

PREDICTION OF CRUDE OIL PRICES IN INDONESIA USING FOURIER SERIES ESTIMATOR AND ARIMA METHOD

Alma Khalisa Rahma¹, Qumadha Zaenal Abidin², Juan Krisfigo Prasetyo³, Berliani Larasati⁴, Dita Amelia^{5*}, Nur Chamidah⁶

^{1,2,3,4,5,6}Mathematics Department, Faculty of Science and Technology, Universitas Airlangga
Jl. Dr. Ir. H. Soekarno, Surabaya, 60116, Indonesia

Corresponding author's e-mail: * dita.amelia@fst.unair.ac.id

ABSTRACT

Article History:

Received: 10th January 2024

Revised: 15th March 2024

Accepted: 29th June 2024

Published: 1st September 2024

Keywords:

Crude Oil;

ARIMA;

Forecasting;

Fourier Series Estimator.

Crude oil is one of the non-renewable natural resources that is crucial for countries around the world in driving economic development. However, the availability of crude oil is decreasing over time. The high demand for crude oil results in scarcity which causes price fluctuations. Low oil prices can reduce state revenues, disrupt development programs, and even trigger budget deficits. On the other hand, an increase in crude oil prices can make a positive contribution to state revenues. Crude oil exports become more profitable, which can increase state revenue through royalties and taxes levied on the oil and gas sector. This additional revenue can be used to support infrastructure development, social programs, and investment in key sectors of the economy. High oil prices can also harm the economy. With the many impacts that can be caused by crude oil prices, the government must be able to anticipate and prepare for it. The data used in this study are data on crude oil prices in Indonesia for monthly periods from January 2018 to October 2023 sourced from the official website of the Ministry of Energy and Mineral Resources (ESDM) of the Republic of Indonesia. The researcher tried to compare two analysis methods, namely the Fourier series and the ARIMA estimator. The results of this study show that the best method in predicting crude oil prices in Indonesia is the Fourier series estimator with Cos-Sin function which produces RMSE and MAPE values of 7.93 and 8.4%. The prediction results can be used as a reference for the government to anticipate and make programs or policies that are more focused and targeted toward the impacts that can be caused by changes in crude oil prices.



This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution-ShareAlike 4.0 International License](https://creativecommons.org/licenses/by-sa/4.0/).

How to cite this article:

A. K. Rahma, Q. Z. Abidin, J. K. Prasetyo, B. Larasati, D. Amelia and N. Chamidah., "PREDICTION OF CRUDE OIL PRICES IN INDONESIA USING FOURIER SERIES ESTIMATOR AND ARIMA METHOD," *BAREKENG: J. Math. & App.*, vol. 18, iss. 3, pp. 1673-1682, September, 2024.

Copyright © 2024 Author(s)

Journal homepage: <https://ojs3.unpatti.ac.id/index.php/barekeng/>

Journal e-mail: barekeng.math@yahoo.com; barekeng_journal@mail.unpatti.ac.id

Research Article · **Open Access**

1. INTRODUCTION

Crude oil is a non-renewable natural resource that is very important for every country to support economic development. However, the availability of crude oil is decreasing as time goes by. The high demand for crude oil results in shortages which cause price fluctuations [1]. Based on data from the Organization of the Petroleum Exporting Countries (OPEC) website, world crude oil prices have fluctuated over the last ten years. The average price of Indonesian crude oil in May 2023 decreased by US\$9.22/barrel from US\$79.34/barrel in April 2023 to US\$70.12/barrel [2]. The decline in the price of primary crude oil on the international market was influenced, among other things, by market concerns over world economic conditions due to inflation and interest rates [3]. Low oil prices can reduce state revenues, disrupt development programs, and even trigger budget deficits. On the other hand, an increase in crude oil prices can make a positive contribution to state revenues. Crude oil exports have become more profitable, which can increase state revenues through royalties and taxes imposed on the oil and gas sector [4]. This additional revenue can be used to support infrastructure development, social programs, and investment in key sectors of the economy. However, high oil prices can also harm the economy, namely by increasing production costs in various economic sectors, especially the transportation and manufacturing sectors [5]. This can lead to higher inflation, which in turn can reduce people's purchasing power and harm domestic consumers and producers.

Many impacts can be caused by crude oil prices, so the government must be able to anticipate and prepare for this. One way is to make predictions or forecasting using time series analysis [6]. Time series analysis is a series of observations taken based on time sequence and between adjacent and correlated observations, so it is said that in a time series, each observation taken from a variable is correlated with the variable itself at the previous time [7]. One method that is often used in time series analysis is Autoregressive Integrated Moving Average (ARIMA) [8]. Then, because the price of crude oil in Indonesia is very volatile, a non-parametric time series analysis method, namely the Fourier series estimator, can also be used [9].

