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RISK ANALYSIS OF GOOGL & AMZN STOCK CALL OPTIONS USING DELTA GAMMA THETA NORMAL APPROACH

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ABSTRACT

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Stocks, as investment products, tend to carry risks due to fluctuations. The tendency of stock prices to rise over time leads investors to opt for call options, which are one of the derivative investment products. However, call options are influenced by several factors that can pose risks and have nonlinear dependence on market risk factors. Therefore, methods are needed to measure the risk of call options, such as Delta Normal Value at Risk and Delta Gamma Normal Value at Risk. Delta and Gamma are part of Option Greeks, parameters that measure the sensitivity of options to various factors used in determining option prices with the Black-Scholes model. This study uses an approach with the addition of Theta, which can measure the sensitivity of options to time. This study aims to analyze Value at Risk with the Delta Gamma Theta Normal approach for call options on Google (GOOGL) and Amazon (AMZN) stocks. The analysis uses closing stock price data from September 7, 2022, to September 7, 2023, and three in-the-money and out-of-the-money call option prices. The study begins by collecting closing stock prices and call option contract components, testing the normality of stock returns, calculating volatility, $d_1, d_2, N(d_1), N(d_2)$*, Delta, Gamma, and Theta, then calculating the Value at Risk. Based on the analysis, it is found that GOOGL and AMZN call options have a Value at Risk of \$0.89588 and \$0.92760, respectively, at a 99% confidence level with a strike price of \$120. Furthermore, based on the comparison of Value at Risk between in-the-money and out-of-the-money call options, it can be concluded that out-of-the-money call options tend to have larger estimated losses.*

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1. INTRODUCTION

Investment is an activity of capital placement to gain profit. Investing in stocks is one of the important aspects of a company's financial system. A company listed on the stock exchange provides opportunities for investors to invest in stocks. Investing in stocks gives investors a source of income through the growth of stock values [1]. Although investing in stocks appears profitable, stock prices experience fluctuations [2]. Fluctuations in stock prices are a natural part of stock market activity. One strategy investors can use as a form of protection (hedging) against stock price declines is call options [3].

Options are contracts granting the owner or holder the right to buy or sell a company's stocks at a specific price within a certain expiration time [4]. Stock options are derivative instruments because the value of stock options is based on the value and characteristics of the underlying stock asset. Options that grant the right to investors to buy stocks at a certain agreement are called call options. Generally, call options tend to be executed if the stock price at expiration time is higher than the strike price. Conversely, suppose the stock price is lower than the strike price at expiration time. In that case, call options tend to expire without execution. Factors influencing the value of call options include the underlying asset's price, the stock, the strike price, volatility, time to maturity, and the risk-free interest rate [5].

The risk measurement of stock options can be conducted through Value at Risk analysis. Value at Risk (VaR) is defined as an estimate of potential losses that may occur within a certain time and a certain level of confidence [6]. Previous research on stock options VaR has been conducted using the Delta Normal and Delta Gamma Normal approaches based on the Black-Scholes option model [7]. The Delta Normal approach measures option risk using Option Greeks, namely Delta. It approximates changes in stock options based on the Taylor Polynomial return of stock prices. Furthermore, the Delta Gamma Normal approach measures option risk using Delta and Gamma, approximating changes in stock options based on the Taylor Polynomial return of stock prices. The risk measurement of stock options with VaR analysis in this study uses the Delta Gamma Theta Normal approach. The Delta Gamma Theta Normal approach approximates changes in stock options based on the second-order Taylor Polynomial return of stock prices. Adding Theta in this approach allows VaR to measure the sensitivity of options to time.

