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ALGORITHM FOR CONSTRUCTING TRIPLE IDENTITY GRAPH OF RING $\mathbb{Z}_n$ USING PYTHON

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ABSTRACT

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Keywords:
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Let R be a commutative ring. The triple identity graph of ring R is denoted by TE(R) with sets of vertices $R-\{0,1\}$. Two different vertices u and v are adjacent if and only if there is an element w in $R-\{0,1\}$ such that $u\cdot v\neq 1, u\cdot w\neq 1, v\cdot w\neq 1$, and $u\cdot v\cdot w=1$. To easily visualize the triple identity graph, a program is needed to represent it briefly. Python can easily manipulate, analyze, and visualize data. Therefore, this study uses Python to construct the algorithm for $TE(\mathbb{Z}_n)$. In this research, some examples will be given and then be observed for new characteristics of the triple identity graph of ring $\mathbb{Z}_n$ such as the connectedness, the diameter, and the girth. And we find the characterize n for which graph $TE(\mathbb{Z}_n)$ is empty, connected, or Hamiltonian.

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1. INTRODUCTION

The graph theory approach in algebraic structures relates to the representation of mathematical objects through graphs. The graph theory approach to algebraic structures focuses on using graphs to represent algebraic structures such as groups, rings, and fields. These graphs help visualize the relationships between elements in algebraic structures and understand the properties of those structures. In ring theory, graphs are used to represent the relationships between elements in a ring.

In 1988, Beck [1] introduced the concept of coloring of a commutative ring. This concept establishes a connection between graph theory and commutative ring theory. The research on graphs associated with commutative rings was also studied by Anderson *et al*. in [2]. In 1999, Anderson and Livingston [3] introduced a zero-divisor graph of a commutative ring and then studied it again in [4], [5], [6], and [7]. Furthermore, in 2021, Çelikel [8] developed the concept of a triple zero graph in a commutative ring R denoted by $TZ\Gamma(R)$. The authors also studied a concept of identity graph by Smarandache and Kandasamy [9] who defined group and semigroup identity graphs. Suppose G is a group and $x, y \in G, x$ is adjacent to y if and only if $x \cdot y = 1$. Furthermore, if x is adjacent to y, it is denoted by $x \sim y$ or $x - y$.

In 2022, Mohammad and Shuker [10], introduced a new kind of divisor graph which is called idempotent divisor graph. Then, Kurniawan et al. [11] defined the connectivity of the triple idempotent graph of ring $\mathbb{Z}_n$ which developed the concept of the idempotent divisor graph [10]. Based on these two concepts, this research defined the triple identity graph of ring $\mathbb{Z}_n$ from developing the concept of identity graph of the group in [9].

In this research, with the combination of algebraic concepts and graph concepts, a new graph called the triple identity graph of a commutative ring R is developed, denoted by $TE(R)$. $TE(R)$ is a graph with a vertex set $R^* = R - \{0, 1\}$, two different vertices u and v is adjacent if there is an element $w \in R - \{0, 1\}$ with $u \neq v$ such that $u \cdot v \neq 1, u \cdot w \neq 1, v \cdot w \neq 1$, and $u \cdot v \cdot w = 1$. It is an adjacency rule for every two distinct vertices in $TE(R)$.

Within the algebraic structure, Python can perform mathematical operations such as addition, subtraction, multiplication, and division. In graph theory, Python can perform graph analysis using libraries such as NetworkX and Matplotlib. These libraries are used in constructing graphs quickly and efficiently, it is useful to shorten the time when compared to creating graphs manually which takes more time. A programming language is a computer language engineered to communicate instructions to a machine [12]. Programs are created through programming languages to control the output through accurate algorithms. There are previous studies in [13], [14], and [15] discussing constructing a graph of ring using Python. With the Python programming language that can be used to construct a graph, this research developed an algorithm using Python to build $TE(R)$ from a ring of integers modulo *n*. In this research, constructing a graph with an algorithm in a Python program will take a very short time compared to making a graph manually. From the examples given in this research, new properties will be observed in the $TE(\mathbb{Z}_n)$ graph. This approach has not been extensively explored in the study of algebraic graph theory, making it a novel and promising avenue for research.