Previous research by Dirgantari, R [10] regarding predictions of Indonesian crude oil prices obtained results from the ESTAR (1.1) model with a MAPE value of 8.318409% and the AR (1) model with a MAPE value of 7.711869%. This means that predictions using the ARMA method are better than the STAR method. The research conducted by Setiadi Y.K [11] regarding forecasting iron prices at PT. Wijaya Karya using the Fourier series method. The results of this research produced a model with a MAPE value of 0.48%.

Based on this description, the author will compare the predictions of the ARIMA method and the Fourier series estimator on crude oil price data in Indonesia to find the best model between the two methods and look for forecasting results that are closer to the original data.

2. RESEARCH METHODS

2.1 Data and Data Sources

The data used in this study are crude oil prices in Indonesia for the monthly period from January 2018 to October 2023 sourced from the official website of the Ministry of Energy and Mineral Resources (ESDM) of the Republic of Indonesia, accessible at esdm.go.id [3]. The total data used in this study were 70 observations.

2.2 Research Variable

This research uses one variable, namely global crude oil prices. The data period is monthly data starting from January 2018 to October 2023. The division of training and testing data is done in a ratio of 90:10. The training data used is the 1st data to the 63rd data and the testing data used is the 64th to the 70th data.

2.3 Analysis Procedure

The analysis procedure to be carried out in this research is as follows:

1. Knowing the characteristics of crude oil price data in Indonesia in the period January 2018 to October 2023 with the following steps:

- a. Making a time series plot of crude oil price data in Indonesia to see the trend of the data.
 - b. Dividing the data into training and testing, where the training data starts in January 2018 to March 2023, while the testing data starts in April 2023 to October 2023.
2. Build a model and predict crude oil prices in Indonesia based on training data using the Fourier series estimator method with the following steps:
- a. Define the response variable, namely the price of crude oil in Indonesia, and the predictor variable, namely time in months.
 - b. Divide the research data into training data to be modeled with nonparametric regression of the Fourier series estimator and testing data to be predicted from the results of the best model criteria of the Fourier series estimator.
 - c. Determining the function $f(x_i)$ to be used based on the training data.

$$f(x_i) = \frac{\alpha_0}{2} + \gamma x_i + \sum_{k=1}^K (\alpha_k \cos(kx_i) + \beta_k \sin(kx_i)) \quad (1)$$

With,

$$\hat{\alpha}_k = \frac{2}{n} \sum_{i=1}^n y_i \cos\left(\frac{2\pi k(i-1)}{n}\right)$$

$$\hat{\beta}_k = \frac{2}{n} \sum_{i=1}^n y_i \sin\left(\frac{2\pi k(i-1)}{n}\right)$$

Where:

$f(x_i)$: Nonparametric regression function of the Fourier series on time series data

$\frac{\alpha_0}{2}, \gamma, \alpha_k, \beta_k$: Regression parameters

k : The Fourier series oscillation parameter

x_i : the input values or independent variables

y_i : The observed values or dependent variables

- d. Input the value of the Fourier series oscillation parameter (k).
- e. Calculates the MSE value using the training data. By using the WLS (Weighted Least Square) method, the Mean Square Error (MSE) equation is obtained as follows [12]:

$$MSE[\mathbf{K}] = \frac{1}{N} (\mathbf{y} - \hat{\mathbf{f}})' \mathbf{W} (\mathbf{y} - \hat{\mathbf{f}}); \mathbf{K} = (k_1, k_2, \dots, k_n) \quad (2)$$

With,

$$\mathbf{W} = \frac{\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}_i'}{n_i} = \frac{(\mathbf{y}_i - \mathbf{f}_i)(\mathbf{y}_i - \mathbf{f}_i)'}{n_i}$$

If consider nonparametric regression function estimator

$$\mathbf{f} = \mathbf{Z}\boldsymbol{\gamma} + \mathbf{X}\boldsymbol{\beta} = \mathbf{H}\mathbf{y}_i$$

Then,

$$\mathbf{W} = \frac{(\mathbf{y}_i - \mathbf{H}\mathbf{y}_i)(\mathbf{y}_i - \mathbf{H}\mathbf{y}_i)'}{n_i}$$

$$\mathbf{W} = \frac{(\mathbf{I}_{n_i} - \mathbf{H})\mathbf{y}_i\mathbf{y}_i'(\mathbf{I}_{n_i} - \mathbf{H})'}{n_i}$$

Where:

\mathbf{I}_{n_i} : The identity matrix of size $n_i \times n_i$

\mathbf{Z}, \mathbf{X} : Matrices representing the explanatory variables

\mathbf{H} : Combination of \mathbf{Z} and \mathbf{X} , can be written with matrix $[\mathbf{Z} \ \mathbf{X}]$

