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RAINBOW VERTEX CONNECTION NUMBER OF BULL GRAPH, NET GRAPH, TRIANGULAR LADDER GRAPH, AND COMPOSITION GRAPH $(P_n[P_1])$

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ABSTRACT

The rainbow connection was first introduced by Chartrand in 2006 and then in 2009 Krivelevich and Yuster first time introduced the rainbow vertex connection. Let graph $G = (V, E)$ be a connected graph. The rainbow vertex-connection is the assignment of color to the vertices of a graph G, if every vertex on the graph G is connected by a path graph that has interior vertices in different colors. The minimum number of colors from the rainbow vertex coloring in the graph G is called rainbow vertex connection number which is denoted $rvc(G)$. The results of the research are the rainbow vertex connection number of bull graph, net graph, triangular ladder graph, and graph composition ($P_n[P_1]$).

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1. INTRODUCTION

A graph G, which is defined as a set pair of $(V(G), E(G))$, consists of a non-empty finite set $V(G)$ of elements called vertices and a finite family $E(G)$ of unordered pairs of (not necessarily distinct) elements of order $\{e, v\}$, where e and v are elements (not necessarily distinct) of the ordered set, with $u, v \in V(G)$. Mathematically, a graph G is denoted by $G(V, E)$. Therefore, a graph should have at least a vertex and still possibly have no edge [1], [2]. The number of vertices and edges of a graph is called the vertex cardinality and edge cardinality. Mathematically the vertex cardinality is denoted by $|V(G)|$ and the edge cardinality is denoted by $E(G)$ [2].

The concept of rainbow connection was first introduced by Chartrand in 2006 [3], [4], [5], [6], and subsequently, in 2009 by Krivelevich and Yuster which divided the concept of rainbow connection into two types called rainbow vertex-connection and rainbow edge-connection [7], [8]. The rainbow vertex-connection is known as the coloring vertex of a graph G, if every two vertices are connected by a path graph that has interior points with distinct edge colors. Whereas rainbow edge connection is coloring edge of the graph G, If for any vertex of the graph G is connected by paths which have distinct colors [9], [10]. The rainbow connection number is denoted by $rc(G)$ and rainbow vertex connection number is denoted by $rvc(G)$ [1], [7], [11].

In the study of rainbow vertex connections, several graphs remain unexplored, including the bull graph, net graph, triangular ladder graph, and composition graph (P_n [P_1]). These graphs were chosen because the four graphs have almost the same characteristics. The bull graph has almost the same characteristics as the net graph, while the triangular ladder graph has almost the same characteristics as the composition graph $(P_n[P_1])$.

2. RESEARCH METHODS

In this study, the method used was a pattern recognition method, Pattern recognition method is a method used to find the rainbow vertex connection labeling pattern so that it meets the definition of rainbow vertex connection coloring on the pre-determined graph define and axiomatic deductive method, The axiomatic deductive method is a research approach that applies deductive proof principles found in mathematical logic, employing existing axioms, lemmas, and theorems to solve problems related to rainbow vertex connection on predetermined graphs[5], [12]. The steps used are as follows:

1. Formulate the research problem for discussion.
2. Study and comprehend literature sources about rainbow vertex connection numbers.
3. Determine the pattern of the rainbow vertex connection number in the specified graphs.
4. Prove the rainbow vertex connection number theorem in the specified graphs.
5. Formulate the conclusions drawn from the obtained results.

Next, some definitions and theorems about graphs that will be used in this research are presented.

Definition 1. [13] The denotation of Graf Bull represented as $B_{3,m}$ is a composite graph of order 3 of the cycle graph with a path graph P_n ordered by n that is connected by an elongated edge of the vertex x_2 and x_3 in a circular graph C_3. The bull graph is denoted by $(B_{3,m})$ where m is the total number of points on the path graph with $m \geq 2$.

