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VALUE AT RISK ESTIMATION FOR STOCK PORTFOLIO USING THE ARCHIMEDEAN COPULA APPROACH

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ABSTRACT

Investment is one of the many ways to achieve future profits. One form of investment that is widely made is stocks. The return obtained in investing in stocks is potentially higher than other investment alternatives, but the risks borne are amplified, so it is necessary to analyze these risks that may occur. In this study, the Archimedean copula method is used to estimate the Value at Risk on shares of PT Bank Rakyat Indonesia Tbk (BBRI) and PT Telekomunikasi Indonesia Tbk (TLKM) for the period September 1, 2021, to August 31, 2023. The stock data is used to determine the Archimedean copula model and calculate the estimated value of Value at Risk (VaR) on the stock return portfolio using the Archimedean copula approach. The Archimedean copula models used are the Clayton copula model, Gumbel copula, and Frank copula. Of the three Archimedean copula models, the best model was selected by looking at the largest Maximum Likelihood Estimation (MLE) value. In this study, the log-likelihood value of Clayton copula is 7.958, Gumbel copula is 6.663, and Frank copula is 8.398. Therefore, Frank copula is the best Archimedean copula model with the largest log-likelihood value of 8.398 for the said data. Then the VaR estimation is done with the Frank copula model. The Value at Risk estimation results based on the Frank copula model show maximum loss rates of -0.0277 at the 90% confidence level, -0.0363 at the 95% confidence level, and -0.0516 at the 99% confidence level.

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1. INTRODUCTION

Investment is the strategic positioning of capital in a company with the aim of obtaining potentially high profits in the future. It can be conceptualized as a commitment to allocate a set amount of funds to one or more assets (at this time) that are expected to provide benefits in the future. According to the Indonesian Central Securities Depository (KSEI), one of the most popular investment instruments today is the allocation of funds in shares [1]. Stocks offer a high profit or return, albeit accompanied by an elevated degree of risk. Risk can be minimized by diversifying. Diversification is done by combining a few assets and determining the proportion of each asset in a portfolio [2].

Stock portfolios exhibit significant volatility, reflected in frequent and substantial price fluctuations which causes the variance of the residuals to change every time and is not constant. For this reason, time series modeling is carried out, namely with the Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) processes. This process aims to minimize risk and uncertainty factors [3]. This risk can be used in capture dynamic volatility patterns and predict future price movements using the Value at Risk (VaR) method.

In the case of stock portfolios, the normality property often deviates significantly. In addition, the dependency structure between stocks is challenging to quantify due to non-linear dependency. These deviations result in the invalidation of VaR estimation so that the risk obtained is leading to a potentially significant underestimation of risk. Therefore, a VaR estimation method using copula was developed. Copula's flexibility lies in its ability to not have the assumption requirement of data normality, so this method is considered appropriate without necessitating normality assumptions. In addition, Copula can also combine several marginal distributions into a combined distribution [4]. One of the copula families, it’s called Archimedean copula, is quite well used for financial assets [5]. This is because the Archimedean copula function can be used in financial asset applications to determine the dependency structure of financial asset returns, model the distribution of financial asset returns, and simulate a more adequate distribution of financial assets. Archimedean copula is divided into several parts, namely Clayton copula, Gumbel copula, and Frank copula [6].

The Copula method has been widely used for financial and actuarial applications. Cases in the financial sector are generally found to have non-normally distributed data. Research that uses this approach in the financial industry, among others, analyzes the relationship between macroeconomic factors and the level of world gold prices. This research uses the Archimedean copula approach whose parameters are estimated using Tau Kendall [7]. [8] has also investigated estimating claim reserves in several lines of business using Archimedean Copula and the Generalized Linear Model. Then [9] conducted the previous work on the application of the copula-GARCH method with a single index model approach which is used as a method to find the optimal portfolio shares from several stock data obtained which will then be estimated Value at Risk with the copula-GARCH method. The types of copulas used are Archimedean copula and elliptical copula. The use of the copula-GARCH method aims to overcome the problems commonly found in time series data, namely autocorrelation and heteroscedasticity. Then copula modeling is done to describe the dependency relationship between stocks without requiring normal assumptions. In addition, in the formation of a stock portfolio using the copula-GARCH method to calculate the estimated value at risk on stock data that has high fluctuations. Then, the research conducted by [10] examined how the application of one type of Archimedean copula family, namely the Gumbel copula. Gumbel copula is used in estimating value at risk on Telecommunication stocks and then backtesting was carried out to check the results of the VaR calculation.

