

T-IDEAL AND α -IDEAL OF BP-ALGEBRAS

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ABSTRACT

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This paper explores the characteristics of two distinct ideal types within BP-algebra, specifically T-ideal and α -ideal. Initially, we elucidate the characteristics of the T-ideal in BP-algebra, establishing its connections with the perfect, normal, and normal ideal in BP-algebra. Subsequently, we demonstrate that the kernel of a homomorphism in BP-algebra constitutes a T-ideal. Moving forward, we delineate the properties of α -ideal in BP-algebra, highlighting its relationships with ideal and filter in the context of BP-algebra. Additionally, we explore the characteristics of α -ideal and subalgebra in 0-commutative BP-algebra. Finally, it is proven that the kernel of a homomorphism in 0-commutative BP-algebra can be identified as an α -ideal.



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1. INTRODUCTION

Abstract algebra, also called modern algebra, is a field of study within algebra that focuses on investigating algebraic structures, including groups, rings, fields, and modules. The significance of abstract algebra lies in its practical application across various scientific disciplines, such as physics, chemistry, biology, and computer science. For instance, the concept of rings finds relevance in cryptographic theory, coding theory, and statistics, particularly in balanced incomplete block design, commonly known as balanced incomplete random group design. The rapid growth of abstract algebra stems from the immense utility of these algebraic forms in diverse fields.

Abstract algebra has witnessed the discovery of various new algebras, one of which is the B -algebra [1]. A B -algebra is defined as a non-empty set M equipped with a binary operation denoted by $*$ and a constant 0 , satisfying the following axioms: (B1) $a * a = 0$, (B2) $a * 0 = a$, and (B3) $(a * b) * c = a * (c * (0 * b))$ for all $a, b, c \in M$. In 2013, C. B. Kim and H. S. Kim [2] introduced the concept of a BN -algebra, which satisfies the axioms (B1), (B2), and (BN) $(a * b) * c = (0 * c) * (b * a)$ for all $a, b, c \in M$. Similarly, in the same year, Ahn and Han [3] defined a BP -algebra, which is a non-empty set M equipped with a binary operation denoted by $*$ and a constant 0 , fulfilling the axioms (B1), (BP2) $a * (a * b) = b$, dan (BP3) $(a * c) * (b * c) = a * b$ for all $a, b, c \in M$. The presence of shared axioms among BP -algebra, B -algebra, and BN -algebra gives rise to certain resemblances and connections between these algebraic concepts.

Fitria et al. [4] have explored the notion of prime ideal within B -algebra, presenting definitions and various properties of ideal and prime ideal in B -algebra. An ideal in B -algebra M is a non-empty subset A that satisfies two conditions: $0 \in M$ and for any $b \in A$, $a * b \in A$ holds $a \in A$ for every $a, b \in A$. On the other hand, a prime ideal of A is an ideal A that satisfies the condition $P \cap Q \subseteq A$, where P and Q are two ideals in M , implying that either $P \subseteq A$ or $Q \subseteq A$. In 2020, Gemawati et al. [5] extended this discussion to introduce the concept of complete ideal (c -ideal) and n -ideal in BN -algebra. This research revealed interesting characteristics that illustrate the relationships between ideals, c -ideals, and n -ideals, as well as the connections between subalgebra and normal with c -ideals and n -ideals in BN -algebra. Additionally, the study delved into the concepts of c -ideal and n -ideal within the homomorphism of BN -algebra and BM -algebra. Furthermore, in 2022, Gemawati et al. [6] explored the concepts of r -ideal and m - k -ideal in BN -algebra and examined their properties within homomorphism BN -algebra. Also, numerous papers pertaining to ideals in algebraic structures have been explored by researchers in references [7], [8], [9], [10] and [11].

