BAREKENG: Journal of Mathematics and Its Applications
December 2024 Volume 18 Issue 4 Page 2137–2152
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol18iss4pp2137–2152

MODELING THE BENEFITS OF A MARRIAGE REVERSE ANNUITY CONTRACT WITH DEPENDENCY ASSUMPTIONS USING ARCHIMEDEAN COPULA

Arnhilda Aspasia Lundy [1*], Mila Novita[2], Ida Fithriani[3]

[1,2,3] *Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia*
Jl. Prof. Dr. Mahar Mardjono, Kampus UI, Depok 16424, Indonesia

*Corresponding author's e-mail: * **arnhilda.aspasia@gmail.com***

Article History:

Received: 28th December 2023
Revised: 8th May 2024
Accepted: 4th July 2024
Published: 14th October 2024

Keywords:

Clayton,
Canonical Maximum Likelihood,
Equity Release,
Frank,
Gumbel

ABSTRACT

Social security benefits may not be enough for retirement. Equity release products like marriage reverse annuities can boost retirement income for older couples. Marriage reverse annuity's contract convert all or part of the real estate value of elderly spouses while they are living (joint life status) or after one partner dies (last survivor status). Since husband and wife face the same death risk, the chance of death between spouses is believed to be dependent for realism. Thus, copula models the future dependency model of a husband and wife. Sklar's theorem states that copulas link bivariate distribution and marginal cumulative functions. One of the most common copulas is Archimedean copula. Clayton, Gumbel, and Frank are Archimedean copula that will be used in this investigation. The Indonesian Mortality Table IV data is used to obtain the marginal distribution of the male and female which will then be used to construct copulas (Clayton, Gumbel, and Frank) that combine two marginal distributions into a joint distribution. The marginal distribution of Indonesian Mortality Table IV is uncertain, hence Canonical Maximum Likelihood parameter estimation is utilized to estimate the parameter of copulas. Multiple-state models depict the marriage reverse annuity model for joint life and last survivor status. The probability structure is based on Sklar's theorem and copula survival function. The contract benefits calculation utilizing copulas (Clayton, Gumbel, and Frank) shows that joint life status benefits are higher than last survivor status. Joint life status uses the dependence assumption with Frank's copula to calculate the smallest annual benefit value of a marriage reverse annuity contract, while last survivor status uses the independence assumption (without copula).

How to cite this article:

A. A. Lundy, M. Novita, and I. Fithriani., "MODELING THE BENEFITS OF A MARRIAGE REVERSE ANNUITY CONTRACT WITH DEPENDENCY ASSUMPTIONS USING ARCHIMEDEAN COPULA," *BAREKENG: J. Math. & App.*, vol. 18, iss. 4, pp. 2137-2152, December, 2023.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

The increase in life expectancy and the aging population give hope that people will live a long life and be able to enjoy their retirement benefits to support their old age together with their spouse. However, sometimes the retirement benefits provided by social security are quite low and cannot cover retirement living. According to the Social Security Administration, social security benefits are only meant to replace about 40% of pre-retirement income, yet retirees need roughly 70% to live comfortably. Moreover, the World Bank data showing a steady increase in life expectancy underscores the need for retirement funds to last longer. Retirees are also at an age where they are vulnerable to critical illnesses, so the cost of living in retirement tends to be quite high. Fidelity Investments estimates that a 65-year-old couple retiring in 2021 will need approximately $300,000 saved (after tax) to cover health care expenses in retirement. Therefore, it is necessary to have an additional funding source for retirement so as not to experience a lack of funds in retirement. The substantial equity in homes owned by people aged 62 and older, which reached a record $7.14 trillion in 2020 as reported by the National Reverse Mortgage Lenders Association, highlights the potential of housing wealth as a retirement asset, supporting the relevance of equity release products like marriage reverse annuity contracts.

A possible strategy for retirees who have property assets in the form of their own house or apartment is to purchase an equity release product. Equity release products allow elderly homeowners to convert their homes into liquid wealth without having to move. The two main forms of equity release products are the loan model, known as reverse mortgages or lifetime mortgages, and the sale model, known as home reversions **[1]**. Financial institutions in several countries provide equity release products to retired individuals **[2]**. These programs allow retirees to get additional income by surrendering their real estate. A wide range of such products may be found in the largest markets in the world, the United States of America and Australia. The equity release contracts market in the United Kingdom is the largest in Europe **[3]**. Furthermore, these contracts are present in numerous other European nations such as Spain, Ireland, France, Germany, Italy, and Poland. This study focuses on the sale model equity release products in Poland, the reverse annuity contract.

A reverse annuity contract is a contract offered to elderly people who own property to relinquish their property rights in exchange for receiving annuity benefits and the rights to live in the property until his/her death (he/she is not the formal owner of the property). This contract is offered in individualized form in Poland. However, there are times when it is found that the owner of the property is not just one person, but a married couple. Therefore, a variation of the contract is proposed, namely the marriage reverse annuity contract.

Marriage reverse annuity contract provides annuity benefits while the spouses are still alive and sometimes even after the death of one of the spouses. Therefore, the contract will be divided into two types: the joint life contract and the last survivor contract. The joint life contract pays benefits only until the first death of the spouse, while the last survivor contract pays benefits until the death of the other spouse or until the death of both. These two cases can be explained using a marriage life annuity, which is a reversionary annuity **[4]**.

A stochastic tool is used in modeling and implementing the marriage reverse annuity contract, namely multiple-state modeling. There are four states used in the marriage reverse annuity contract, (1) both spouses are alive, (2) the husband is dead, (3) the wife is dead, and (4) both spouses are dead. The multiple-state model applies to joint life and last survivor status. The marriage reverse annuity contract pays annuities at specific time units, so the model is focused on discrete time with the assumption that the evolution of the contract risk is described using a non-homogeneous time Markov chain, which has transition probabilities that are not constant over time.

The mortality risk of spouses is often assumed to be independent of each other. However, spouses are exposed to the same risks. These risks include similar lifestyles, infectious diseases, natural disasters, external events such as car or plane crashes (called “shock”), and even broken heart syndrome can occur in these situations. As a result of the same risks that spouses may experience, the future lifetime of spouses is calculated using the dependency assumption.

The dependency structure of spouses’ future lifetime is modeled using a copula. Various studies have been conducted in modeling dependencies using copula in spouses. Spreeuw **[5]** investigated the types of dependencies (momentary, long-term, and short-term) and time-dependent association effects between two lifetimes in an Archimedean copula model with a single parameter. Luciano, Spreeuw, & Vigna **[6]** first modeled the mortality risk of individual couples based on a stochastic intensity approach with dependencies between the survival times of couple members described using Archimedean copulas. Heilpern [7] modeled the dependency structure induced by Markov chains and used Archimedean copulas in multiple life insurance. Luciano, Spreeuw, & Vigna [4] studied dependency with copula on married couples from different generations and its effect on reversionary annuity pricing.