In this study, VaR estimation was carried out using the Archimedean copula. The Archimedean copula is a type of copula family that is quite well used in analyzing financial data because it has a unique generator for each sub-copula in its part. In Archimedean copulas, the generator function is very important. This is because the generator function is used to determine the cumulative distribution function, the probability density function, and the log-likelihood function. These functions are used to model the Archimedean copula and to determine its loglikelihood value. The Archimedean copula has a wide range of applications. It is widely used because it can be constructed easily, has a wide range of dependency structures, models tail dependency, and is a simple bivariate copula in describing dependencies [11]. Then, from the Archimedean copula model, the best Archimedean copula is selected based on the largest log-likelihood value found using the Kendall Tau value. After that, VaR estimation is calculated based on the best Archimedean copula model and backtesting is conducted to check the accuracy of VaR calculations on stock return portfolios with 90%, 95%, and 99% confidence levels.

Thus, this study examined how to calculate the amount of VaR value estimation and backtesting of VaR results on the stock return portfolio with the best Archimedean copula approach based on the largest log-likelihood value. The data used in this study are daily stock closing price data incorporated in the LQ45 index for the period September 1, 2021, to August 31, 2023, namely stock data for PT Bank Rakyat Indonesia Tbk (BBRI) and PT Telekomunikasi Indonesia Tbk (TLKM).

2. RESEARCH METHODS

2.1 Stock Return

Stock return is the rate of return for a stock and is the cash payment received because of a stock at the time of initial investment. Stock returns can be calculated daily, weekly, monthly, and annually using **Equation (1)**.

$$R_{it} = \frac{P_t - P_{(t-1)}}{P_{(t-1)}} \tag{1}$$

where R_{it} is stock return i period t, P_t is stock price i period t, and $P_{(t-1)}$ is stock price period $t-1$**[12]**.

2.2 Data Identification

The initial step in identifying BBRI and TLKM stock return data is to test the normality of stock return data using the Kolmogorov-Smirnov test. The interpretation of the Kolmogorov-Smirnov test results is if the test value is more than $\alpha = 5\%$ then the data distribution is declared to fulfill the normality assumption, and if the test value is less than $\alpha = 5\%$ then it is interpreted as abnormal **[13]**. Then testing the stationarity of stock return data using time series plots and the Augmented Dickey-Fuller (ADF) test. Furthermore, testing the autocorrelation properties to see whether there is a correlation effect on the data using the Ljung-Box test and looking at the ACF and PACF plots formed from the observation data. Then the identification of the appropriate ARIMA model is carried out and a heteroscedasticity test is carried out to find out whether the data has a very diverse variant that can result in unstable residues by looking at the ACF plot squared data and the ARCM LM test. After that, GARCH modeling is performed to eliminate the effects of heteroscedasticity on the data.

2.3 Stages of Data Analysis

The stages in analyzing the data are as follows.

1. Describe the data to obtain an overview based on the descriptive statistics of the research data.
2. Normality test on stock return data using the Kolmogorov-Smirnov test.
3. Identify the appropriate ARIMA model.
   - Testing the stationarity of stock return data using time series plots and the Augmented Dickey-Fuller (ADF) test.
   - Autocorrelation test on BBRI and TLKM stock return data using the Ljung-Box test.
   - Heteroscedasticity test on BBRI and TLKM stock return data using the Lagrange Multiplier (LM).
4. Calculate the Tau Kendall correlation coefficient used in the Copula parameter estimation.
5. Establishment of Archimedean copula model.
6. The best Archimedean copula model was selected.

