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MAXIMUM EXPONENTIALLY WEIGHTED MOVING AVERAGE WITH MEASUREMENT ERROR (USING COVARIATE METHOD) USING AUXILIARY INFORMATION FOR CEMENT QUALITY CONTROL

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ABSTRACT

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The main quality characteristic at XYZ Inc. that should be observed is Compressive Strength. Cement production quality control is carried out on the average and process variability jointly with the Max-EWMA control chart. Measurement error can be found in the Compressive Strength. It can affect the sensitivity of the control chart, so quality control will be carried out by considering the presence of measurement error. Handling measurement errors can be done through three approaches (covariate method, multiple measurements, and linearly increasing variance). This research only focuses on the covariate method. Auxiliary variables also explain variance in the production process, so they are also considered in this research, with Blaine used as an auxiliary variable. Therefore, the control chart that will be formed is the Max-EWMA ME (Covariate) AI. The Max-EWMA and Max-EWMA ME (Covariate) AI control charts show that the XYZ Inc. cement production process based on variability and process averages is simultaneously statistically controlled. The controlled Max-EWMA control chart has an upper control limit of UCL=1.503018, and parameters $\mu_y = 252.5823$ *and* $\sigma_y^2 = 970.1596$. *Max-EWMA ME (Covariate) AI has in-control parameters* $\mu_x = 251.49; \sigma_x^2 = 975.809; A = 198.14$; $B = 0.2165$; $\sigma_m^2 = 917.798$; $\mu_w = 341.05$; $\sigma_w^2 = 163.0266$; $\rho = 0.2550$; $\rho^* = -0.144$. *The Max-EWMA ME (Covariate) AI control chart is more sensitive than the Max-EWMA control chart. Cement production capabilities based on Compressive Strength have a* C_{pl} *and* C_{pk} *capability index of 1.54, which means that the cement production process is capable, consistent, and has high accuracy so that the quality has reached the target.*

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1. INTRODUCTION

Quality became an essential target for the industry because improving the quality can increase productivity, reduce deficiencies (such as rework, wasted materials, and increased product costs due to errors), and increase sales. Quality is inversely proportional to variability, so decreasing the variability of the phenomenon can affect increasing the quality of the products and services produced by the industry [1]. Optimizing quality dimensions also means controlling and reducing variability in processes and products [1]. Variability can only be explained in statistics, so statistical methods have a role in quality improvement.

Cement quality characteristics are measured based on chemical and physical standards. One physical measurement of cement quality is cement compressive strength, which measures the material's ability to withstand compressive loads with the influence of the main mineral composition [2]. Building raw materials are expected to have high load-bearing strength, so cement's compressive strength should be the main characteristic that will be controlled. Auxiliary variables can explain quality characteristics and variance in the production process so that its existence can increase the efficiency of the control chart [3]–[5]. The auxiliary variable is not a contact variable (variable with a direct relationship to the response) and is not an identifier of constituents of an observed variable [6]. According to physical standards, the other quality characteristic is Blaine or cement fineness. The compressive strength of cement was measured using a compressive strength machine on a compact mixed cement that had been soaked for three days. Blaine was measured using an automatic Blaine immediately after the sample was taken. Blaine can increase the interaction between cement and water. Then, a strong interaction between cement and water can increase cement density and strengthen the bonds in cement. The strong cement bonds and high density of cement can increase the compressive strength of cement [7]. Because of the different measurement treatments and indirect relationship between compressive strength and Blaine, Blaine is considered beneficial as an auxiliary variable rather than carrying out multivariate analysis in the statistical process control.

Control charts are one of the quality control techniques using statistical methods (statistical process control or SPC). Initially, control charts were memoryless charts that only considered the last observation. Then, the control chart was developed into a memory-based control chart, including an Exponentially Weighted Moving Average (EWMA) control chart introduced by Robert in 1959 [8]. By Xie in 1999, the EWMA control chart was developed to control the average and variability process jointly (jointly monitoring process) and has equivalent sensitivity to independent monitoring. From now on, it is referred to as Maximum Exponentially Weighted Moving Average or Max-EWMA [9]. The max-EWMA control chart has the required assumption that there is no measurement error. In the compressive strength measurement, there is an uncertain test value because the compressive strength measurement is based on the uncertainty of the force applied and the uncertainty of the cross-sectional area, so it can be confirmed that there is a measurement error in the compressive strength measurement [7]. In addition, previous research has shown that measurement error can reduce the efficiency of control charts [4], [10]–[12]. Handling the assumption of measurement error can be overcome with three approaches, namely using the covariate method, multiple measurements, and cases of measurement error in data with non-constant variance (linearly increasing variance) [12]. In this research, we want to handle measurement error using a covariate approach and auxiliary variables in the control chart. So, the control chart used is the maximum weighted moving average considering measurement error, covariate approach using auxiliary information, or Max-EWMA ME (Covariate) AI.

This research aims to obtain the results of controlling the compressive strength of cement with Blaine as an auxiliary variable in the cement production process at PT XYZ based on the variability and average process simultaneously with the Max-EWMA ME (Covariate) AI control chart. This chart is appropriate because cement's compressive strength has a measurement error and a small process shift. Furthermore, the sensitivity of the proposed method is compared to the Max-EWMA control chart. The comparison is only to the Max-EWMA control chart because this research wants to see the effect of the measurement error on the sensitivity of the Max-EWMA chart. This research is limited by the data used in the final milling stage data. The observed variables are the three-day compressive strength values, and the auxiliary variable used is Blaine, which was used in the observation period from January 1st, 2023, to November 30th, 2023. According to Noor ul-Amin (2020), in formulating the Max-EWMA ME AI control chart, parameter values $L=2.709$ and $\lambda=0.05$ to produce a controlled ARL of $ARL_0 \approx 370$ [3]. Therefore, this research will use the parameter values $L=2.709$ and $\lambda=0.05$ in creating the Max-EWMA ME (Covariate) AI control chart.

2. RESEARCH METHODS

2.1 Literature Review

2.1.1 Estimation of Individual Variance Parameters (σ²)

Max-EWMA control chart measurements are based on the assumption of mean population μ and variance population σ^2 are known. If μ and σ^2 are unknown, one method for estimating variance is to use a moving average with the following equation [13].

$$\hat{\sigma}^2_{\overline{MR}} = \left(\frac{\overline{MR}}{d_2}\right)^2 \tag{1}$$

With $\hat{\sigma}^2_{\overline{MR}}$ is the estimated variance value, $\overline{MR}$ is the average moving range with $\overline{MR} = \frac{\sum_{i=1}^{m-1} MR_i}{m-1} = \frac{\sum_{i=1}^{m-1}(x_{i+1}-x_i)}{m-1}$ and d_2 is an adjustment factor for estimated bias with a value of d_2=1.128 for individual data that is normally distributed.