The following is the cardinality of the vertex and the cardinality of the edges in a bull graph:

$$V(B_{3,m}) = \{x_i; 1 \leq i \leq 3\} \cup \{x_{i,j}; 2 \leq i \leq 3, 1 \leq j \leq m\}$$

$$E(B_{3,m}) = \{x_1x_i; 2 \leq i \leq 3\} \cup \{x_2x_3\} \cup \{x_ix_{i,j}; 2 \leq i \leq 3, 1 \leq j \leq m\}$$

Definition 2. The Graph net is the combination of a cycle graph of order 3 with a path graph P_m ordered by m at each point of the cycle graph C_3. The net graph is denoted by $(N_{3,m})$ where m is the total number of points of the path graph with $m \geq 2$ [13].

The following is the cardinality of the vertex and the cardinality of the edges in a net graph:

$$V(N_{3,m}) = \{x_i; 1 \leq i \leq 3\} \cup \{x_{i,j}; 1 \leq i \leq 3, 1 \leq j \leq m\}$$

$$E(N_{3,m}) = \{x_1x_i; 2 \le i \le 3\} \cup \{x_2x_3\} \cup \{x_ix_{i,j}; 1 \le i \le 3, 1 \le j \le m\}$$

Definition 3. Triangular Ladder Graph (TL_n) is an undirected graph and planar graph with vertex cardinality $2n$ and edge cardinality $4n - 3$.

The following is the cardinality of the vertex and the cardinality of the edges in a triangular ladder graph:

$$V(TL_n) = \{x_iy_i; 1 \le i \le n\}$$

$$E(TL_n) = \{x_ix_{i+1}, y_iy_{i+1}, x_iy_{i+1};\ 1 \le i \le n-1\} \cup \{x_iy_i; 1 \le i \le n-1\}$$

Definition 4. The Composition graph is a graph constructed of graphs P_n and P_1 with the disjoints of the vertices $V(P_n)$ and $V(P_1)$ and edges E_1 and E_2. A Composition Graph is a graph with $V(P_n) \times V(P_1)$ and $v = (v_1, v_2, \dots, v_n)$ adjacent to $u = (u_1, u_2)$ when $[v_1\ adj\ u_1]$ or $[v_1 = u_1\ and\ v_2\ adj\ u_2]$ and so on. A composition graph is denoted by $P_n[P1]$ [14].

The following is the cardinality of the vertex and the cardinality of the edges in a composition graph $P_n[P_1]$:

$$V(P_n[P_1]) = \{x_i; 1 \le i \le n; n \le 3\} \cup \{y_i; 1 \le i \le n; n \le 3\}$$

$$E(P_n[P_1]) = \{x_iy_i; 1 \le j \le n; n \le 2\} \cup \{x_ix_{i+1}; 1 \le j \le n; n \le 2\} \cup \{x_iy_{i+1}; 1 \le j \le n; n \le 2\} \cup \{x_ix_{i+1}; 1 \le j \le n; n \le 2\}$$

Theorem 1. If graph G is connected with $diam(G)$, means $rvc(G) \ge diam(G) - 1$ [7], [9], [10], [15].

3. RESULTS AND DISCUSSION

This research produces four theorems on the rainbow vertex connection number in the bull graph, net graph, triangular ladder graph, and composition graph $P_n[P_1]$, as follows:

3.1 Rainbow Vertex Connection Number Of Bull Graph $(B_{3,m})$

The rainbow vertex connection is referred to as the vertex coloring of graph bull, if every two vertices in the bull graph are connected by a path that has interior points with different side colors. The following is a proof of the rainbow vertex connection theorem on the bull graph.

Theorem 2. The Rainbow vertex connection number of the bull graph $(B_{3,m})$ for $m \ge 2$ is $rvc(B_{3,m}) = 2m$.