This research utilizes the Delta Gamma Theta Normal Value at Risk ($\mathrm{VaR}_{\mathrm{DGTN}}$) as a nonlinear method to estimate the risk of stock call options. The risk measurement of call options using $\mathrm{VaR}_{\mathrm{DGTN}}$ leverages the use of Option Greeks. Generally, Option Greeks are sensitivity metrics for the option value in the Black-Scholes model. Prices of options under the Black-Scholes model have nonlinear dependence on market risk factors, assuming that stock price returns follow a normal distribution, there are no dividend payments, and stock price volatility and risk-free interest rates are constant [8]. The results of this research are expected to provide a deeper understanding of risk estimation in call options, thus enhancing the effectiveness of risk management in investments.

2. RESEARCH METHODS

2.1 Stocks and Stock Return

Stocks can be considered as a form of capital participation by individuals or businesses in a company. By providing capital, they can own a portion of the company's assets and income. An important aspect of stock investment is stock return. Stock return is the rate of return or yield obtained from stock investments. The calculation of stock return can be formulated as follows [9]:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

with R_t representing the stock return for period t, S_t denoting the stock price at time t, and S_{t-1} representing the stock price at time $t-1$. **Equation (1)** represents the log return of the stock.

2.1 Stock Options

Stock options are derivative products known as contracts that grant the holder of the stock option contract the right to buy or sell a company's asset to the option issuer at a specific price (strike price) within a certain period. Based on the type of right granted to the holder, stock options consist of call and put options. Call options grant the right to buy a company's stock at a specified price and time. On the other hand, put options grant the right to sell a company's stock at a specified price and time. The formula for the intrinsic value of a call option is expressed as follows [9]:

$$C_t = max(S_t - K, 0) \tag{2}$$

A stock call option will have zero value if the strike price (K) is higher than the stock price at (S_t). Stock call options are divided into three types, as follows [10]:

a. Out-of-the-money is an option where the stock price at the transaction is lower than the strike price.

b. In-the-money is an option where the stock price at the transaction is higher than the strike price.

c. At-the-money is an option where the stock price at the transaction is equal to the strike price.

Furthermore, based on the expiration period and the rights held by the contract holder, options consist of American and European option types. American options allow the contract holder to exercise the call option before or at expiration. In contrast, European options only allow the contract holder to exercise the call option at expiration. The price of the underlying stock, the strike price, the interest rate, the expiration time, and the stock price volatility are the factors that impact the price of stock options [5].

2.2 Black-Scholes Option Model

Fischer Black & Myron Scholes introduced a model for option pricing in 1973. It's known as the Black-Scholes model. Only European option types can be used with the Black-Scholes model. The Black-Scholes model assumes that the underlying stock price of the option follows a lognormal distribution and the return of the stock follows a normal distribution. The expression for the Black-Scholes call option with a time indicator $\tau = T - t$ is as follows [8]:

$$C_t = S_t N(d_1) - Ke^{-r\tau} N(d_2) \tag{3}$$

with C_t as the call option's value, S_t representing the stock price at time t, K as the strike price, r as the risk-free interest rate. The cumulative normal standards for d_1 and d_2 are denoted by $N(d_1)$ and $N(d_2)$, respectively. The values of d_1 and d_2 are formulated as follows:

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \tag{4}$$

$$d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \tag{5}$$

The calculation of $N(d_1)$ and $N(d_2)$ also uses the "pnorm()" function in the RStudio software.

2.3 Normality Test

A normality test is a recognized process for determining if the data distribution is normal. Normality tests can be conducted using the Kolmogorov-Smirnov test (KS test). KS test compares the data distribution (to be tested for normality) with the standard normal distribution. The following is the hypothesis that was used in the KS test.

H_0: data is normally distributed

H_1: data is not normally distributed

Furthermore,

$$D = maximum|F_o(X) - S_n(X)|$$

with $F_o(X)$ as the cumulative frequency distribution and $S_n(X)$ as the observed cumulative frequency distribution. If $D < D_{table}$, then H_1 is rejected, it tells us that the distribution of the data is normal [11]. The criterion for testing is if the p-value is greater than the specified significance level (p-value $> a$), then we cannot reject H_0 [12].

