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BAYESIAN NEURAL NETWORK RAINFALL MODELLING: A CASE STUDY IN EAST JAVA

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ABSTRACT

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Rainfall is an important parameter in meteorology and hydrology, and it measures the amount of rain that falls from the atmosphere to the ground surface in liquid form. However, in the process of measuring rainfall, changes in the rainfall cycle sometimes occur due to climate change, global warming, and other factors. Therefore, this research aims to model daily rainfall using the Bayesian Neural Network (BNN) approach, combining the Bayesian Method and Artificial Neural Network (ANN). ANN is suitable for rainfall models that have intermittent characteristics. Meanwhile, the Bayesian method provides advantages in producing model parameter inferences that provide uncertainty measurements in predictions. BNN is expected to deliver better daily rainfall predictions than ANN. This research used daily rainfall data in East Jawa, and the results show that the Bayesian Neural Network produces better rainfall predictions when describing rainfall in East Java. These predictions will be very useful for the government and the people of East Java province to prevent flooding. Also, with rainfall predictions, people will know more about what crops should be planted during the rains.

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1. INTRODUCTION

Indonesia is a country that has two seasons, namely, the dry season and the rainy season. During the rainy season, the amount of rainwater that falls to the earth's surface can be small or large and cause flooding. The amount of air rain in a certain area during a specific period can be called rainfall. Rainfall is an important parameter in meteorology and hydrology, and it measures the amount of air rain that falls from the atmosphere to the ground surface in liquid form. Rainfall can be measured in various units, such as millimeters (mm) or inches (inches). It can be expressed as amounts per unit area or overall totals within a given area. However, in measuring rainfall, changes in the rainfall cycle sometimes occur due to climate change, global warming, and other factors. Therefore, it is important to model rainfall, especially in East Java, so that it can predict rainfall in that area. These predictions will be beneficial for the government and the people of East Java province to prevent flooding. Also, with rainfall predictions, people will know more about what crops should be planted during that period.

In modeling rainfall, various techniques can be used, namely: Astutik et al. [1] developed a rainfall model with a Bayesian state space approach to spatio temporal data, Astutik et al. [2], [3]; Kumari et al. [4] developed models rainfall with Geographically Weighted Regression (GWR), [5] discuss Generalized Space Time Autoregressive (GSTAR) on rainfall data in East Java; Holsclaw et al. [6] developed a Bayesian Hidden Markov Model (HMM) for downscaling at several locations daily rainfall data in South and East Asia; Astutik et al. [7], [8], developed a rainfall model in climate change prediction with Spatial HMM Bayesian approach. Astutik, et al. [9] developed a rainfall model with Artificial Neural Network (ANN). Nourani et al. [10] developed a method for estimating prediction intervals on ANN in the field of hydrology. Sarasa-Cabezuelo [11] predicted rainfall in Australia with ANN. Bayesian methods can produce model parameter inferences that measure of uncertainty in predictions. Markov Chain Monte Carlo (MCMC) algorithm in the Bayesian method provides advantages in applying inference to ANN [12].

Bayesian Neural Network (BNN) is an ANN design based on Bayesian concepts. BNN is an approach that combines Bayesian probability principles into neural networks for statistical modeling and machine learning. In a BNN, each model parameter (e.g., weights and biases in neurons) is considered as a random variable distributed according to a probability distribution. In other words, not only does it have a single value for each parameter, but it also has a probability distribution that describes that value. When a BNN is drilled, it not only tries to minimize the prediction error as in a conventional neural network but also to update the probability distribution of the model parameters based on the given data. In this way, Bayesian neural networks can produce predictions accompanied by robust estimates. The research aims to integrate the Bayesian method and ANN for implementation in modeling rainfall patterns in East Java. Understanding rainfall patterns in East Java is fundamental because some areas also flood, but there are droughts. By understanding this pattern, disasters can be prevented.

2. RESEARCH METHODS

2.1 Bayesian Analysis

According to Gelman [13], Bayesian inference is the process of fitting a probability model to a data group and summarizing the results with a probability distribution on model parameters and unobserved quantities, such as predictions for new observations. In the Bayesian approach, all parameters are indeed considered as random variables. This is one of the fundamental principles of Bayesian statistics. Bayesian methods are a statistical framework for analyzing data and making inferences based on Bayesian probability concepts. This method is known as "Bayesian" because it refers to the work of Thomas Bayes, an 18th-century mathematician and theologian who developed Bayes' theorem, which is the basis of this approach. In Bayesian analysis, there are several important concepts, including:

- Prior Distribution, namely initial beliefs about the phenomenon being observed before new data comes in. Prior distributions can be based on previous knowledge, experience, or reasonable assumptions.
- Likelihood function is a statistical model describing the relationship between observed data and the parameters you want to estimate. Likelihood measures the degree to which a parameter might explain the data found.

- Posterior Distribution, namely a probability distribution that describes our beliefs about parameters after entering new data. The posterior Distribution is calculated by combining the prior and likelihood distributions using Bayes' Theorem.
- Bayes' Theorem
  Bayes' theorem states that the posterior distribution is proportional to the product of the prior and probability distributions. This theorem is used as the basis of the method estimating the parameters of a distribution or a model.

