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APPLIED MODIFIED EXPONENTIAL APPROACH METHOD TO DETERMINE THE OPTIMAL SOLUTION

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ABSTRACT

PT. IGM distributes vaccines to several cities within and outside West Kalimantan. Distribution can be carried out directly or through CV. XYZ. To maintain vaccine quality, an effective and efficient vaccine management plan is required, especially for storage and distribution, to prevent any deviations in these processes This is done to ensure the vaccine's potency remains intact until it is ready for use. Distribution routes are chosen to be as efficient as possible. Therefore, this article discusses the application of the transportation method to manage vaccine distribution and minimize distribution costs. The distribution problem is formulated into a mathematical model and solved using the modified exponential approach method. This method is improvement on the improved Exponential Approach, focusing on the determination of initial solution and table revisions. Allocation is based on selecting cells with the smallest reduced cost entries. Based on research findings, PT IGM distributes vaccines to CV. XYZ, Pontianak and Kuburaya in amounts of 209.000 units, 151.000 units and 310.000 units, respectively. CV. XYZ distributes vaccines to Ketapang, Singkawang, Sintang and Bengkayang in amount of 40.000 units, 55.000 units, 45.000 units, and 9.000 units, respectively.

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1. INTRODUCTION

Vaccines are antigens in the form of microorganisms that are dead, still alive but weakened, intact, or processed, such as microbial toxins that have been processed into toxoids and recombinant proteins. When administered to a person, these vaccines induce specific active immunity against certain infectious diseases [1]. The health department is responsible for the timely storage and distribution of vaccines evenly and systematically to primary health care units. To maintain vaccine quality, an effective and efficient vaccine management plan is required for storage and distribution to prevent deviations in storage or distribution, ensuring the vaccine's potency is maintained until use [2].

PT. IGM, a subsidiary of Indofarma, is engaged in the distribution of medicines and medical equipment This company has several branches, including one in West Kalimantan tan tasked with distributing vaccines to several cities within and outside of West Kalimantan, including Pontianak, Ketapang, Singkawang, Sintang, Kubu Raya and Jakarta. In March, 670.000 units of vaccines at PT IGM scheduled for distribution to Pontianak, Ketapang, Singkawang, Sintang, Bengkayang and Kubu Raya. Shipping may be done directly or through CV. XYZ. Distribution routes are selected to be as efficient as possible. A distribution route is considered efficient if it meets several indicators, including distribution objectives, demand volume, distribution costs, distribution sources, and inventory levels at each source. The distribution cost from the source to the marketing destination is a significant topic because distribution costs influence product prices in the market. One factor affecting transportation costs is the geographic location or different distances from one source to another destination, which can result in varying transportation costs. Therefore, considerations are necessary to manage the distribution of goods to minimize total distribution costs.

The transportation problems can be solved using the transportation method. The transportation method consists of three stages: determining the initial Feasible Solution, Optimality Test, and Table Revision. Based on its stages, the transportation method is divided into two types: direct and indirect methods. The indirect method is a transportation method that seeks a solution in two stages: determining the initial feasible solution and determining the optimal solution (optimality test and table revision) [3] [4]. Several methods can be used to find the initial feasible solution, including the North West Corner Method, the Least Cost Method, and Vogel's Approximation Method (VAM). To achieve an optimal solution, optimization tests can be conducted using the Stepping Stone Method and the MODI Method [5] [6] [7]. Thus, a lengthy process is required to obtain an optimal solution. Therefore, a new method is needed to accelerate the steps in solving transportation problems.

In 2012, [8] introduced a direct method known as the ASM method, and in 2016, [9] introduced its revised version. However, [10] stated that the ASM method only yields the best initial feasible solution and is not a direct method, indicating that this method only provides an initial solution and does not offer an optimal solution in some cases. Existing methods continue to evolve and produce updates to create direct methods that provide optimal solutions. In 2013, [11] proposed the Exponential Approach method. However, this method has weaknesses in unbalanced cases where the solutions obtained are not always optimal. Therefore, in 2016, a better version of the Exponential Approach method was proposed, the Improved Exponential Approach method [12]. However, this method cannot yet be considered optimal because the allocation of the equal exponent penalties is chosen based on the minimum average. In 2021 [13], the Modified Exponential Approach method was proposed. However, this method does not first check whether each row and column already have a zero element. This step is crucial as it determines the initial feasible solution. The number of basic cell (filled cells should be $m + n - 1$, with m being the number of sources and n being the number of columns. Thus, the initial solution is considered feasible [7]. Additionally, in the optimal solution determination stage, when allocating the cell values with the maximum possible amount, the cell with the largest reduced cost is chosen. This affects the selection of the cell with the highest cost, whereas it should choose the cell with the smallest reduced cost to ensure that the obtained cost is minimized. This aligns the dual problem of the transportation problem is sought, and the optimal solution is achieved if $R_i + K_j - c_{ij} \geq 0$ or $c_{ij} - \left(R_i + K_j\right) \leq 0$ in the modified distribution method [14] [15]. Here, the selection of the incoming cell in the modified distribution method is based on the primal dual relationship of the simplex method. Therefore, this research will revise this method by adding an initial feasible solution determination stage and refining the table revision stage. The modified Exponential Approach method includes all three stages of the transportation method, namely the initial feasible solution, table revision, and optimization testing, making it a direct method.