2.1.2 Correlation Coefficient

The correlation coefficient by Karl Pearson 1990 states the relationship between two continuous variables. The Pearson correlation coefficient can be calculated using the formula [14].

$$r = \frac{n\sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \cdot \sum_{i=1}^{n} Y_i}{\sqrt{\left[n\sum_{i=1}^{n} {X_i}^2 - \left(\sum_{i=1}^{n} X_i\right)^2\right]\left[n\sum_{i=1}^{n} {Y_i}^2 - \left(\sum_{i=1}^{n} Y_i\right)^2\right]}} \tag{2}$$

Pearson correlation has a value between -1 to 1, meaning that if the value $|r| = 1$, then the two variables have a very strong correlation relationship, and if the value $r = 0$, then the two variables have no correlation relationship [14]. If the correlation value is negative, then an increase in one variable is followed by a decrease in another variable, whereas if the correlation value is positive, then an increase in one variable is followed by an increase in another variable. The significance of the relationship based on the Pearson correlation value can be tested using the *t-test* with the following test hypothesis [15].

H_0 : $\rho = 0$ or there is no significant correlation between variables
H_1 : $\rho \neq 0$ or there is a significant correlation between variables
with t-test statistics, it can be calculated using equation [15]

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \tag{3}$$

where t is the Pearson correlation significance test statistic, *r* is the Pearson correlation value, and *n* is the number of observations. The critical region for rejecting H_0 is when the $|t_{hitung}| > t_{(\alpha;n-2)}$ or p-*value* <α [15].

2.1.3 Normality Test

The normality assumption test is divided into two assumptions.

a. Kolmogorov-Smirnov Univariate Normality Test

The Kolmogorov-Smirnov test is to calculate whether there is an absolute difference between [S(x)] (sample cumulative distribution function) and $[F_0(x)]$ (theoretical cumulative distribution function) at each interval so that the test hypothesis obtained is as follows [16].

H_0 : $[F_0(x_i)] = [S(x_i)]$ or normally distributed data
H_1 : $[F_0(x)] \neq [S(x)]$ or Data is not normally distributed.

Kolmogorov-Smirnov test statistic is the maximum value of the absolute difference between the theoretical cumulative distribution function $[F_0(x)]$ and the sample cumulative distribution function $[S(x)]$, denoted by *D* or maximum deviation. The following equation can calculate the test statistical value [16].

$$D_{hitung} = max\{|F_0(x_i) - S(x_i)| \tag{4}$$

with a sample size of *n* and a significance level of α, the critical region rejects H_0 when the calculated *D value* > $D_{\alpha,n}$ or the *p-value* < α [16].

b. Shapiro-Wilk Multivariate Normality Test

An observation of several p-variables is said to have a multivariate normal distribution, $\mathbf{X} \sim \mathbf{N_p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, when it has a density function $f(\mathbf{X}) = \frac{1}{(2\pi)^{p/2}\,|\boldsymbol{\Sigma}|^{1/2}} e^{-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu})}$ with $\mathbf{X} = [\mathbf{X}_1\ \mathbf{X}_1 \ldots \mathbf{X}_p]^T$ is the data matrix, $\boldsymbol{\mu}$ is the mean matrix, and $\boldsymbol{\Sigma}$ is the covariance matrix. Therefore, the normal distribution test with Shapiro Wilk has the following test hypothesis [16].

H_0 : Data has a multivariate normal distribution.
H_1 : The data is not normally distributed in a multivariate manner.

with test statistics W^* following the equation

$$W^* = \frac{1}{p}\sum_{i=1}^{p} \frac{[\sum_{j=1}^{n} a_{ij} x_{ij}]^2}{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2} \tag{5}$$

with W^* is a multivariate normal test statistic, p is the number of variables, and n is the number of observations. The critical area for rejecting H_0 is when $W^* < C_{\alpha;n;p}$ or the p-value < α [16].

2.1.4 Covariate Model

A covariate is defined as an independent variable that has a relationship with a dependent variable or observed variable in a study. The covariate variable can be denoted as X, and the dependent variable can be denoted as Y. The linear covariate model can be written in the form $Y = A + BX + \varepsilon$, with ε is the error model [17]. One method for estimating linear covariate model parameters is to use the regression method. The covariate model can be reduced to a simple linear regression model for n observations in the form $y_i = A + Bx_i + \varepsilon_i$ [17]. By using a random sample of n observations $y_1, y_2, \ldots, y_n$ with the inclusion of fixed variables $x_1, x_2, \ldots, x_n$, parameters A and B can be estimated using the least-squares approach. The least-squares approach method minimizes the sum of square error values for the parameters so that the regression estimator for parameters A and B can be written in the following equations [17].

$$\hat{B} = \frac{\sum_{i=1}^{n} x_i \cdot y_i - n\bar{x}\bar{y}}{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} \tag{6}$$

$$\hat{A} = \bar{y} - \hat{B}\bar{x} \tag{7}$$

where $\hat{A}$ is the estimator of parameter A, $\hat{B}$ is the estimator of parameter B, $\bar{x} = \sum_{i=1}^{n} x_i/n$ is the average of the covariate variable X and $\bar{y} = \sum_{i=1}^{n} y_i/n$ is the average of the observed variable Y [17].

2.1.5 Max-EWMA

Denote the observation quality characteristics as Y, then Y has a normal distribution with a mean μ_y and variance σ_y^2 then $a = 0$ and $b = 1$ under controlled conditions (In-control) [18]. The process mean and variance are mutually independent, following a normal distribution with a mean of 0 and variance of 1 [19]. The statistical transformation for the mean (U_i) and variance (V_i) can be calculated so that they follow the Standard normal distribution by this equations [20].

$$U_i = \frac{(\bar{y}_i - E(\bar{y}))}{\sqrt{var(\bar{y})}} = \frac{(\bar{y}_i - \mu_y)}{\sqrt{\frac{var(y)}{n}}} = \frac{(\bar{y}_i - \mu_y)}{\frac{\sigma_y}{\sqrt{n}}} \tag{8}$$

$$V_i = \Phi^{-1}\left\{H\left(\frac{(n_i-1)S_i^2}{\sigma_y^2}; (n_i - 1)\right)\right\} \tag{9}$$

where $\Phi(z) = P(Z \le z)$ with $Z \sim N(0,1)$ so $\Phi^{-1}(.)$ is the inverse function of the standard normal distribution, and $H(c; v) = P(C \le c|v)$ with C follows the Chi-Square distribution with degrees of freedom v($C \sim \chi_v^2$) [20].