Bayesian analysis is a statistical approach to estimate decisions based on probability distributions. This analysis uses probability theory to update beliefs about an event or phenomenon as additional information arrives. The posterior distribution is comparable or proportional to the product of the prior distribution and the probability function, which can be written as follows [14]:

$$f(\theta|y,\alpha) = \frac{f(y|\theta)f(\theta|\alpha)}{f(y|\alpha)} \propto f(y|\theta)f(\theta|\alpha) \tag{1}$$

$$\text{where } f(y|\theta) = \prod_{i=1}^{n} f(y_i|\theta) \text{ dan } f(y|\alpha) = \int f(y|\theta)f(\theta|\alpha)d\theta \tag{2}$$

Information :
$f(\theta|y,\alpha)$: posterior distribution of model parameters θ
$f(\theta|\alpha)$: prior distribution of model parameter θ with hyper-parameters α.
$f(y|\theta)$: multiplication of the probability density function evaluated at each observation value.
$f(y|\alpha)$: marginal distribution of y

Bayesian methods' advantages include overcoming communications, formally combining initial information with new data, and producing precise probability estimates. Bayesian methods are used in various scientific disciplines, including statistics, computer science, natural sciences, social sciences, and many more. Their applications include data analysis, statistical modeling, Bayesian machine learning, and decision-making under uncertainty.

2.2 Bayesian Neural Network

ANN is a kind of artificial intelligence [15] that pursues reproducing the network of neurons to make up the human brain so that processers may recognize the brain signals and make decisions like a human being in the computing system. Neurons are the building blocks of the brain, central nervous system [16], spinal cord, and peripheral nervous system ganglia. When utilizing spatial information, ANN can be called spatial analysis neural network (SANN). SANN is a nonparametric spatial analysis model based on a neural network computational scheme. It has been developed [17] for point estimation and classification of spatial data.

BNN is an ANN that integrates Bayesian probability concepts into the neural network architecture. The main goal of BNN is to overcome uncertainty in machine learning by treating neural network model parameters as random variables distributed according to a probability distribution. In BNNs, each parameter, such as weights and biases in neurons, not only has a single value but also has a probability distribution that describes the uncertainty about that value. Following are some of the characteristics and advantages of BNN.

a. Measurable Uncertainty: BNN allows us to measure and manage uncertainty in the model. For example, in situations where there is little training data or when the data used for training is not entirely reliable.
b. Automatic Regularization: By treating parameters as random variables with probability distributions, BNNs naturally generate regularization in the model. This can help reduce overfitting because the model tends to be more conservative in making predictions.
c. Predictions Accompanied by Uncertainty: BNNs provide predictions and produce uncertainty estimates that can be used to measure the extent to which we can trust those predictions. This is useful in making decisions related to risk, such as in financial or health applications.
d. Management of Incomplete or Noisy Data: BNN can overcome problems where the data used for training could be perfect and more convenient. By modeling uncertainty, BNN can handle situations where the data has outliers or lacks information.

However, BNNs also have challenges, including higher computational complexity and the need to use MCMC or variational inference methods to estimate parameter probability distributions. This can make such analyses more difficult to implement and train than conventional neural networks.

In particular, we show: (1) BNNs can achieve significant performance gains over standard training and deep ensembles; (2) a single long Hamiltonian Monte Carlo (HMC) chain can provide a comparable performance to multiple shorter chains; (3) in contrast to recent studies, we find posterior tempering is not needed for near-optimal performance, with little evidence for a "cold posterior" effect, which we show is largely an artifact of data augmentation; (4) Bayesian Model Averaging (BMA) performance is robust to the choice of prior scale and relatively similar for diagonal Gaussian, mixture of Gaussian, and logistic priors overweights. This result highlights the importance of architecture relative to parameter priors in specifying the prior over functions. (5) While BNNs have good performance for out-of-distribution (OOD) detection, they show surprisingly poor generalization under domain shift; (6) while cheaper alternatives such as deep ensembles and Stochastic Gradient Markov Chain Monte Carlo (SGMCMC) can provide good generalization, their predictive distributions are distinct from HMC. Notably, deep ensemble predictive distributions are similar to HMC as standard Stochastic Gradient Langevin Dynamics (SGLD) and closer than standard variational inference **[18]**.

2.3. Data

Data of this research is daily rainfall data at 11 East Java rainfall stations in 2022, which is then averaged at each station. This secondary data was obtained from the official website of the Meteorology, Climatology and Geophysics Agency (BMKG) **[19]**. This research involves spatial elements, namely the areas of the eleven rain stations. The research variables are rainfall (mm), month, latitude, and longitude.

2.4. Research Steps

- Data Preprocessing: data cleaning, normalization, filling in missing data, and converting categorical data to an appropriate form.
- Data Division: Data is divided into training, validation, and test sets. The training set is used to train the neural network, the validation set is used to optimize parameters, and the test set is used to test the network's performance.
- Model Selection: choosing the appropriate type of ANN as there are many different types of ANN, such as feedforward neural networks, recursive neural networks, or convolutional neural networks, depending on the type of data and problem at hand. One way to get the best model is to look for the model with the smallest Root Mean Square Error (RMSE) value.
- Model Initialization: The ANN model must be initialized with appropriate initial weights and biases. This initialization method can affect the training results.
- Model Training: uses training algorithms such as Backpropagation to optimize weights and biases so that the model can produce accurate predictions.
- Validation and Tuning: After training, the model needs to be validated using the validation set, and adjustments can be made to parameters such as the number of layers and neurons in the network to improve performance.
- Model Evaluation: tests its final performance on a separate test set and can use evaluation metrics such as accuracy, precision, recall, and F1-score to measure model performance.

3. RESULTS AND DISCUSSION

3.1 Descriptive Analysis

The result of rainfall data exploration was showed on **Table 1**.

Table 1. Descriptive Statistics

	Latitude	Longitude	Rainfall
Minimum	2657398	8607799	0.000
Quarter 1	2717202	8728131	0.000
Median	2766599	8737946	1.300
Mean	2767358	8737920	8.944
Quarter 3	2790703	8785327	10.200
Maximum	2909045	8806747	160.000

Data source: *BMKG* **[19]**

Based on **Table 1**, the average daily rainfall of the 11 East Java stations in 2022 is 8.944 mm. The highest rainfall is 160 mm, while the lowest is 0 mm. Suppose the rainfall is divided into 3 quartiles. In that case, the first quartile has a total rainfall of 0 mm, the second quartile (median) is 1.30 mm, and the third quartile is 10.20 mm.