Proof. For example $B_{3,m}$ is a bull graph with the vertex set $V(G) = \{x_i; 1 \le i \le 3\} \cup \{x_{i,j}; 2 \le i \le 3, 1 \le j \le m\}$ and set edges $E(G) = \{x_1x_i; 2 \le i \le 3\} \cup \{x_2x_3\} \cup \{x_ix_{i,j}; 2 \le i \le 3, 1 \le j \le m\}$. Firstly, the lower bound of the bull graph $B_{3,m}$ will be shown. The bull graph $B_{3,m}$ has $diam(B_{3,m}) = 2m + 1$. Based on Theorem 1 it is found $rvc(B_{3,m}) \ge diam(B_{3,m}) - 1 = 2m$. As we know, in the graph bull cardinality point x_2 dan x_3 has a path graph as long as m point. Based on this condition, it obtains $rvc(B_{3,m}) \ge 2m$. Second, there will be shown the upper bound of the bull graph $rvc(B_{3,m}) \ge 2m$. The rainbow vertex coloring function of bull $B_{3,m}$ as follows:

$$f(x_i) = 2m; i \in odd\ number$$
$$f(x_i) = 2m - 1; i \in even\ number$$
$$f(x_{2,j}) = 1; j = m$$
$$f(x_{3,j}) = 2m; j = m$$
$$f(x_{2,j}) = j; 1 \le j \le m - 1$$

Based on the rainbow vertex coloring functions above, it is found that the rainbow path from $u - v$ can be seen in Table 1.

Table 1. Rainbow Vertex Connection $u - v$ in Net Graph $N_{3,m}$

Case	*u*	*v*	Condition	*Rainbow Vertex Connection u-v*
1	$x_{2,j}$	x_i	$i \in Odd\ and\ j = m$	$x_{2,j-1}, x_{2,j-2}, \dots, x_{2,1}, x_2$
2	$x_{3,j}$	x_i	$i = 1, \quad j = m$	$x_{3,j-1}, x_{3,j-2}, \dots, x_{3,1}, x_3$
3	$x_{2,j}$	$x_{3,j}$	$2 \le i \le 3\ and\ j = m$	$x_{2,j-1}, x_{2,j-2}, \dots, x_{2,1}, x_2, x_3, x_{3,1}, \dots, x_{3,j-2}, x_{3,j-1}$

According to **Table 1**, it is evident that each path $u - v$ has a rainbow path, or in other words, every path in the bull graph that contains interior vertices with different colors satisfies the concept of rainbow vertex connection. So, based on the upper and lower bounds, it is proven that $rvc(B_{3,m}) = 2m$. An illustration as an example of a rainbow vertex connection can be found in **Figure 1** which is an example of a rainbow vertex connection of bull graph $B_{3,5}$. The rainbow vertex connection of the bull graph $B_{3,5} = 2m$ or equal to $2m = 2 \cdot 5 = 10$ can also be found in **Figure 1**.

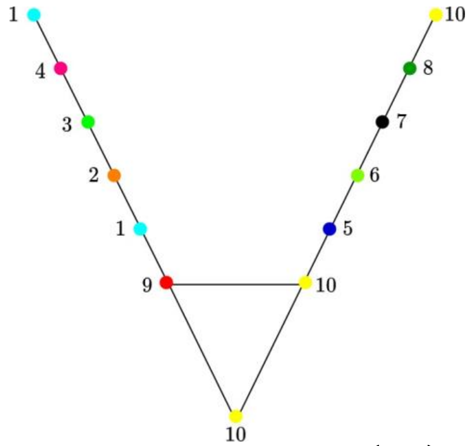

Figure 1. Illustration for Example $rvc(B_{3,5}) = 10$

3.2 Rainbow Vertex Connection Number of Net Graph $(N_{3,m})$

The rainbow vertex connection is referred to as vertex coloring of the net graph, if every vertex in the net graph is connected by a path that has interior points with different side colors. The following is a proof of the rainbow vertex connection theorem of the net graph.

Theorem 3. The Rainbow vertex connection number of the net graph $N_{3,m}$ for $m \ge 2$ is $rvc(N_{3,m}) = 3m$.