2. RESEARCH METHODS

The method used in this research is a literature study that examines references in the form of books, journals, and papers related to graph theory, algebra, Python programming, and especially the structure of triple identity graphs of a commutative ring. The steps in this research are as follows.

a. Learn basic definitions and theorems related to graph theory and the structure of algebra.
b. Define the triple identity graph of ring $\mathbb{Z}_n$.
c. Build an algorithm in Python to construct the triple identity graph of the ring $\mathbb{Z}_n$.
d. Observe the properties of the triple identity graph of the ring $\mathbb{Z}_n$.
e. Conclude.

The research process is shown in **Figure 1**.

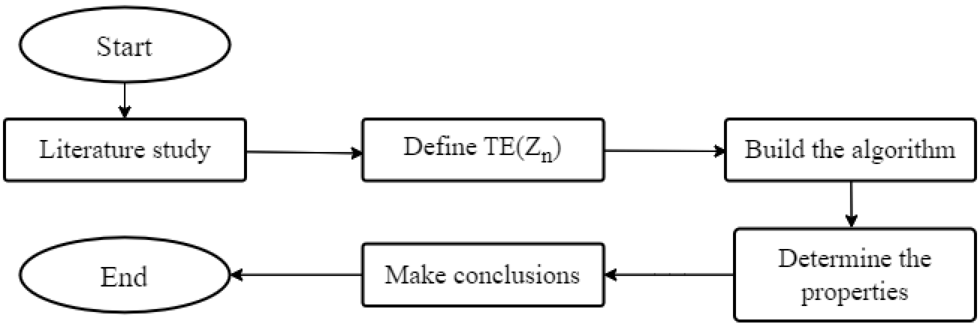


Figure 1. Research Process Chart

The purpose of this article is to determine the properties of the triple identity graphs of ring $\mathbb{Z}_n$. Therefore, it is necessary to outline some definitions that will form the basis of this research.

2.1 Basic Concepts of Graphs

Some basic definitions of graphs theory are cited from Chartrand [16], Skiena [17], Mazorodze and Mukwembi [18], and Chartrand *et al.* [19].

Definition 1. Graph G is a finite non-empty set $V(G)$ of objects called vertices and a set $E(G)$ of 2-element subsets of $V(G)$ called edges. The number of vertices in a set $V(G)$ is called order, while the number of edges of a set $E(G)$ is called size.

As an example, the illustration of graph G is shown in **Figure 2**.

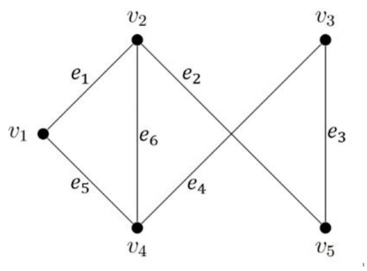


Figure 2. Graph G

In **Figure 2**, the graph G has order 5 with a vertex set $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ and size 6 with an edge set $E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

Definition 2. A $u - v$ walk of a graph G is a sequence of vertices in G, starting with u and ending at v. A $u - v$ trail is a $u - v$ walk in which no edge is traversed more than once. A circuit is a $u - v$ trail of length 3 or more that begins and ends at the same vertex. A circuit that repeats no vertex, except the first and the last vertex is called a cycle.

For example, graph G in **Figure 2** contain $v_1 - v_3$ walk : $v_1, e_1, v_2, e_6, v_4, e_4, v_3$. Example of $v_1 - v_3$ trail: $v_1, e_1, v_2, e_2, v_5, e_3, v_3$. Example of $v_2 - v_2$ circuit: $v_2, e_6, v_4, e_5, v_1, e_1, v_2$. Cycle in graph G is $v_2 - v_2$ cycle: $v_2, e_2, v_5, e_3, v_3, e_4, v_4, e_6, v_2$.

Definition 3. A graph of G is said to be connected if every two vertices of G are connected.

Figure 2 is an example of a connected graph because every two vertices of the graph G are connected.

Definition 4. A Hamiltonian cycle in a graph G refers to a cycle that includes every vertex in G, and a graph that has a Hamiltonian cycle is called a Hamiltonian graph.

In **Figure 2**, the graph G is a Hamiltonian graph because it contains a Hamiltonian cycle. Hamiltonian cycle in the graph G is $v_2, e_1, v_1, e_5, v_4, e_4, v_3, e_3, v_5, e_2, v_2$.

Definition 5. Girth is the length of the shortest cycle in the graph G denoted by $gr(G)$.