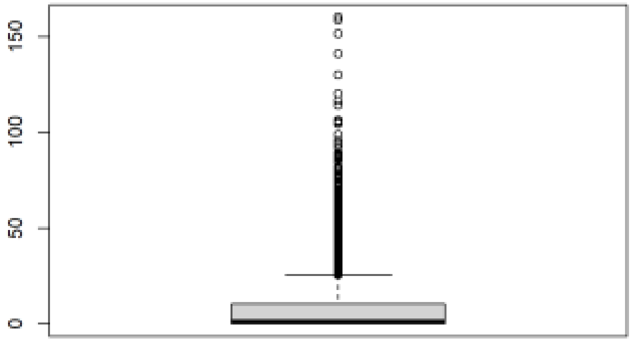


Figure 1. Boxplot of Rainfall Data at 11 East Java Stations in 2022

Figure 1 shows two outliers of the rainfall data at 11 East Java stations in 2022. These outliers will not affect this analysis because Bayesian analysis is robust to assumptions and outliers.

3.2 Bayesian Neural Network Modeling

A total of 92 observations on training data result in the number of Markov chains, iterations, burn-in, and thin are 4, 2000, 1000, and 1, respectively. Based on the 4000 posterior iterations obtained, the following group-level effects are presented in **Table 2**.

Table 2. Group Level Effects for the Month: Station Variable

	Estimate	**Est. Error**	**l-95% CI**	**u-95% CI**	**Rhat**	**Bulk_ESS**	**Tail_ESS**
sd (Intercept)	68.82	35.07	3.24	107.23	2.21	5	24

Table 2 shows that the estimation standard deviation (sd(Intercept)) and estimation error (Est.Error) are 68.82 and 35.07, respectively. Meanwhile, this estimation's 95% confidence interval is 3.24 to 107.23. Rhat value is 2.21 (Rhat > 1), which means that the chain has not been mixed properly. Bulk-Effective Sample Size (Bulk-ESS) is useful as a diagnostic tool for sampling efficiency in the bulk of the posterior. The estimated mass effective sample size (in most of the distribution) is 5 samples, and the estimated effective sample size in the tail of the distribution is 24 samples. The estimation result of each station can be seen in **Table 3**.

Table 3. Population Level Effects

	Estimate	**Est.Error**	**l-95% CI**	**u-95% CI**	**Rhat**	**Bulk _ESS**	**Tail _ESS**
Intercept	2817365.60	6735510.00	-4990509.59	16200789.37	2.92	5	18
Nganjuk Geophysical Station	-25623.48	52074.17	-129700.78	36858.71	3.05	5	18

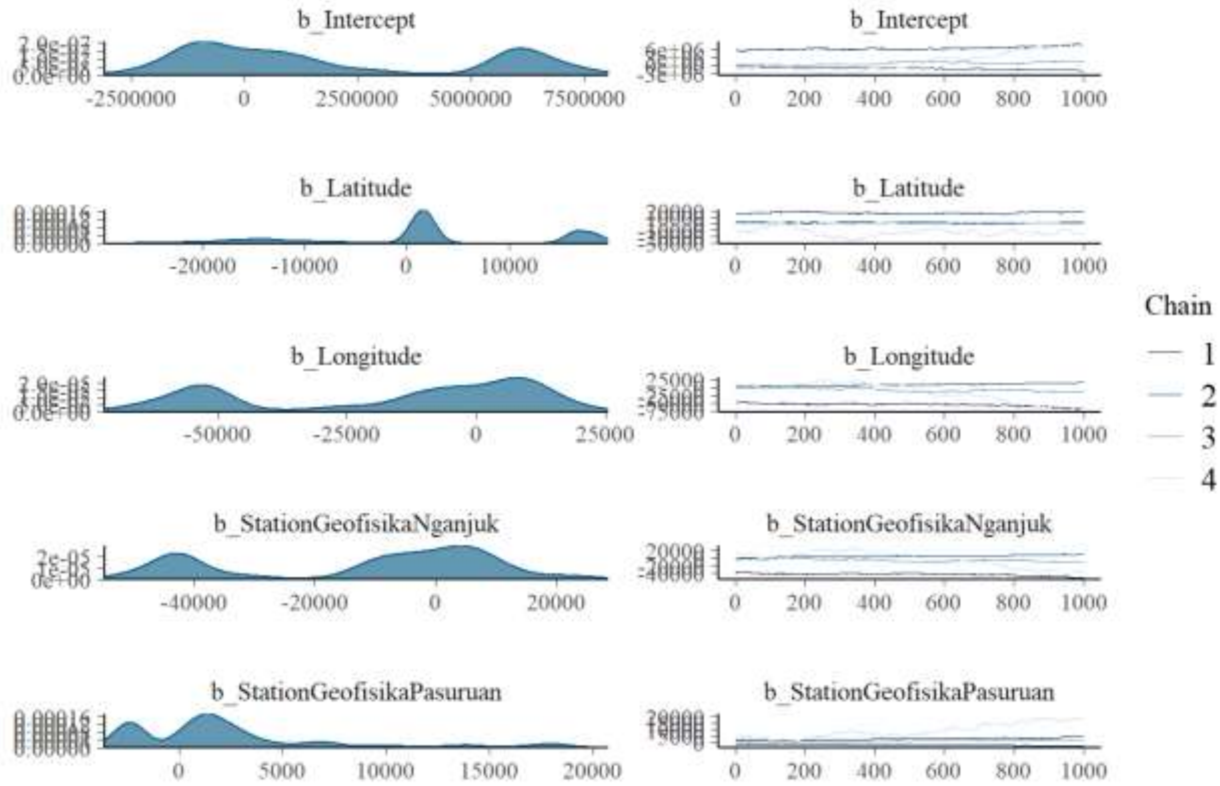
	Estimate	Est.Error	l-95% CI	u-95% CI	Rhat	Bulk _ESS	Tail _ESS
Pasuruan Geophysical Station	-6116.91	4439.28	-14414.33	479.10	2.91	5	11
East Java Climatology Station	-2361.27	2185.55	-6499.59	525.81	2.18	5	14
Banyuwangi Meteorological Station	45878.35	111907.46	-83542.59	267984.00	2.90	5	18
Juanda Meteorological Station	-10188.22	7134.98	-22999.95	318.68	2.71	5	11
Maritim Tanjung Perak Meteorological Station	-15584.86	14156.47	-43571.26	4459.28	3.27	4	12
Perak I Meteorological Station	-15404.57	14167.31	-43426.96	4645.27	3.30	4	12
Sangkapura Meteorological Station	-49036.36	60208.29	-168732.95	32272.66	3.46	4	15
Trunojoyo Meteorological Station	8083.80	51655.72	-52011.15	108078.20	2.52	5	26
Tuban Meteorological Station	-41879.92	67346.13	-176218.44	43302.05	3.21	4	15
April	169.02	64.87	43.83	293.41	1.78	6	20
December	242.90	54.05	144.29	353.83	1.67	7	18
February	213.03	62.54	122.33	345.21	1.67	7	33
Januari	356.29	50.57	274.11	467.54	1.38	10	32
July	75.11	53.03	5.49	206.01	1.53	7	22
Juni	129.75	61.34	19.35	231.56	1.73	6	45
March	270.11	48.96	189.18	386.79	1.73	6	17
May	154.54	66.48	49.63	283.59	1.95	6	41
November	302.52	52.12	223.23	422.11	1.75	6	22
October	303.53	48.06	210.04	400.94	1.50	8	20
September	27.31	58.77	-90.41	141.26	1.60	7	14