- f. Calculate the Generalized Cross Validation (GCV) value using the training data. The equation for calculating the GCV value is as follows [12]:

$$GCV[\mathbf{K}] = \frac{MSE[\mathbf{K}]}{(N^{-1} \text{trace}(\mathbf{I}_N - \mathbf{H}))^2} \quad (3)$$

Where:

N^{-1} : The inverse of the number of observations

- g. Based on Step f, the optimal Fourier series oscillation parameter value (k) with the minimum GCV value of looping is the selected.
- h. Selecting the best model based on MSE, GCV and R^2 values. The best model is selected based on the minimum GCV value at step f and the largest R^2 value [12].

$$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y}_i)^2} \tag{4}$$

Where:

- y_i : The observed values of the dependent variable
- \hat{y}_i : The predicted values of the dependent variable
- \bar{y}_i : The mean of the observed values of the dependent variable

- i. Calculating the estimated value of \hat{y}_i using training data.

$$\hat{y}_i = \frac{\hat{\alpha}_0}{2} + \hat{\gamma}x_i + \sum_{k=1}^K (\hat{\alpha}_k \cos(kx_i) + \hat{\beta}_k \sin(kx_i)) \tag{5}$$

- j. Calculating the Mean Absolute Percentage Error (MAPE) value for training data [12].

$$MAPE = \frac{\sum_{i=1}^n \left| \frac{y_i - \hat{y}_i}{y_i} \right|}{n} \times 100\% \tag{6}$$

Where:

- y_i : The observed values of the dependent variable
- \hat{y}_i : The predicted values of the dependent variable
- n : The number of observations

- k. Predict the crude oil price data in Indonesia based on the best model based on step h.
- l. Calculate the Root Mean Square Error (RMSE) and MAPE of the predicted value of the testing data [12].

$$RMSE = \sqrt{\frac{\sum_{t=1}^n (y_t - \hat{y}_t)^2}{n}} \tag{7}$$

Where:

- y_t : The observed values of the dependent variable
- \hat{y}_t : The predicted values of the dependent variable
- n : The number of observations

- m. Make a comparison table of actual data and prediction data from the model that has been obtained based on the best model criteria of the Fourier series estimator.
- 3. Build a model and predict crude oil prices in Indonesia based on training data using the ARIMA method with the following steps:

- a. Identify the stationarity of the data in average and variance through Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots, if the data is not stationary in average, then differencing is performed [13]. The mathematical formula is as follows:

$$\Delta Z_t = Z_t - Z_{t-1} \text{ and } \Delta^2 Z_t = Z_t - 2Z_{t-1} + Z_{t-2} \tag{8}$$

If the data is not stationary in variance, then the Box-Cox transformation with the mathematical formula is as follows [13].

$$Z_t^\lambda = \frac{Z_t^{(\lambda)}}{\lambda}, \text{ with } -1 < \lambda < 1 \tag{9}$$

- b. Make ACF and PACF plots to determine the model. The ACF can be written as follows [13]:

$$\rho_k = \frac{Cov(Z_t, Z_{t-k})}{\sqrt{Var(Z_t)}\sqrt{Var(Z_{t-k})}} = \frac{\gamma_k}{\gamma_0} \tag{10}$$

γ_k is autocovariance at the k_{th} lag, γ_0 is variance of Z_t , and ρ_k is autocorrelation at the k_{th} lag. While the PACF can be written as follows [13]:

$$\phi = Corr(Z_t, Z_{t-k} | Z_{t-1}, \dots, Z_{t-k-1})$$

With autocorrelation function:

$$\rho_j = \phi_{k1}\rho_{j-1} + \phi_{k2}\rho_{j-2} + \dots + \phi_{kj}\rho_{j-k}$$

Then ϕ_{kk} can be written mathematically with the equation:

$$\phi_{kk} = \frac{\rho_k - \sum_{j=1}^{k-1} \phi_{k-j}\rho_{k-j}}{1 - \sum_{j=1}^{k-1} \phi_{k-j}\rho_j} \tag{11}$$

- c. Perform diagnostic checks on the model. Residual diagnostic checks include white noise assumption test and normality test to prove the adequacy of the ARIMA model. White noise

testing on the model can also be seen using the Ljung-Box statistical test with the following hypothesis:

$H_0 : \rho_1 = \rho_2 = \dots = \rho_k = 0$, (qualified)

$H_1 : \text{At least one } \rho_i \neq 0 \text{ for } i = 1, 2, 3, \dots, k$ (not yet qualified)

Test Statistics:

$$Q = n(n+2) \sum_{k=1}^K \frac{\hat{\rho}_k^2}{n-k}, n > K \quad (12)$$

Where:

K : maximum lag

n : number of data (observations)

$\hat{\rho}_k$: residual autocorrelation for the k_{th} lag

Testing Criteria:

If $Q < \chi^2_{(\alpha; df=K-p-q)}$, or p -value $> \alpha$, Then H_0 is accepted and it is concluded that the residuals fulfill the white noise assumption [14].