2.4 Volatility

Volatility can be expressed as the standard deviation of the log returns of stocks over an annual period. Volatility represents the magnitude of stock price fluctuations. One method that can be used to estimate volatility is historical volatility. Historical volatility is volatility that has been calculated from historical stock price data [13]. Estimating historical volatility from stock prices begins with calculating the stock return values. After that, it continues calculating the mean and standard deviation of the stock returns. The calculation of the mean and standard deviation of stock returns can be done using **Equation (6)** and **Equation (7)** [13].

$$\overline{R_t} = \frac{\sum_{t=1}^{n} R_t}{n} \tag{6}$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}(R_t - \overline{R_t})^2}{n-1}} \tag{7}$$

with $\overline{R_t}$ being the average of stock returns, $\hat{\sigma}$ being the standard deviation of stock returns, R_t being the return of stock at time t, and n being the number of analyzed-stock-return data. Furthermore, the annual historical volatility can be calculated using **Equation (8)**.

$$\sigma_{hist} = \hat{\sigma}\ x\ \sqrt{\mathrm{n}} \tag{8}$$

2.5 Option Greeks

Option Greeks measure the sensitivity of call option prices to the underlying factors that impact the price. Delta, Gamma, and Theta are Option Greeks. Delta measures the sensitivity of call option price changes relative to stock price changes [14]. The Delta equation is denoted as **Equation (9)** [15].

$$\delta = \frac{\partial C_t}{\partial S_t} \tag{9}$$

The Delta equation for European call options can be derived as follows [8]:

$$\begin{aligned}\delta &= \frac{\partial C_t}{\partial S_t}\\ &= \frac{\partial\left(S_t N(d_1) - K e^{-r\tau} N(d_2)\right)}{\partial S_t}\\ &= N(d_1)\end{aligned} \tag{10}$$

Furthermore, Gamma measures the change in Delta concerning changes in the stock price [14]. An option has maximum Gamma when its position approaches or is at the At-the-money. However, Gamma decreases when a call option is in the In-the-money or Out-of-the-money position, further away from the stock price. Gamma can be written as **Equation (11)** [15].

$$\gamma = \frac{\partial \delta}{\partial S_t} \tag{11}$$

Based on **Equation (11)**, the Gamma's value for European call options can be derived as follows [8]:

$$\begin{aligned}\gamma &= \frac{\partial \delta}{\partial S_t}\\ &= \frac{\partial(N(d_1))}{\partial S_t}\\ &= \frac{1}{S_t \sigma \sqrt{2\pi\tau}} e^{-\frac{d_1^2}{2}}\end{aligned} \tag{12}$$

Furthermore, Theta measures the sensitivity of call option prices to the derivative value to the passage of time [14]. If the time to maturity decreases by one day, the call option price will change by the amount of Theta. Theta can be written as **Equation (13) [15]**.

$$\theta = -\frac{\partial C}{\partial \tau} \tag{13}$$

where τ is the time difference from time t to maturity time T. Theta represents the option value influenced by the time difference to maturity time T and time t, thus $\theta = \frac{\partial C}{\partial t} = -\frac{\partial C}{\partial \tau} < 0$. Theta can be derived into the following **Equation (14) [15]**:

$$\begin{aligned} \theta &= -\frac{\partial C}{\partial \tau} \\ &= -\frac{\partial\left(S_t N(d_1) - Ke^{-r\tau}N(d_2)\right)}{\partial \tau} \\ &= -\frac{S_t\sigma}{2\sqrt{\tau}}N'(d_1) - rKe^{-r\tau}N(d_2) \end{aligned} \tag{14}$$

