)$: individual expected return

σ_i^2 : individual stock risk

σ_{ij} : portfolio stock risk

N : the number of returns that occurred during the observation period

3. RESULTS AND DISCUSSION

3.1 Data Description

The object of this research is to use the adjusted closing value and dividend value at PT. Astra International Tbk (ASII) and PT. Indo Tambangraya Megah Tbk (ITMG) in January 2022 – December 2022. PT. Astra International Tbk (ASII) and PT. Indo Tambangraya Megah Tbk (ITMG) was chosen from the 10 companies analyzed because PT. Astra International Tbk (ASII) and PT. Indo Tambangraya Megah Tbk (ITMG) has a negative covariance with a very different expected return rate. The results of calculating total return, expected return, variance and covariance are presented in **Table 1** and **Table 2**.

Table 1. Expected Return and Variance

i	Stock	Expected return ($E(R_i)$)	Variance (σ_i^2)
1	ASII	1.305%	0.681%
2	ITMG	9.832%	2.358%

Table 1 shows that the expected return of ASII is 1.305% and ITMG is 9.832%. It shows that the expected return of ASII is smaller than ITMG. The variance of ASII is 0.681%, and ITMG is 2.358%. It shows that the variance of ASII is smaller than ITMG.

Table 2. Stock Portfolio Covariance Value

i	Stock	Covariance (σ_{ij})
1	ASII/ITMG	-0.870%

Table 2 shows that the stock portfolio covariance value of ASII/ITMG is -0.870%. A negative value means the data of ASII and ITMG tend to change simultaneously in opposite directions.

3.2 Calculate Portfolio Return and Risk Expectations.

Expected return is the expected return from the investment made. The portfolio is composed of two companies. If it is assumed that each company gets the same proportion, namely 50%, then the expected return value of each stock portfolio can be determined. Return expectations of ASII and ITMG can be seen in **Equation (11)**.

$$\begin{aligned} E(R_{ASII\&ITMG}) &= \sum_{i=1}^{N}(k_i E(R_i)) \\ &= (50\% \cdot 1.305\%) + (50\% \cdot 9.832\%) \\ &= (0.5 \cdot 0.01305) + (0.5 \cdot 0.09832) \\ &= 0.05569 = 5.569\% \end{aligned} \tag{11}$$

3.3 Calculate Portfolio Risk

Portfolio risk is the variance in returns from the securities that make up a portfolio so that the portfolio risk may be smaller than the weighted average risk of each single security. Portfolio risk of ASII and ITMG can be seen in **Equation (12)**.

$$\begin{aligned}\sigma^2_{ASII\&ITMG} &= \sum_{i=1}^{N} k_i^2\sigma_i^2 + \sum_{j=1}^{N} k_j^2\sigma_j^2 + 2\sum_{i=1}^{N}\sum_{j=1}^{N} k_i k_j \sigma_{ij} \\ &= ((50\%)^2 \cdot 0.681\%) + ((50\%)^2 \cdot 2.358\%) \\ &\quad + \left(50\% \cdot 50\% \cdot (-0.01374\%)\right) \\ &= (0.5^2 \cdot 0.00681) + (0.5^2 \cdot 0.02358) \\ &\quad + \left(0.5 \cdot 0.5 \cdot (-0.0001374)\right) \\ &= 0.00725 = 0.725\%\end{aligned} \tag{12}$$

3.4 Quadratic Programming Model Based on Markowitz's Mean Variance Model

This research discusses a stock portfolio model with characteristics, namely a stock portfolio with negative covariance and a very different expected return rate, namely the stock portfolio. PT. Astra International Tbk (ASII) and PT. Indo Tambangraya Megah Tbk (ITMG). Below is a quadratic programming model, namely ASII and ITMG, where the covariance value of the two shares is -0.0087 with an expected return from ASII of 0.01305 and an expected return from ITMG of 0.09832. Therefore, a quadratic programming model is obtained, as shown in **Equation (13)**.

$$\text{Minimize } Z = 0.00681x_1^2 + 0.02358x_2^2 - 0.00137x_1x_2 \tag{13}$$

with constraints

$$\begin{aligned} x_1 + x_2 &= 1 \\ 0.01305x_1 + 0.09832x_2 &\le 0.01305 \\ x_1, x_2 &\ge 0 \end{aligned}$$