The next residual diagnostic check is testing the assumption that the residuals are normally distributed. This test can be done with a statistical test, namely the Kolmogorov-Smirnov

Test with the following hypothesis:

$H_0 : F(x) = F_0(x)$ for all x (residuals are normally distributed)

$H_1 : F(x) \neq F_0(x)$ for all x (residuals are normally distributed)

Test Statistics:

$$D = \sup_x |S(x) - F_0(x)| \quad (13)$$

Where:

D : maximum deviation

\sup : maximum value for all x of the absolute difference of $S(x)$ and $F_0(x)$

$F_0(x)$: normally distributed cumulative probability function or hypothesized function

$S(x)$: cumulative distribution function of the sample data

Testing Criteria:

If $D < D_{\alpha, n}$ or P -value $> \alpha$, then H_0 is accepted and it is concluded that the residual model is normally distributed [14].

- d. Determine the best ARIMA model based on diagnostic checks and the smallest MAPE and RMSE values [13].
 - e. Predict the testing data using the best model and calculate the accuracy without sample MAPE.
4. Comparing the prediction results of crude oil prices in Indonesia on training and testing data from ARIMA and Fourier series estimators with the following steps:
 - a. Make a comparison plot of testing data with the results of ARIMA and Fourier series estimator testing predictions.
 - b. Comparing the prediction results of testing data from ARIMA and Fourier series estimator using MAPE and RMSE.

3. RESULTS AND DISCUSSION

3.1 Descriptive Analysis of Crude Oil Prices in Indonesia

The first step to determine the characteristics of the data is to carry out a descriptive analysis. The following is a plot image of crude oil price data in Indonesia.

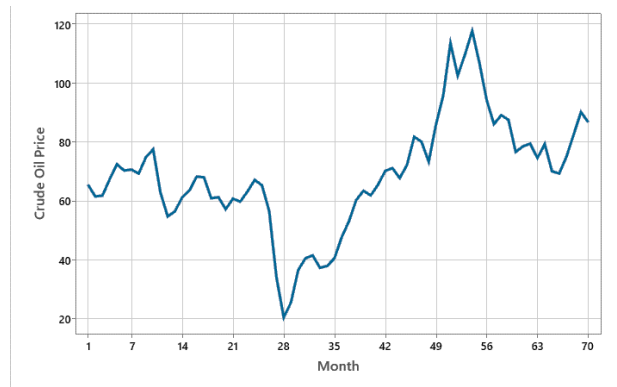


Figure 1. Time series plot of crude oil price data in Indonesia

In **Figure 1**, it can be observed that the prices experienced fluctuations. This is made clear in the table below which displays the results of a descriptive analysis of crude oil prices in Indonesia.

Table 1. Description of Crude Oil Price Data in Indonesia

Variable	N	Mean	Minimum	Maximum
Price	70	68.80	20.66	117.62

Table 1 shows that the average global crude oil price from January 2018 to August 2023 was \$68.22. The lowest crude oil price was recorded in April 2020 at \$20.66, which occurred due to no countries intending to purchase delivery contracts in May because the COVID-19 pandemic led many nations to halt their economic activities. This caused an oversupply of crude oil, sharply driving down its price [15]. The highest crude oil price reached \$117.62 in June 2022, attributed to uncertainties in crude oil supply from OPEC to meet production quotas. This inability stemmed from several factors, notably the imposition of sanctions to abstain from purchasing crude oil from Russia, despite Russia itself contributing over 36% of Europe's crude oil needs. On the other hand, the U.S. crude oil production did not display a significant increase [16].

3.2. Modeling and Predicting Crude Oil Prices in Indonesia using the Fourier Series Estimator Method

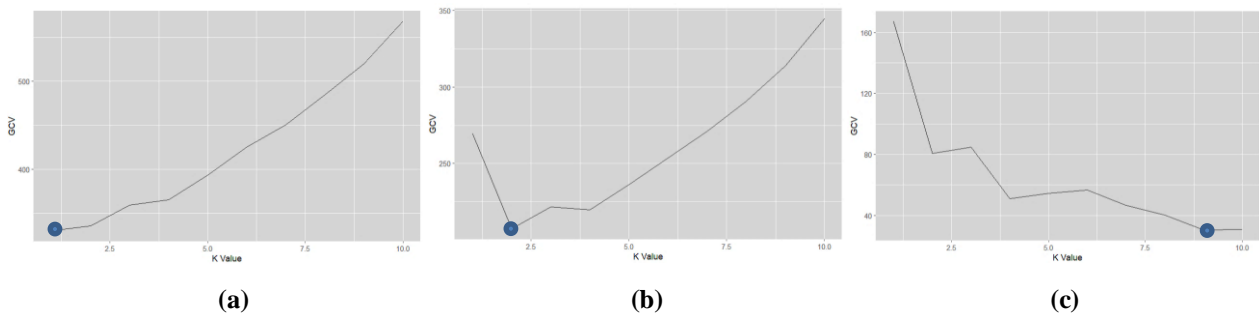


Figure 2. Selection of oscillation function parameters, (a) Sin function oscillation parameters, (b) Cos function oscillation parameters, (c) Cos-sin function oscillation parameters