   The next step is to determine the Archimedean copula function that can show the dependency relationship of the function that can show the dependency relationship of the two stocks well. The method of estimating copula parameters The Archimedean copula parameter estimation method used is the Maximum Likelihood Estimation (MLE) method.

7. Calculation of Value at Risk (VaR) value using Monte Carlo simulation of copula models. Monte Carlo simulation of the best Archimedean copula model for BBRI and TLKM stocks.
8. Backtesting test to test the accuracy of the VaR value.
9. Interpretation of the VaR value assuming the weight of the two stocks is the same.
10. Drawing conclusions.

2.4 Kendall Tau Correlation

There are $\binom{n}{2}$ pairs (X_i, Y_i) and (X_j, Y_j) of observations in the sample, each pair is either concordant (pairs of agreements) or discordant (pairs of disagreements). C denotes the number of concordant pairs and D denotes the number of discordant pairs.

If there is bivariate data (X_i, Y_i), $i = 1,2, \ldots, n$ Where X and Y are at least ordinal scale. Then for each pair of observed values (X_i, Y_i) and (X_j, Y_j) for $i \neq j$ can be defined if the pair (X_i, Y_i) and (X_j, Y_j) are concordant, if $(X_i - X_j)(Y_i - Y_j) > 0$ means that if $X_i > X_j$ then $Y_i > Y_j$ or if $X_i < X_j$ then $Y_i < Y_j$ so that$(X - X)$ and $(Y - Y)$ have the same sign, which is equally positive or equally negative with a product that is always positive. If the pairs (X_i, Y_i) and (X_j, Y_j) are discordant, if $(X_i - X_j)(Y_i - Y_j) < 0$ it means that if $X_i > X_j$ then $Y_i < Y_j$ or if $X_i < X_j$ then $Y_i > Y_j$ so that $(X - X)$ and $(Y - Y)$ have opposite signs with a product that is always negative. The sample-based Kendall Tau equation can be seen in **Equation (2) [14]**.

$$\tau = \frac{C - D}{C + D} = \frac{C - D}{\binom{n}{2}} \tag{2}$$

2.5. Copula

Copulas are generalized distribution functions of several marginal distribution functions **[15]**. One of the popular copula families used is the Archimedean copula. Archimedean copulas are divided into several parts including Clayton, Gumbel, and Frank **[16]**. Find the correlation coefficient using Kendall's Tau Correlation **[17]** for the Archimedean copula in **Equation (3)**.

$$\tau = 1 + 4 \int_0^1 \frac{\varphi(t)}{\varphi'(t)} dt \tag{3}$$

2.6. Archimedean Copula

The Archimedean copula is a continuous multivariate copula that has a simple form, but it has a wide range of dependency structures that are easy to implement. The Archimedean copula is defined by **Equation (4) [18]**.

$$C(u, v) = \varphi^{-1}\{\varphi(u) + \varphi(v)\} \,\forall\, u, v \,\epsilon\, [0, 1] \tag{4}$$

Where:
$\varphi(u)$: Archimedean copula generator function u.
$\varphi(v)$: Archimedean copula generator function v.
φ^{-1} : inverse of φ, with φ^{-1}: [0,1].

The type of Archimedean copula used is as follows.

1. Clayton Copula
   The cumulative distribution function of the Clayton copula is defined in **Equation (5)**.

$$C(u, v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{5}$$

2. Gumbel Copula
   The cumulative distribution function of the Gumbel copula is defined in **Equation (6)**.

$$C(u,v) = \exp\left(-\left(\left(-ln(u)\right)^{\theta} + \left(-ln(v)\right)^{\theta}\right)^{\frac{1}{\theta}}\right) \tag{6}$$

3. Frank Copula
   The cumulative distribution function of the Frank copula is defined in **Equation (7)**.

$$C(u,v) = -\frac{1}{\theta}\ln\left(1 + \frac{\left(e^{-\theta u} - 1\right) + \left(e^{-\theta v} - 1\right)}{e^{-\theta} - 1}\right) \tag{7}$$

For each type of Archimedean copula family which includes Clayton copula, Gumbel copula, and Frank copula, the dependency relationship with Kendall Tau is as follows.