In **Figure 2**, the girth of the graph G is 3, in cycle $v_2 - v_2$: $v_2, e_6, v_4, e_5, v_1, e_1, v_2$.

Definition 6. Diameter is defined as the maximum distance over all pairs of vertices u and v in G denoted by $diam(G)$.

In **Figure 2**, the diameter of the graph G, $diam\ (G) = 2$ because the longest length of all shortest paths connecting two vertices in the graph G is no more than 2.

Definition 7. A degree of a vertex v in a graph G denoted by $deg_G\ v$ is the number of edges incident with v.

As an example, in the graph G v_1 have 2 degrees, v_2 have 3 degrees, v_3 have 2 degrees, v_4 have 3 degrees, and v_5 have 2 degrees.

2.2 The Triple Identity Graph of Commutative Ring

Before we define the triple identity graph, here is the definition of the triple zero graph of the commutative ring from Çelikel [8] and the identity graph of the group concept from Kandamasy and Smarandache [9].

Definition 8. The triple zero graph of the commutative ring R is an undirected graph of R denoted by $TZ\Gamma(R)$ with vertices $TZ(R) = \{a \in Z(R)^*$: there exists $b, c \in R\backslash\{0\}$ such that $abc = 0$, $ab \neq 0$, $ac \neq 0$, $bc \neq 0\}$, and two distinct vertices, a and b, are adjacent if and only if $ab \neq 0$, and there exists a nonzero element c of R such that $ac \neq 0$, $bc \neq 0$, and $abc = 0$.

Definition 9. Suppose G is a group and $x, y \in G, x$ is a neighbor of y if and only if $x.y = e$, where e is the identity element of G. Furthermore, if x is adjacent to y, it is denoted by $x \sim y$ or $x - y$. By this adjacency, all elements of the group take place in the forming graph is called the identity graph of the group G.

By these definitions, this research developed a new type of graph called the triple identity graph of commutative ring denoted by $TE(R)$ that combined the concept of the triple zero graph of the commutative ring in **Definition 8** and the concept of the identity graph of the group in **Definition 9**. The definition of the triple identity graph of ring R is presented in **Definition 10**.

Definition 10. The Triple identity graph of the commutative ring R denoted by $TE(R)$ is a graph with the set of vertices $R^* = R - \{0, 1\}$. Two different vertices $u\ and\ v$ are adjacent if there is an element $w \in R - \{0,1\}$ such that $u.v \neq 1, u.w \neq 1, v.w \neq 1$, and $u.v.w\ =\ 1$.

In the vertex set of $TE(R)$, element 0 and 1 are not included in the set because the element 0 will be an isolated vertex in all $TE(R)$ and element 1 will always be adjacent to every vertex that has inverse in $TE(R)$.

Example 1. The following is an example of $TE(\mathbb{Z}_5)$ formed by ring $R = \mathbb{Z}_5$. Where $\mathbb{Z}_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$.

Table 1. Multiplication Table of $\mathbb{Z}_5$

u	v	w	$u \cdot v$	$u \cdot w$	$v \cdot w$	$u \cdot v \cdot w$
$\bar{2}$	$\bar{4}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{3}$	$\bar{1}$
$\bar{3}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	$\bar{4}$	$\bar{2}$	$\bar{1}$

By the definition of the triple identity graph of the commutative ring R in **Definition 10**, we get the vertex set of $TE(\mathbb{Z}_5) = \{\bar{2}, \bar{3}, \bar{4}\}$ and by the rule of adjacency in $TE(R)$ we found that if $u = \bar{2}, v = \bar{4}$, and $w = \bar{2}$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 1$. Then, there is an edge between $\bar{2}$ and $\bar{4}$. Found again in $u = 3, v = 4$, and $w = 3$ satisfy the condition. So, $TE(\mathbb{Z}_5)$ obtained $E\big(TE(\mathbb{Z}_5)\big) = \{(\bar{2}, \bar{4}), (\bar{3}, \bar{4})\}$. The $TE(\mathbb{Z}_5)$ is shown in **Figure 3**.

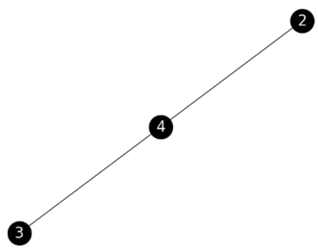


Figure 3. Triple Identity Graph of $\mathbb{Z}_5$

3. RESULT AND DISCUSSION

This section is separated into two different subs. The first sub describes the algorithm for constructing graph $TE(\mathbb{Z}_n)$ in Python, while the second sub contains observations and some characteristics of graph $TE(\mathbb{Z}_n)$.