We have discussed other concepts of BP -algebra like f_q -derivation in BP -algebra [12]. Jefferson and Chandramouleeswaran [13] introduce the notion of ideals in fuzzy BP -ideals and T -ideals in fuzzy T -ideals within BP -algebra [14]. However, these articles do not delve into the properties of ideals or T -ideals in BP -algebra. Similarly, El-Gendy [15] defines the concept of α -ideals in BP -algebra but only discusses the properties of bipolar fuzzy α -ideals in BP -algebra. It is evident that a more comprehensive and in-depth investigation into the characteristics of ideals, T -ideals, and α -ideals in BP -algebra could lead to intriguing characterizations of these concepts.

Based on the description of the relevant research, this study aims to explore and characterize various properties of ideals, T -ideals, and α -ideals in BP -algebra.

2. RESEARCH METHODS

The following provides the basic concepts needed in the construction of the concept of ideal, T -ideal, and α -ideal, and filter in BP -algebras.

Definition 1. [3] BP -algebra was defined as a non-empty set $(D; *, 0)$ satisfying the following axioms:

$$(BP1) \quad a * a = 0,$$

$$(BP2) \quad a * (a * b) = b,$$

$$(BP3) \quad (a * c) * (b * c) = a * b,$$

for all $a, b, c \in D$.

Some of properties about BP -algebra needed in this research is given in the following theorem.

Theorem 1. [3] If $(H; *, 0)$ is a BP-algebra, then for every $a, b \in H$:

- (i) $0 * (0 * a) = a$,
- (ii) $0 * (b * a) = a * b$,
- (iii) $a * 0 = a$,
- (iv) If $a * b = 0$, then $b * a = 0$,
- (v) $0 * (0 * a) = a$,
- (vi) $0 * (b * a) = a * b$,

Let $(H; *, 1)$ be a BP-algebra. We can define a relation \leq on H as follows: for any elements a and b in H , $a \leq b$ if and only if $a * b = 0$.

Definition 2. [3] A BP-algebra $(H; *, 0)$ is 0-commutative if fulfill $a * (0 * b) = b * (0 * a)$ for every $a, b \in H$.

Proposition 1. [3] If $(H; *, 0)$ is a 0-commutative BP-algebra, then for every $a, b, c \in H$:

- (i) $(a * c) * (b * c) = (c * b) * (c * a)$,
- (ii) $a * b = (0 * b) * (0 * a)$.

Definition 3. [13] A non-empty subset M of a BP-algebra $(H; *, 0)$ is called a subalgebra of H if $a * b \in M$ for all $a, b \in M$.

Definition 4. A non-empty subset M of a BP-algebra $(H; *, 0)$ is called a normal in H if $(m * a) * (n * b) \in M$ for any $m * n, a * b \in M$.

Let $(M; *, 0)$ and $(N; *, 0)$ are two BP-algebras. A map $f: M \rightarrow N$ is called a homomorphism if $f(a * b) = f(a) * f(b)$ for all $a, b \in M$. The kernel of f is defined to be $\ker f = \{a \in M : f(a) = 0\}$.

Definition 5. [13] A non-empty subset A of a BP-algebra $(H; *, 0)$ is called an ideal of H if for all $a, b \in H$:

- (i). $0 \in A$,
- (ii). $b \in A$ and $a * b \in A$ imply $a \in A$.

Definition 6. [14] An ideal A of a BP-algebra $(H; *, 0)$ is said to be closed if $0 * a \in A$ for all $a \in A$.

Definition 7. [14] Let $(H; *, 0)$ be a BP-algebra. A non-empty subset A of H is called a T -ideal of H if it satisfies the following conditions:

- (i). $0 \in A$,
- (ii). $(a * b) * c \in A$ and $b \in A$ implies $a * c \in A$ for all $a, b, c \in H$.

Definition 8. [15] Let $(H; *, 0)$ be a BP-algebra. A non-empty subset A of H is called a α -ideal of H if it satisfies the following conditions

- (i). $0 \in A$,
- (ii). $a * c \in A$ and $a * b \in A$ implies $b * c \in A$ for all $a, b, c \in H$.