2.6 Value at Risk with Normal Distribution

Value at Risk (VaR) assuming that profit/loss (stock return) follows a normal distribution, at a confidence level of 1-α is as follows [16]:

$$VaR = Z_{1-\alpha}\sigma_{P/L_{t+\Delta t}} - \mu_{P/L_{t+\Delta t}} \tag{15}$$

with $\mu_{P/L_{t+\Delta t}}$ and $\sigma_{P/L_{t+\Delta t}}$ respectively being the mean and standard deviation of stock returns. $Z_{1-\alpha}$ is the value of the standard normal variate (Z-Score) such that $1-\alpha$ of the probability density mass lies on the left side, and α of the probability density mass lies on the right side. If the confidence level used is 95%, then the value of $Z_{1-\alpha} = Z_{0.95}$ is 1.645. $\mu_{P/L_{t+\Delta t}}$ and $\sigma_{P/L_{t+\Delta t}}$ are unknown in application. Therefore, VaR will be estimated based on the estimation of these parameters. $\mu_{P/L_{t+\Delta t}}$ and $\sigma_{P/L_{t+\Delta t}}$ are estimated by the formula of mean and variance of profit/loss (stock return) in this research [17].

2.7 Delta Gamma Theta Normal Value at Risk

VaR with the Delta Gamma Theta Normal approach is a calculation to measure the risk of an option with a nonlinear dependence on market risk factors. Measuring the risk of options using Delta Gamma Theta Normal Value at Risk requires an option pricing model. In this case, the option pricing model used is the Black-Scholes model.

Delta Gamma Theta Normal Value at Risk ($\mathrm{VaR_{DGTN}}$) is a nonlinear method to estimate the risk of stock call options. Measuring VaR with the Delta Gamma Theta Normal approach uses Taylor Polynomials to approximate changes in option value based on stock returns. It is assumed that options with time t are functions around S_t [17]. The equation for the change in option value can be expressed as follows [18]:

$$\Delta C_t = \left(\frac{\partial C_t}{\partial S_t}\right)(\Delta S_t) + \frac{1}{2}\left(\frac{\partial \delta}{\partial S_t}\right)(\Delta S_t)^2 + \frac{\partial C}{\partial t}\Delta t \tag{16}$$

with $\frac{\partial C}{\partial S_t}$, $\frac{\partial \delta}{\partial S_t}$, and $\frac{\partial C}{\partial t}$ are respectively Delta (δ), Gamma (γ), and Theta (θ) of an option, so that **Equation (16)** can be written as follows [6]:

$$\Delta C_t = \delta(\Delta S_t) + \frac{\gamma}{2}(\Delta S_t)^2 + \theta\Delta t \tag{17}$$

The profit/loss (return) of the stock is assumed to follow a normal distribution with zero mean and volatility $\sigma_{\Delta S_t}$. Therefore, the mean and variance of **Equation (17)** are obtained as follows [17]:

$$\begin{aligned} E[\Delta C_t] &= E\left[\delta(\Delta S_t) + \frac{\gamma}{2}(\Delta S_t)^2 + \theta\Delta t\right] \\ &= E[\delta(\Delta S_t)] + E\left[\frac{\gamma}{2}(\Delta S_t)^2\right] + E[\theta\Delta t] \end{aligned}$$

$$= \delta E[\Delta S_t] + \frac{\gamma}{2} E[(\Delta S_t)^2] + \theta \Delta t$$

$$= \delta(0) + \frac{1}{2}\gamma(0) + \theta \Delta t$$

$$= \theta \Delta t \tag{18}$$

and

$$Var[\Delta C_t] = Var\left[\delta(\Delta S_t) + \frac{\gamma}{2}(\Delta S_t)^2 + \theta \Delta t\right]$$

$$= Var[\delta(\Delta S_t)] + Var\left[\frac{\gamma}{2}(\Delta S_t)^2\right] + Var[\theta \Delta t]$$

$$= \delta^2 Var[\Delta S_t] + \frac{\gamma^2}{4} Var[(\Delta S_t)^2] + 0$$

$$= \delta^2 \sigma_{\Delta S_t}^2 + \frac{\gamma^2}{4}\sigma_{\Delta S_t}^4 \tag{19}$$