Based on the related research relevance, the probability structure for joint life and last survivor status is illustrated through a multiple-state model on marriage reverse annuity contract with dependency assumptions using copulas from the Archimedean copula family, namely Gumbel and Ali-Mikhail-Haq (AMH) [8]. However, in this study, the Archimedean copula family will be used are Clayton, Gumbel, and Frank. Several reasons Archimedean copulas are used, namely because they are easy to construct, have a great variety of families of copulas which belong to this class [9], and allow for a wider variety of dependence structures [10]. The parameter estimation method used is also different from related research relevance, where the previous study used Kendall's tau coefficient of correlation. This study uses the parameter estimation method for each copula with the Canonical Maximum Likelihood method. In addition, the calculation of benefits with the copula model will also be determined on the marriage reverse annuity contract for joint life and last survivor status based on the Indonesian Mortality Table IV with the assumption of a constant interest rate.

2. RESEARCH METHODS

In this section, the methods used to model marriage reverse annuity benefits with dependencies using Archimedean copulas will be discussed. First, the copulas, survival copulas, and Archimedean copula families, namely Clayton, Gumbel, and Frank, will be explained. Next, the copula parameter estimation method that will be used is the canonical maximum likelihood method. Then, the probability structure and benefit calculation of a marriage reverse annuity contract on joint life and last survivor status with dependency assumption using Archimedean copulas will be constructed.

Before proceeding to the explanation, here is a flowchart of research methods that will be carried out in this study.

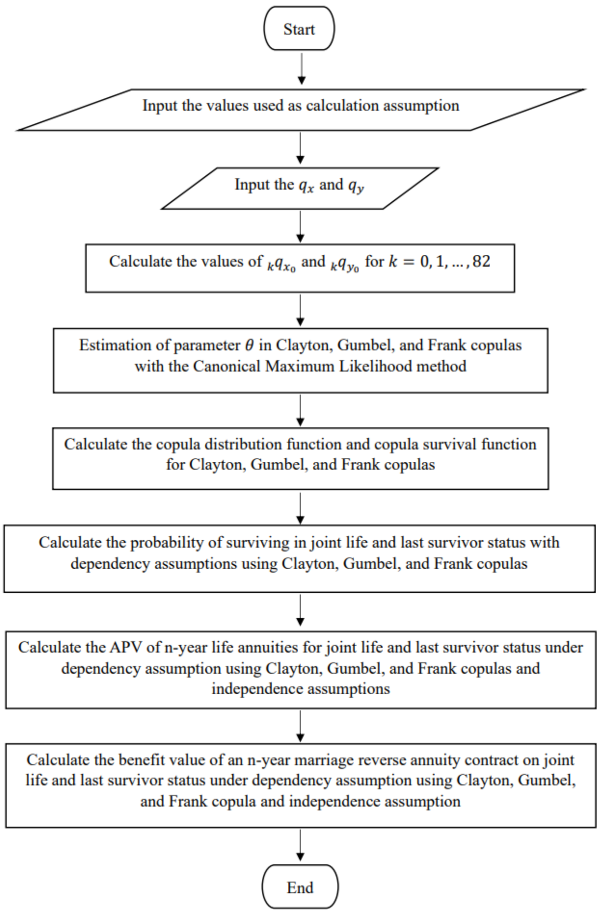

Figure 1. Flow Chart of Term Marriage Reverse Annuity Contract Benefit Calculation Procedure

2.1 Copulas

According to Cassel's Latin Dictionary, the word copula is a noun of Latin origin with the meaning "a link, tie, bond". The word copula was first used in a mathematical or statistical sense by Abe Sklar (1959) on a theorem known as Sklar's theorem [9]. Copulas are helpful tools in constructing joint distributions. Copulas are functions that join or couple multivariate distribution functions to their one-dimensional marginal distribution functions that are uniformly distributed on $\mathbf{I} = [0, 1]$.

A bivariate copula is a function C with domain $\mathbf{I}^2$ and range $\mathbf{I}$ which is defined as $C: \mathbf{I}^2 \to \mathbf{I}$ and satisfies the following properties.

a. For every u, v in $\mathbf{I}$,

$$C(u, 0) = 0 = C(0, v) (\text{grounded}) \tag{1}$$

And

$$C(u, 1) = u \text{ and } C(1, v) = v; \tag{2}$$

b. For every u_1, u_2, v_1, v_2 in $\mathbf{I}$ such that $u_1 \leq u_2$ and $v_1 \leq v_2$,

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \ (2 - \text{increasing}) \tag{3}$$

Assuming that C is differentiable with respect to its both arguments, Equation (3) is satisfied if [11]

$$c(u, v) \equiv \frac{\partial^2 C(u, v)}{\partial u \partial v} \geq 0 \tag{4}$$

where $c(u, v)$ is the copula density function.

Sklar's theorem is the core of the copula theory and is the foundation of several statistical theories. It explains the role of the copulas in the relationship between the bivariate distribution function and its univariate marginal distribution function. As stated by Sklar's theorem, suppose H is the joint distribution function of random variables X and Y with marginal functions $F(x)$ and $G(y)$, then there exists a copula function C such that for all $x, y \in \overline{\mathbb{R}}$,

$$H(x,y) = C\big(F(x), G(y)\big) = C(u,v) \tag{5}$$

where $u = F(x)$ and $v = G(y)$.

There are many types of copulas, here is a concise table summarizing the general types of copulas and their advantages and disadvantages.

Table 1. The General Types of Copulas with Their Advantages dan Disadvantages

Type of Copula	Description	Advantages	Disadvantages
Archimedean	Simple form based on a generator function (e.g. Clayton, Gumbel, and Frank).	▪ Simple and flexible ▪ Can model various dependency structures ▪ Easy to construct and interpret	▪ Limited in capturing complex dependencies ▪ Each type has specific tail dependence limitations
Gaussian (Normal)	Based on the multivariate normal distribution, capturing linear dependencies.	▪ Suitable for capturing linear dependencies ▪ Widely used	▪ Cannot capture tail dependence ▪ Assumes symmetry in dependencies
t-Copula	Extension of Gaussian copula with heavier tails, capturing tail dependencies.	▪ Capture tail dependencies better than Gaussian ▪ Flexible in modeling various dependence structures	▪ More complex to estimate parameters ▪ May require large datasets for accurate parameter estimation
Elliptical	Based on elliptical distributions (e.g. Gaussian and t-copula).	▪ Can model symmetric dependencies ▪ Captures a range of dependence structures	▪ Limited in capturing asymmetric dependencies ▪ More complex than Archimedean copulas
Archimedean Mixture	Combines multiple Archimedean copulas to capture different dependence structures.	▪ More flexible than single Archimedean copulas ▪ Can capture a wider range of dependencies	▪ Increased complexity ▪ More difficult to estimate and interpret
Extreme Value	Designed to capture extreme value dependencies, useful in risk management.	▪ Effective for modeling extreme co-movements ▪ Useful in finance and insurance for risk analysis	▪ May be complex to estimate ▪ Requires large datasets to accurately model extreme events
Vine Copula	Constructed from a sequence of bivariate copulas, offering high flexibility.	▪ Highly flexible and can model complex dependencies ▪ Can capture tail dependencies effectively	▪ Highly complex and computationally intensive ▪ Parameter estimation can be challenging

To apply the copula method in the data, there are several steps to execute it. Here's a simple algorithm for applying a copula to data

1. Do data preparation by inputting the bivariate (or multivariate) data set (e.g. $X = \{X_1, X_2, \dots, X_d\}$) and preprocessing it.
2. For each variable X_i, estimate the marginal distribution $F_i(x_i)$ using either parametric methods or non-parametric methods.
3. Transform each variable X_i to uniform marginals U_i using the estimated marginal cumulative distribution functions $U_i = F_i(x_i)$. The transformed data $U = \{U_1, U_2, \dots, U_d\}$ will lie in the unit interval.
4. Select a copula family based on the characteristics of the data and the nature of dependencies.
5. Estimate the parameters of the chosen copula.