EWMA statistics for statistical transformation results for the mean (U_i) and variance (V_i), respectively, are given by P_i and Q_i in the following equations.

$$P_i = \lambda U_i + (1-\lambda)P_{i-1};\ i = 1,2,\ldots,m \tag{10}$$

$$Q_i = \lambda V_i + (1-\lambda)Q_{i-1};\ i = 1,2,\ldots,m \tag{11}$$

Because the EWMA statistics for the mean (P_i) and the EWMA statistics for the variance (Q_i) are influenced by the results of statistical transformations for the mean and variance as well as the EWMA statistical values of previous observations, then P_i and Q_i has distribution $P_i \sim N\left(0, \frac{\lambda}{2-\lambda}\right)$ and $Q_i \sim N\left(0, \frac{\lambda}{2-\lambda}\right)$ [18]. $Q_0 = P_0 = 0$ is the starting value/initial value, and λ is the smoothing constant value $0 < \lambda \leq 1$ [18]. The following equation can form the Max-EWMA statistic (Mi) [18].

$$\mathrm{M_i} = \max\{|\mathrm{P_i}|, |\mathrm{Q_i}|\} \tag{12}$$

Because M_i has a non-negative value, the lower control limit (LCL) equals zero, and the upper control limit Max-EWMA can be written in the following equation [9].

$$\mathrm{UCL_{MAX-EWMA}} = 1{,}128379 + 0{,}602810 \cdot \mathrm{L}\sqrt{\frac{\lambda}{2-\lambda}} \tag{13}$$

2.1.6 Max-EWMA ME (Covariate) AI

The covariate variable X is defined as the actual value of a quality characteristic, and the value is unknown, it is assumed that $X \sim N(\mu_x, \sigma^2)$ and $\varepsilon \sim N(0, \sigma_m^2)$ in the covariate model $Y=A+BX+\varepsilon$ that affects the quality characteristics of Y so that it has a normal distribution with $E(Y) = \mu_y = A + B\mu_x$ dan $Var(Y) = \sigma_y^2 = B^2\sigma^2 + \sigma_m^2$ [19]. It is defined that the auxiliary variable W has a relationship with the quality characteristic variable Y with a correlation value ρ_{YW} and has the assumption that the basic process (Yj, Wj) follows a bivariate normal distribution with *mean* (μ_Y, μ_W) and variance (σ_Y^2, σ_W^2) [21]. With this information, a differentiation estimator can be formed for the *mean* $({M_{YW}}_j^{(1)})$ and variance $({V_j}^{(1)})$, respectively, as follows [3].

$$ {M_{YW}}_j^{(1)} = \bar{Y}_j + \rho\left(\frac{\sqrt{B^2\sigma^2 + \sigma_m^2}}{\sigma_W}\right)(\mu_W - \overline{W}_j) \tag{14}$$

$$ {V_j}^{(1)} = \Phi^{-1}\left[H\left\{\frac{(n-1)S_{Y,j}^2}{B^2\sigma^2+\sigma_m^2}, (n-1)\right\}\right] - \rho^* \,\Phi^{-1}\left[H\left\{\frac{(n-1)S_{W,j}^2}{\sigma_W^2}, (n-1)\right\}\right] \tag{15}$$

with the expectation and variance for the mean differentiation estimator expressed by $E\left({M_{YW}}_j^{(1)}\right) = \mu_Y$, $Var\left({M_{YW}}_j^{(1)}\right) = \frac{1}{n}(B^2\sigma^2 + \sigma_m^2)(1 - \rho_{YW}^2)$ and the expectation and variance for the variance differentiation estimator expressed by $E({V_j}^{(1)}) = 0$, $Var({V_j}^{(1)}) = 1 - {\rho^*}^2$, where the values $\bar{Y}_j = \sum_{i=1}^{n} y_{ij}/n$, $\overline{W}_j = \sum_{i=1}^{n} W_{ij}/n$, $S_{Y,j}^2 = \frac{\sum_{i=1}^{n}(Y_{ij}-\bar{Y}_j)^2}{n-1}$ and $S_{W,j}^2 = \frac{\sum_{i=1}^{n}(W_{ij}-\bar{W}_j)^2}{n-1}$ and the function $H(\xi, v)$ follow the *Chi-square distribution* with degrees of freedom v and $\Phi^{-1}(.)$ are the inverse of the standard normal distribution function [3].

The transformed estimators for *the mean* $({M_{je}}^{(1)})$ and variance $({V_{je}}^{(1)})$ follow the equations, respectively.

$$ {M_{je}}^{(1)} = \frac{{M_{YW}}_j - (A + B\mu_x)}{\sqrt{\frac{1}{n}(B^2\sigma^2+\sigma_m^2)\,(1-\rho_{YW}^2)}} \tag{16}$$

$$ {V_{je}}^{(1)} = \frac{V_j}{\sqrt{1-{\rho^*}^2}} \tag{17}$$

Using values of $({M_{je}}^{(1)})$ and $({V_{je}}^{(1)})$, then the EWMA statistic for the population *mean* ($P_i^{(1)}$) and the EWMA statistic for the variance population ($Q_i^{(1)}$) follow the equations.

$$P_i^{(1)} = \lambda M_{je} + (1-\lambda)P_{i-1};\ i = 1,2,\ldots,m \tag{18}$$

$$Q_i^{(1)} = \lambda V_{je} + (1-\lambda)Q_{i-1};\ i = 1,2,\ldots,m \tag{19}$$

Plotted *Max-EWMAMEAI* statistics follow **Equation (12)** with $\{P_i; Q_i\} = \{P_i^{(1)}; Q_i^{(1)}\}$ and the control chart control limits follow **Equation (13)**.

2.1.7 Process Capability Analysis

Process capability analysis is a statistical method in quality control to estimate process capability. The capability index used has the criteria of being capable if it has a value of more than 1.33 [22]. The capability index for a process in control (in-control) is C_p and C_{pk}, while the capability index for processes in a state of statistical uncontrol (out of control) is P_p and P_{pk}, which is the process performance index [22].

C_p value can be calculated using the following equation [22].

$$C_p = \frac{USL - LSL}{6\sigma} \tag{20}$$

where σ is the standard deviation, *USL* is the upper specification limit, and *LSL* is the lower specification limit. C_{pk} is an improvement of the C_p to show accuracy and precision with calculations based on the following equation [22].

$$C_{pk} = min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right) \tag{21}$$

For quality characteristics with one-sided specification limits (only having a minimum value or only a maximum value) then, the C_{pk} can still be calculated, but the value of C_p is unknown, and an index can be used is $C_{pl} = \left(\frac{\mu-LSL}{3\sigma}\right)$ (if there is *a lower control limit/* LCL) or $C_{pu} = \left(\frac{USL-\mu}{3\sigma}\right)$ (if there is *an upper control limit/* UCL) [22].

2.1.8 Cement Quality Characteristics

Two characteristics of cement quality will be used in this research.

1) Compressive Strength

Compressive Strength is a measure of a material's ability to withstand compressive loads with the influence of the main mineral composition. Compressive strength values are generally measured on the 3rd, 7th, and 28th days. Compressive Strength was tested using a compressive strength machine by pressing the immersion mortar (for a certain test life) that dried for 24 hours, containing 740 grams of cement sample, 2035 Ottawa sand, and 260 ml of water [23].

2) Blaine (Fineness of cement)

Blaine has a relationship with compressive strength with the effect that the finer the cement, the more reactive (the speed at which it interacts with water) the cement, and the higher the compressive strength [24]. The test was carried out by weighing 113.2 grams of cement used as a sample and calculating the Blaine value with air permeability of the compacted cement sample under certain conditions [25]. The Blaine tool has the basic principle of drawing air through a cement base prepared with a shaft as a function of the cement grain size and determining the airflow speed [2].