Table 3 shows that the estimated intercept (line intercept) is 2817365.60. It means that the estimated rainfall is 2817365.60 mm if all predictor variables are 0. The estimated coefficient of the Station variable shows the rainfall difference of station "X" compared to other stations. The estimated coefficient of the Month variable shows the rainfall difference of month "A" compared to other months. The estimated standard deviation of the sigma parameter is shown in **Table 4**.

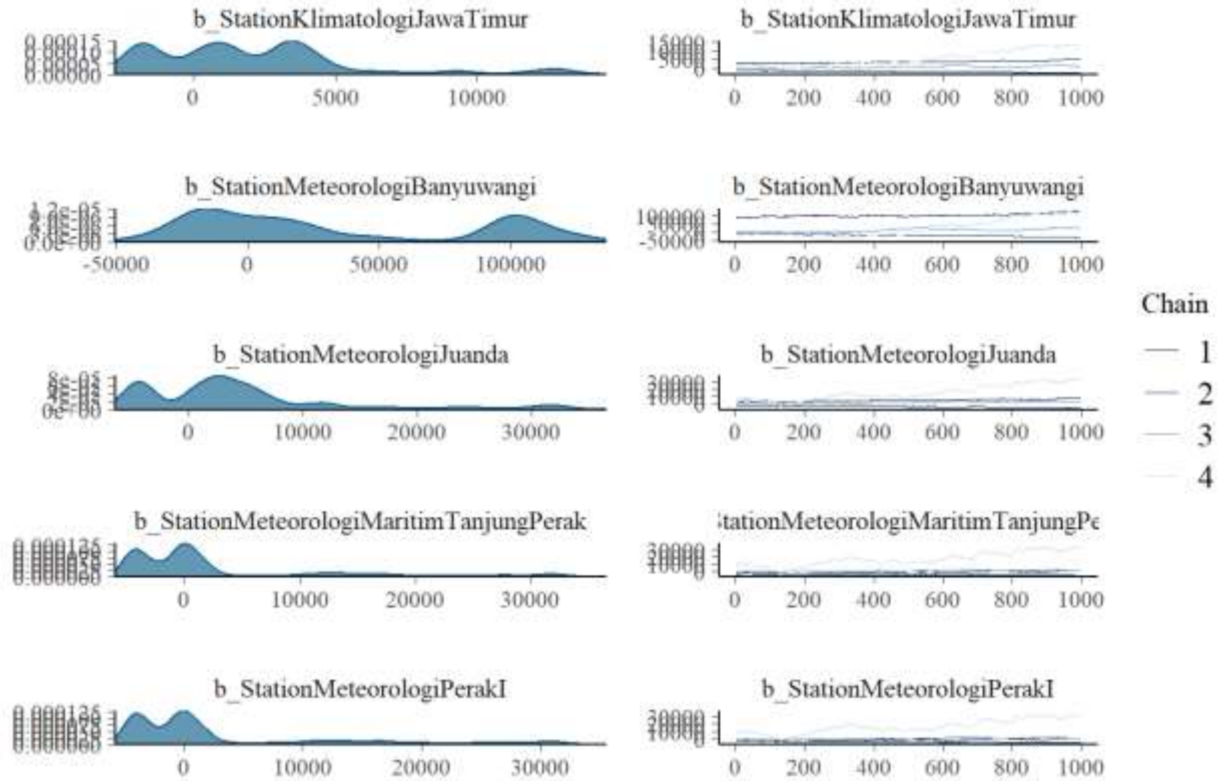
Table 4. Specific Parameters of Distribution Families