6. Using the estimated parameters, fit the copula model to the transformed data $\boldsymbol{U}$.

2.2 Survival Copulas

In the context of copulas and survival analysis, the marginal function typically refers to the marginal distribution function of a single random variable. For example, a random variable T representing the time until an event (e.g. death), the marginal distribution function is $F_T(t) = \Pr(T \leq t)$, which gives the probability that the event occurs by time t. The univariate survival function denoted as $\bar{F}_T(t)$, is related to the marginal distribution function but focuses on the probability that the event has not occurred by time t. It is defined as $\bar{F}_T(t) = \Pr(T > t) = 1 - F_T(t)$. Thus, the marginal function and the survival function are directly related.

For a pair of random variables (X, Y) with joint distribution function H, the univariate survival functions of X and Y are $\bar{F}$ and $\bar{G}$, respectively. Suppose the copula of X and Y is C, then for every $(x, y) \in \mathbb{R}$ the joint survival function $\bar{H}$ is obtained as follows

$$\begin{aligned}\bar{H}(x,y) &= 1 - F(x) - G(y) + H(x,y) \\ &= 1 - \left(1 - \bar{F}(x)\right) - \left(1 - \bar{G}(y)\right) + H(x,y) \\ &= \bar{F}(x) + \bar{G}(y) - 1 + H(x,y).\end{aligned} \tag{6}$$

Based on Sklar's theorem on defining the copula function C through **Equation (5)**, the joint survival function $\bar{H}$ can be written as

$$\bar{H}(x,y) = \bar{F}(x) + \bar{G}(y) - 1 + C\left(F(x), G(y)\right) \tag{7}$$

$$\bar{H}(x,y) = \bar{F}(x) + \bar{G}(y) - 1 + C\left(1 - \bar{F}(x), 1 - \bar{G}(y)\right) \tag{8}$$

If a function $\bar{C}$ is defined from the domain $\mathbf{I}^2$ to range $\mathbf{I}$ by

$$\bar{C}(u,v) = u + v - 1 + C(1-u, 1-v) \tag{9}$$

then the joint survival function $\bar{H}$ can be rewritten as

$$\bar{H}(x,y) = \bar{C}\left(\bar{F}(x), \bar{G}(y)\right) \tag{10}$$

The function $\bar{C}$ is also called the copula survival function of X and Y because $\bar{C}$ is a function that relates the joint survival function with its univariate marginal survival function.

The copula distribution function can be expressed in the copula survival function using **Equation (8)** and **Equation (10)** as follows

$$\bar{C}\left(\bar{F}(x), \bar{G}(y)\right) = \bar{F}(x) + \bar{G}(y) - 1 + C\left(1 - \bar{F}(x), 1 - \bar{G}(y)\right) \tag{11}$$

$$\bar{C}\left(\bar{F}(x), \bar{G}(y)\right) = \bar{F}(x) + \bar{G}(y) - 1 + C\left(F(x), G(y)\right) \tag{12}$$

$$C\left(F(x), G(y)\right) = 1 - \bar{F}(x) - \bar{G}(y) + \bar{C}\left(\bar{F}(x), \bar{G}(y)\right) \tag{13}$$

Given that the copula is differentiable based on **Equation (4)**, the copula density function can be defined as

$$c\left(F(x), G(y)\right) = \frac{\partial^2 C\left(F(x), G(y)\right)}{\partial F(x) \partial G(y)} = \frac{\partial^2 \bar{C}\left(\bar{F}(x), \bar{G}(y)\right)}{\partial \bar{F}(x) \partial \bar{G}(y)} = \bar{c}\left(\bar{F}(x), \bar{G}(y)\right) \tag{14}$$

2.3 Archimedean Copulas

Archimedean copulas are one of the copula families that is often found in various applications in finance and insurance. Archimedean copulas are constructed using generator function [12].

Theorem 1. Let a function $\varphi: \mathbf{I} = [0,1] \to [0, \infty]$ is continuous and strictly decreasing such that $\varphi(1) = 0$. The pseudo-inverse of φ is the function $\varphi^{[-1]}$ with Dom $\varphi^{[-1]} = [0, \infty]$ and Ran $\varphi^{[-1]} = \mathbf{I}$ defined as [9]

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{-1}(t), & 0 \leq t \leq \varphi(0) \\ 0, & \varphi(0) \leq t \leq \infty \end{cases} \tag{15}$$

Given the generator function and the pseudo-inverse of the generator function, the general form of the Archimedean copula can be defined. Let $\varphi: \mathbf{I} = [0,1] \to [0, \infty]$ be continuous and strictly decreasing function such that $\varphi(1) = 0$, and let $\varphi^{[-1]}$ be the pseudo-inverse of φ defined by **Equation (15)**. Then, the function $C: \mathbf{I}^2 \to \mathbf{I}$ which is defined in **Equation (16)** below is a copula if and only φ is convex.

$$C(u,v) = \varphi^{[-1]}\big(\varphi(u) + \varphi(v)\big) \tag{16}$$

The copula form in **Equation (16)** is the generalized form of the Archimedean bivariate copula. The parameter t represents the aggregate dependence measure among the variables u and v. By summing the generator-transformed marginals, the overall dependency structure imposed by the generator function is effectively captured. This aggregate measure is then mapped back to the unit interval by the pseudo-inverse function $\varphi^{[-1]}$, producing the copula value.

From the above, it can be said that The Archimedean copula converts the distribution function into a function that can be summed together, i.e. with a generator function. According to **Theorem 1**, the generator function produces an output number in the interval $[0,\infty]$. If the two generator functions of the u and v are summed, then the resulting output is in the interval $[0,\infty]$ as well. By using the inverse (pseudo-inverse) of the generator function, the resulting output will be in the interval [0,1], which is the definition of the Archimedean copula.

In this study, the bivariate Archimedean copula will be used with one parameter constructed using the generator function $\varphi_\theta(t)$ where θ is a copula parameter called a dependency parameter that measures the dependence between marginal functions. Several types of one-parameter Archimedean copula parameters that will be used in this study are Clayton, Gumbel, and Frank.

2.3.1 Clayton Copula

The Clayton copula has a generator function that is **[9]**

$$\varphi_\theta^{Clayton}(t) = \frac{1}{\theta}\big(t^{-\theta} - 1\big), \qquad \theta \in (0,\infty) \tag{17}$$

with copula distribution function **[13]**

$$C_\theta^{Clayton}(u,v) = \big(u^{-\theta} + v^{-\theta} - 1\big)^{-\frac{1}{\theta}}, \qquad \theta \in (0,\infty) \tag{18}$$

and copula density function **[13]**

$$c_\theta^{Clayton}(u,v) = (1+\theta)\big(u^{-\theta} + v^{-\theta} - 1\big)^{-\left(\frac{1}{\theta}+2\right)}(uv)^{-(\theta+1)}, \qquad \theta \in (0,\infty) \tag{19}$$

The Clayton copula shows asymmetric dependencies and cannot account for negative dependence. Clayton copula shows strong left tail dependence and relatively weak right tail dependence **[14]**. The Clayton copula is suitable for applications where outcomes or outputs tend to be strongly correlated at low values, but weakly correlated at high values.