Table 1. **Quality Specification Limits**

Quality Characteristics	**Unit**	**Type Requirements**	
		IP-U	**IP-K**
Subtlety with *Blaine tools*	m^2/kg	Min. 280	Min. 280
Compressive strength at 3 days	Kg/cm^2	Min. 130	Min. 110

2.2 Data Structure

Data were taken from cement samples produced in the period 1 January 2023 to 30 November 2023 in the form of compressive strength test values for 3-day-old cement (in Kg/cm^2) as the observed quality characteristic variable (Y) and Blaine (in m^2/Kg) as the *auxiliary variable* (W). The research data structure can be written in the following table.

Table 2. **Research Data Structure**

Subgroup	**Subgroup Units**	**Quality Characteristics**	
		Y	**W**
1	1	Y_{11}	W_{11}
	2	Y_{22}	W_{22}
	…	…	…
	n	Y_{n1}	W_{n1}
…	…	…	…
I	1	Y_{1m}	W_{1m}
	2	Y_{2m}	W_{2m}
	…	…	…
	n	Y_{nm}	W_{nm}

2.3 Analysis Steps

The steps for the analysis of this research are explained below.

1. Collecting data on the cement quality characteristics test from 1 January 2023 to 30 November 2023.
2. Conducting data exploration on each quality characteristic, namely, the compressive strength, and Blaine.
3. Dividing the data into phase I data for 1 January 2023 to 31 August 2023 and phase II data for 1 September 2023 to 30 November 2023.
4. Calculating the correlation coefficient with **Equation (2)** and carry out a significance test of the Pearson correlation coefficient with **Equation (3).**
5. Running the Kolmogorov-Smirnov univariate normality test for the observed quality characteristics of Cement Compressive Strength using **Equation (4)** and the Shapiro-Wilk normal multivariate test for Cement Compressive Strength and the auxiliary Blaine variable using **Equation (5)** for Phase I data.
6. Controlling cement production using the Maximum Exponentially Weighted Moving Average control chart without handling measurement error.
7. Controlling cement production using Maximum Exponentially Weighted Moving Average control chart with Measurement Error using Covariate approach with Auxiliary Variable hereinafter referred to as Max-EWMA ME (Covariate) AI for Phase I data by these steps.
   1) Get the values of regression parameters A and B in the covariate model $Y = A + BX + \varepsilon$ using **Equation (6)** and **Equation (7)**.
   2) Considering the desired ARL_0, the parameter values are determined to be L=2.709 and λ=0.05.
   3) Carry out Max-EWMAMEAI test statistical calculations with steps:
      - Calculate the mean ($\overline{Y}_j$) and variance ($S^2_{Y,j}$) of quality characteristics Y for each i^{th} subgroup j, as well as the mean ($\overline{W}_j$) and variance ($S^2_{W,j}$) of the auxiliary variable W for each subgroup j. These values follow the equations $\overline{Y}_j = \sum_{i=1}^{n} y_{ij}/n$, $\overline{W}_j = \sum_{i=1}^{n} W_{ij}/n$, $S^2_{Y,j} = \frac{\sum_{i=1}^{n}(Y_{ij}-\overline{Y}_j)^2}{n-1}$, and $S^2_{W,j} = \frac{\sum_{i=1}^{n}(W_{ij}-\overline{W}_j)^2}{n-1}$.
      - Calculate differentiation estimators for the mean (${M_{YW_j}}^{(1)}$) and variance (${V_j}^{(1)}$) with **Equation (14)** and **Equation (15)**.
      - Calculate the transformation estimator for the mean (${M_{je}}^{(1)}$) and variance (${V_{je}}^{(1)}$) with **Equation (16)** and **Equation (17)**.
      - Calculate the EWMA statistic for the population mean (${P^{(1)}}_i$) following **Equation (18)** and the EWMA statistic for the variance population (${Q^{(1)}}_i$) follows **Equation (19)**.
      - Calculate Max-EWMAMEAI statistics, which are plotted using **Equation (12)** with $\{P_i; Q_i\} = \{{P^{(1)}}_i; {Q^{(1)}}_i\}$.
   4) Calculate the Upper Control Limit (UCL) with **Equation (13)** with previously determined L and λ parameter values.
   5) If the overall test statistical value is within the control limits, then the process can be statistically controlled. However, if not, the cause of the out-of-control will be sought. Suppose the cause of out-of-control in one of the observations is a cause that can be determined/known (assignable causes). In that case, the observation can be removed, and stages 4 and 5 are carried out again until control limits are obtained and the entire observation is statistically controlled (in control).
   6) Obtain control limit values for the Max-EWMA ME (Covariate) control chart, which has been statistically controlled using Phase I data.
   7) Perform a Max-EWMAMEAI control chart using a covariate approach for Phase II data.
   8) Plot test statistical results for Phase II data using control limits on Phase I data that has been in control.
8. Comparing the effectiveness of the Max-EWMA ME (Covariate) AI with Max-EWMA
9. Conducting process capability analysis.
10. Drawing conclusions from research results.

3. RESULTS AND DISCUSSION

3.1 Descriptive Statistics of Two Characteristics of Cement

3.1.1 Compressive Strength

Based on **Table 3** below, it is found that the average compressive strength of cement in Phase I was 252.5823 kg/cm^2 and increased on average in Phase II to 256.3538 kg/cm^2. The variance of cement compressive strength in Phase I was less than that of cement compressive strength in Phase II, which means that the compressive strength data in Phase I tended to be more collected or homogeneous than in Phase II data. **Table 1** shows that the minimum specification limit for compressive strength is 130 Kg/cm^2 and data has reached the standard. Data have a small skewness value, so the data tends to be centered on the average data. So, data tends to have a centralized distribution (has mean and median values that are close together and can be assumed to be normally distributed), so a Max-EWMA control chart can be formed.

Table 3. Descriptive Statistics of Cement Compressive Strength

Period	*Mean*	*Var*	*StDev*	*Min*	*Max*	*Skew*
Phase I	252.58	737.66	27.16	180.6	335.6	0.461
Phase II	256.35	1015.7	31.87	194.4	320.2	0.290

3.1.2 Blaine

Based on **Table 4** below, it is found that the average of Blaine in Phase I was 341.0365 kg/cm^2 and the average increase in Phase II was 344.6416 kg/cm^2. The variance of Blaine in Phase I was more significant than that of Blaine in Phase II, which means that the Blaine data in Phase II tended to be more homogeneous compared to Phase II data. Based on **Table 1**, it is known that the minimum specification limit for Blaine is 280 Kg/cm^{2,} and data has reached the standard. Data have a small skewness value, so the data tends to be centered on the average data. Then, data tends to have a centralized distribution (can be assumed to be normally distributed), so a Max-EWMA control chart can be formed.

Table 4. Blaine Descriptive Statistics

Period	*Mean*	*Var*	*StDev*	*Min*	*Max*	*Skew*
Phase I	341.04	140.27	11.84	302.99	390.41	0.36
Phase II	344.64	139.75	11.82	323.73	369.91	0.06

3.2 Correlation Value

Denote the cement compressive strength variable Y and the auxiliary variable Blaine as W. Define ρ as the Pearson correlation value for the compressive strength and Blaine. It is also defined that ρ^* represents the Pearson correlation value for the variance of compressive strength and Blaine variance. The correlation results and significance using **Equations (2)** and **(3)** are as follows.