1. Clayton Copula
   The relationship of the Clayton copula with Kendall Tau's dependency is measured by **Equation (8)**.

$$\theta = \frac{2\tau}{1-\tau}, \theta \geq 0 \tag{8}$$

2. Gumbel Copula
   The relationship of the Gumbel copula with Kendall Tau's dependency is measured by **Equation (9)**.

$$\theta = \frac{1}{1-\tau}, 1 \leq \theta \leq \infty \tag{9}$$

3. Frank Copula
   The relationship of the Frank copula with Kendall Tau's dependency is measured by **Equation (10)**.

$$D_k(\theta) = \frac{k}{\theta^k}\int_0^{\theta} \frac{t^k}{e^t - 1} dt, k = 1,2, \dots \tag{10}$$

2.7. Maximum Likelihood Estimation

The estimated value of copula parameters is obtained by Maximum Likelihood Estimation (MLE) and the MLE value for copula is obtained by maximizing the log-likelihood function. If $\aleph = \{x_{1t}, x_{2t}, \dots, x_{dt}\}_{t=1}^{T}$ is a sample data matrix that can be formulated in **Equation (11) [19]**.

$$l(\theta) = \sum_{t=1}^{T} \ln\, c\left(F_1(x_{1t}), F_2(x_{2t}), \dots, F_d(x_{dt})\right) + \sum_{t=1}^{T}\sum_{j=1}^{d} \ln f_j\left(x_{jt}\right) \tag{11}$$

with θ are the parameters of the marginals and copula, c is the probability density function of the copula. The maximum likelihood estimator can be expressed in **Equation (12)**.

$$\hat{\theta}_{MLE} = \max_{\theta \in \Theta} l(\theta) \tag{12}$$

2.8. Value at Risk

Value at Risk is a statistical risk measurement method that estimates the maximum loss that may occur on a portfolio at a certain level of confidence **[20]**. In portfolios, VaR is defined as an estimate of the maximum loss that a portfolio will experience in a certain period with a certain level of confidence so that there is a possibility that a loss that will be suffered by the portfolio during the ownership period will be lower than the limit formed by VaR **[21]**. The VaR value at the confidence level $(1-\alpha)$ in a period of t days on the single return and portfolio can be calculated with **Equation (13) [22]**.

$$VaR_{1-\alpha}(t) = W_0 R^* \sqrt{t} \tag{13}$$

where
W_0 : Initial investment fund portfolio.
R^* : The α-quantile value of the portfolio return distribution.
t : Time period.

2.9. Backtesting

The main problem in building a risk model is to validate the model. When a model is formed, it is important to validate it beforehand. Backtesting is a statistical procedure in which actual returns are systematically compared to corresponding VaR estimates [23]. The method used in validating risk models is known as the backtesting method [24]. Testing backtesting on the VaR method can use the Kupiec Test [25]. The test is conducted with the Proportional of Failures (POF) method based on the proportion of violations, namely by comparing the Likelihood Ratio (LR) value with the Critical Value (CV) based on the Chi-Square distribution ($\chi^2_{(df;\alpha)}$) with a degree of freedom of 1. If the LR value < ($\chi^2_{(df;\alpha)}$), then accept H_0 which means that the VaR method is valid. The chi-square value for a 90% confidence level is 2.706, for a 95% confidence level is 3.841, and for a 99% confidence level is 6.635. The Likelihood Ratio (LR) value is obtained based on Equation (14).

$$LR = -2ln\left(\frac{(1-p)^{(T-N)}p^N}{\left[1-\left(^N/_T\right)\right]^{(T-N)}\left(^N/_T\right)^N}\right) \tag{14}$$

where:
N : Number of failures between VaR value and actual loss,
T : Number of observation data,
p : Probability $(1-\alpha)$, α is the confidence level.