3.1 Algorithm for Constructing $TE(\mathbb{Z}_n)$

The algorithm for constructing the triple identity graphs in Python uses parameters according to the number of each order in the ring $\mathbb{Z}_n$. The following table is an algorithm for constructing triple identity graphs of ring $\mathbb{Z}_n$.

Table 2. Algorithm for Constructing $TE(\mathbb{Z}_n)$

Algorithm 1 Triple Identity Graph Algorithm

```
Require: n ≥ 0
    Import library networkx and mathplotlib.pyplot
    Step 1:
    for u, v, w ∈ ℤn − {0,1} do
        add u to V(TE(ℤn));
        if u ≠ v then
            if u ∗ v, u ∗ w, v ∗ w ≠ 1 and u ∗ v ∗ w = 1 then
                add u~v to E(TE(ℤn));
            end if;
        end if;
    end for;
    Step 2:
    Draw TE(ℤn);
    end;
```

In step 1, the algorithm checks with the looping process for each element $u, v, w \in \mathbb{Z}_n - \{0,1\}$. Then u is added to the set $V(TE(\mathbb{Z}_n))$. Next, if $u \neq v$ then, continue. If not, go back to the looping process in step 1. Next, if $uv \neq 1, uw \neq 1, vw \neq 1$, and $uvw = 1$, then add an edge between u and v. If not, go back to the looping process in step 1. Continue to step 2, draw a triple identity graph of ring R using the vertex and edge elements that were obtained in step 1.

Some examples will be given to illustrate how the algorithm works on the $TE(\mathbb{Z}_n)$ ring. Note that the triple identity graphs of ring $\mathbb{Z}_n$ only existed for $n \geq 3$ because for $n = 1,2$ no graph will be formed since it has no vertices.

Example 2.
Given $\mathbb{Z}_n$ for $n = 3, 4$, and 6.

For $\mathbb{Z}_3$, input value $n = 3$. In step 1, the algorithm checks with the looping process for each element $u, v, w \in \mathbb{Z}_3 - \{0, 1\}$. Then, u is added to the set $V(TE(\mathbb{Z}_3))$. So, we get $V(TE(\mathbb{Z}_3)) = \{2\}$. Because $|V(TE(Z3)| < 2$ then adjacency is not found, so for $TE(\mathbb{Z}_3)$ it does not have an edge. Continue to step 2, draw the $TE(\mathbb{Z}_3)$ graph using the vertex element that was obtained in step 1.

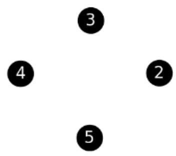
For $\mathbb{Z}_4$, input value $n = 4$. Step 1, the algorithm checks with the looping process for each element $u, v, w \in \mathbb{Z}_4 - \{0, 1\}$. Then, u is added to the set $V(TE(\mathbb{Z}_4))$. So, we get $V(TE(\mathbb{Z}_4)) = \{2, 3\}$. Continue, if $u \neq v$ then continue. If not, go back to the looping process in step 1. At $u = 2, v = 3$, and $w = 2$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 0 \neq 1$ then adjacency is not found. In the same way, at $u = 2, v = 3$, and $w = 3$, there is also no adjacency so $TE(\mathbb{Z}_4)$ does not have an edge. Continue to step 2, draw the $TE(\mathbb{Z}_4)$ graph using the vertex element that were obtained in step 1.

For $\mathbb{Z}_6$, input value $n = 6$. Step 1, the algorithm checks with the looping process for each element $u, v, w \in \mathbb{Z}_6 - \{0, 1\}$. Then, u is added to the set $V(TE(\mathbb{Z}_6))$. So, $V(TE(\mathbb{Z}_6)) = \{2, 3, 4, 5\}$. Continue, if $u \neq v$ then continue. If not, go back to the looping process in step 1. At $u = 2, v = 3$, and $w = 4$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 0 \neq 1$ then adjacency is not found. In the same way, the adjacency is also not found for the other vertex combinations u, v, w so $TE(\mathbb{Z}_6)$ does not have an edge. Continue to step 2, draw the $TE(\mathbb{Z}_6)$ graph using the vertex element that were obtained in step 1.