The aim of this research is to find the optimal solution for vaccine distribution problems by applying the modified exponential approach method. The research begins by transforming the transshipment problem into a transportation problem. The existing problem is structured into a transportation model. Next, an initial feasible solution is sought, and its optimality is tested. If it is optimal, then the calculations are complete. If not, the table is revised, and the optimality test is repeated.

2. RESEARCH METHODS

PT. Indofarma Global Medika (PT. IGM) is a subsidiary of Indofarma, specializing in the distribution of medicines and medical devices. The company was established on January 4, 2000, with 99.99% of its shares held by Indofarma and the remaining shares held by the Indofarma employee cooperative. This company, which has been operating for over 20 years, has 29 branch offices spread across Indonesia. One of its branches in Indonesia is the West Kalimantan branch. In this research, the issue of vaccine distribution at PT IGM is examined. The literature study involved collecting information and data from PT IGM, various reference books, the internet, and other sources related to the research. Additionally, observations and interviews were conducted to obtain primary data. This data includes vaccine inventory data, vaccine demand, costs incurred by each source during distribution, distribution destinations, and distribution sources. From this data, a mathematical model was created, which includes determining decision variables, constraint functions, objective functions, and a transportation model. This model was then solved using the transportation method.

Every month, PT. Indofarma Global Medika distributes vaccines to several cities in West Kalimantan and outside West Kalimantan, namely Pontianak, Ketapang, Bengkayang, Singkawang, Sintang, Kubu Raya, and Jakarta. In March, there were 670,000 units of goods at PT. Indofarma Global Medika that were to be distributed to Pontianak, Ketapang, Singkawang, Sintang, Bengkayang, and Kubu Raya. Each destination requires 151,000 units, 40,000 units, 55,000 units, 45,000 units, 69,000 units and 310,000 units. Shipments can be made directly or through CV. XYZ. The vaccine distribution network can be seen in **Figure 1**.

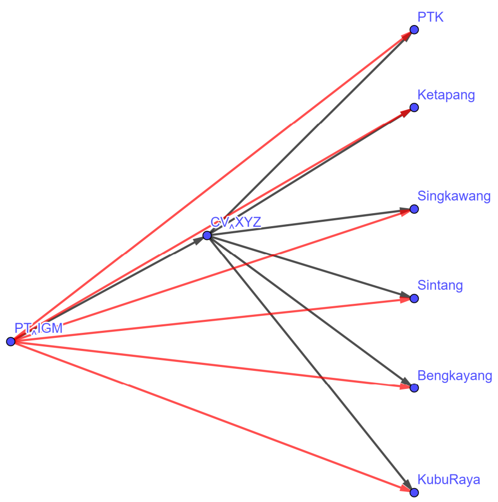

Figure 1. PT. IGM Vaccine Distribution Network.

In **Figure 1** the red line represents the direct distribution of vaccines from PT IGM to several cities in West Kalimantan. Meanwhile, the blue arrows indicate the distribution of goods from PT. IGM through CV. XYZ.

The shipping cost per unit from the source to the destination or intermediary is shown in **Table 1.**

Table 1. Vaccine Distribution from Source to Destination

	CV XYZ	PTK	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya
PT IGM	5,000	6,000	340,000	90,000	172,000	112,000	12,000
CV XYZ	0	15,000	40,000	55,000	45,000	69,000	20,000

Source: PT Indofarma Global Medika Vaccine Shipping Cost Data, 2023

Based on **Table 1**, the distribution costs of vaccines from PT. IGM to CV. XYZ, PTK, Ketapang, Singkawang, Sintang, Bengkayang and kubu Raya are Rp 5,000; Rp 6,000; Rp 340,000; Rp 90,000; Rp 172,000; Rp 112,000 and Rp 12,000, respectively. The vaccine distributions costs from CV XYZ to CV. XYZ, PTK, Ketapang, Singkawang, Sintang, Bengkayang dan Kubu Raya are Rp 0; Rp 15,000; Rp 40,000; Rp 55,000; Rp 45,000; Rp 69,000; and Rp 20,000 respectively.