The first step in constructing a Fourier series estimator model is to determine the optimal oscillation parameters for the *Sin*, *Cos*, and *Cos – Sin* functions [12]. Determining the optimal oscillation parameters is based on identifying the minimum GCV value from each input oscillation parameter. From **Figure 2**, it is found that the best oscillation parameter value for the *Sin* function is $k = 1$ with a GCV value of 329.972, for the *Cos* function is $k = 2$ with a GCV value of 207.377, and for the *Cos – Sin* function is $k = 9$ with a GCV value of 30.390.

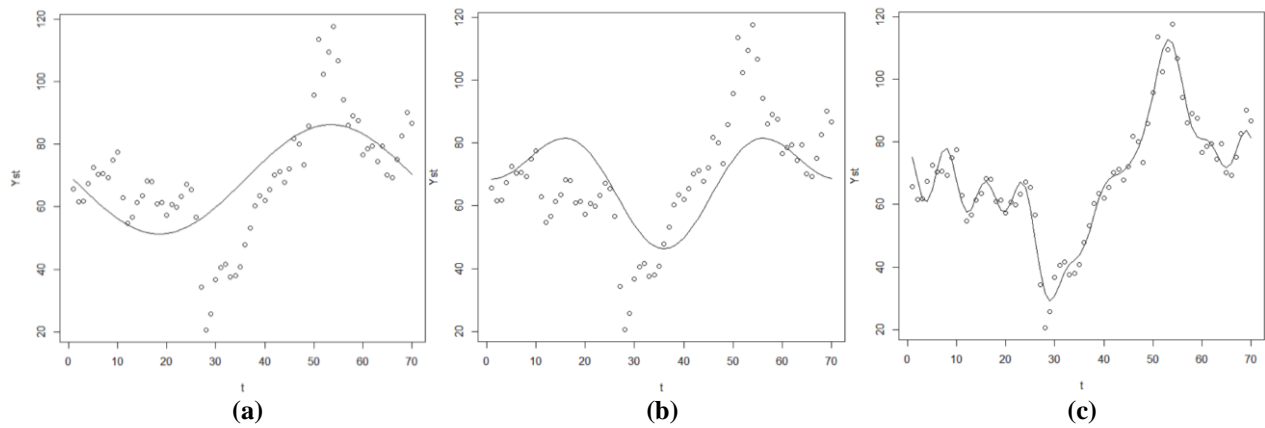


Figure 3. Calculation of Estimated Values for Each Function, (a) Sin Function Estimation Results, (b) Cos Function Estimation Results, (c) Cos-sin Function Estimation Results

After obtaining the optimal oscillation parameter values for each function, the next step is to calculate the estimated values from the training data for each function. Here are the calculated estimation values for the *Cos*, *Sin*, and *Cos – Sin* functions [12]. From **Figure 3**, it is observed that the estimation results moderately represent the data well with an *R*-square value of 42%, MSE of 3.62, and MAPE of 8.5%. The estimation results reasonably represent the data well with an *R*-square value of 35%, MSE of 4.03, and MAPE of 9.4%. The estimation results highly represent the data well with an *R*-square value of 95%, MSE of 0.35, and MAPE of 0.83%.

Following the estimation calculations, the next step is to determine the best model by comparing the *R*-square, MSE, and MAPE values obtained from the estimation results of the training data for each function as follows.

Table 2. Selection of the Best Model for Fourier Series Estimator

Function	<i>k</i>	GCV	MSE	MAPE	<i>R</i> ²
<i>Sin</i>	1	329.9717	3.62	8.5%	42%
<i>Cos</i>	2	207.3769	4.03	9.4%	35%
<i>Cos – Sin</i>	9	30.38993	0.35	0.83%	95%

Based on **Table 2**, it is determined that the model using the *Cos – Sin* function is the most optimal for the crude oil price data in Indonesia, as it exhibits the smallest GCV, MSE, and MAPE values, and also the highest *R*-square value. Based on the optimal model selected, the *Cos – Sin* function model can be formulated for the crude oil price data in Indonesia as follows.