Here are the output images of triple identity graphs using the Python algorithm for constructing $TE(\mathbb{Z}_3), TE(\mathbb{Z}_4)$, and $TE(\mathbb{Z}_6)$ shown in **Figure 4**.

$(a)\ TN(\mathbb{Z}_3)$

(b) $TE(\mathbb{Z}_4)$



(c) $TE(\mathbb{Z}_6)$

Figure 4. Triple Identity Graph of $\mathbb{Z}_n$ for $n = 3, 4, 6$

Example 3.

Construct the triple identity graph of $\mathbb{Z}_{10}$. Input value $n = 10$. Step 1, the algorithm checks with the looping process for each element $u, v, w \in \mathbb{Z}_{10} - \{0, 1\}$. Then, u is added to the set $V(TE(\mathbb{Z}_{10}))$. So, $V(TE(\mathbb{Z}_{10})) = \{2, 3, 4, 5, 6, 7, 8, 9\}$. Continue, if $u \neq v$ then, continue. If not, go back to the looping process in step 1. Found at u$= 3, v = 9$, and $w = 7$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 1$. Then add an edge between 3 and 9. Go back to the looping process in step 1. Found at $u = 7, v = 9$, and $w = 3$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 1$. Then add an edge between 7 and 9. Go back to the looping process in step 1 and no longer found u, v, w that fulfills, so the looping process stops. Therefore, $E(TE(\mathbb{Z}_{10})) = \{e_{3,9}, e_{7,9}\}$. Continue to step 2, draw the graph $TE(\mathbb{Z}_{10})$ using the vertex and edge elements that were obtained in step 1. The output of triple identity graphs using Python algorithm for constructing $TE(\mathbb{Z}_{10})$ are shown in **Figure 5**.

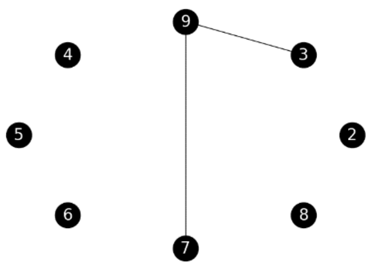


Figure 5. Triple Identity Graph of $\mathbb{Z}_{10}$

Table 3 shows the number of degrees at each vertex in $TE(\mathbb{Z}_{10})$ below.

Table 3. Number of degrees $V(TE(\mathbb{Z}_{10}))$

Vertex	Number of Degrees
v_2	0
v_3	1
v_4	0
v_5	0
v_6	0
v_7	1
v_8	0
v_9	2

Example 4.

Construct the triple identity graph of $\mathbb{Z}_{11}$. Input value $n = 11$. Step 1, the algorithm checks with the looping process for each element $u, v, w \in \mathbb{Z}_{11} - \{0, 1\}$. Then, u is added to the set $V(TE(\mathbb{Z}_{12}))$. So, $V(TE(\mathbb{Z}_{11})) = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Continue, if $u \neq v$ then, continue. If not, go back to the looping process in step 1. Found at $u = 2, v = 3$, and $w = 2$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 1$. Then add an edge between 2 and 3.

Go back to the looping process in step 1. Found at $u = 3, v = 5$, and $w = 3$, where $uv \neq 1, vw \neq 1, uw \neq 1$, and $uvw = 1$. Then add an edge between 3 and 5. By returning the loop process, there is another adjacency between the vertices. The result of the algorithm of $\mathbb{Z}_{11}$ in step 1 is shown in **Table 4** below.

Table 4. Result of Looping Process in Step 1 for Graph $TE(\mathbb{Z}_{11})$

u	v	w	$u \cdot v \neq 1$	$u \cdot w \neq 1$	$v \cdot w \neq 1$	$u \cdot v \cdot w = 1$
2	3	2	6	6	4	1
2	4	7	8	6	3	1
2	5	10	10	6	9	1
2	7	4	3	6	8	1
2	8	9	5	6	7	1
2	9	8	7	6	5	1
2	10	5	9	6	10	1
3	5	3	4	4	9	1
3	6	8	7	4	2	1
3	7	10	10	4	8	1
3	8	6	2	4	7	1
3	9	9	5	4	5	1
3	10	7	8	4	10	1
4	5	5	9	3	9	1
4	6	6	2	3	2	1
4	7	2	6	3	8	1
4	8	10	10	3	7	1
4	9	4	3	3	5	1
4	10	8	7	3	10	1
5	6	7	8	9	2	1
5	7	6	2	9	8	1
5	8	8	7	9	7	1
5	10	2	6	9	10	1
6	7	5	9	2	8	1
6	8	3	4	2	7	1
6	9	10	10	2	5	1
6	10	9	5	2	10	1
7	9	7	8	8	5	1