Table 5. Correlation Coefficient

Phase	Notation	Pearson Correlation (r)	Statistics	*p-value*	*Information*
I	ρ	0.2550732	2.611479	0.01043	Significant
	ρ^*	-0.1448688	-0.70217	0.4896	Not significant
II	ρ	0.5541474	4.891932	0.000009386	Significant
	ρ^*	0.1207401	0.42134	0.681	Not significant

Based on **Table 5**, it is known that there is a significant positive relationship between compressive strength and Blaine, which means that an increase in Blaine also follows an increase in compressive strength. Then, Blaine becomes an auxiliary variable in the Max-EWMA ME AI control chart.

3.3 Normality Assumption Test

3.3.1 Kolmogorov-Smirnov Test

In the Max-EWMA control chart and Max-EWMAMEAI control chart, the observed quality characteristic (compressive strength) assumes a normal distribution. Even though this is a type of control chart, the normal assumption is needed because, in simultaneous charts, it is necessary to calculate the inverse of the derivative of the normal distribution. Additionally, control limits were calculated using the assumption of a normal distribution.

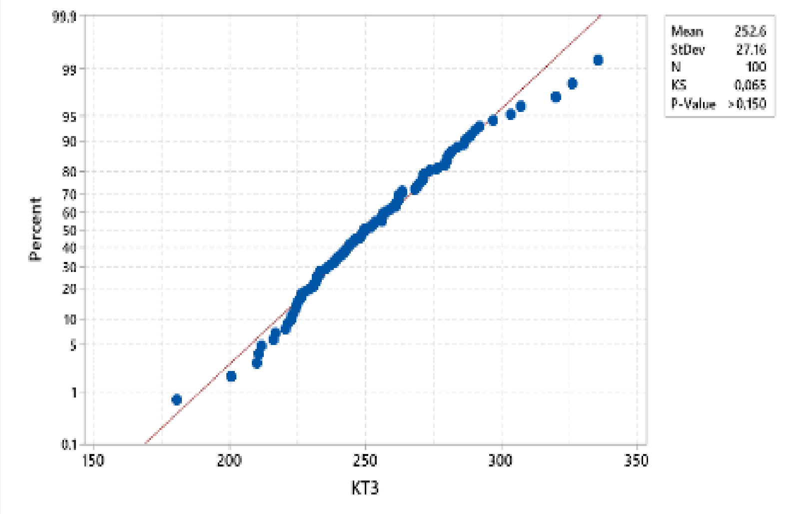

Figure 1. Kolmogorov-Smirnov *Normality Test*

Based on **Figure 1**, the test results have a *p-value* > 0.05 so that the decision to fail to reject H_0 so that it can be concluded that the compressive strength data is normally distributed and meets the assumptions.

3.3.2 Shapiro-Wilk Test

In the Max-EWMAMEAI control chart, cement compressive strength (Y) and auxiliary variables Blaine (W) have the assumption that the data has a bivariate normal distribution. Therefore, multivariate normality testing using the Shapiro-Wilk test with the test hypothesis can be carried out.

H_0 : Data has a bivariate normal distribution.
H_1 : Data does not have a bivariate normal distribution.

Table 6. Shapiro-Wilk Multivariate Normality Test

W*	***p-value***
0.97825	0.05427

Based on **Table 6**, the test results have a p-value > 0.05, so the decision to fail to reject H_0 so that the compressive strength and Blaine data have a normal bivariate distribution and meet the assumptions.

3.4 Max-EWMA Control Chart

3.4.1 Max-EWMA Control Chart Phase I

The Max-EWMA control chart was formed using phase I data, data from 1 January 2023 to 31 August 2023, a number of $m = 25$ subgroups, with each subgroup having a number $n = 4$ of observations. Denote the cement compressive strength variable as Y, with Y having a normal distribution with a mean $\mu_y = 252.5823$ and variance $\sigma_y^2 = 970.1596$. Calculations for statistical transformations (U_i) and (V_i) are calculated using **Equations (8)** and **(9)**. EWMA statistics for mean (P_i) and EWMA statistics for variance (Q_i) can be calculated using **Equations (10)** and **(11)** with $Q_0 = P_0 = 0$ the starting value/initial value. A summary of the results of statistical calculations is obtained as follows.

Table 7. Max-EWMA Phase I Control Chart Statistics

Subgroup (i)	U_i	V_i	P_i	Q_i	Max-EWMA $_i$
1	-0.12407	-1.31444	-0.00620	-0.06572	0.065722
2	-1.26061	-1.62827	-0.06892	-0.14385	0.143849
3	-1.70366	-0.38292	-0.15066	-0.15580	0.155803
4	-0.49329	-0.9588	-0.16779	-0.19595	0.195953
5	-1.47571	-1.56732	-0.23319	-0.26452	0.264521
6	1.67704	0.968753	-0.13768	-0.20286	0.202857
7	-0.03418	-1.28062	-0.13250	-0.25675	0.256746
8	-0.25892	2.088105	-0.13882	-0.13950	0.139503
9	1.527939	-1.57242	-0.05548	-0.21115	0.211149
10	-0.04381	-0.65303	-0.05490	-0.23324	0.233243
11	-0.72766	-0.17240	-0.08854	-0.23020	0.230201
12	-0.28460	0.083983	-0.09834	-0.21449	0.214492
13	0.14240	0.475062	-0.08630	-0.18001	0.180014
14	2.697992	0.528140	0.052910	-0.14461	0.144606
15	0.462039	-1.08824	0.073367	-0.19179	0.191788
16	0.771667	-1.42438	0.108282	-0.25342	0.253418
17	0.473086	-0.24956	0.126522	-0.25322	0.253225
18	-1.34408	-1.90968	0.052992	-0.33605	0.336048
19	-0.50292	-1.59199	0.025196	-0.39884	0.398845
20	0.890457	0.310755	0.068459	-0.36336	0.363365
21	0.004347	-0.79675	0.065254	-0.38503	0.385034
22	-0.74191	-0.82094	0.024895	-0.40683	0.406830
23	0.935176	0.784151	0.070409	-0.34728	0.347281
24	-0.13692	0.206133	0.060043	-0.31961	0.319610
25	-0.44975	-1.27906	0.034550	-0.36758	0.367580

The Max-EWMA phase I control chart with parameters L=2.709 and λ=0.05 has an upper control limit of 1.503018 and a lower control limit of 0. The Max-EWMA control chart for phase 1 data is depicted in **Figure 2** below.