2.3.2 Gumbel Copula

The Gumbel copula has a generator function that is **[9]**

$$\varphi_\theta^{Gumbel}(t) = (-\ln t)^\theta, \qquad \theta \in [1,\infty) \tag{20}$$

with copula distribution function **[13]**

$$C_\theta^{Gumbel}(u,v) = exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{1}{\theta}}\right\}, \qquad \theta \in [1,\infty) \tag{21}$$

and copula density function **[13]**

$$c_\theta^{Gumbel}(u,v) = \begin{matrix} C_\theta^{Gumbel}(u,v) \cdot \dfrac{\big((-\ln u)(\ln v)\big)^{\theta-1}}{uv} \cdot \left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{2}{\theta}-2} \\ \cdot \left[1 + (\theta-1)\big((-\ln u)^\theta + (-\ln v)^\theta\big)^{-\frac{1}{\theta}}\right], \end{matrix} \qquad \theta \in [1,\infty) \tag{22}$$

The Gumbel copula is similar to the Clayton copula that shows asymmetric dependencies and cannot account for negative dependence. However, in contrast to the Clayton copula, the Gumbel copula shows strong right tail dependence and relatively weak left tail dependence. The Gumbel copula is suitable for applications where outcomes or outputs are strongly correlated at high values, but weakly correlated at low values **[14]**.

2.3.3 Frank Copula

The Frank copula *has* a generator function that is [9]

$$\varphi_\theta^{Frank(t)} = -ln\left((e^{-\theta t}-1)/(e^{-\theta}-1)\right), \qquad \theta \in (-\infty,\infty)\{0\} \tag{23}$$

with copula distribution function [13]

$$C_\theta^{Frank}(u,v) = -\frac{1}{\theta} ln\left(1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right), \qquad \theta \in (-\infty,\infty)\{0\} \tag{24}$$

and copula density function [13]

$$c_\theta^{Frank}(u,v) = -\frac{\theta e^{-\theta(u+v)}(e^{-\theta}-1)}{[e^{-\theta(u+v)}-e^{-\theta u}-e^{-\theta v}+e^{-\theta}]^2}, \qquad \theta \in (-\infty,\infty)\{0\} \tag{25}$$

The Frank copula is widely used because it shows symmetrical dependencies on both tails and allows for negative dependence. The Frank copula shows strong left tail dependence and relatively weak right tail dependence between marginals. The Frank copula can be used to model data that exhibits weak tail dependence [14].

2.4 Canonical Maximum Likelihood

Estimation of copula parameters using the Canonical Maximum Likelihood (CML) method is a semiparametric method that does not require assumptions of the marginal distribution, but rather the marginal distribution is estimated nonparametrically with its distribution function. The estimation using the CML method consists of two steps. In the first step of this method, the series of interest are transformed into uniform variates using the empirical probability integral transform [15], which the empirical distribution function is defined as

$$\hat{F}(\cdot) = \frac{1}{T}\sum_{t=1}^{T} \mathbf{I}_{\{X_t \le \cdot\}} \tag{26}$$

where $\mathbf{I}_{\{X_t \le \cdot\}}$ is the indicator function. The parameters of copula can be estimated by maximizing the log-likelihood function of copula density using transformed variables [15]. The log-likelihood function of copula density is as follows

$$\mathcal{L}_C(\theta) = \sum_{t=1}^{T} \ln\left(c\left(\hat{F}(x_t), \hat{G}(y_t)\right)\right) = \sum_{t=1}^{T} \ln\left(c(\hat{u}_t, \hat{v}_t)\right) \tag{27}$$

Then, the semi parametric estimator given by

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}}\, \mathcal{L}_C(\theta) \tag{28}$$

In this study, the first step is to transform the empirical dataset which is the probability that the husband (x) dies before the age of $x+1$ year or denoted by q_x and the probability that the wife (y) dies before the age of $y+1$ year or denoted by q_y into an empirical distribution function with uniform distribution [0,1]. The transformed dataset produces the probability that the husband (x) dies before the age of $x+t$ years or denoted by ${}_tq_x$ and the probability that the wife (y) dies before the age of $y+t$ years or denoted by ${}_tq_y$. The next step is to estimate the parameters of the copula using **Equation (29)** as follows

$$\hat{\theta} = \underset{\theta}{\operatorname{argmax}} \sum_{t=1}^{T} \ln c_\theta({}_tq_x, {}_tq_y; \theta) \tag{29}$$

where c is the density function of the copula used and θ is the parameter of the copula used.

Estimating the parameter using the canonical maximum likelihood method can be done with the help of R Studio software which has a special package for copula. Here are the general steps that can be done

1. Install and load the copula package in R Studio by using install.packages("copula").
2. Define the copula model that will be used.
3. Transform the data to uniform margin using the empirical cumulative distribution function (ECDF) as shown in the **Equation (26)** to transform the data to the unit interval $[0, 1]$.
4. Define the log-likelihood function using the copula density as shown in the **Equation (27)**.
5. Optimize the log-likelihood function using the optim function in the package to find the parameters that maximize the log-likelihood.

2.5 Marriage Reverse Annuity Contract

Under a marriage reverse annuity contract, the annuity benefits are payable when both spouses are alive and sometimes after the death of whichever spouse. Therefore, marriage reverse annuity contracts can be divided into two types, namely the joint life status contract (where benefits are paid only until the first death of the spouse) and the last survivor status contract (in which the benefits are paid until the death of the other spouses) [8].

The modeling of the marriage reverse annuity contract adapts the theory that developed from multistate insurance contracts. The modified multistate model of the marriage reverse annuity contract is based on four states, namely (1) both spouses are alive, (2) husband is dead, (3) wife is dead, and (4) both spouses are dead. The following is an illustration of the multistate model on the marriage reverse annuity contract.

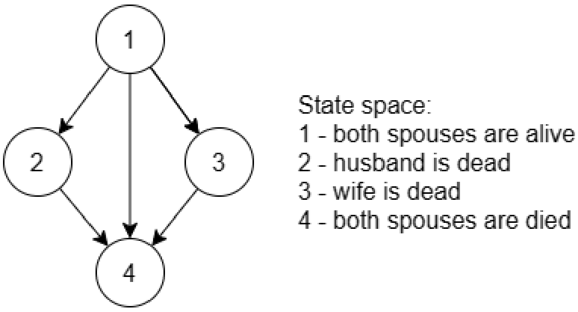

Figure 2. Multiple state model for marriage reverse annuity contract

Let $S = \{1, 2, 3, 4\}$ be the state space and $T = \{(1,2), (1,3), (1,4), (2,4), (3,4)\}$ denotes the set of direct transitions between states in the state space according to **Figure 2**. The pair (S, T) is called the modified multiple state model and describes all possible contracted risk events up to the end of the marriage reverse annuity contract.

Also, let that $X(k)$ denotes the state of the contract at time k where $k = 0, 1, 2, \dots, n$ such that $X(k) = i$ means that the couple is in state i at time k. Therefore, $\{X(k), k = 0, 1, 2, \dots, n\}$ is a discrete time process that describes the evolution of the insured's risk within the contract period, and it is assumed that $\{X(k), k = 0, 1, 2, \dots, n\}$ is a nonhomogeneous Markov chain.