$$\begin{aligned}
 \hat{y}_i = & 68.21794 + 13.75385 \cos 2\pi x_i - 16.7223 \sin 2\pi x_i - 13.2287 \cos 4\pi x_i \\
 & + 2.459978 \sin 4\pi x_i + 0.5543227 \cos 6\pi x_i + 1.0823 \sin 6\pi x_i \\
 & + 0.4309314 \cos 8\pi x_i + 5.379201 \sin 8\pi x_i + 1.219468 \cos 10\pi x_i \\
 & - 4.478632 \sin 10\pi x_i - 0.1756481 \cos 12\pi x_i - 1.88986 \sin 12\pi x_i \\
 & - 2.481296 \cos 14\pi x_i + 1.293085 \sin 14\pi x_i + 2.462208 \cos 16\pi x_i \\
 & - 2.734585 \sin 16\pi x_i + 2.672633 \cos 18\pi x_i - 0.0075 \sin 18\pi x_i
 \end{aligned} \tag{13}$$

3.3 Modeling and Predicting Crude Oil Prices in Indonesia Using the ARIMA Method

To determine whether the acquired original data is stationary, Time Series Plot, Plot ACF, and Plot PACF are displayed. The obtained plots are as follows:

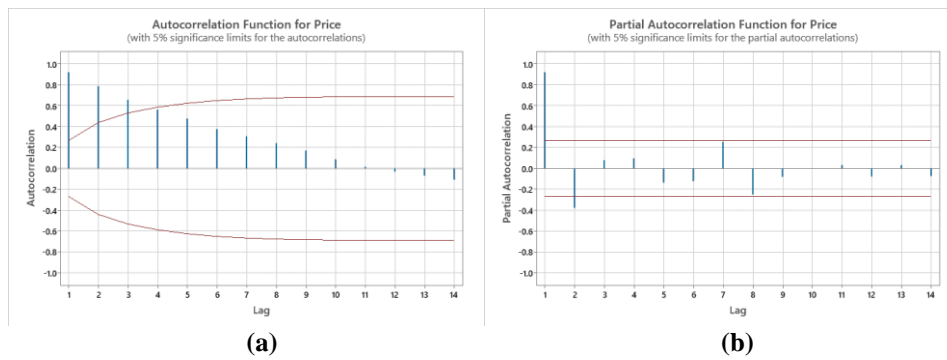


Figure 4. Identification of Stationarity, (a) ACF Plot Preliminary Data on Crude Oil Prices, (b) PACF Plot Preliminary Data on Crude Oil Prices

Based on **Figure 4**, it can be observed that the data is still non-stationary in both mean and variance. The resulting ACF plot forms waves and shows a decrease at certain lags, while the PACF plot extends beyond certain lags, indicating that the data is still non-stationary [13]. Before proceeding with data processing, it's necessary to perform Box-Cox transformation and differencing to achieve stationarity in the data. The initial step in making the data stationary involves stabilizing its variance. The data needs to undergo Box-Cox transformation first, followed by differencing. This is because, in practice, data that is non-stationary in variance often remains non-stationary in mean as well.

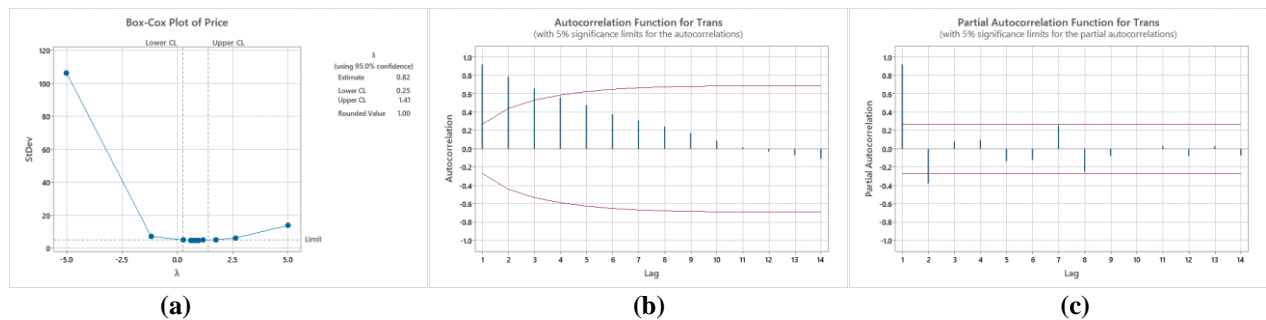


Figure 5. Transformation Process, (a) Box-Cox Transformation, (b) ACF of transformed Data, (c) PACF of Transformed Data

Based on **Figure 5**, it can be seen that the result of the Box-Cox transformation indicates a Rounded Value (λ) of 1,00, meaning that the data has become stationary in variance. After performing the Box-Cox transformation, the next step is to perform differencing at lag 1 to make the data stationary in terms of the mean and display its time series plot, ACF plot, and PACF plot.