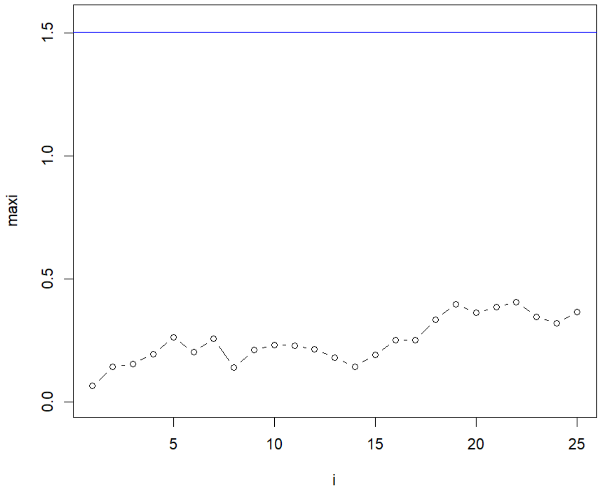

Figure 2. Phase I Max-EWMA Control Chart

Based on **Figure 2**, it is known that in the Max-EWMA phase 1 control chart, all observations are within the control limits. So, it can be concluded that the process average and variance of the cement production process are controlled together using the Max-EWMA control chart and have been statistically controlled.

3.4.2 Max-EWMA Control Chart Phase II

Phase II uses data on cement quality characteristics from 1 September 2023 to 30 November 2023 for a number of $m = 14$ subgroups, with each subgroup having a number $n = 4$ of observations. The Max-EWMA control chart for phase II data was formed using the parameters obtained in phase I so that the upper control limit value is 1.503018, $\mu_y = 252.5823$, and variance $\sigma_y^2 = 970.1596$. Calculations for statistical transformations for mean (U_i) and variance (V_i) are calculated using **Equations (8)** and **(9)**. EWMA statistics for mean (P_i) and EWMA statistics for variance (Q_i) can be calculated using **Equations (10)** and **(11)**, $Q_0 = Q_{25}$ and $P_0 = P_{25}$ as starting values/initial values with statistics P_{25} and Q_{25} obtained from the results of the phase I control chart.

Table 8. Max-EWMA Phase II Control Chart Statistics

Subgroup (i)	U_i	V_i	P_i	Q_i	Max-EWMA $_i$
26	-1.51103	-0.78418	-0.04273	-0.38841	0.38841
27	-2.86588	-1.09553	-0.18388	-0.42377	0.42377
28	-0.08375	-0.78155	-0.17888	-0.44166	0.44166
29	0.82281	0.87121	-0.12879	-0.37601	0.37601
30	-0.30708	-2.56233	-0.13771	-0.48533	0.48533
31	-0.27497	-0.84171	-0.14457	-0.50315	0.50315
32	-0.17576	1.47789	-0.14613	-0.40410	0.40410
33	-0.91331	-1.13247	-0.18449	-0.44052	0.44052
34	-0.64551	-0.26171	-0.20754	-0.43158	0.43158
35	2.01061	-0.18486	-0.09663	-0.41924	0.41924
36	2.15890	-0.68387	0.01615	-0.43247	0.43247
37	2.35369	-0.73979	0.13302	-0.44784	0.44784
38	2.00667	0.61642	0.22670	-0.39462	0.39462
39	0.81501	-1.89843	0.25612	-0.46981	0.46981

Using the statistical values in **Table 8**, a Max-EWMA control chart for phase II data is formed in **Figure 3**.

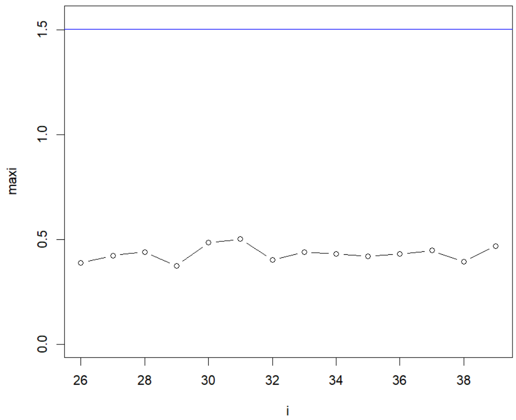

Figure 3. Max-EWMA Phase II Control Chart

The overall Max-EWMA statistics data for phase I and II control charts can be depicted in **Figure 4**.

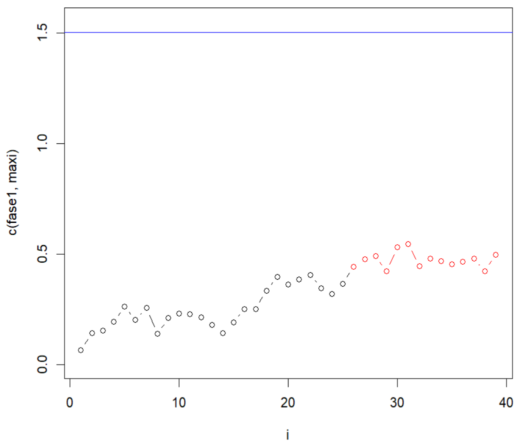

Figure 4. Max-EWMA Phase I and Phase II Control Chart

Based on **Figure 3**, it is known that all observation points are within the control limits, so it can be concluded that the process average and variance of the cement production process have been statistically controlled. Based on **Figure 4**, it is known that there is a pattern of tendency for EWMA statistical values to increase monotonically from phase I data to phase II data that indicated a small shift.

3.5 Max-EWMA ME (Covariate) AI Control Chart

3.5.1 Max-EWMA ME (Covariate) AI Control Chart Phase I

Max-EWMA ME (*Covariate*) AI control chart was formed using phase I data for a number of $m = 25$ subgroups, with each subgroup having a number $n = 4$ of observations. The covariate variable X is denoted as the actual value of cement compressive strength (Y), which has a normal distribution with mean $\mu_x = 251.4909$ and variance $\sigma_x^2 = 975.8091$. So, with the regression approach using **Equations (6)** and **(7)**, the covariate model has parameters $A = 198.143$ and $B = 0.2164664$ in covariate model $Y = A + BX + \varepsilon$, and ε has normal distribution with mean $\mu_\varepsilon = 0$ and variance $\sigma_m^2 = 917.798$. The covariate model influences the quality characteristics of Y so that it has a normal distribution with expectation values and variances that can be calculated using the equations below.

$$E(Y) = \mu_y = A + B\mu_x = 198.143 + 0.2164664 \times 251.4909 = 252.5823 \text{ and}$$

$$Var(Y) = \sigma_y^2 = B^2\sigma^2 + \sigma_m^2 = 0.216466^2 \times 975.8091 + 917.798 = 963.5222 .$$

The Blaine is used as an auxiliary variable with the notation W having a normal distribution, with mean $\mu_w = 341.0465$ and variance $\sigma_w^2 = 163.0266$. The variables W and Y have the relationship written in **Table 5**, notate as $\rho = 0.2550732$ and $\rho^* = -0.1448688$.

With those parameters, differentiation estimators can be formed for mean ($M_{YW_j} = M_{YW_j}^{(1)}$) and variance ($V_j = V_j^{(1)}$), respectively, using **Equations (14)** and **(15)**. Calculations for statistical transformations for *mean* ($M_{je}^{(1)}$) and variance ($V_{je}^{(1)}$) so that they follow the Standard normal distribution and are calculated with equations **Equations (16)** and **(17)**. EWMA statistics for mean (P_i) and EWMA statistics for variance (Q_i) can be calculated using **Equations (18)** and **(19)** with $Q_0 = P_0 = 0$ is the starting value/initial value. So, the statistical results of the jth subgroup test are obtained as follows.