The marriage reverse annuity contract is issued at time 0 which is defined as the contract issue time and expires at time n which is referred to as the term of contract. The graphic representation of multiple state model with joint life and last survivor status for marriage reverse annuity contract of n years is illustrated as follows

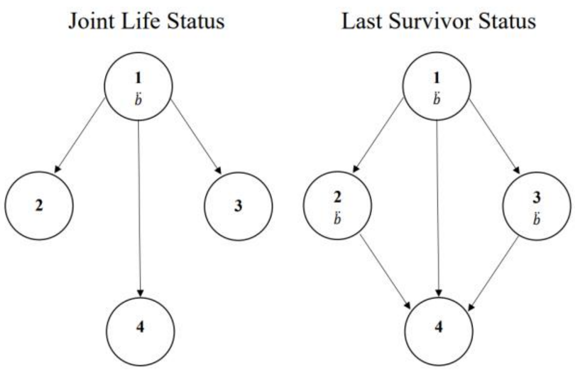

Figure 3. Marriage Reverse Annuity Contract for Joint Life Status and Last Survivor Status

where $\ddot{b}$ denotes the life annuity due payable for the period $[k, k+1)$, if both spouses are alive $(X(k) = 1)$ or one of the spouses is alive (wife is alive $X(k) = 2$ and husband is alive $X(k) = 3$) at time k $(k = 0, 1, \dots, n-1)$.

Note that the n-year marriage reverse annuity contract involves two individuals, which are husband and wife. Suppose the x-year old husband is denoted by (x) and the y-year old wife is denoted by (y) with future lifetime random variables T_x^M and T_y^W respectively, where $T_x^M \in [0, \omega_x^M]$ and $T_y^W \in \left[0, \omega_y^W\right]$. The notation ω_x^M (or ω_y^W) denotes the difference between the age limit ω of the man (or woman) and the man's age (or woman's age) when entering the contract (based on the Indonesian Mortality Table, $\omega = 111$). The term of the marriage contract reverse annuity contract term is determined based on the type of contract status and ages at the entry of the husband and wife, which is explained as follows

- $n = \min\{\omega_x^M, \ \omega_y^W\}$ for joint life status (JLS),
- $n = \max\{\omega_x^M, \ \omega_y^W\}$ for last survivor status (LLS).

In contrast to the general assumption of future lifetime of spouses which is assumed to be independent, in this study the future lifetime of spouses assumed to be dependent using copula model. Dependency of spouses's future lifetime starts from age x_0 and y_0, where x_0 and y_0 are the reference ages for man and woman respectively, such that $x = x_0 + s$ and $y = y_0 + t$. It is because members of a couple don't have the common genes and usually didn't grow up in the same environment. Therefore, it is realistic to assume that the future lifetimes of spouses are independent until they first meet.

It is assumed that the joint cumulative distribution function $H(w,z)$ of the pair $\left(T_{x_0}^M, T_{y_0}^W\right)$ and based on the **Equation (5)**, the copula $C(u,v)$ that link the joint cumulative distribution function with its marginal distribution function as follows

$$H(w,z) = C\big(F(w), G(z)\big) \tag{30}$$

where $F(w) = \Pr\left(T_{x_0}^M \leq w\right) = u$ and $G(z) = \Pr\left(T_{y_0}^W \leq z\right) = v$ are marginal distribution functions of lifetime $T_{x_0}^M$ and $T_{y_0}^W$. Other than joint cumulative distributive function, the joint survival function is also required for actuarial calculations and can be defined as

$$\bar{H}(w,z) = \Pr\left(T_{x_0}^M > w, T_{y_0}^W > z\right) = \bar{C}\big(\bar{F}(w), \bar{G}(z)\big) \tag{31}$$

where $\bar{F}(w) = \Pr\left(T_{x_0}^M > w\right)$, $G(z) = \Pr\left(T_{y_0}^W > z\right)$, and the survival copula form in **Equation (9)** holds.

Let $p_i(k) = P_i(X(k) = i)$ for $i = 1, 2, 3, 4$ and $\mathbf{P}(k) = \big(p_1(k), p_2(k), p_3(k), p_4(k)\big)$ be the probabilities of the staying process $\{X(k), k = 0, 1, 2, \dots, n\}$ at the moment k in one of the states belonging to the state space. To obtain the probability structure $p_i(k) = P_i(X(k) = i)$, a survival copula will be used.

Assume that the survival copula $\bar{C}(w,z)$ is the function that connects the joint survival function and the marginal survival for x_0-year old man and y_0-year old woman. In addition, $\{X(k), t \in T\}$ is a nonhomogeneous Markov chain hat describes the evolution of the insured's risk in the multistate model $(S,T) = (\{1,2,3,4\}, \{(1,2),(1,3),(1,4),(2,4),(3,4)\})$ for marriage reverse annuity contract. Then, the elements of the vector $\mathbf{P}(k) = \big(p_1(k), p_2(k), p_3(k), p_4(k)\big)$ contain the joint probability of a man aged $x + k$ years old and woman aged $y + k$ years old with the following form

$$p_1(k) = \frac{\bar{C}\big(\bar{F}(s+k), \bar{G}(t+k)\big)}{\bar{C}\big(\bar{F}(s), \bar{G}(t)\big)} \tag{32}$$

$$p_2(k) = \frac{\bar{C}\big(\bar{F}(s), \bar{G}(t+k)\big) - \bar{C}\big(\bar{F}(s+k), \bar{G}(t+k)\big)}{\bar{C}\big(\bar{F}(s), \bar{G}(t)\big)} \tag{33}$$

$$p_3(k) = \frac{\bar{C}\big(\bar{F}(s+k), \bar{G}(t)\big) - \bar{C}\big(\bar{F}(s+k), \bar{G}(t+k)\big)}{\bar{C}\big(\bar{F}(s), \bar{G}(t)\big)} \tag{34}$$

$$p_4(k) = \frac{\bar{C}\big(\bar{F}(s), \bar{G}(t)\big) - \bar{C}\big(\bar{F}(s), \bar{G}(t+k)\big) - \bar{C}\big(\bar{F}(s+k), \bar{G}(t)\big) + \bar{C}\big(\bar{F}(s+k), \bar{G}(t+k)\big)}{\bar{C}\big(\bar{F}(s), \bar{G}(t)\big)} \tag{35}$$

Proof. The joint survival function of future lifetimes T_x^M and T_y^W in state $i = 1$ where both spouses are alive and denoted by $p_1(k)$ can be formulated as follows.

$$p_1(k) = \Pr(T_x^M > k, T_y^W > k)$$

Note that T_x^M and T_y^W are the future lifetime random variables of man aged x and woman aged y respectively. Also defined $T_{x_0}^M$ and $T_{y_0}^W$ which are the future lifetime random variables of man aged x_0 and woman aged y_0 respectively, where x_0 and y_0 are the reference ages when the future lifetime of spouse is assumed to be dependent. It is known that $x = x_0 + s$ and $y = y_0 + t$. The distribution of $T_{x_0}^M$ can be derived through the distribution of T_x^M. Under the condition that (x_0) survives until age x, i.e. $T_{x_0}^M > s$, then the age at death is expressed as

$$s + T_x^M \equiv T_{x_0}^M | T_{x_0}^M > s$$
$$T_x^M \equiv T_{x_0}^M - s | T_{x_0}^M > s.$$