Proof. For example $N_{3,m}$is a net graph with vertex set $V(G) = \{x_i; 1 \le i \le 3\} \cup \{x_{i,j}; 1 \le i \le 3, 1 \le j \le m\}$ and edge set $E(G) = \{x_1x_i; 2 \le i \le 3\} \cup \{x_2x_3\} \cup \{x_ix_{i,j}; 1 \le i \le 3, 1 \le j \le m\}$. Firstly, the lower bound of the net graph will be shown. The net Graph $N_{3,m}$ has diameter $diam(N_{3,m}) = 2m + 1$. Based on **Theorem 1** it is found that $rvc(N_{3,m}) \ge diam(N_{3,m}) - 1 = 2m$. As we know regarding the net graph cardinality, vertex x_1, x_2 and x_3 has a long path as much as m point. Based on this condition, it is found that $rvc(N_{3,m}) \ge 3m$.

Secondly, There will be the upper bound of the net graph $rvc(N_{3,m}) \ge 3m$. The rainbow coloring vertex function of the net graph $N_{3,m}$ as follows:

$$f(x_{i,j}) = 1; j = m\ and\ 1 \le i \le 3$$
$$f(x_{1,j}) = j; 1 \le j \le m - 1$$
$$f(x_{2,j}) = m - 1 + j; 1 \le j \le m - 1$$
$$f(x_{3,j}) = 2m - 2 + j; 1 \le j \le m - 1$$
$$f(x_i) = 3m + 1 - i; 1 \le i \le 3$$

In conclusion, based on the rainbow coloring vertex function, it can be found (rainbow path) of $u - v$, where $u, v \in V(N_{3,m})$ can be found in **Table 2**.

Table 2. Rainbow Vertex Connection $u - v$ in net graph $N_{3,m}$

Case	u	v	Condition	*Rainbow Vertex Connection u-v*
1	$x_{1,j}$	$x_{2,j}$	$j=m$	$x_{1,j-1}, x_{1,j-2}..., x_{1,1}, x_1, x_2, x_{2,1}..., x_{2,j-2}, x_{2,j-1}$
2	$x_{1,j}$	$x_{3,j}$	$j=m$	$x_{1,j-1}, x_{1,j-2}...x_{1,1}, x_1, x_3, x_{3,1}..., x_{3,j-2}, x_{3,j-1}$
3	$x_{2,j}$	$x_{3,j}$	$j=m$	$x_{2,j-1}, x_{2,j-2}...x_{2,1}, x_2, x_3, x_{3,1}..., x_{3,j-2}, x_{3,j-1}$

Referring to **Table 2,** it is known that each path of $u - v$ has rainbow connection or in other words, each path or net graph has interior vertex with distinct colors, so it can meet the concept of rainbow vertex connection. So, based on the upper and lower bounds, it is proven that $rvc(N_{3,m}) = 3m$. An illustration as an example of rainbow vertex connection can be found in **Figure 2** which is an example of rainbow vertex connection of net graph $N_{3,3}$. The rainbow vertex connection of graph net $N_{3,3} = 3m$ or equal to $3m = 3 \cdot 3 = 9$ can also be found in **Figure 2**.

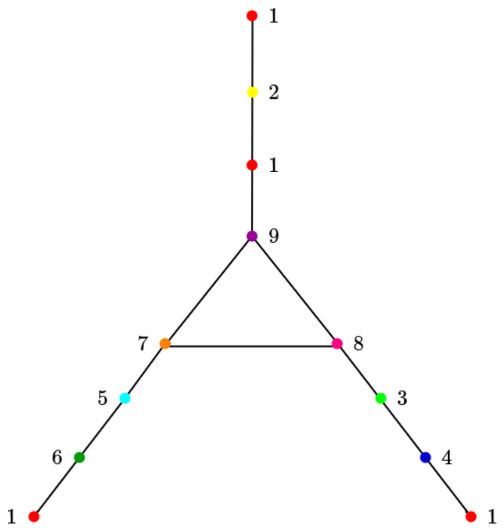

Figure 2. Illustration for Example $rvc(N_{3,3}) = 9$

3.3 Rainbow Vertex Connection Number of Triangular Ladder Graph (TL_n)

The rainbow vertex connection is referred to as the vertex coloring of a triangular ladder graph, if every vertex in the net graph is connected by a path that has interior points with different side colors. The following is a proof of the rainbow vertex connection theorem for the triangular ladder graph.