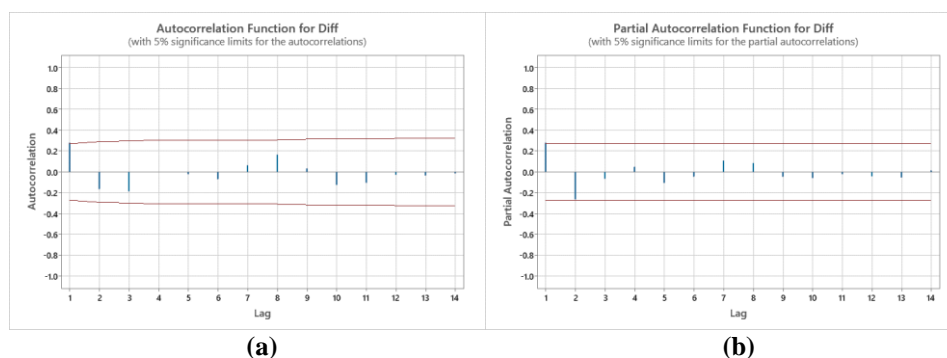


Figure 6. Differentiating Process, (a) ACF of Differenced Data, (b) PACF of Differenced Data

After the differencing, the seasonal lags have disappeared. This implies that the seasonal component of the model is now stationary after applying differencing at lag 1. Therefore, the model in place is a non-seasonal multiplicative ARIMA model, non-stationary in the non-seasonal mean. We can refer back to the ACF and PACF plots of the differenced data at lag 1 to proceed to the next step. The next step is to determine the best ARIMA model which is listed in **Table 3** below.

Table 3. Selection of the Best Model for ARIMA

ARIMA	Type	Parameter Significance	White Noise	MSE
(1,1,1)	Deterministic	Not Significant	White Noise	49.7404
	Probabilistic	Not Significant	White Noise	48.9168
(0,1,1)	Deterministic	Not Significant	White Noise	48.8086
	Probabilistic	Significant	White Noise	48.0139
(1,1,0)	Deterministic	Not Significant	White Noise	50.5876
	Probabilistic	Significant	White Noise	49.7293

Based on **Table 3**, the examination results indicate that the best model is the probabilistic ARIMA (0,1,1) model, which meets the criteria as all its parameters are significant with a P – value (0.002) < 0.05. Additionally, the model conforms to the white noise concept with a P – value > 0.05. Moreover, in terms of MSE, this model has the smallest value of 48.0139. After obtaining the best model, the ARIMA (0,1,1) probabilistic, the next stage of analysis involves writing down the equation of the model as follows.

$$Z_t = Z_{t-1} + a_t + 0.376Ba_{t-1} \quad (14)$$

3.4 Comparison of Prediction Results of the ARIMA Method and the Fourier Series Estimator

Once the appropriate model for estimating the training data is obtained, the next step is to forecast using that model and compare it with the testing data. The forecast results and testing data are depicted in **Figure 7** below.

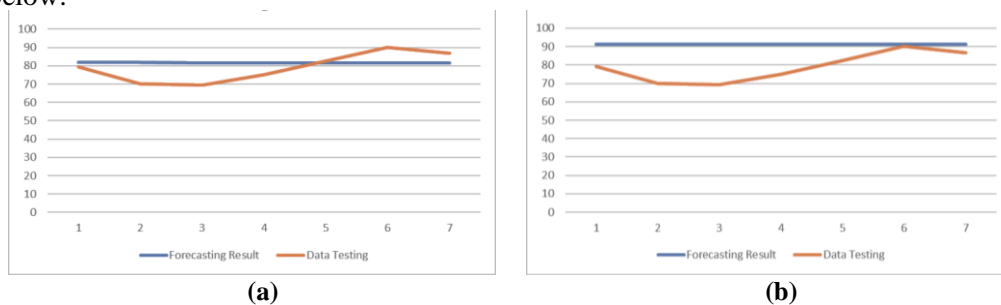


Figure 7. Forecasting result, (a) Forecasting result with Fourier series estimator, (b) Forecasting result with ARIMA

Based on the results from **Figure 7**, it's evident that the forecast results differ significantly from the testing data. The comparison of prediction results between the ARIMA method and Fourier Series Estimator can be observed based on the RMSE and MAPE values shown in the following **Table 4**.

Table 4. Comparison of Results

Model	RMSE	MAPE
Fourier Series Estimator	7.93	8.4%
ARIMA	14.18	13.2%

According to **Table 4**, the RMSE and MAPE values obtained using the ARIMA method are 14.18 and 13.2%, respectively. Meanwhile, the RMSE and MAPE values obtained using the Fourier Series Estimator are 7.93 and 8.4%, respectively. This indicates that predictions made using the Fourier Series Estimator have better accuracy compared to the ARIMA method, as they exhibit smaller RMSE and MAPE values. Thus, it can be concluded that the model is suitable for forecasting crude oil prices in Indonesia [17].