3.5 Wolfe's method and Particle Swarm Optimization

Wolfe's Method

The quadratic programming model in **Equation (13)** is transformed into a linear programming model using the Wolfe's method as follows:

$$\text{Minimize } Z = z_1 + z_2 + z_3 + z_4 \tag{14}$$

with constraints

$$\begin{gathered} 0.01362x_1 - 0.00137x_2 + \lambda_1 - \lambda_2 - 0.01305\lambda_3 - \mu_1 + z_1 = 0 \\ -0.00137x_1 + 0.04716x_2 + \lambda_1 - \lambda_2 - 0.01305\lambda_3 - \mu_1 + z_1 = 0 \\ x_1 + x_2 + y_1 = 1 \\ x_1 + x_2 - y_1 + z_3 = 1 \\ 0.01305x_1 + 0.09832x_2 - y_3 + z_4 = 0.01305 \end{gathered}$$

and fulfill complementary requirements

$$\lambda_1 y_1 = 0,\ \ \lambda_2 y_2 = 0,\ \lambda_3 y_3 = 0, \mu_1 x_1 = 0,\ \mu_2 x_2 = 0$$

with

$$x_1, x_2, y_1, y_2, \lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2\ \ge 0$$

The linear programming model in **Equation (14)** is solved using the Wolfe's method. The flowchart of Wolfe's method can be seen in **Figure 2**.

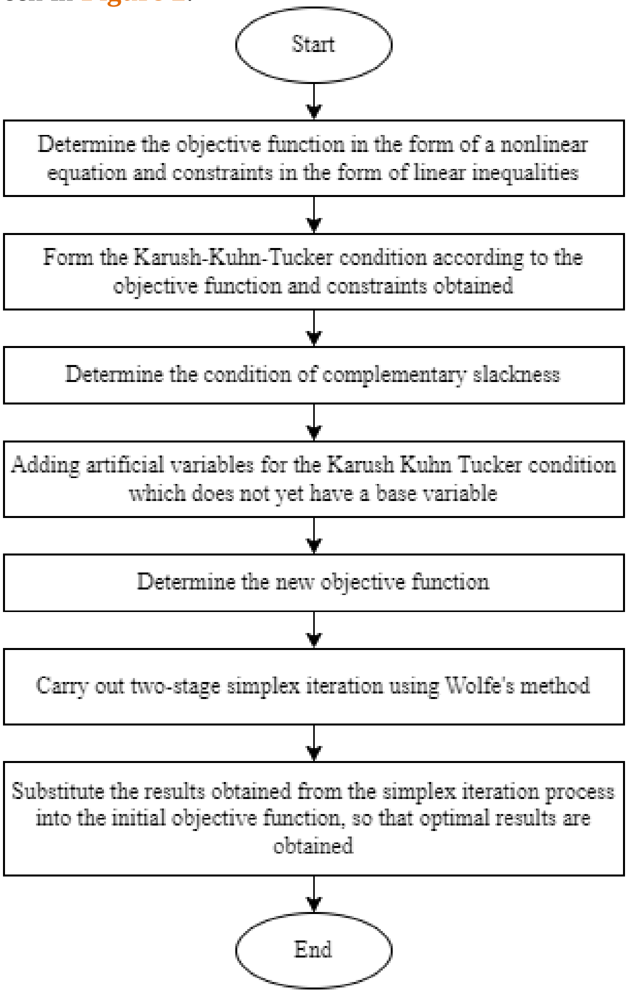

Figure 2. The flowchart of the Wolfe's method

Wolfe's method iteration process is stated in **Table 3**.

Table 3. Iteration Wolfe's Method on Linear Programming Models

Iteration	Base Variabel	Right Value	Input Variable	Output Variable
1	$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ y_1 \end{bmatrix}$	1.01305	x_2	z_2
2	$\begin{bmatrix} z_1 \\ x_2 \\ z_3 \\ z_4 \\ y_1 \end{bmatrix}$	1.01305	μ_2	z_1
3	$\begin{bmatrix} x_1 \\ x_2 \\ z_3 \\ z_4 \\ y_1 \end{bmatrix}$	1.01305	x_1	μ_2
4	$\begin{bmatrix} x_1 \\ x_2 \\ z_3 \\ z_4 \\ y_1 \end{bmatrix}$	1.01305	λ_2	z_4
5	$\begin{bmatrix} x_1 \\ x_2 \\ z_3 \\ \lambda_2 \\ y_1 \end{bmatrix}$	0.60660	y_3	z_3

Iteration	Base Variabel	Right Value	Input Variable	Output Variable
6	$\begin{bmatrix} x_1 \\ x_2 \\ y_3 \\ \lambda_2 \\ y_1 \end{bmatrix}$	0	Already optimized	Already optimized

The solution to the linear programming problem is:

$$\begin{bmatrix} x_1 \\ x_2 \\ y_3 \\ \lambda_2 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0.76401 \\ 0.23598 \\ 0.02012 \\ 0.01008 \\ 0 \end{bmatrix}$$

The optimal proportion using Wolfe's method for ASII shares = 0.76401 or 76.401% and ITMG shares = 0.23598 or 23.598% with Z values = 4709.52.

Particle Swarm Optimization

The flowchart of the PSO algorithm in solving portfolio problems according to **Equation (13)** can be seen in **Figure 3**.