Table 9. Max-EWMA ME (Covariate) AI Phase I Control Chart Statistics

j	$M_{YW_j}^{(1)}$	$V_j^{(1)}$	$M_{je}^{(1)}$	$V_{je}^{(1)}$	$P_i^{(l)}$	$Q_i^{(l)}$	Max $_i$
1	255.8386	-1.6116	0.2170	-1.6288	0.0108	-0.0814	0.0814
2	241.3177	-1.9213	-0.7506	-1.9418	-0.0272	-0.1745	0.1745
3	226.1275	-0.6815	-1.7628	-0.6887	-0.1140	-0.2002	0.2002
4	248.6839	-1.2252	-0.2598	-1.2383	-0.1213	-0.2521	0.2521
5	234.8089	-2.0091	-1.1844	-2.0305	-0.1745	-0.3410	0.3410
6	279.4645	1.0487	1.7913	1.0599	-0.0762	-0.2710	0.2710
7	253.9941	-1.1666	0.0941	-1.1791	-0.0677	-0.3164	0.3164
8	251.0153	1.8507	-0.1044	1.8705	-0.0695	-0.2070	0.2070
9	276.5034	-1.7959	1.5940	-1.8150	0.0137	-0.2874	0.2874
10	252.4919	-0.7407	-0.0060	-0.7486	0.0127	-0.3105	0.3105
11	250.5965	-0.0445	-0.1323	-0.0450	0.0055	-0.2972	0.2972
12	253.0245	0.0333	0.0295	0.0336	0.0067	-0.2807	0.2807
13	263.7124	0.3742	0.7417	0.3782	0.0434	-0.2477	0.2477
14	288.7264	0.4107	2.4085	0.4151	0.1617	-0.2146	0.2146
15	253.3684	-0.6708	0.0524	-0.6779	0.1562	-0.2377	0.2377
16	260.1155	-1.5678	0.5020	-1.5845	0.1735	-0.3051	0.3051
17	255.8240	-0.5363	0.2160	-0.5421	0.1756	-0.3169	0.3169
18	230.4171	-1.8214	-1.4770	-1.8408	0.0930	-0.3931	0.3931
19	242.7622	-1.8474	-0.6544	-1.8671	0.0556	-0.4668	0.4668
20	261.9919	0.1030	0.6270	0.1041	0.0842	-0.4383	0.4383
21	249.7719	-1.0594	-0.1873	-1.0707	0.0706	-0.4699	0.4699
22	236.1984	-1.0440	-1.0918	-1.0551	0.0125	-0.4992	0.4992
23	261.7444	0.5218	0.6105	0.5274	0.0424	-0.4478	0.4478
24	243.3709	0.1272	-0.6138	0.1285	0.0096	-0.4190	0.4190
25	242.6881	-1.1038	-0.6593	-1.1155	-0.0239	-0.4538	0.4538

The Max-EWMA ME (Covariate) AI phase I control chart with parameters L=2.709 and λ=0.05 has an upper control limit of 1.503018 and a lower control limit of 0. The max-EWMA ME (*Covariate*) AI control chart is depicted in **Figure 5** below.

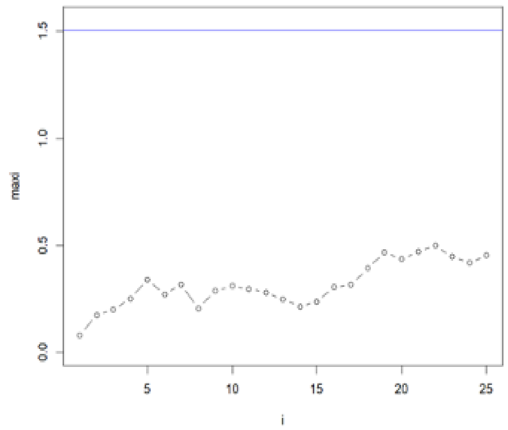

Figure 5. Phase I Max-EWMA ME (Covariate) AI Control Chart

Based on **Figure 5**, it is known that in the Max-EWMA ME (Covariate) AI phase 1 control chart, all observations are within the control limits. So, it can be concluded that the process average and variance of the cement production process are controlled together using the Max-EWMA ME (Covariate) AI control chart and have been statistically controlled. Therefore, control chart observations can be continued for phase II data.

3.5.2 Max-EWMA ME (Covariate) AI Control Chart Phase II

Phase II data is data on cement quality characteristics from 1 September 2023 to 30 November 2023 for a number of $m = 14$ subgroups, with each subgroup having a number $n = 4$ of observations. The Max-EWMA ME (Covariate) AI control chart for phase II data was formed using the parameters obtained in the phase I control chart, including $UCL = 1.503018$; $\mu_x = 251.4909$; $\sigma_x^2 = 975.8091$; $A = 198.143$; $B = 0.2164664$; $\sigma_m^2 = 917.798$; $\mu_w = 341.0465$; $\sigma_w^2 = 163.0266$; $\rho = 0.2550732$; $\rho^* = -0.1448688$. Starting value/initial value for EWMA statistics for mean (${P_i}^{(1)}$) and EWMA statistics for variance (${Q_i}^{(1)}$) uses the EWMA statistical value in the last observation phase 1 data so that $P_0 = {P_{25}}^{(1)} = -0.02386$ and $Q_0 = {Q_{25}}^{(1)} = -0.45384$. With the same calculation steps as phase I, using R software, a summary of the results of statistical calculations and a Max-EWMA ME (Covariate) AI control chart are obtained as follows.

Table 10. Max-EWMA ME (Covariate) AI Phase II Control Chart Statistics

j	${M_{YW_j}}^{(1)}$	${V_j}^{(1)}$	${M_{je}}^{(1)}$	${V_{je}}^{(1)}$	${P_i}^{(1)}$	${Q_i}^{(1)}$	Max_i
26	229.2963	-0.7847	-1.5517	-0.7930	0.0103	-0.5221	0.5221
27	213.4805	-1.0097	-2.6056	-1.0204	-0.1205	-0.5470	0.5470
28	254.2661	-0.9025	0.1122	-0.9121	-0.1089	-0.5653	0.5653
29	260.5101	1.1345	0.5283	1.1466	-0.0770	-0.4797	0.4797
30	239.0567	-2.8587	-0.9013	-2.8892	-0.1182	-0.6001	0.6001
31	246.7692	-0.7945	-0.3874	-0.8030	-0.1317	-0.6103	0.6103
32	255.2952	1.1661	0.1808	1.1786	-0.1161	-0.5208	0.5208
33	239.4217	-1.2825	-0.8770	-1.2962	-0.1541	-0.5596	0.5596
34	239.2227	-0.5947	-0.8902	-0.6011	-0.1909	-0.5617	0.5617
35	279.2152	-0.5085	1.7747	-0.5139	-0.0926	-0.5593	0.5593
36	276.2304	-0.6845	1.5758	-0.6918	-0.0092	-0.5659	0.5659
37	284.4274	-0.9144	2.1220	-0.9241	0.0974	-0.5838	0.5838
38	278.0709	0.5522	1.6985	0.5581	0.1774	-0.5267	0.5267
39	262.4805	-1.7995	0.6596	-1.8187	0.2015	-0.5913	0.5913

Using the statistical values in **Table 10**, a Max-EWMA control chart for phase II data can be formed in **Figure 6**.