Definition 9. Consider a BP-algebra $(H; *, 0)$ and let F be a non-empty subset of H . F is defined as a filter of H if it satisfies the following conditions:

- (F1) $0 \in F$,
- (F2) $a \in F$ and $a * b \in F$ imply $b \in F$.

3. RESULTS AND DISCUSSION

This section presents the properties derived from the concepts of ideal, T -ideal, and α -ideal in both BP-algebras and 0-commutative BP-algebras.

3.1 T-Ideal of BP-algebras

We will commence the discussion by exploring pertinent instances that highlight the properties of the T -ideal in BP -algebra.

Example 1. Let $M = \{0, a, b, c\}$ which is

Table 1. Table for $(M; *, 0)$

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

The structure $(M; *, 0)$ represents a BP -algebra. Let's explore the set A , which consists of all the ideals in M : $A = \{\{0\}, \{0, a\}, \{0, b\}, \{0, c\}, \{0, a, b, c\}\}$. We can verify that A is both a closed ideals and T -ideals in the BP -algebra M . But, $B = \{0, a, b\}$ not an ideal in M because it does not satisfy the properties of an ideal. Specifically, for $a \in B$ and $c * a = b \in B$, but c is not an element of B . Similarly, B is not a T -ideal in M because for $a \in B$, $(0 * a) * c = a * c = b \in B$, but $0 * c = c$ is not an element of B , which means the T -ideal property is not fulfilled.

Example 2. Consider the set of integers \mathbb{Z} equipped with the subtraction operation and constant 0, denoted as $(\mathbb{Z}; -, 0)$. It can be readily demonstrated that $(\mathbb{Z}; -, 0)$ qualifies as a BP -algebra. Let us now focus on a specific non-empty subset of \mathbb{Z} , denoted as $J = \mathbb{Z}^+ \cup \{0\}$. If $b \in J$, then it must be greater than or equal to zero ($b \geq 0$). Moreover, if $a - b \in J$, then it also be greater than or equal to zero ($a - b \geq 0$). These conditions hold true under the assumption that a is greater than or equal to b , which in turn implies that $a \in J$. This demonstrates that J is an ideal of \mathbb{Z} . However, J is not a closed ideal in \mathbb{Z} due to the following reason: $2 \in J$, but $0 - 2 = -2 \notin J$. J is additionally a T -ideal in \mathbb{Z} . For $(a - b) - c \in J$ we have $(a - b) - c = (a - c) - b \geq 0$ and for $b \in J$ we get for $b \geq 0$ such that $a - c \geq b \geq 0$. Hence, $a - c \in J$. This proof is complete.

Theorem 2. Let $(H; *, 0)$ is a BP -algebra. If A is a T -ideal of H , then A is an ideal of H .

Proof. Suppose $(H; *, 0)$ is a BP -algebra. Since A is a T -ideal of H , then $0 \in A$. Let $(a * b) * c \in A$, $b \in A$, and $c = 0$, by using **Theorem 1** (iii) we have $(a * b) * 0 = a * b \in A$ and $a * c = a * 0 = a \in A$. Hence, A is an ideal of H .

Corollary 1. Let $(H; *, 0)$ is a BP -algebra. If A is a normal T -ideal of H , then A is a normal ideal of H .

Proof. Let $(H; *, 0)$ be a BP -algebra and A be a T -ideal of H . By using **Theorem 2** we obtain A is an ideal of H . Since A is a normal, then A is a normal ideal of H .

Theorem 3. Let $(H; *, 0)$ is a BP -algebra. If A is an ideal of H , then A is a T -ideal of H .

Proof. Suppose $(H; *, 0)$ is a BP -algebra. Let A is an ideal of H we have $0 \in A$ and if $b \in A$, $a * b \in A$ implies $a \in A$ such that if $(a * b) * c \in A$, then $a * c \in A$. Hence, A is a T -ideal of H .

Theorem 4. Let $(H; *, 0)$ is a BP -algebra and A is a non-empty subset of H . A is a T -ideal of H if and only if it is an ideal of H .

Proof. This is directly proven by using **Theorem 2** and **Theorem 3**.