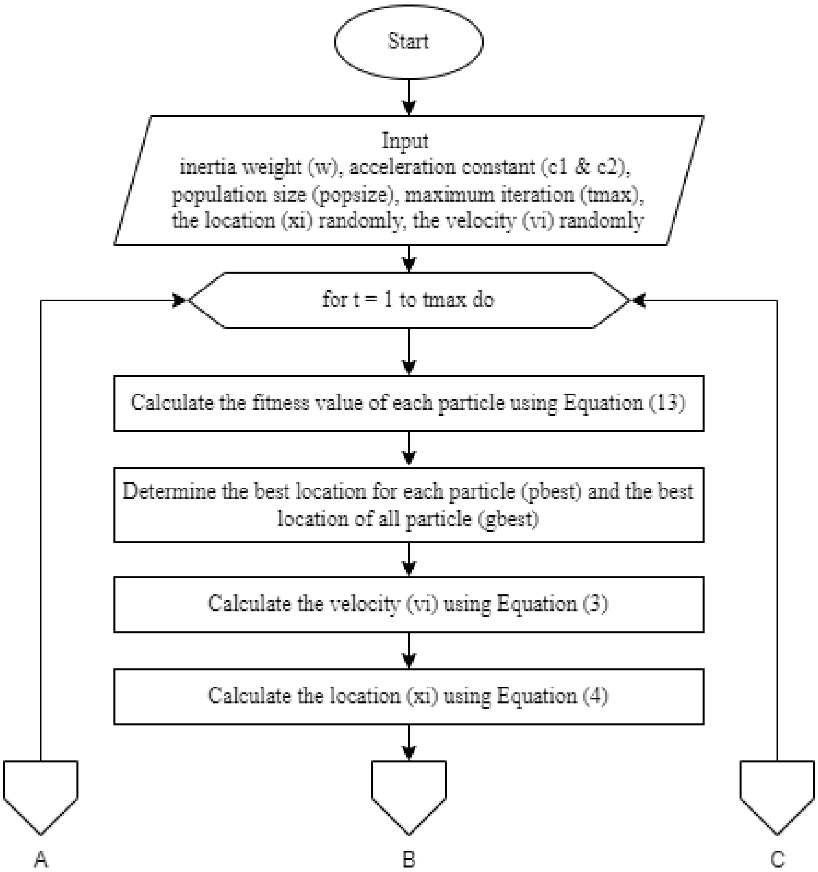

Figure 3. The Flowchart of the PSO Algorithm

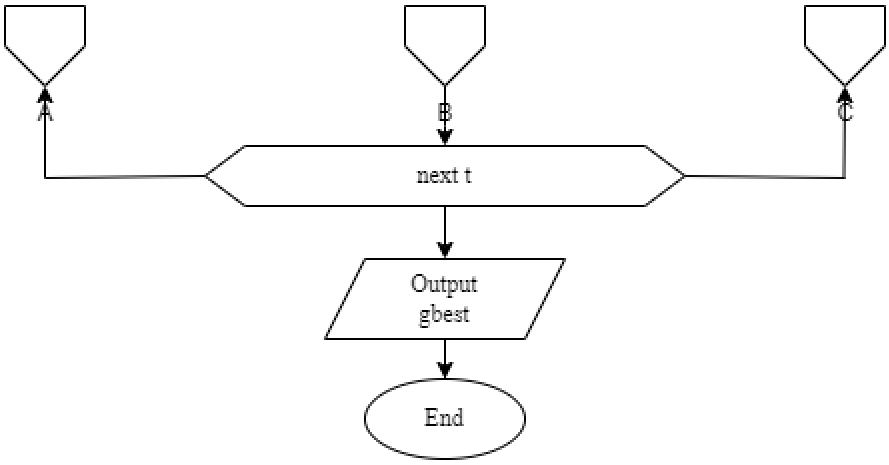

Figure 3. **The Flowchart of the PSO Algorithm (continued)**

Based on Figure 3, the PSO algorithm will be run 1 time with the PSO parameter settings seen in Table 4.

Table 4. **PSO Parameter Settings**

Parameter	Value
w	0.5
c_1	2
c_2	2
$popsize$	10
$tmax$	100

Table 4 shows the parameter of the PSO algorithm that is used. The inertia value (w) is 0.5, acceleration constant 1 (c_1) and 2 (c_2) is 2, population size $(popsize)$ is 10, and maximum iteration $(tmax)$ is 100. The run results for solving the quadratic programming problem in Equation (13) can be seen in Figure 4.