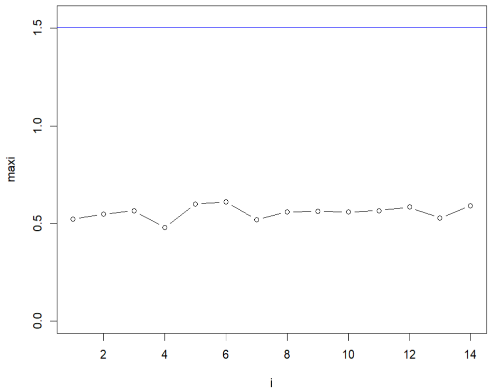

Figure 6. **Max-EWMA ME (Covariate) AI Phase II Control Chart**

Based on **Figure 6**, it is known that all observation points in phase II data are within the control limits, so it can be concluded that the process average and variance of the cement production process are controlled together. Overall, Max-EWMA ME statistics data for phase I and phase II control charts can be depicted in **Figure 7** below.

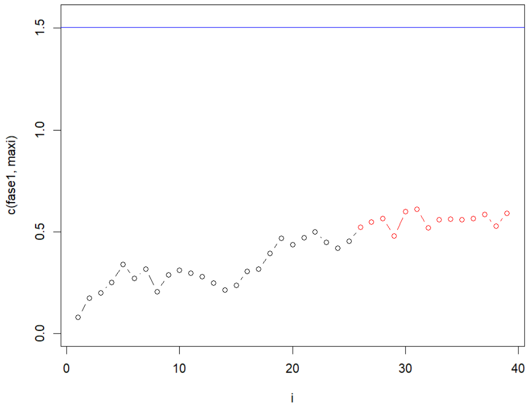

Figure 7. **Max-EWMA ME (Covariate) AI Phase I and Phase II Control Chart**

Based on **Figure 7**, it is known that there is a pattern of tendency for EWMA statistical values to increase monotonically from phase I data to phase II data. This indicates that there has been a change in the average and variance of the cement production process, which needs to be investigated for the cause.

3.6 Sensitivity Comparison

The Max-EWMA control chart on phase II data does not have observations outside the control limits, nor does the Max-EWMA ME (Covariate) AI control chart. The Max-EWMA phase I and phase II control charts in **Figure 4** have an upward monotonic trend pattern that can be observed since the 14^{th} subgroup. This indicates that the Max-EWMA control chart detected a shift in the average and variance of the process. However, the change that occurred was not significant enough and was still statistically controlled. The Max-EWMA ME (Covariate) AI Phase I and Phase II control charts in **Figure 7** also have an upward monotonic trend pattern from the 14^{th} subgroup to the Phase II data. The EWMA statistical value on the Max-EWMA ME (Covariate) AI control chart experiences a higher shift than the statistical value formed on the Max-EWMA control chart. This indicates that the method of handling measurement error with covariate and auxiliary information can detect shifts in the average and variance of the production process better than control charts without handling measurement error. It can be concluded that the Max-EWMA ME (Covariate) AI control chart is more sensitive than the Max-EWMA control chart.

3.7 Process Capability Analysis

Because data is statistically in-controlled, the production capability determination can use the C_p and C_{pk} index. Because there is only a minimum specification limit, *the* C_p value can be represented by the C_{pl} value. The results of process capability analysis for compressive strength can be described as follows.

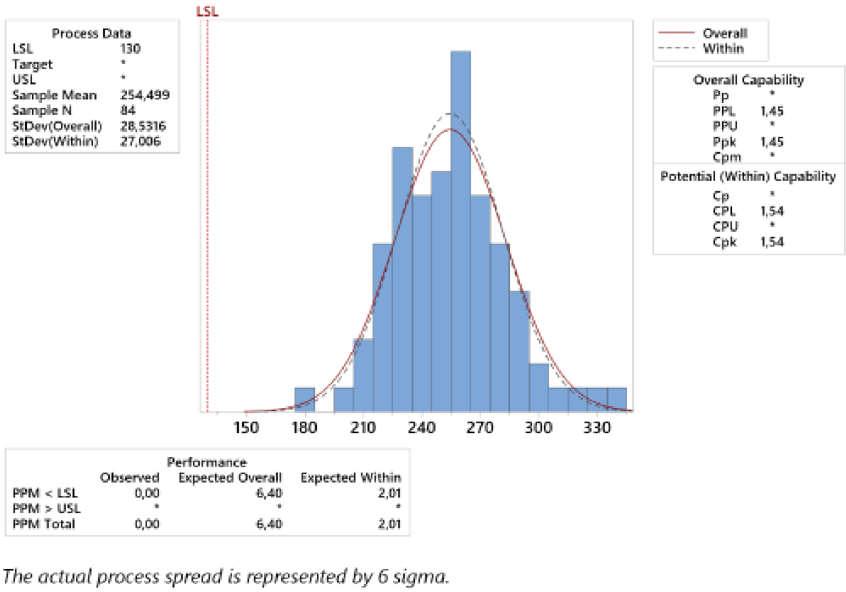

Figure 8. Process Capability of Compressive Strength of Cement Aged 3 Days

Based on **Figure 8**, the compressive strength variable's process capability ratio (Cpl) is 1.54. This shows that the cement production process based on the compressive strength is capable because it has a $C_{pl} >$ 1.33. The C_{pk} value is 1.54 (also greater than 1.33), which means the production process is quite consistent, has high accuracy, and is in accordance with the desired target.

4. CONCLUSIONS

Based on the results and discussion, the following conclusions were obtained.

1. Max-EWMA and Max-EWMA ME (Covariate) AI state that the cement production process at PT XYZ is statistically in control. Max-EWMA control chart, control chart has an upper control limit UCL = 1.503018 and several parameters, $\mu_y = 252.5823$ and $\sigma_y^2 = 970.1596$. Max-EWMA ME (*Covariate*) AI control chart has an upper control limit UCL = 1.503018, and several parameters include: $\mu_x = 251.4909$; $\sigma_x^2 = 975.8091$; $A = 198.143$; $B = 0.21647$; $\sigma_m^2 = 917.798$; $\mu_w = 341.0465$; $\sigma_w^2 = 163.0266$; $\rho = 0.2550732$; $\rho^* = -0.1448688$.
2. The results of the control chart sensitivity comparison show that the Max-EWMA ME (*Covariate*) AI control chart is more sensitive than the Max-EWMA control chart, as evidenced by the shift in the EWMA statistical values, which can show a monotonically increasing pattern of changes in the production process.
3. Cement production process capabilities at PT XYZ (Persero), Tbk. has a C_{pl} capability index of 1.54, which means that the cement production process based on cement's compressive strength quality characteristics is capable. C_{pk} capability index of 1.54 indicates that the production process is consistent and accurate, so the quality characteristics have reached the desired target.

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