The same is true for $T_{y_0}^W$ and T_y^W. Furthermore, the concept of conditional probability is used to measure the likelihood of an event occurring given that another event has already occurred. If there are two events C_1 and C_2, the conditional probability of C_2 given C_1 (denoted as $P(C_2|C_1)$ is defined as

$$P(C_2|C_1) = \frac{P(C_1 \cap C_2)}{P(C_1)}.$$

Using the relationship between $T_{x_0}^M$ and T_x^M, $T_{y_0}^W$ and T_y^W, the conditional probability concept, and the relation of the joint survival function and copula survival on **Equation (31)**, then **[8]**

$$\begin{aligned} p_1(k) &= \Pr(T_x^M > k, T_y^W > k) \\ &= \Pr(T_{x_0}^M > s + k, T_{y_0}^W > t + k \mid T_{x_0}^M > s, T_{y_0}^W > t) \\ &= \frac{\Pr(T_{x_0}^M > s + k, T_{y_0}^W > t + k)}{\Pr(T_{x_0}^M > s, T_{y_0}^W > t)} \\ &= \frac{\bar{H}(s + k, t + k)}{\bar{H}(s, t)} \\ &= \frac{\bar{C}(\bar{F}(s + k), \bar{G}(t + k))}{\bar{C}(\bar{F}(s), \bar{G}(t))}. \end{aligned}$$

In a similar way, the joint survival function of future lifetimes T_x^M and T_y^W in state $i = 2$ where the husband is dead and denoted by $p_2(k)$ can be obtained as follows **[8]**

$$\begin{aligned} p_2(k) &= \Pr(T_x^M \le k, T_y^W > k) \\ &= \Pr(T_{x_0}^M \le s + k, T_{y_0}^W > t + k \mid T_{x_0}^M > s, T_{y_0}^W > t) \\ &= \frac{\Pr(s < T_{x_0}^M \le s + k, T_{y_0}^W > t + k)}{\Pr(T_{x_0}^M > s, T_{y_0}^W > t)} \\ &= \frac{\Pr(T_{x_0}^M > s, T_{y_0}^W > t + k) - \Pr(T_{x_0}^M > s + k, T_{y_0}^W > t + k)}{\Pr(T_{x_0}^M > s, T_{y_0}^W > t)} \\ &= \frac{\bar{H}(s, t + k) - \bar{H}(s + k, t + k)}{\bar{H}(s, t)} \\ &= \frac{\bar{C}(\bar{F}(s), \bar{G}(t + k)) - \bar{C}(\bar{F}(s + k), \bar{G}(t + k))}{\bar{C}(\bar{F}(s), \bar{G}(t))} \end{aligned}$$

Analogously to $p_2(k)$, $p_3(k)$ can be obtained corresponding to the **Equation (34)**. Then, using the equality $p_4(k) = 1 - p_1(k) - p_2(k) - p_3(k)$, proof of **Equation (35)** can be obtained. Therefore, it is proven that **Equation (32)** till **Equation (35)** are true.

Observe that if $w > \omega_{x_0}^M$, then $\bar{F}(w) = 0$ and if $z > \omega_{y_0}^W$, then $\bar{G}(z) = 0$. Based on these properties, it is clear that the survival copula satisfies

$$\bar{C}(\bar{F}(w), \bar{G}(z)) = \begin{cases} \bar{C}(\bar{F}(w), \bar{G}(z)), & \text{for } w < \omega_{x_0}^M \text{ and } z < \omega_{y_0}^W \\ 0, & \text{in other cases} \end{cases} \quad (36)$$

Using **Equation (36)**, form of vector $\mathbf{P}(k) = (p_1(k), p_2(k), p_3(k), p_4(k))$ depending on spouses' age and the contract period can be obtained as shown in the **Table 2**.

Table 2. Form of Vector $\mathbf{P}(k) = (p_1(k), p_2(k), p_3(k), p_4(k))$ Depending on Spouses' Age and Contract Period

(x, y)	$k = 1, 2, \dots, \min\{\omega_x^M, \omega_y^W\} - 1$ $(x + k > \omega_x^M$ and $y + k > \omega_y^W)$	$k = \min\{\omega_x^M, \omega_y^W\}, \dots, \max\{\omega_x^M, \omega_y^W\}$ $(x + k > \omega_x^M$ and/or $y + k > \omega_y^W)$
$x = y$	$\mathbf{P}(k) = (p_1(k), p_2(k), p_3(k), p_4(k))$	$\mathbf{P}(k) = (0, 0, 0, 1)$
$x > y$	$\mathbf{P}(k) = (p_1(k), p_2(k), p_3(k), p_4(k))$	$\mathbf{P}(k) = (0, p_2(k), 0, p_4(k))$
$x < y$	$\mathbf{P}(k) = (p_1(k), p_2(k), p_3(k), p_4(k))$	$\mathbf{P}(k) = (0, 0, p_3(k), p_4(k))$

The marriage reverse annuity contract is determined based on the value of the real estate (property consisting of land and buildings on it, ex: homes or apartments) denoted by W and the percentage of the real estate value W paid to the spouses denoted by β, where $\beta \in (0,1]$. The real estate is sold for less than its market price, so the spouses receive a discount amount on the real estate sold, the amount of which depends on their age (typically between 35% and 60%) [16]. Using the equivalence principle, the benefits of the marriage reverse annuity contract can be determined as follows [17]

$$E(\text{PV of benefit outflows}) = E(\text{PV of premium inflows})$$
$$E(b \cdot Z) = E(\beta \cdot W)$$
$$b = \frac{\beta \cdot W}{E(Z)} \tag{37}$$

where b is the contractual benefit paid and $E(Z)$ is the expected discounted value of the benefit. Note that $E(\beta) = \beta$ and $E(W) = W$ because β and W is not a random variable, it is only used as an assumption variable whose input value will be a number. Since the stream of benefits arising during the contract period is paid at the beginning of each year, the benefits are paid in the form of life annuity due. Life annuity due for joint life status is defined as

$$\ddot{a}_{xy:\overline{n}|} = \sum_{k=0}^{n-1} v(k) \cdot {}_k p_{xy} \tag{38}$$

and life annuity due for last survivor status defined as

$$\ddot{a}_{\overline{xy}:\overline{n}|} = \sum_{k=0}^{n-1} v(k) \cdot {}_k p_{\overline{xy}} \tag{39}$$

where $v(k) = \frac{1}{(1+i)^k}$ is the discount factor, i is interest rate, and $k = 0, 1, 2, \dots, n-1$. Based on Denuit [18], the notation used for marriage life insurance can be written using actuarial notation with the following definition

$${}_k p_{xy} = \Pr\left(T_x^M > k \wedge T_y^W > k\right) = p_1(k) \tag{40}$$
$${}_k p_{\overline{xy}} = \Pr\left(T_x^M > k \vee T_y^W > k\right) = p_1(k) + p_2(k) + p_3(k) \tag{41}$$

However, in the calculation of the benefits of marriage reverse annuity contracts, the notation ${}_k p_{\overline{xy}}$ in **Equation (41)** is defined as

$${}_k p_{\overline{xy}} = \Pr\left(T_{y_0}^W > t + k \middle| T_{x_0}^M > s, T_{y_0}^W > t\right) + \Pr\left(T_{x_0}^M > s + k \middle| T_{x_0}^M > s, T_{y_0}^W > t\right) - {}_k p_{xy} \tag{42}$$