u	v	w	$u \cdot v \neq 1$	$u \cdot w \neq 1$	$v \cdot w \neq 1$	$u \cdot v \cdot w = 1$
7	10	3	4	8	10	1
8	9	2	6	7	5	1
8	10	4	3	7	10	1
9	10	6	2	5	10	1

So, the looping process shows for all $u, v, w \in V\big(TE(\mathbb{Z}_{11})\big)$ and $u \neq v$ there is w such that $uv \neq 1, uw \neq 1, vw \neq 1$ and $uvw = 1$, then all vertices u and v are adjacent. Now there is no more vertex in the loop process. Continue to step 2, the output of triple identity graphs using Python algorithm for constructing $TE(\mathbb{Z}_{11})$ are shown in **Figure 6**.

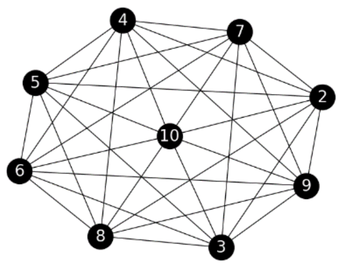


Figure 6. Triple Identity Graph of $\mathbb{Z}_{11}$

Table 5 shows the number of degrees at each vertex in $TE(\mathbb{Z}_{11})$ below.

Table 5. Number of degrees $V\big(TE(\mathbb{Z}_{11})\big)$

Vertex	Number of Degrees
v_2	7
v_3	7
v_4	7
v_5	7
v_6	7
v_7	7
v_8	7
v_9	7
v_{10}	8

Example 5.

Last, here is an example of a triple identity graph with a higher n, and it shows how to easily construct a triple identity graph using the Python program in a very short time. Construct the triple identity graph of $\mathbb{Z}_{41}$, and the output of $TE(\mathbb{Z}_{41})$ is shown in **Figure 7**.

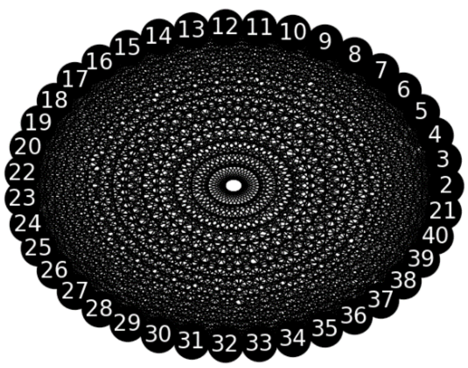


Figure 7. Triple Identity Graph of $\mathbb{Z}_{41}$

3.2 Observation of The Characteristics of $TE(\mathbb{Z}_n)$

After constructing the triple identity graph using the Python algorithm, here are the observations we found some characteristics of the triple identity graph of ring $\mathbb{Z}_n$ shown above.

(i) If $n \le 6$ and $n \ne 5$, it is an empty graph, because it obtained vertices with no edge.
(ii) If n is prime ≥ 5, then it is a connected graph.
(iii) If n is prime ≥ 7, then it is a Hamiltonian graph.
(iv) If n is prime ≥ 5, then $gr\big(TE(\mathbb{Z}_n)\big) = 3$.
(v) If n is prime ≥ 5, then $diam\big(TE(\mathbb{Z}_n)\big) = 2$.

The results of these observations become the basis for determining the properties of the identity triple graph of ring $\mathbb{Z}_n$ which still needs to be proven algebraically.

4. CONCLUSIONS

In this research, we developed a new graph called the triple identity graph, a combination of identity graph and triple zero divisor graph. Python program is used to create an algorithm that makes it easier for us to visualize the triple identity graph of the ring $\mathbb{Z}_n$. The observation results for the properties from the triple identity graph of the ring $\mathbb{Z}_n$ is shown and readers who are interested can prove the observation mathematically. Last, constructing a graph with an algorithm in a Python program will take a very short time compared to making a graph manually.

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