	Estimate	Est.Error	l-95% CI	u-95% CI	Rhat	Bulk_ESS	Tail_ESS
sigma	45.78	34.77	4.16	104.51	2.36	5	32

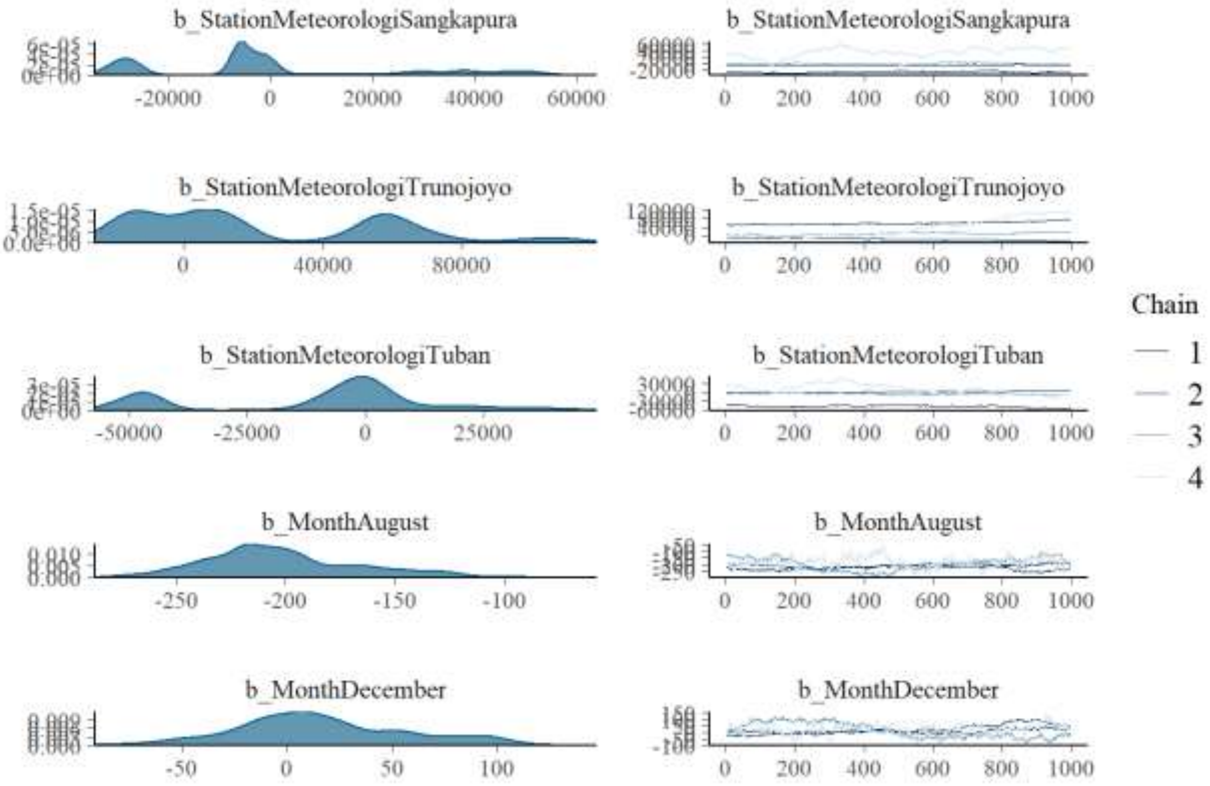
Table 4 shows the estimated standard deviation of the sigma parameter in a Gaussian (normal) distribution is 45.78. These estimates illustrate the variability in the rainfall data. The estimation error in this data is 34.77. Meanwhile, this estimate's 95% confidence interval is 4.16 to 104.51. Rhat value is 2.36 (Rhat > 1), meaning the chain has not been mixed properly. The estimated mass effective sample size (in most of the distribution) is 5 samples, and the estimated effective sample size in the tail of the distribution is 32 samples. The estimated error of the sigma parameter (**Table 2**) is less than the group level effect (**Table 4**). The value of sigma and sd (Intercept) have Rhat more than 1, but Tail_ESS of sigma is higher than sd (Intercept). The higher the Tail_ESS value, the better the model. Based on the model, some plots can be constructed (**Figure 2**).


b_Intercept
b_Latitude
b_Longitude
b_StationGeofisikaNganjuk
b_StationGeofisikaPasuruan
Chain
1
2
3
4