Theorem 5. Let $(M; *, 0)$ and $(N; *, 0)$ are two BP -algebras. If $f: M \rightarrow N$ is a homomorphism, then $\ker f$ is a T -ideal of M .

Proof. Suppose $(M; *, 0)$ and $(N; *, 0)$ are two BP -algebra and f is a homomorphism of M to N . By using axiom **BPI**, then $f(0) = f(0 * 0) = f(0) * f(0) = 0$ such that $0 \in \ker f$. Let $b \in \ker f$ and $(a * b) * c \in \ker f$, then $f(b) = 0$ and by using **Theorem 1** (iii) we obtain $0 = f((a * b) * c) = (f(a) * f(b)) * f(c) = (f(a) * 0) * f(c) = f(a) * f(c) = f(a * c)$. It means $a * c \in \ker f$. Thus, $\ker f$ is a T -ideal of M .

Theorem 6. Let $(D; *, 0)$ is a BP -algebra and A is a T -ideal of D . If $b \in A$ and $a * b \leq c$, then $a * c \in A$.

Proof. Suppose $(D; *, 0)$ is a BP -algebra. Since $a * b \leq c$, then $(a * b) * c = 0$. Since A is a T -ideal of D , then $0 \in A$, it means $(a * b) * c = 0 \in A$. Thus, if $b \in A$, and $a * b \leq c$, then $a * c \in A$.

3.2 α -Ideal of BP -algebras

This section discusses the properties of α -ideal in BP -algebra.

Theorem 7. Let $(J; *, 0)$ is a BP -algebra. If A is an α -ideal of J , then A is a filter of J .

Proof. Suppose $(J; *, 0)$ is a BP -algebra. Since A is a α -ideal of J , then $0 \in A$, let $a * c \in A$ and $a * b \in A$ implies $b * c \in A$. Now, let $c = 0$, by using **Theorem 1** (iii) obtained $a * c = a * 0 = a \in A$ and $a * b \in A$ implies $b * c = b * 0 = b \in A$. This shows that A is a filter of J .

Example 3. Consider $(\mathbb{Z}; -, 0)$ is the set of integers \mathbb{Z} equipped with the subtraction operation and constant 0. It can be readily demonstrated that $(\mathbb{Z}; -, 0)$ qualifies as a BP -algebra. Consider a set $K = 2\mathbb{Z}$ is a non-empty subset of \mathbb{Z} . Then, $0 = 2 \times 0 \in K$. Also, for every $a - c \in K$ and $a - b \in K$ we have $(a - c) - (a - b) = b - c$. On the other hand, we get $(a - c) - (a - b) = 2p - 2q = 2(p - q)$. This means that $b - c \in K$. Therefore, it has been showed that K is an α -ideal of \mathbb{Z} . We can also prove that K is a closed ideal and T -ideal in \mathbb{Z} . Let the set $J = \mathbb{Z}^+ \cup \{0\}$. In Example 2, we have demonstrated that it satisfies the conditions of being both an ideal and a T -ideal of \mathbb{Z} . But, J is not an α -ideal of \mathbb{Z} because $9 - 5 = 4 \in J$ and $9 - 3 = 6 \in J$, however $3 - 5 = -2 \notin J$.

Theorem 8. Let $(J; *, 0)$ is a BP -algebra and A is a α -ideal of J . If $a \leq c$ and $a \leq b$, then $b * c \in A$.

Proof. Suppose $(J; *, 0)$ is a BP -algebra. If $a \leq c$ and $a \leq b$ such that $a * c = 0 \in A$ and $a * b = 0 \in A$. Since A is a α -ideal of J , then it is clear that $b * c \in A$.

Considering a 0-commutative BP -algebra, we can examine the characteristics of an α -ideal within this context.

Theorem 9. Let $(K; *, 0)$ is a 0-commutative BP -algebra. If B is a subalgebra of K , then B is an α -ideal of K .