3. RESULTS AND DISCUSSION

The data used in this study are secondary data on the closing price of daily shares of PT Bank Rakyat Indonesia Tbk (BBRI) and PT Telekomunikasi Indonesia Tbk (TLKM) for the period September 1, 2021, to August 31, 2023, obtained from www.finance.yahooo.com.

3.1. Data Normality Testing

The initial step in the data processing process is to calculate BBRI and TLKM stock returns. Then normality testing is carried out to see whether the data is normally distributed or not. This normal distribution test is carried out on BBRI and TLKM stock return data using the Kolmogorov-Smirnov test with the following hypothesis.

Hypothesis:
H_0: Return data is normally distributed.
H_1: Return data is not normally distributed.

Table 1. Normal Distribution Test of Stock Returns

Stock Return	D_{Count}	p-value	Decision
BBRI	0.059562	0.0002908	Reject H_0
TLKM	0.062046	0.0001227	Reject H_0

The results in Table 1 show the D_{Count} and p-value on both stock *returns* when compared to the Kolmogorov-Smirnov table value of 0.0612, which would result in a decision to reject H_0. This is because the p-value is smaller than the Kolmogorov-Smirnov table value at $\alpha = 0.05$, which means that both stock returns are not normally distributed.

3.2. ARIMA Modeling Process

The identification of the ARIMA model is carried out through the process of checking stationarity and estimating the ARIMA model from the BBRI and TLKM stock return data. This data stationarity check is carried out using the Augmented Dickey-Fuller (ADF) test shown in Table 2.

Table 2. Stationary Test of Stock Return

Stock Return	p-value
BBRI	0.01
TLKM	0.01

Table 2 shows that the p-value of each data is less than $\alpha = 0.05$ which means reject H_0. This means that the data is stationary. Because the data is stationary, there is no need to transform or differentiate the two BBRI and TLKM stock return data. Stationary testing can also be done by looking at the plot on each stock return data. If the plot graph has a constant average and variance, then the data is stationary.

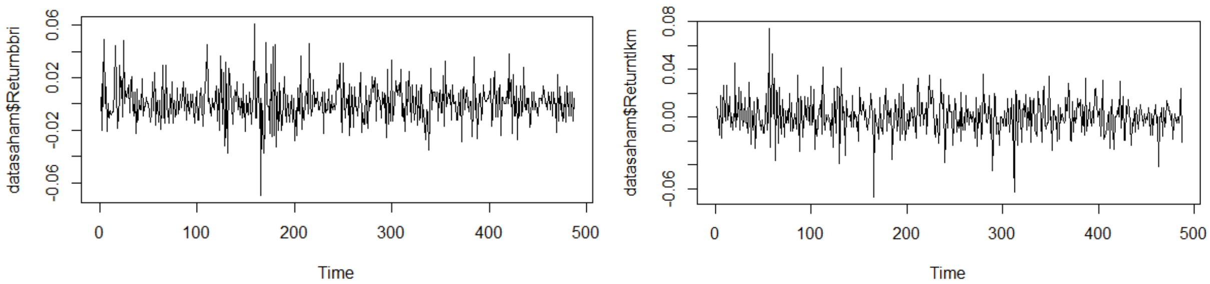

Figure 1. Stationary Data Plot of BBRI and TLKM Stock Returns

After the data is stationary, the next stage is to determine the presence of autocorrelation in the data through the Ljung-Box test with the results in Table 3.

Table 3. Ljung-Box Test

Stock Return	Ljung-Box				
	Lag	**2**	**4**	**6**	**8**
BBRI	**p-value**	0.0044	0.0223	0.0331	0.0303
TLKM	**p-value**	0.0001	0.0010	0.0032	0.0002

In Table 3, the p-value on all lags is less than $\alpha = 0.05$, thus, it can be concluded that H_0 is rejected, so there is autocorrelation in the BBRI and TLKM stock return data. The next step is to identify the ARIMA model with ACF and PACF plots.