The mean and variance are substituted as parameters in VaR with the normal distribution approach. Thus, the calculation of VaR based on the Delta Gamma Theta Normal approach ($\mathrm{VaR}_{\mathrm{DGTN}}$) to measure the risk of an option is as follows [17]:

$$VaR_{DGTN} = Z_{1-\alpha}\sqrt{\delta^2 \sigma_{\Delta S_t}^2 + \frac{\gamma^2}{4}\sigma_{\Delta S_t}^4} - (\theta \Delta t) \tag{20}$$

with $Z_{1-\alpha}$ representing the value of the standard normal variate (Z-Score), δ representing the Delta value, γ representing the Gamma value, θ representing the Theta value, and $\sigma_{\Delta S_t}$ representing the annual standard deviation value (volatility (σ_{hist})).

3. RESULTS AND DISCUSSION

The data used in the $\mathrm{VaR}_{\mathrm{DGTN}}$ analysis consists of call option data for the companies Google and Amazon. Google and Amazon are listed on the Nasdaq stock index with their respective ticker codes being GOOGL and AMZN. The analysis process utilizes daily closing stock price data from September 7, 2022, to September 7, 2023, obtained from Yahoo Finance at the website address *http://www.finance.yahoo.com*. During this period, there were a total of 252 closing stock prices.

In addition to the closing stock price data, three In-the-money and three Out-of-the-money call option contract data are also used. The call option contracts used are listed in Table 1 and Table 2. The strike price (K) used consists of three In-the-money call options and three Out-of-the-money call options, all with an expiration date of September 15, 2023. The stock prices at the time of the call option transactions for GOOGL and AMZN were \$135.26 and \$137.85 respectively, while the risk-free interest rate used was 5.5%.

Table 1. The contract of Googl's stock call option

Types of Options	The Contract of Options	Strike Price
In The Money	GOOGL230915C00120000	120
	GOOGL230915C00125000	125
	GOOGL230915C00130000	130
Out of The Money	GOOGL230915C00140000	140
	GOOGL230915C00145000	145
	GOOGL230915C00150000	150

***Data source**: http://www.finance.yahoo.com*

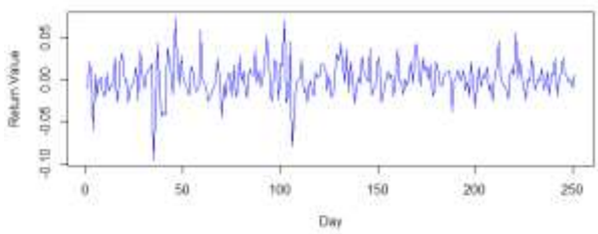
Table 2. **The contract of Amazon's stock call option**

Types of Options	The Contract of Options	Strike Price
In The Money	AMZN230915C00120000	120
	AMZN230915C00125000	125
	AMZN230915C00130000	130
Out of The Money	AMZN230915C00145000	145
	AMZN230915C00150000	150
	AMZN230915C00155000	155

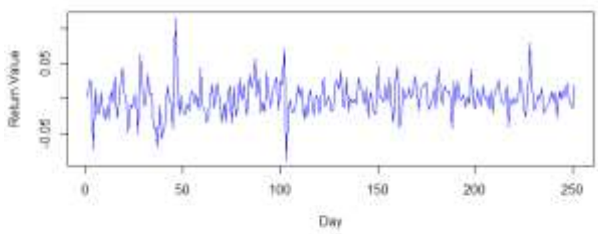
Data source: *http://www.finance.yahoo.com*

3.1 Calculation of Stock Returns

The stock return values are obtained using **Equation (1)**, where the plots of the stock returns for GOOGL and AMZN are presented in **Figure 1**.



(a)



(b)

Figure 1. (a) Plot of the stock return data of GOOGL, (b) Plot of the stock return data of AMZN

Based on **Figure 1**, it can be seen that the obtained return values are both positive and negative. Positive returns indicate profit/gain in investments. From the stock return plot, it can be observed that the stock price experienced fluctuating movements during the observation period.