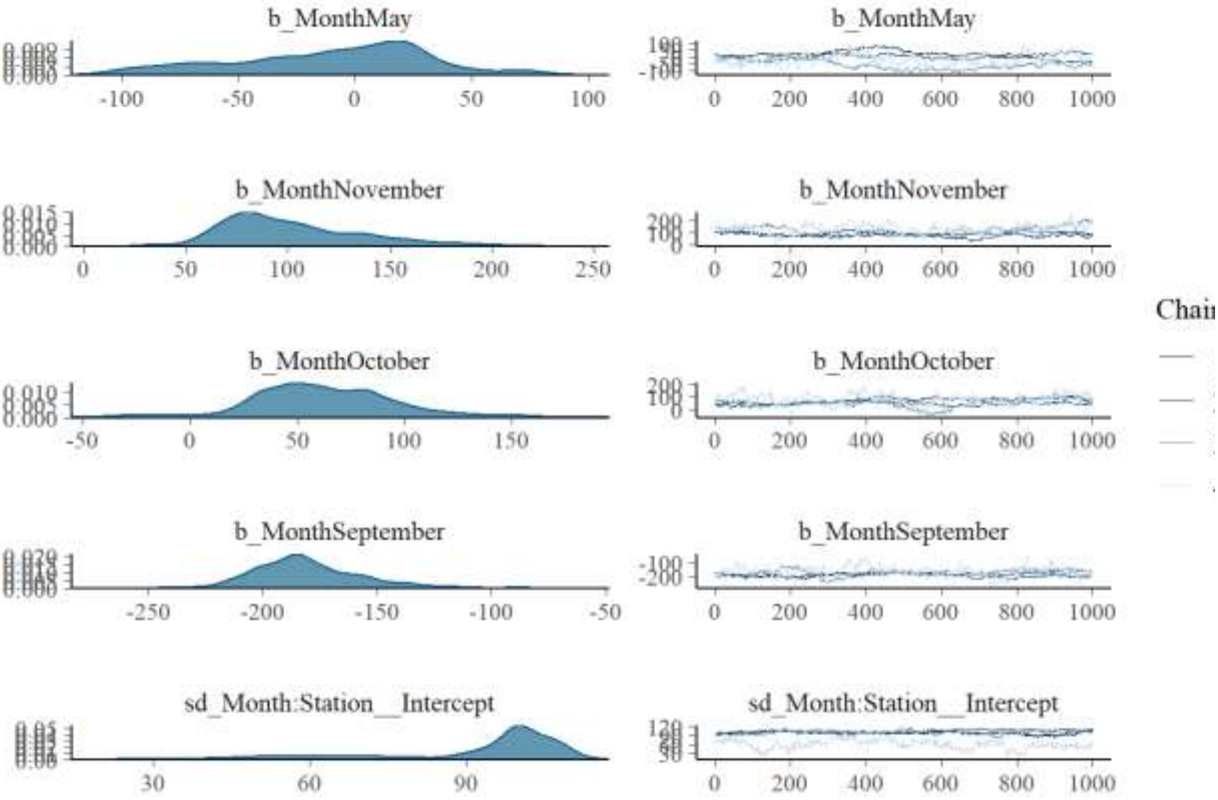
(a)


b_StationKlimatologiJawaTimur
b_StationMeteorologiBanyuwangi
b_StationMeteorologiJuanda
b_StationMeteorologiMaritimTanjungPerak
b_StationMeteorologiPerakI
Chain
1
2
3
4

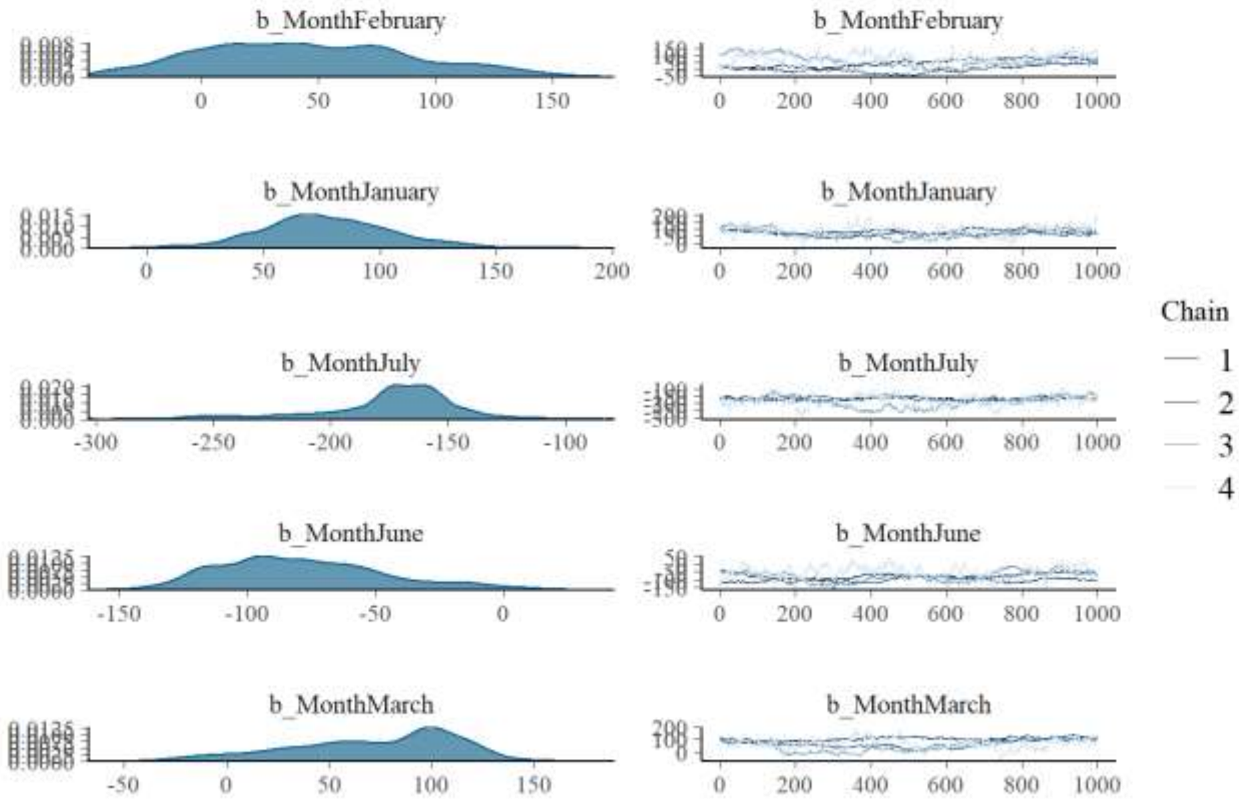
(b)


b_StationMeteorologiSangkapura
b_StationMeteorologiTrunojoyo
b_StationMeteorologiTuban
b_MonthAugust
b_MonthDecember
Chain
1
2
3
4

(c)


b_MonthMay
b_MonthNovember
b_MonthOctober
b_MonthSeptember
sd_Month:Station__Intercept

(d)



(e)