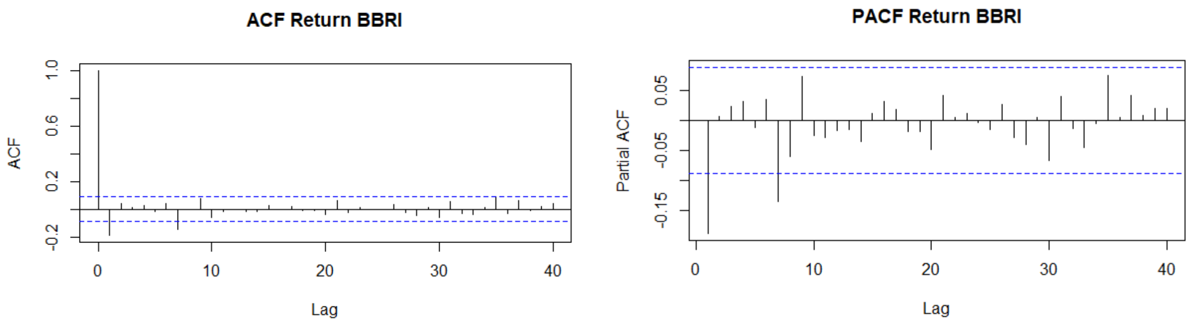

Figure 2. ACF and PACF Plot of BBRI Stock Return

The ACF and PACF plots of BBRI stock returns can be seen in Figure 2. The ACF and PACF patterns of BBRI stock returns cut off at lag 1, so the initial model estimation for BBRI stock is ARIMA(1,0,0) or ARIMA(0,01).

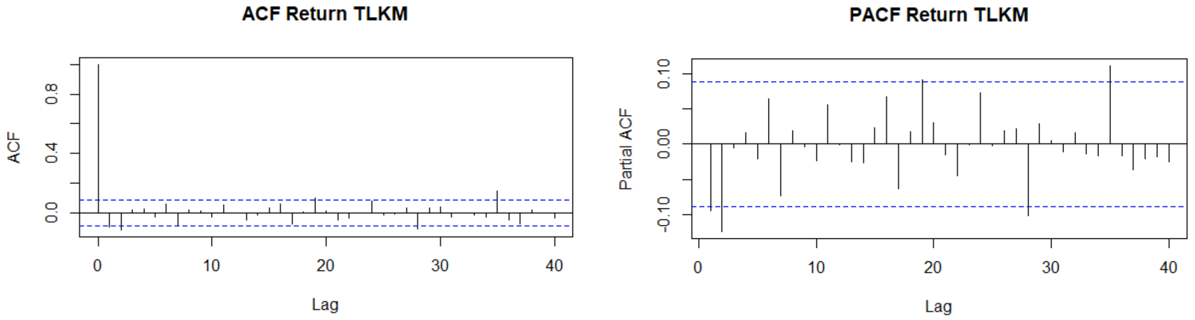

Figure 3. ACF and PACF Plot of TLKM Stock Return

The ACF and PACF plots of TLKM stock returns can be seen in **Figure 3**. The ACF and PACF patterns of TLKM stock returns cut off at lag 1 or 2, so the initial model estimation for TLKM is ARIMA(1,0,0), ARIMA(1,0,2), or ARIMA(0,0,2). After determining the possible models, a model verification test is conducted and the best model with the smallest AIC value is selected. Then it produces the best ARIMA model for two stock returns.

BBRI: ARIMA(1, 0, 0)

$$r_t = 0.00133 - 0.19738 r_{t-1} + a_t.$$

TLKM: ARIMA(2, 0, 0)

$$r_t = 0.00057 - 0.12448 r_{t-1} - 0.12886 r_{t-2} + a_t.$$

3.3. GARCH Modeling

After getting the best ARIMA model for BBRI and TLKM stock returns, the next step is to conduct a Lagrange Multiplier test on the squared residuals to determine the presence of heteroscedasticity effects with the hypothesis:

H_0: $\alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$ or there is no ARCH/GARCH effect.
H_1: there is at least one $\alpha_i \neq 0$ for $i = 1,2,\dots,m$ or there is an ARCH-GARCH effect.