Theorem 4. The rainbow vertex connection number in the triangular ladder graph TL_n for $n \geq 2$ is $rvc(TL_n) = n - 1$.

Proof. for example TL_n is the triangular ladder graph of the vertex set $V(TL_n) = \{x_i y_i; 1 \leq i \leq n\}$ and edge set $E(TL_n) = \{x_i x_{i+1}, y_i y_{i+1}, x_i y_{i+1}; 1 \leq i \leq n\} \cup \{x_i y_i; 1 \leq i \leq n\}$. Firstly, there will be shown the lower bound of the triangular ladder graph TL_n. The triangular ladder graph TL_n has a diameter formulated by $diam(TL_n) = n$. Refers to **Theorem 1** we can find $rvc(TL_n) \geq diam(TL_n) - 1 = n - 1$. Based on this condition, it can be found that $rvc(TL_n) \geq n - 1$.

Secondly, the lower bound of the triangular ladder graph will be shown $rvc(TL_n) \geq n - 1$. The rainbow vertices coloring function of the triangular ladder graph TL_n is as follows:

$$f(x_i) = i; 1 \leq i \leq n - 1$$
$$f(y_i) = i; 1 \leq i \leq n - 1$$
$$f(x_i) = 1; i = n$$
$$f(y_i) = 1; i = n$$

In conclusion, based on the rainbow coloring vertex function, it can be found (rainbow path) of $u - v$ where $u, v \in V(TL_n)$ can be seen in **Table 3**.

Table 3. Rainbow Vertex Connection $u - v$ in Triangular Ladder Graph TL_n

Case	u	v	*Rainbow Vertex Connection* $u - v$
1	x_1	x_n	$x_2, x_3, \dots, x_{n-1}$
2	x_1	y_n	$y_2, y_3, \dots, y_{n-2}, y_{n-1}$
3	y_1	x_n	$x_1, x_2, x_3, \dots, x_{n-2}, x_{n-1}$
4	y_1	y_n	$y_2, y_3, \dots, y_{n-1}$

Based on Table 3, we can see that every path $u - v$ has a different color or so-called rainbow path or in other words, every path in the triangular ladder graph has an interior vertex with a different color, this means that the graph satisfies the concept of rainbow vertex connection. Thus, it is concluded based on the lower bound and upper bound, it is proved that $rvc(TL_n) = n - 1$. An illustration of the rainbow vertex connection can be found in Figure 3 which is an example of rainbow vertex connection in the triangular ladder graph TL_n. In Figure 3, there is also a rainbow vertex connection on the triangular ladder graph $TL_4 = n - 1$ or equal to $rvc(TL_4) = n - 1 = 4 - 1 = 3$.

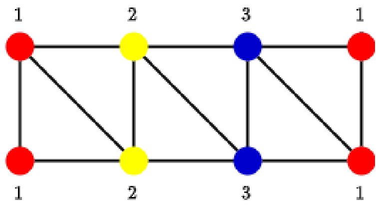

Figure 3. Illustration for example $rvc(TL_4) = 3$

3.4 Rainbow Vertex Connection Number of Composition Graph $(P_n[P_1])$.

The rainbow vertex connection is referred to as vertex coloring of Composition Graph $(P_n[P_1])$ graph, if every vertex in the Composition Graph $(P_n[P_1])$ is connected by a path that has interior points with different side colors. The following is a proof of the rainbow vertex connection theorem on Composition Graph $(P_n[P_1])$.