Depending on the contract status, the annuity benefit from the marriage contract reverse annuity contract can be paid in any $S^b \subset S$ where $S^b = \{1\}$ for JLS and $S^b = \{1, 2, 3\}$ for LSS. Therefore, equation (34) can be rewritten with the contract benefit is denoted as $\ddot{b}_{S^b}$, where the benefit is paid during the whole period has the following form

$$\ddot{b}_{S^b} = \begin{cases} \dfrac{\beta W}{\sum_{k=0}^{n-1} v(k) \cdot {}_k p_{xy}}, & \text{for } S^b = \{1\} \text{ and } n = \min\{\omega_x^M, \omega_y^W\} \text{ (JLS)} \\ \dfrac{\beta W}{\sum_{k=0}^{n-1} v(k) \cdot {}_k p_{\overline{xy}}}, & \text{for } S^b = \{1,2,3\} \text{ and } n = \max\{\omega_x^M, \omega_y^W\} \text{ (LSS)} \end{cases} \tag{43}$$

The calculation of annuity benefits of marriage reverse annuity contract will be carried out in the next chapter using **Equation (43)** with the probabilities ${}_k p_{xy}$ and ${}_k p_{\overline{xy}}$ determined by **Equation (40)** and **Equation (42)** respectively.

3. RESULTS AND DISCUSSION

In this section, numerical calculations of marriage reverse annuity contract for joint life and last survivor status will be simulated with assumption of dependency in the future lifetime of spouse using Archimedean copula. Assume that the real estate value W is equal to $Rp1.000.000.000,00$ and the percentage β of real estate value W is equal to 50%. It is assumed that the future lifetime of the husband and wife is dependent when the husband is $x_0 = 30$ years old and the wife is $y_0 = 30$ years old. Since this contract for retirees, the entry age of the husband and wife (x, y) is assumed to be (60, 60) years old.

3.1 Data

In this study, the data used is Indonesian Mortality Table IV which is a scheme that provides information about the probability of death of the Indonesian population of male and female sexes at a certain age. It's the last mortality table launched in 2019 with data exposure of 52 life insurance companies, where this number is greater than the previous version which is 40 companies [19].

Before estimating the copula parameters (θ), first calculate the values of ${}_kq_{x_0}$ and ${}_kq_{y_0}$ for $x_0 = y_0 = 30$ which is the reference age when the lifetimes of husband and wife is assumed to be dependent. Also calculate the values of ${}_kq_x$ and ${}_kq_y$ for $x = y = 60$ that will be used in the calculation of the independent assumption. The values of ${}_kq_{x_0}, {}_kq_{y_0}, {}_kq_x$, and ${}_kq_y$ can be obtained using Indonesian Mortality Table IV and the formulas below

$$ {}_{t+u}p_x = {}_kp_x \cdot {}_up_{x+t} \tag{44} $$

$$ {}_tp_x + {}_tq_x = 1 \tag{45} $$

3.2 Copula Parameter Estimation

After obtaining the values of ${}_kq_{x_0}$ and ${}_kq_{y_0}$, the next step is to perform estimation of the parameter θ in copula using CML method in **Equation (29)** with copula density function for Clayton, Gumbel, and Frank shown in **Equation (19)**, **Equation (22)**, and **Equation (25)** respectively. Through R software, the estimated value of parameter θ in copula are obtained as follows.

Table 3. Estimated Value of Parameter θ in Copula

Copula	$c_\theta(u,v)$	$\hat{\theta}$
Clayton	$(1+\theta)\left(u^{-\theta}+v^{-\theta}-1\right)^{-\left(\frac{1}{\theta}+2\right)}(uv)^{-(\theta+1)}$	4.910293
Gumbel	$C_\theta^{Gumbel}(u,v)^* \cdot \frac{\left((-\ln u)(\ln v)\right)^{\theta-1}}{uv} \cdot \left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{2}{\theta}-2} \cdot \left[1+(\theta-1)\left((-\ln u)^\theta+(-\ln v)^\theta\right)^{-\frac{1}{\theta}}\right]$	3.101762
Frank	$-\frac{\theta e^{-\theta(u+v)}\left(e^{-\theta}-1\right)}{\left[e^{-\theta(u+v)}-e^{-\theta u}-e^{-\theta v}+e^{-\theta}\right]^2}$	25.44457

* $C_\theta^{Gumbel}(u,v) = \exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{1}{\theta}}\right\}$

3.3 Marriage Reverse Annuity Contract Benefit Calculation

The contract period for joint life status is defined as $n^{JLS} = min\{111 - x + 1, 111 - y + 1\}$ and for the last survivor status is $n^{LSS} = max\{111 - x + 1, 111 - y + 1\}$, where x, y is the entry age of husband and wife that is $(x, y) = (60, 60)$ years old. Then, it is found that the contract period for both statuses is the same, which is 52 years. In adddition, it is known that the value of real estate $W = Rp1.000.000.000{,}00$, percentage of real estate value $\beta = 50\%$, and constant interest rate $i = 5\%$.

First, calculate the copula distribution function and survival copula function for Clayton, Gumbel, and Frank copula using estimated value from **Table 3**. The survival copula function that is obtained from the first step is used to calculate the survival probability for each copula (Clayton, Gumbel, and Frank) in joint life status using **Equation (40)** and **Equation (32)**, and the survival probability for each copula (Clayton, Gumbel, and Frank) in last survivor status using **Equation (42)**.

In the independent assumption of marriage reverse annuity contract, after obtaining the values of ${}_kp_x$ and ${}_kp_y$ for $x = y = 60$, the survival probability in joint life status is calculated using the formula

$$_kp_{xy} = {_kp_x} \cdot {_kp_y} \tag{46}$$

and the survival probability in last survivor status is calculated using

$$_kp_{\overline{xy}} = {_kp_x} + {_kp_y} - {_kp_x} \cdot {_kp_y} \tag{47}$$

Next, calculate the life annuity due for joint life and last survivor status with dependent assumption using Clayton, Gumbel, and Frank copula, and independent assumption without using copula. The numerical result of life annuity due for joint life and last survivor status is presented in Table 4 below

Table 4. Joint Life and Last Survivor Status Life Annuity Due Calculation

Copula	Joint Life Status	Last Survivor Status
	$\sum_{k=0}^{n-1} v(k) \cdot {_kp_{xy}}$	$\sum_{k=0}^{n-1} v(k) \cdot {_kp_{\overline{xy}}}$
Clayton	13,55859483	15,22473351
Gumbel	13,46175942	15,61506094
Frank	13,77001809	12,26644257
Independent	12,21174131	16,04892851

where $v(k) = \frac{1}{(1+i)^k} = \frac{1}{(1+0{,}05)^k}$ is discount factor with $k = 0, 1, 2, \dots, 51$.

Finally, the annuity benefit of marriage reverse annuity contract in joint life and last survivor status for 60 years old husband and 60 years old wife can be calculated using Equation (43) and results of life annuity due from Table 4. Note that it is assumed that the value of real estate $W = Rp1.000.000.000{,}00$ and percentage $\beta = 50\%$.