3.2 Normality Test of Stock Returns

Based on the KS test results, the statistical p-value for the GOOGL stock returns is 0.41490, while the statistical p-value for the AMZN stock returns is 0.68880.

Table 3. **Normality test**

Characteristic	Value	
	GOOGL	AMZN
Sample Observation	251	251
Significance level	0.05	0.05
P-value	0.41490	0.68880

The decision-making criterion for the KS test is if the p-value > a, then the stock returns are normally distributed. Both p-values for the stock returns are greater than the significance level of 5% (0.05). Based on **Table 3**, it can be concluded that H_0 is accepted, indicating that the GOOGL and AMZN stock return data are normally distributed.

3.3 Calculation of Volatility

Based on **Equation (6)** and **Equation (7)**, it is known that the mean of GOOGL stock return is 0.00084, while for the AMZN is 0.00025. Furthermore, the standard deviation of the GOOGL stock return is 0.02176 and the standard deviation of the AMZN stock return is 0.02465. Thus, the calculation of the volatility of GOOGL and AMZN stocks computed with **Equation (8)** is given as follows:

GOOGL σ_{hist} =0.02176 $* \sqrt{252} = 0.34541$

AMZN σ_{hist} =0.02465 $* \sqrt{252} = 0.39130$

3.4 The Calculation of $d_1, N(d_1), d_2$, and $N(d_2)$

The results of the calculations for $d_1, N(d_1), d_2$, and $N(d_2)$ for the GOOGL and AMZN call option are presented in **Table 4** and **Table 5**.

Table 4. The calculation results of $d_1, N(d_1), d_2$, and $N(d_2)$ for GOOGL

Types of Options	Strike Price	d_1	$N(d_1)$	d_2	$N(d_2)$
In The Money	120	2.00422	0.97748	1.94267	0.97397
	125	1.34092	0.91003	1.27937	0.89962
	130	0.70363	0.75917	0.64209	0.73959
Out of The Money	140	-0.50052	0.30836	-0.56206	0.28704
	145	-1.07070	0.14215	-1.13225	0.12877
	150	-1.62156	0.05245	-1.68310	0.04618

Table 5. The calculation results of $d_1, N(d_1), d_2$, and $N(d_2)$ for AMZN

Types of Options	Strike Price	d_1	$N(d_1)$	d_2	$N(d_2)$
In The Money	120	2.04891	0.97976	1.97919	0.97610
	125	1.46340	0.92832	1.39368	0.91829
	130	0.90086	0.81617	0.83114	0.79705
Out of The Money	145	-0.66539	0.25290	-0.73511	0.23114
	150	-1.15164	0.12474	-1.22136	0.11098
	155	-1.62194	0.05241	-1.69166	0.04536

Based on **Table 4** and **Table 5**, the strike prices of GOOGL and AMZN call options on In-the-money options have a higher likelihood of being lower than the stock prices at maturity.

3.5 Estimation of Delta, Gamma, and Theta

The next analysis process involves estimating the values of the Greeks, namely Delta, Gamma, and Theta. The estimation of the Greek values is carried out using **Equation (10)**, **Equation (12)**, and **Equation (14)**. The estimated values of the Greeks for GOOGL and AMZN are presented in **Table 6** and **Table 7**.

Table 6. The result of delta, gamma, and, theta for GOOGL

Types of Options	Strike Price	Delta	Gamma	Theta
In The Money	120	0.97748	0.00643	-13.43620
	125	0.91003	0.01950	-27.46038
	130	0.75917	0.03742	-46.11397
Out of The Money	140	0.30836	0.04228	-48.35340
	145	0.14215	0.02702	-30.51035
	150	0.05245	0.01287	-14.42674