Table 4. Lagrange Multiplier Test

Stock Return	p-value
BBRI	0.000
TLKM	0.000

The LM test results in **Table 4** show that BBRI and TLKM stock returns have a p-value = 0.000 so it can be concluded that all ARIMA models have heterogeneous residuals or have ARCH / GARCH effects. Furthermore, GARCH modeling is carried out to eliminate the effects of heteroscedasticity. GARCH model selection based on the smallest AIC value with GARCH models used are GARCH(1,0), GARCH(1,1), GARCH(1,2), and GARCH(2,1). From the model, the identification of each parameter value and AIC is carried out with the help of R software. The best ARIMA-GARCH model for BBRI and TLKM stocks is obtained as follows.

BBRI: ARIMA(1, 0, 0) GARCH(1, 2)

$$r_t = 0.00133 - 0.19738 r_{t-1} + a_t.$$
$$a_t = \sigma_t Z_t.$$
$$\sigma_t^2 = 0.00002 + 0.09622 a_{t-1}^2 + 0.00000 \sigma_{t-1}^2 + 0.80370 \sigma_{t-2}^2.$$
$$\sigma_t^2 = 0.00002 + 0.09622 a_{t-1}^2 + 0.80370 \sigma_{t-2}^2.$$

TLKM: ARIMA(2, 0, 0) GARCH(1, 0)

$$r_t = 0.00057 - 0.12448 r_{t-1} - 0.12886 r_{t-2} + a_t.$$
$$a_t = \sigma_t Z_t.$$
$$\sigma_t^2 = 0.00020 + 0.11940 a_{t-1}^2.$$

3.4. Kendall Tau Correlation

The Kendall Tau test is conducted to determine the correlation value of dependence between stocks, which would then be used in Archimedean copula modeling. The calculation result of Kendall Tau's value on the residual return of BBRI and TLKM shares is 0.124. Based on the Kendall Tau correlation test, the value of $Z_{count} = 4.0927 > Z_{0.05} = 1.96$, it can be concluded that the residuals of BBRI and TLKM shares are correlated.

3.5. Archimedean Copula

After obtaining the Kendall Tau value, modeling with Archimedean copulas consisting of Clayton copula, Gumbel copula, and Frank copula is then performed. By using the Kendall Tau value of 0.124, the value of θ in each copula is obtained, namely the Clayton copula of 0.2206, the Gumbel copula of 1.141, and the Frank copula of 1.135. Then modeling is done on each Archimedean copula as follows.

1. Clayton Copula
$$C(u,v) = (u^{-0.2206} + v^{-0.2206} - 1)^{-\frac{1}{0.2206}}.$$
2. Gumbel Copula
$$C(u,v) = exp\left(-\left(\left(-ln(u)\right)^{1.141} + \left(-ln(v)\right)^{1.141}\right)^{\frac{1}{1.141}}\right).$$
3. Frank Copula
$$C(u,v) = -\frac{1}{1.135} ln\left(1 + \frac{(e^{-1.135u} - 1) + (e^{-1.135v} - 1)}{e^{-1.135} - 1}\right).$$

Furthermore, the calculation of the Log-likelihood value of each Archimedean copula model on the ARIMA residual data of BBRI and TLKM stocks aims to determine the best Archimedean copula model. The selection of the best Archimedean copula model is based on the largest log-likelihood value. With the help of R software, the log-likelihood value is obtained in **Table 5**.

Table 5. Log Likelihood Value of Archimedean Copula

Copula Type	Log Likelihood Value
Clayton	7.958
Gumbel	6.663
Frank	8.398

Based on **Table 5**, the Frank copula has the largest log-likelihood value of 8.398. So, Frank's copula is the best Archimedean copula model to describe the dependency and model the dependency structure on the BBRI and TLKM stock residual data. This means that there is a close relationship between the two stocks when both are of low value or high value.