Theorem 5. The rainbow vertex connection number of graph composition $P_n[P_1]$ for $n \geq 3$ is $rvc(P_n[P_1]) = n - 2$.

Proof. For Example $P_n[P_1]$ is graph composition of set vertices $V(P_n[P_1]) = \{x_i; 1 \leq i \leq n; n \leq 3\} \cup \{y_i; 1 \leq i \leq n; n \leq 3\}$ and set edges $E(P_n[P_1]) = \{x_i y_i; 1 \leq j \leq n; n \leq 2\} \cup \{x_i x_{i+1}; 1 \leq j \leq n; n \leq 2\} \cup \{x_i y_{i+1}; 1 \leq j \leq n; n \leq 2\} \cup \{x_i x_{i+1}; 1 \leq j \leq n; n \leq 2\}$. Firstly, there will be found the lower bound of graph composition $P_n[P_1]$. Composition graph $P_n[P_1]$ has diameter formulated by $diam(P_n[P_1]) = n - 1$. Based on Theorem 1 we find $rvc(P_n[P_1]) \geq diam(P_n[P_1]) - 1 = n - 2$. Based on the condition, it is found that $rvc(P_n[P_1]) \geq n - 2$.

Secondly, there will be indicating the lower bound of graph composition $rvc(P_n[P_1]) \geq n - 2$. The rainbow vertex coloring function of graph composition $P_n[P_1]$ is as follows:

$$f(x_i) = 1; i = 1 \; and \; i = n$$
$$f(y_i) = 1; i = 1 \; and \; i = n$$
$$f(x_i) = i - 1; 2 \leq i \leq n - 1$$
$$f(y_i) = i - 1; 2 \leq i \leq n - 1$$

Based on its rainbow vertex coloring function, the rainbow paths from $u \; to \; v$, where $u, v \in V(P_n[P_1])$ can be seen in Table 4 below.

Table 4. Rainbow Vertex Connection $u - v$ in composition graph $P_n[P_1]$

Case	u	v	*Rainbow Vertex Connection* $u - v$
1	x_1	x_n	$x_2, x_3, \dots, x_{n-1}$
2	x_1	y_n	$y_2, y_3, \dots, y_{n-2}, y_{n-1}$

Case	u	v	Rainbow Vertex Connection $u - v$
3	y_1	x_n	$x_2, x_3, x_4, \dots, x_{n-2}, x_{n-1}$
4	y_1	y_n	$y_2, y_3, \dots, y_n$

According to Table 4, we find that each path $u - v$ has a distinct path color or so-called rainbow path, or in other words in every path of graph composition has a distinct interior vertice color, which means that the graph meets the concept of rainbow vertex connection. Based on the lower and upper bound, it is proven that $rvc(P_n[P_1]) = n - 2$. An illustration of rainbow vertex connection can be seen in Figure 4 which is an example of rainbow vertex connection in the graph composition $P_4[P_1]$. In Figure 4, the rainbow vertex connection of graph composition is also found $rvc(P_4[P_1]) = 4 - 2 = 2$.

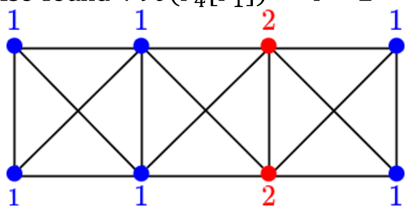

Figure 4. Illustration for Example $rvc(P_4[P_1]) = 2$

4. CONCLUSIONS

In this research, we have determined the correct values of the rainbow vertex connection number in the graph we observed and studied, which includes the bull graph bull graph $B_{3,m}$, net graph $N_{3,m}$, ladder graph L_n and graph composition $P_n\ [P_1]$. The results indicate that this research produces four theorems of rainbow vertex connection numbers. The results show that $rvc(B_{3,m}) = 2m$, $rvc(N_{3,m}) = 3m$, $rvc(L_n) = n - 1$, and $rvc(P_n\ [P_1\ ]) = n - 2$.

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