Table 7. The result of delta, gamma, and, theta for AMZN

Types of Options	Strike Price	Delta	Gamma	Theta
In The Money	120	0.97976	0.00509	-13.83337
	125	0.92832	0.01423	-27.00029
	130	0.81617	0.02766	-45.93599
Out of The Money	145	0.25290	0.03327	-50.23709
	150	0.12474	0.02139	-32.02847
	155	0.05241	0.01114	-16.59330

Based on **Tables 6** and **Table 7**, it is known that the Delta values of each call option tend to be high for In-the-money options. This indicates that In-the-money options are more sensitive to changes in stock prices. Furthermore, we know that the stock prices at the time of the call option transactions for GOOGL and AMZN were $135.26 and $137.85. The Gamma values tend to be higher for call options approaching At-the-money options. Gamma provides information about the speed of Delta changes in measuring the option value response to changes in stock prices. Additionally, the Theta values tend to be negative for each call option. This is because Theta measures how quickly the option value decreases over time. Theta values tend to become more negative as the options approach the At-the-money position.

3.6 Value at Risk Analysis with Delta Gamma Theta Normal Approach

The calculation Delta Gamma Theta Normal Value at Risk (VaR_{DGTN}) is performed using **Equation (20)**. The analysis results of VaR_{DGTN} for call options of GOOGL stock at confidence levels of 80%, 90%, 95%, and 99% are presented in **Table 8**. Furthermore, the results of VaR_{DGTN} analysis for call options of AMZN stock at confidence levels of 80%, 90%, 95%, and 99% are presented in **Table 9**.

Table 8. The result of VaR_{DGTN} for GOOGL stock call options

Types of Options	Strike Price	*Value at Risk*			
		80%	90%	95%	99%
In The Money	120	0.86430	0.87366	0.88139	0.89588
	125	1.74651	1.75522	1.76241	1.77591
	130	2.91881	2.92608	2.93208	2.94334
Out of The Money	140	3.05163	3.05458	3.05702	3.06159
	145	1.92458	1.92594	1.92706	1.92917
	150	0.90976	0.91026	0.91068	0.91146

Table 9. The result of VaR_{DGTN} for AMZN stock call options

Types of Options	Strike Price	*Value at Risk*			
		80%	90%	95%	99%
In The Money	120	0.86430	0.87366	0.88139	0.89588
	125	1.74651	1.75522	1.76241	1.77591
	130	2.91881	2.92608	2.93208	2.94334
Out of The Money	140	3.05163	3.05458	3.05702	3.06159
	145	1.92458	1.92594	1.92706	1.92917
	150	0.90976	0.91026	0.91068	0.91146

Based on **Table 8**, the VaR_{DGTN} value for Out-of-the-money call options on GOOGL stock tends to have larger estimated losses. The lowest VaR value for each confidence level is for call options with a strike price of $120. The VaR value at the 99% confidence level for In The Money call options on GOOGL stock with a strike price of $120 is $0.89588. This means that out of 100 investment occurrences in call options, there is a possibility of 99 occurrences where the loss will not exceed $0.89588.

Based on **Table 9**, the VaR_{DGTN} value for Out-of-the-money call options on AMZN stocks tends to have larger estimated losses. The lowest VaR values for each confidence level are for call options with a strike price of 120 dollars. The VaR value at a 99% confidence level for In The Money call options on GOOGL stocks with a strike price of 120 dollars is 0.92760 dollars. This means that out of 100 investment events in call options, there is a possibility of 99 events where the loss will not exceed 0.92760 dollars.

4. CONCLUSIONS

Based on the analysis results of the Delta Gamma Theta Normal Value at Risk applied to the call options of GOOGL and AMZN stocks, the call option of GOOGL with a strike price of $120 has a VaR value of $0.89588 at a 99% confidence level, whereas the call option of AMZN with the same strike price and confidence level has a VaR value of $0.92760. Additionally, the comparison of VaR values between In-the-money and Out-of-the-money call options tends to have higher estimated losses for Out-of-the-money call options. Therefore, from these estimation results, it can be concluded that risks will be smaller when entering into call option contracts with In-the-money options.

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