3.6. Value at Risk

The selected copula model based on the largest log-likelihood value is used to estimate VaR on BBRI and TLKM stock data. Value at Risk estimation is performed using the Monte Carlo simulation method by generating 1000 random numbers that follow the Frank copula model and using the parameter θ which is 1.135. VaR estimation is carried out for the next 21-day period at 90%, 95%, and 99% confidence levels with the same stock portfolio weight. VaR estimation is done with the help of R software which is shown in **Table 6**.

Table 6. Value at Risk Estimation

Period daily	Level of confidence $(1-\alpha)$		
	99%	**95%**	**90%**
1	-0.0527	-0.0367	-0.0289
2	-0.0541	-0.0370	-0.0295
3	-0.0578	-0.0378	-0.0284
4	-0.0535	-0.0365	-0.0269
5	-0.0524	-0.0354	-0.0266
6	-0.0524	-0.0365	-0.0271
7	-0.0489	-0.0367	-0.0254
8	-0.0510	-0.0384	-0.0277

Period daily	Level of confidence $(1-\alpha)$		
	99%	95%	90%
9	-0.0496	-0.0342	-0.0289
10	-0.0523	-0.0368	-0.0264
11	-0.0508	-0.0352	-0.0279
12	-0.0569	-0.0335	-0.0279
13	-0.0502	-0.0367	-0.0255
14	-0.0535	-0.0363	-0.0283
15	-0.0556	-0.0361	-0.0284
16	-0.0535	-0.0359	-0.0301
17	-0.0539	-0.0324	-0.0262
18	-0.0486	-0.0384	-0.0270
19	-0.0509	-0.0352	-0.0296
20	-0.0533	-0.0363	-0.0269
21	-0.0493	-0.0359	-0.0284
Mean	-0.0516	-0.0363	-0.0277

From the results of the VaR calculation in **Table 6**, VaR at the 90% confidence level is -0.0277, meaning that there is a maximum loss of 2.77% of the invested funds for the period 1 day ahead of August 31, 2023. At the 95% confidence level of -0.0363, it means that there is a maximum loss of 3.63% of the funds invested for the period 1 day ahead of August 31, 2023. And at the 99% confidence level of -0.0516, meaning that there is a maximum loss of 5.16% of the funds invested for the period 1 day ahead of August 31, 2023. So, the higher the level of confidence, the higher the resulting VaR.

3.7. Backtesting

After obtaining VaR, then the backtesting test is carried out to see whether VaR is acceptable (valid) or not (invalid). The results of backtesting on VaR with 90%, 95%, and 99% confidence levels for stock portfolios with the same weight are shown in **Table 7**.

Table 7. VaR Backtesting

VaR	LR	CV	Conclusion	Description
90%	0.662	2.706	Accept H_0	Valid
95%	0.942	3.841	Accept H_0	Valid
99%	0.682	6.635	Accept H_0	Valid

In **Table 7**, it is obtained that the LR value at each confidence level is smaller than the CV value so it can be concluded that VaR with the Frank copula method is valid for use at the 90%, 95%, and 99% confidence levels for BBRI and TLKM stock portfolios.

4. CONCLUSIONS

Based on the results of the analysis and discussion, it is concluded that Frank copula is the best type of Archimedean copula with a log-likelihood value of 8.398. With the Frank copula model, Value at Risk estimation was carried out. The Value at Risk estimation results based on the Frank copula model show a maximum loss of -0.0277 at the 90% confidence level, -0.0363 at the 95% confidence level, and -0.0516 at the 99% confidence level. This shows that the higher the confidence level, the higher the VaR. Through the backtesting procedure, VaR is proven to have very good accuracy in predicting risk, this is evidenced by the Likelihood Ratio (LR) value at each confidence level being smaller than the Critical Value (CV) value.

Suggestions for further research in estimating the Value at Risk value of the portfolio should be made. More VaR calculations so that it can be seen the range of VaR results includes the upper limit and lower limit of the VaR value. In addition, VaR estimation can be done using other copula models, because other copula models may produce better VaR estimates, such as t −student copula, Joe copula, Gaussian copula, and other types of copula models.

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