4. CONCLUSIONS

Based on the results of the analysis that has been done modeling crude oil price data in Indonesia with a Fourier series estimator using the $\text{Cos} - \text{Sin}$ function can make better predictions when compared to ARIMA. The RMSE and MAPE values for modeling crude oil price data in Indonesia with the Fourier series estimator using the $\text{Cos} - \text{Sin}$ function are 7.93 and 8.4%. The prediction results with the Fourier series estimator using the $\text{Cos} - \text{Sin}$ function are very good because they have small RMSE and MAPE values. From these results, it can be concluded that the Fourier series estimator model with the $\text{Cos} - \text{Sin}$ function can be used to predict crude oil prices in Indonesia in the next few periods. So, it can provide early warning to the government regarding fluctuations in crude oil prices and help in preventing the impacts resulting from fluctuations in crude oil prices in Indonesia.

ACKNOWLEDGMENT

The authors would like to thank the Statistics Study Program of Universitas Airlangga for providing opportunities and support to students in carrying out research projects to strengthen the application of learning materials in statistical modeling courses. In addition, the authors also express their gratitude to the Ministry of Energy and Mineral Resources (ESDM) for providing the data used in this study.

REFERENCES

- [1] Y. K. Dalimunthe and C. Rosyidan, "Keterkaitan harga minyak indonesia dengan harga minyak dunia melalui koefisien korelasi," *PETRO: Jurnal Ilmiah Teknik Perminyakan*, vol. 5, no. 1, 2016.
- [2] "DPR RI," 23 Mei 2023. [Online]. Available: <https://www.dpr.go.id/berita/detail/id/44622/t/Asumsi%20Harga%20Minyak%20Mentah%20Indonesia%20Tahun%202024%20yang%20Ditetapkan%20Pemerintah%20Cukup%20Realistis,>
- [3] "Kementrian ESDM," 2023. [Online]. Available: <https://migas.esdm.go.id/post/read/menteri-esdm-tetapkan-icp-minyak-mentah-indonesia-mei-2023-sebesar-us-70-12-per-barel>.
- [4] A. M. Herrera, H. Liang and P. Daniel, "Forecasting crude oil price volatility," *International Journal of Forecasting*, vol. 34, no. 4, pp. 622-635, 2018.
- [5] K. Lang and R. A. Benjamin, "The economic and financial properties of crude oil," *The North American Journal of Economics and Finance*, vol. 52, 2020.
- [6] C. Deng, L. Ma and T. Zeng, "Crude oil price forecast based on deep transfer learning: Shanghai crude oil as an example.," *Sustainability*, vol. 13, no. 24, 2021.
- [7] X. Guo, Y. Gao and D. Zheng, "Short-Term Household Load Forecasting Based on Long-and Short-Term Time Series Network," *Energy Reports*, pp. 58-64, 2021.
- [8] A. Prahutama, Artist, *Model Regresi Nonparametrik dengan Pendekatan Deret Fourier pada Kasus Tingkat Pengangguran Terbuka di Jawa Timur*. [Art]. Universitas Diponegoro, 2013.
- [9] M. Mardianto, S. H. Kartiko and H. Utami, "The Fourier series estimator to predict the number of dengue and malaria sufferers in Indonesia.," *AIP Conference Proceedings*, vol. 2329, no. 1, 2021.
- [10] R. Dirgantari, Artist, *Prediksi harga minyak mentah indonesia menggunakan model Smooth Transition Autoregressive (STAR) dan model Autoregressive Moving Average (ARMA)*. [Art]. UIN Syarif Hidayatullah, 2018.
- [11] Y. K. Setiadi, Artist, *PERAMALAN HARGA BESI PADA PT. WIJAYA KARYA DENGAN METODE DERET FOURIER*. [Art]. Universitas Atma Jaya, 2023.
- [12] M. F. F. Mardianto and H. U. Gunardi, "An Analysis about Fourier Series Estimator in Nonparametric Regression for Longitudinal Data," *Mathematics and Statistics*, vol. 9, no. 4, pp. 501-510, 2021.
- [13] M. Poongodi, V. Vijayakumar and N. Chilamkurti, "Bitcoin price prediction using ARIMA model," *International Journal of Internet Technology and Secured Transactions*, vol. 10, no. 4, pp. 396-406, 2020.
- [14] W. W. Wei, *Multivariate time series analysis and applications*, John Wiley & Sons, 2018.
- [15] S. Supriyanto, "Impact of oil prices and stock returns: Evidence of oil and gas mining companies in Indonesia during the COVID-19 period," *International Journal of Energy Economics and Policy*, 2021.
- [16] A. W. Nyoman, "Bisnis.com," 2022. [Online]. Available: <https://ekonomi.bisnis.com/read/20220704/44/1551050/harga-icp-juni-2022-naik-jadi-us11762-per-barel-ini-penyebabnya>.
- [17] M. J. Moreno, P. A. Pol, S. A. Abad and C. B. Blasco, "Using the R-MAPE index as a resistant measure of forecast accuracy," *Psicothema*, vol. 25, no. 4, pp. 500-506, 2013.