Figure 2. (a) Model plots on b_Intercept, b_Longitude, b_Lintang, b_Nganjuk Geophysical Station, and b_Pasuruan Geophysical Station, (b) Model plots at b_East Java Climatological Station, b_Banyuwangi Meteorological Station, b_Juanda Meteorological Station, b_Tanjung Perak Maritime Meteorological Station, and b_Perak I Meteorological Station, (c) Model plots at b_Sangkapura Meteorological Station, b_Trunojoyo Meteorological Station, b_Tuban Meteorological Station, b_April, and b_December, (d) Plot the model on b_May, b_November, b_October, b_September, and sd_Month:Intercept_Station, (e) Plot the model for b_February, b_January, b_July, b_June, and b_March

Figure 2 (a) shows the distribution of the Intercept, Nganjuk Geophysical Station, and Pasuruan Geophysical Station. The BNN estimation shows that rainfall at the Nganjuk Geophysical Station produces stable iterations in the 1st chain of the 4 Markov chains obtained, and the highest rainfall prediction is in the 4th Markov chain. Meanwhile, the Pasuruan Geophysical Station produces stable iterations in the 1st chain of the 4 Markov chains obtained, and the highest rainfall prediction is in the 3rd Markov chain. The Pasuruan Geophysical Station has higher rainfall than the Nganjuk Geophysical Station. It can be seen from the number of hills formed and the height of the hills.

Figure 2 (b), describes the distribution of the Markov chain on the East Java Climatology Station, Banyuwangi Meteorological Station, Juanda Meteorological Station, Tanjung Perak Maritime Meteorological Station, and Perak I Meteorological Station. The East Java Climatological Station has higher rainfall than the other four stations. The first Markov chain is the chain that has higher rainfall than the other three Markov chains at the East Java Climatology Station. Meanwhile, at the Banyuwangi Meteorological Station, stable iterations in the 1st chain of the 4 Markov chains, but the other chains (chains 3 and 4) have almost the same shape. In addition, the Perak I Meteorological Station and the Tanjung Perak Maritime Meteorological Station have similar graphic shapes in the four chains. The chain with the lowest rainfall predictions is located in chain 3 at the Juanda Meteorological Station.

Figure 2 (c) displays that the shape of the rainfall model graph at the Sangkapura Meteorological Station is similar to the Tuban Meteorological Station, namely the right-skewed. Meanwhile, the Trunojoyo Meteorological Station is left skewed. In December, the distance between chains tends to be close. Meanwhile, in April, the difference in distance between chains was clearly visible.

Figure 2 (d) indicates that the rainfall model in May and November is more the left skewed. Meanwhile, in October and September, the graph is almost normally distributed. In the standard deviation graph of the random effect related to the combination of month and station, it can be seen that chains 1 and 2 are almost similar, and the distance difference between the two chains is very small.

Figure 2 (e) explains that the rainfall model in February, January, July, and March is more the left skewed. Meanwhile, June displays that rainfall is spread evenly. Apart from that, the difference in June that can be seen compared to the other four months is that the distance between the chains in that month is obvious. After the modeling is visualized, a posterior distribution can be visualized in **Figure 3**.