Proof. Suppose $(K; *, 0)$ is a BP -algebra and B is a subalgebra of K . If $a \in B$, by using axiom **BP1** we have $a * a = 0 \in B$. Furthermore, if $a * c \in B$ and $a * b \in B$, by using **Proposition 1** (i) and axiom **BP3** we obtain $(a * c) * (a * b) = (b * a) * (c * a) = b * c \in B$. Thus, B is an α -ideal of K .

Theorem 10. Let $(K; *, 0)$ is a 0-commutative BP -algebra. If B is an α -ideal of K , then B is a subalgebra of K .

Proof. Suppose $(K; *, 0)$ is a BP -algebra. If B is an α -ideal of K , then $0 \in B$. If $a * c \in B$ and $a * b \in B$ implies $b * c \in B$ for all $a, b, c \in K$. Thus, by using **Proposition 1** (i) and axiom **BP3** we obtain $(a * c) * (a * b) = (b * a) * (c * a) = b * c \in B$. Thus, B is a subalgebra of K .

Theorem 11. Let $(D; *, 0)$ and $(E; *, 0)$ are two 0-commutative BP -algebras. If $f: D \rightarrow E$ is a homomorphism, then $\ker f$ is an α -ideal of D .

Proof. Suppose $(D; *, 0)$ and $(E; *, 0)$ are two 0-commutative BP -algebra and f is a homomorphism of D to E . By using axiom **BP1** we get $f(0) = f(0 * 0) = f(0) * f(0) = 0$ such that $0 \in \ker f$. Let $a * c \in \ker f$ and $a * b \in \ker f$, then $f(a * c) = f(a) * f(c) = 0$ and $f(a * b) = f(a) * f(b) = 0$. By using **Proposition 1** (i), axiom **BP1** and **BP3** we obtain:

$$\begin{aligned} (f(a) * f(c)) * (f(a) * f(b)) &= 0 * 0 & (1) \\ (f(b) * f(a)) * (f(c) * f(a)) &= 0 & (2) \\ f(b) * f(c) &= 0 & (3) \\ f(b * c) &= 0 & (4) \end{aligned}$$

This implies that the element $b * c$ belongs to the kernel of f . Thus, $\ker f$ is an α -ideal of D .

Theorem 12. Let $(K; *, 0)$ is a 0-commutative BP -algebra. If C is a normal of K , then C is an α -ideal of K .

Proof. Suppose $(K; *, 0)$ is a 0-commutative BP -algebra. Let $a, b \in C$, by using **Theorem 1** (iii), then $a = a * 0 \in C$ and $b = b * 0 \in C$. Since C is a normal of K , then by using axiom **BP1** and **Theorem 1** (iii) we get

$(a * b) * (0 * 0) = (a * b) * 0 = a * b \in C$. Hence, it has been established that C is a subalgebra of K . Utilizing **Theorem 9**, we can deduce that C is indeed an α -ideal of K .

Theorem 13. Let $(K; *, 0)$ is a 0-commutative BP-algebra. If C is an α -ideal of K , then C is a normal of K .
Proof. Suppose $(K; *, 0)$ is a 0-commutative BP-algebra. Since C is an α -ideal of K and by using **Theorem 1** (iii), if $a * b \in C$ and $a * 0 = a \in C$, then $b * 0 = b \in C$, such that $(a * b) * (0 * 0) = a * b \in C$. Hence, C is a normal of K .

Theorem 14. Let $(K; *, 0)$ is a 0-commutative BP-algebra. C is a normal of K if and only if C is an α -ideal of K .

Proof. This fact is explicitly demonstrated by using **Theorem 12** and **Theorem 13**.

4. CONCLUSIONS

This article presents the construction of T-ideal properties in BP-algebra. The derived properties establish connections between T-ideals, ideals, normals, and kernels in BP-algebra. Additionally, the properties of α -ideal in BP-algebra are examined, specifically focusing on the relationship between α -ideals and filters. Finally, the article explores the relationships between α -ideals and subalgebras, kernels, and normals in 0-commutative BP-algebra.

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