Table 5. Annuity Benefit of Marriage Reverse Annuity Contract

Copula	Joint Life Status	Last Survivor Status
	$\ddot{b}_{S^b=\{1\}}$	$\ddot{b}_{S^b=\{1,2,3\}}$
Clayton	Rp36.876.977,75	Rp32.841.297,34
Gumbel	Rp37.142.247,48	Rp32.020.368,15
Frank	Rp36.310.772,91	Rp32.918.215,74
Independent	Rp40.944.201,76	Rp31.154.727,86

From the calculation results in Table 5, the value of annual benefit in joint life status is always greater than the last survivor status. When viewed on each status, it is found that in the joint life status, the largest value of annual benefit is generated from calculation without using copula and the smallest value of annual benefit is generated from calculation using Frank copula (Independent > Gumbel > Clayton > Frank). Meanwhile, in the last survivor status, the largest value of annual benefit is generated from the calculation using the Frank copula and the smallest value of annual benefit value is generated from calculation without using the copula (Frank > Clayton > Gumbel > Independent). Or it can be said that the order of the benefit value is contrary to the joint life status.

Based on the explanation above, it can be concluded that for the joint life status, the value of annual benefit will be greater without copula than with copula. Except for the last survivor status vice versa. The results of previous research conducted by Debicka, Heilpern, and Marciniuk [8] also concluded the same thing regarding the value of benefits on joint life status contracts and last survivor status even though the parameter estimation method used was different, namely using the Kendall's tau coefficient of correlation method and the copula used was only AMH and Gumbel.

This numerical simulation is also useful as an insight for companies (insurers) that want to sell annuity products in the form of marriage reverse annuity contract. Of course, from the companies' perspective that wants to get more income or profit, the company will choose to pay the benefits for the insured as small as possible to avoid being overpriced. To overcome this, the company will consider again using the assumption of calculating the annual benefit of the contract.

If seen in the joint life status, it is found that to produce the smallest value of annual benefit of the contract, the suitable assumption used is the dependency assumption with Frank copula. Meanwhile, in the last survivor status, it is found that to produce the smallest value of annual benefit of the contract, the suitable assumption used is the independence assumption (without using copula).

4. CONCLUSIONS

1. The marriage reverse annuity contracts for joint life and last survivor status are modeled with multiple states. Joint life status contract pays annual benefits until the death of one of the spouses, while the last survivor status contract pays annual benefits until the death of the other spouse. This study assumes that the future lifetime of the spouse is dependent which is modeled using Archimedean copula. The probability structure of the distribution function and survival function of the spouses' future lifetime are constructed based on Sklar's theorem and survival copula function. The marriage reverse annuity contract is determined based on the real estate value and the percentage of the real estate value which is explained in the **Equation (43)**.
2. Based on simulation of marriage reverse annuity contract benefit calculation with dependency and independence assumption, it is obtained that
    a) the annual benefit received for the marriage reverse annuity contract in joint life status is always greater than in last survivor status,
    b) in the joint life status, the value of the annual benefit of the marriage reverse annuity contract sorted from the largest value to the smallest is Independent > Gumbel > Clayton > Frank,
    c) the last survivor status is contrary to the joint life status, the value of annual benefit of the marriage reverse annuity contract sorted from the largest to the smallest value is Frank > Clayton > Gumbel > Independent, and
    d) for the insurer, if they want to make a profit, then to calculate the annual benefit value of the marriage reverse annuity contract for joint life status can use the dependency assumption with Frank copula, while to calculate the annual benefit value of the marriage reverse annuity contract for last survivor status can use the independence assumption (without copula).

REFERENCES

[1] U. Reifner, S. Clerc-Renaud, E. F. P. Carrillo, A. Tiffe, and M. Knobloch, "Study on equity release schemes in EU. Part I: General Report," *Institut für Finanzdienstleistungen e.V.*, Jan. 2009.

[2] K. HanewaldS M. Sherris, "Portfolio Choice in Retirement-What is The Optimal Home Equity Release Product?," *Journal of Risk and Insurance*, vol. 83, no. 2, pp. 421–446, Jun. 2016.

[3] A. W. Shao, K. Hanewald, and M. Sherris, "Reverse mortgage pricing and risk analysis allowing for idiosyncratic house price risk and longevity risk," *Insurance: Mathematics and Economics*, vol. 63, pp. 76–90, Jul. 2015.

[4] E. Luciano, J. Spreeuw, and E. Vigna, "Spouses' dependence across generations and pricing impact on reversionary annuities," *Risks*, vol. 4, no. 2, Jun. 2016.

[5] J. Spreeuw, "Types of dependence and time-dependent association between two lifetimes in single parameter copula models," *Scandinavian Actuarial Journal*, vol. 2006, no. 5, pp. 286–309, Sep. 2006.

[6] E. Luciano, J. Spreeuw, and E. Vigna, "Modelling stochastic mortality for dependent lives," *Insurance: Mathematics and Economics*, vol. 43, no. 2, pp. 234–244, Oct. 2008.

[7] S. Heilpern, "Dependent Structure Induced by Markov Chain in the Multiple Life Insurance," in 18th International Scientific Conference Application of Mathematics and Statistics in Economics, Sep. 2-6, 2015.

[8] J. Dębicka, S. Heilpern, and A. Marciniuk, "Application of copulas to modelling of marriage reverse annuity contract," *Prague Economic Papers*, vol. 29, no. 4, pp. 445–468, 2020.

[9] R. B. Nelsen, *An Introduction to Copulas*, 2nd ed., New York: Springer, 2006.

[10] C. Schölzel and P. Friederichs, "Multivariate non-normally distributed random variables in climate research-introduction to the copula approach," *Nonlinear Processes Geophysics*, vol. 15, no. 5, pp. 761–772, 2008.

[11] Z. Sharifonnasabi, M. H. Alamatsaz, and I. Kazemi, "A large class of new bivariate copulas and their properties," *Brazilian Journal Probability and Statistics*, vol. 32, no. 3, pp. 497–524, 2018.

[12] U. Cherubini, E. Luciano, and W. Vecchiato, *Copula Methods in Finance*, Chichester: John Wiley & Sons Ltd, 2004.

[13] J. T. Shiau, "Fitting drought duration and severity with two-dimensional copulas," *Water Resources Management*, vol. 20, no. 5, pp. 795–815, Oct. 2006.

[14] P. K. Trivedi and D. M. Zimmer, "Copula modeling: An introduction for practitioners," *Foundations and Trends in Econometrics*, vol. 1, no. 1, pp. 1–111, 2005.

[15] H. Manner, "Estimation and model selection of copulas with an application to exchange rates," *Meteor Research Memorandum*, no. 056, Jan. 2007.

[16] ASIC, "Equity release products," *ASIC,* Nov. 2005.

[17] A. Marciniuk, “Marriage Reverse Annuity Contract and Reverse Mortgage – Application of a Generalized Model of Reversionary Annuity,” in 20th International Scientific Conference Application of Mathematics and Statistics in Economics, Aug. 30 - Sept. 3, 2017.

[18] M. Denuit, J. Dhaene, C. Le Bailly De Tilleghem, and S. Teghem, “Measuring the Impact of a Dependence Among Insured Lifelengths,” *Belgian Actuarial Bulletin*, vol. 1, no. 1, pp. 18–39, Jan. 2001.

[19] AAJI, “TABEL MORTALITAS INDONESIA IV,” *AAJI,* Dec. 2019.