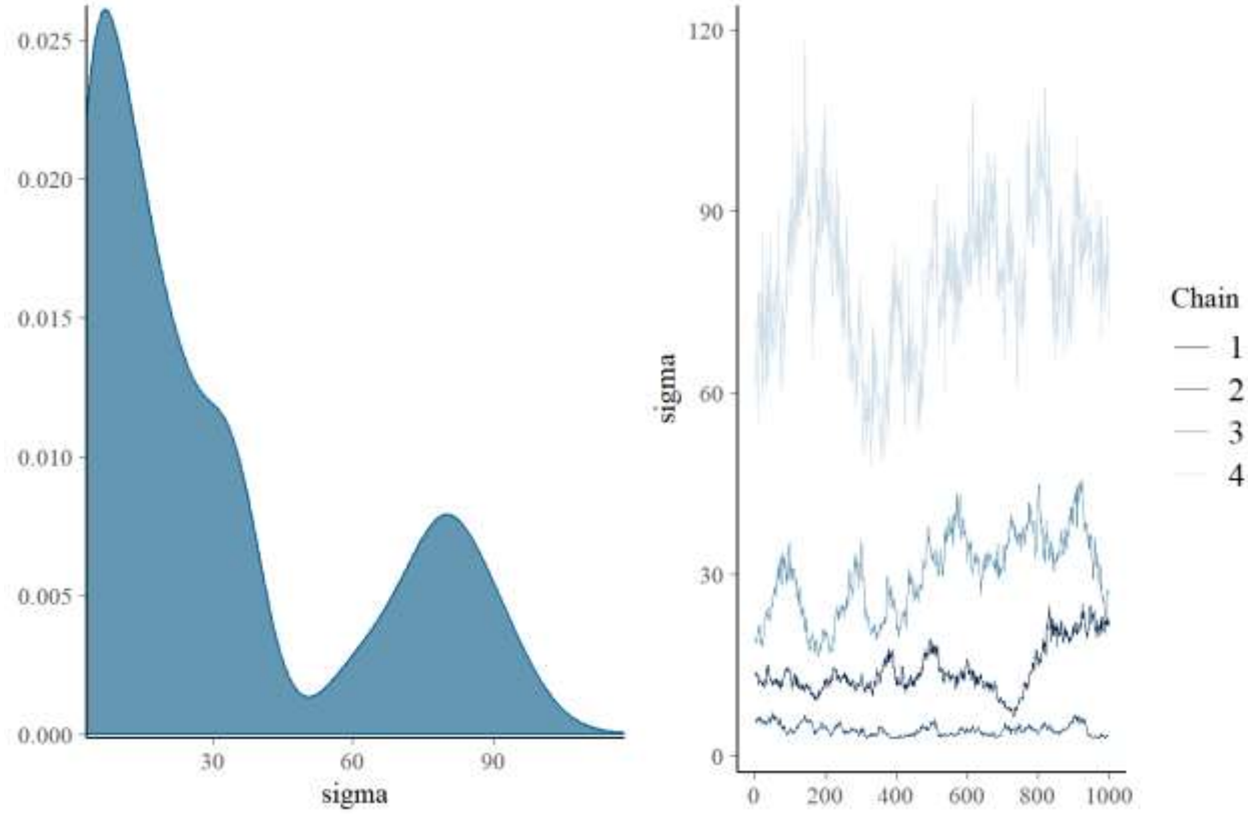


Figure 3. Visualization of Posterior Predictions Based on Testing Data

Based on **Figure 3**, the shape of the distribution of the sigma parameters is the Gaussian distribution. Besides that, it can also be seen that the higher the sigma, the more stationary than average. Among the four chains, the 3rd chain is stationary compared to the other three Markov chains

4. CONCLUSIONS

Bayesian analysis and ANN can be integrated to modeling rainfall patterns in East Java. From this modeling, the rainfall patterns can be predicted to prevent flooding in East Java. In BNN modeling, continuous iteration of the Markov chain is required until convergence is reached. However, in this research, the convergence obtained was good enough but can be increased. In other words, the values from comparing estimates between and within chains for model parameters and other univariate quantities of interest do not match. Therefore, it is recommended that in future research, the number of iterations be increased or stronger priors set.

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REFERENCES

[1] S. Astutik, N. Iriawan, N. G. Nair, Suhartono and Sutikno, “Bayesian State Space Modeling for Spatio-Temporal Rainfall Disaggregation,” *International Journal of Applied Mathematics and Statistics*, vol. 37, no. 7, pp. 26-37, 2013.

[2] S. Astutik, H. Pramoedyo and Solimun, “ZIG Model on Intermittent and Positively Skewed Rainfall Data,” in *International Conference on Sciences (ICOS)*, Universitas Hasanuddin, 2014.

[3] S. Astutik, “Pemodelan Data Curah Hujan Harian Menggunakan Zero Inflated Gamma Dengan Kovariat,” in *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika IndoMS Wilayah Sumatera Bagian Tengah, Universitas Negeri Padang*, Padang, 2015.

[4] S. K. S. M. Kumari, O. Bakimchandrad and A. Basisthae, “Geographically weighted regression based quantification of rainfall–topography relationship and rainfall gradient in Central Himalayas,” *International Journal Of Climatolog. https://doi.org/10.1002/joc.4777. 2016..*

[5] R. E. Caraka, M. Ulhusna, B. D. Supatmanto, N. E. Goldameir, B. Hutapea, G. Darmawan, D. Novitasari and B. Pardamean, “Generalized Spatio Temporal Autoregressive Rainfall-Enso Pattern In East Java Indonesia,” in *The 1st 2018 INAPR International Conference*, Jakarta, 2018.

[6] T. G. Holsclaw, A. Robertson and W. P. Smyth, “A Bayesian Hidden Markov Model of Daly Precipitation over South and East Asia,” *Journal of Hydrometeorororology*, vol. 17, 2016.

[7] S. Astutik, U. Sa'adah, Darmanto and B. Anoraga, “Algorithm of Hidden Markov Model with Bayesian Approach on Rainfall Data,” in *IORA International Conference on Operations Research*, Manado, 2019.

[8] S. Astutik, A. B. Astuti, Darmanto, B. Anoraga and D. Irsandy, “Algorithm of HMM's Parameter Estimation for Spatial Data Using Markov Chain Monte Carlo,” in *International Confetence on Mathematics, Statistics, and Data Science (ICMSDS)*, Bogor, Indonesia, 2020.

[9] S. Astutik, H. Pramoedyo, N. S. Rahmi and D. Irsandy, “Rainfall Data Modeling with Artificial Neural Networks Approach,” *Journal of Physics Conference Series 2123(1):012029*, 2021.

[10] V. Nourani, N. Paknezhad and H. Tanaka, “Prediction Interval Estimation Methods for Artificial Neural Network (ANN)-Based Modeling of the Hydro-Climatic Processes, a Review,” *Sustainability*, 2021.

[11] A. Sarasa-Cabezuelo, “Prediction of Rainfall in Australia Using Machine Learning,” *Information*, vol. 13, no. 163, 2022.

[12] R. C. R and Y. He, “Bayesian neural networks for stock price forecasting before and during COVID-19 pandemic,” *PLoS ONE*, vol. 16, no. 7, 2021.

[13] A. Gelman, J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari and D. B. Rubin, Bayesian Data Analysis, New York: CRC press, 2013.

[14] I. Ntzoufras, Bayesian Modeling Using WinBUGS, John Wiley & Sons, Inc, 2009.

[15] F. Amato, A. López., E. M. Peña-Méndez, P. Vaňhara., A. Hampl and J. Havel, “Artifcial neural networks in medical diagnosis,” *Journal of Applied biomedicine*, vol. 11, no. 2, pp. 47-58, 2013.

[16] A. Baliyan, K. Gaurav and S. K. Mishra, “A review of short-term load forecasting using artifcial neural network models,” *Procedia Computer Science*, vol. 48, pp. 121-125, 2015.

[17] H. S. Shin and J. D. Salas, “Spatial analysis and neural network modeling for hydrological and environmental data,” *Water Resources Research, sumitted.*

[18] P. Izmailov, S. Vikram, M. D. Hoffman and A. G. G. Wilson, “What are Bayesian neural network posteriors really like?,” in *International conference on machine learning*, 2021.

[19] BMKG. [Online]. Available: https://www.bmkg.go.id. [Accessed September 2023].