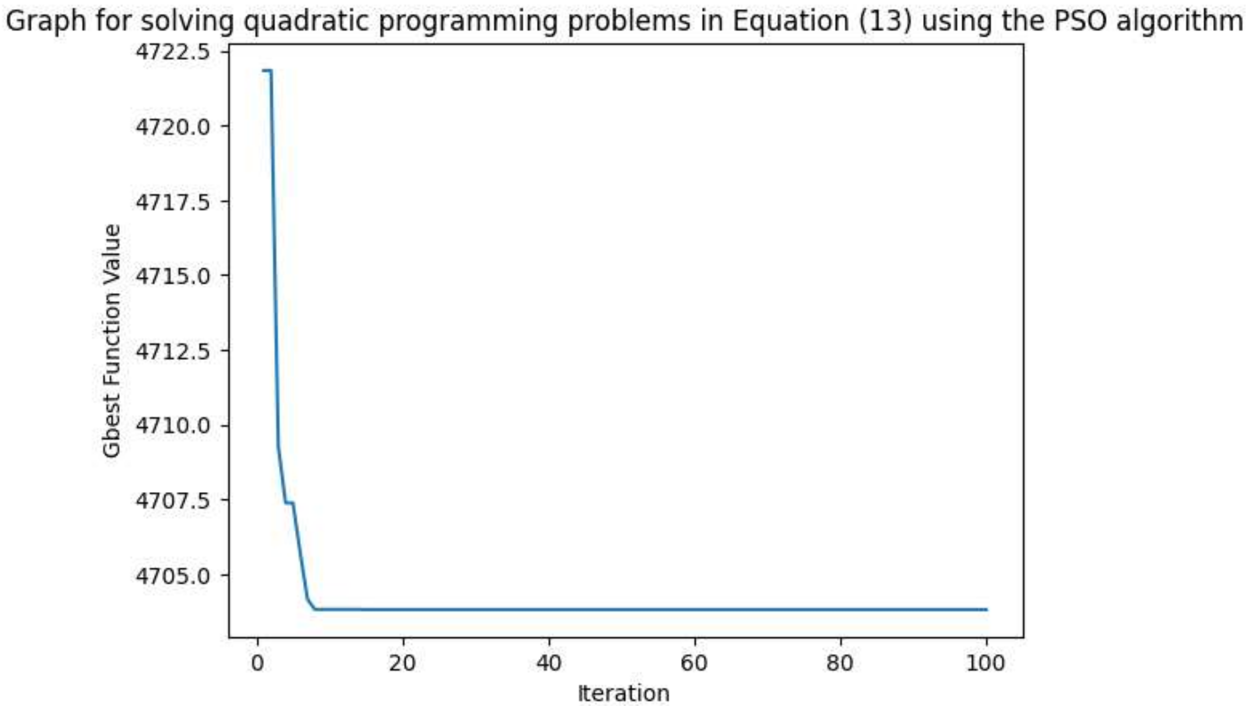

Figure 4. **The Run Results for Solving the Quadratic Programming Problem using PSO Algortihm**

Figure 4 shows that in iteration 1 the gbest function value equal to 4721.84. It is obtained where the values ASII shares (x_1) = 77.44% and ITMG shares (x_2) = 22.55%. The best function value decreases and converges in the 8th iteration with a value of 4703.82. It is obtained when the values ASII shares (x_1) =

75.02% and ITMG shares (x_2) = 24.98%. Because the gbest function value has converged in the 8th iteration, it can be concluded that the optimal proportion using PSO algorithm for ASII shares (x_1) = 75.02% and ITMG shares (x_2) = 24.98% with Z values = 4703.82.

The final results of the proportional distribution of ASII and ITMG shares using Wolfe's method and PSO algorithm can be seen in **Table 5**.

Table 5. PSO Parameter Settings

Method	Optimal proportion of ASII shares	Optimal proportion of ITMG shares	Z Values
Wolfe's method	76.401%	23.598%	4709.52
PSO algorithm	75.02%	24.98%	4703.82

Table 5 shows the optimal proportion of ASII shares using Wolfe's method = 76.401% and PSO algorithm = 75.02%. The optimal ASII share proportion using the PSO algorithm is 1.381% smaller than Wolfe's method. The optimal proportion of ITMG shares using Wolfe's method = 23.598% and PSO algorithm = 24.98%. The optimal ITMG share proportion using the PSO algorithm is 1.382% bigger than Wolfe's method. The Z value of the PSO algorithm is 5.7 smaller compared to Wolfe's method.

4. CONCLUSIONS

Results of stock portfolio optimization based on quadratic programming using Wolfe's method and the PSO algorithm: Using Wolfe's method, the ASII and ITMG stock portfolios are obtained, namely the optimal proportion of ASII shares = 0.76401 or 76.401% and ITMG shares = 0.23598 or 23.598% while the PSO algorithm obtains a portfolio of ASII and ITMG shares, namely ASII shares = 75.02% and ITMG shares = 24.98%. Compared to Wolfe's method, the PSO algorithm has a smaller Z value of 5.7.

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