The Modified Exponential Approach method is related to the determination of the exponent penalty associated with the minimum shipping cost. This method proposes first checking whether each row and column has a zero element. Additionally, in the optimal solution determination stage, cell values are allocated with the maximum possible amount by selecting the cell with the smallest reduced cost. The steps in the Modified Exponential Approach method are as follows [4] [13].

1. Form a transportation model (table) from the given transportation problem. If the total demand is equal to the total supply, or $\sum_{j=1}^{n} b_j = \sum_{i=1}^{n} a_i$ then the table is balanced, and the process continues to Step 3. However, if not, there are two possibilities, either $\sum_{j=1}^{n} b_j < \sum_{i=1}^{n} a_i$ or $\sum_{j=1}^{n} b_j > \sum_{i=1}^{n} a_i$. If $\sum_{j=1}^{n} b_j < \sum_{i=1}^{n} a_i$, add a dummy column and if $\sum_{j=1}^{n} b_j > \sum_{i=1}^{n} a_i$, add a dummy row. Then continue the process to Step 2.
2. If a dummy column (row) is added, subtract the minimum entry c_{ij} in each column (row). Replace the cost in the dummy column (row) with the largest entry c_{ij} from the reduced table. If a dummy column is added, proceed to Step 3 and then Step 5. If a dummy row is added, proceed to Step 4 and then Step 5.
3. Subtract the minimum entry c_{ij} in each row from every entry c_{ij} in the transportation table.
4. Subtract the minimum entry c_{ij} in each column from every entry c_{ij} in the transportation table.
5. Check if each row and column already have a zero element? If yes, proceed to Step 6. If not, check whether the row (column) without a zero element is a row; if yes, perform Step 3 and then Step 6. If not, perform Step 4 and then Step 6.
6. Check if each column b_j is less than or equal to a_i. This is done by looking at the column with reduced cost entries c_{ij} equal to zero. Then, check if each row a_i is less than or equal to b_j by looking at the row with reduced cost entries c_{ij} equal to zero. If this condition is met, proceed to Step 9. If not, proceed to Step 7.
7. Draw the minimum number of horizontal and vertical lines through all rows and columns with zero cost entries c_{ij}. This is done without covering any non-zero cost entries c_{ij} that do not meet Step 6.
8. Select the smallest entry c_{ij} in cells not covered by lines. Subtract the selected c_{ij} from all uncovered entries c_{ij} and add the selected c_{ij} to all entries c_{ij} at the intersection of two lines. Return to Step 6.
9. Assign exponent penalties (number of zero entries c_{ij} from each row i and column j), denoted as by 0_n, to cells with zero cost entries c_{ij}, excluding the c_{ij} whose exponent penalty is to be determined. Repeat Step 9 for all zero cost entries c_{ij} in the table. Then proceed to Step 10.
10. Allocate cell values with the maximum possible amount from the supply row and demand column, denotes as x_{ij}, with $x_{ij} = \min(a_i, b_j)$, considering the following allocation priorities.
    a. Zero cost entries c_{ij} with an exponent penalty of 1
    b. Zero cost entries c_{ij} with an exponent penalty of 2
    c. Select the cell with the smallest reduced cost entry c_{ij}. Allocate on row i or column j with the minimum c_{ij} entry when reduced. Except for dummy c_{ij} entries.
11. Check if $a_i = 0$ and $b_j = 0$ for the selected cell. If yes, proceed to Step 12. If not, check if $a_i = 0$? if yes, find a zero element in column j and allocate $x_{ij} = b_j$. If not, find a zero element in row i and allocate $x_{ij} = a_i$. Then proceed to Step 12.
12. Check if $a_i = 0$ for each i and $b_j = 0$ for each j. If yes, the solution is optimal and process to Step 13. If not, proceed to Step 5.

13. Calculate the optimal cost and finish.

In the Modified Exponential Approach, there are three stages in the transportation method: determining the initial feasible solution, table revision and optimality test. The initial feasible solution stage is found in steps 1 to 5. The table revision stage is found in step 6 to 8 and steps 9 to 12 constitute the optimality test stage.

3. RESULTS AND DISCUSSION

PT. Indofarma Global Medika (PT IGM) is a subsidiary of Indofarma, specializing in the distribution of medicines and medical devices. The problem faced by PT. IGM is a transshipment problem. There is one source point, PT IGM; one intermediate point CV. XYZ; and six destination points: PTK, Ketapang, Singkawang, Sintang, Bengkayang, and Kubu Raya. The transshipment problem needs to be transformed into a transportation problem. In this transformation, the source and intermediate points act as sources, and the intermediate and destination points act as destinations. Therefore, PT IGM and CV XYZ are considered sources, while CV.XYZ, PTK, Ketapang, Singkawang, Sintang, Bengkayang, and Kubu Raya are considered destinations. Subsequently, the Modified Exponential Approach method is applied.

Solution by the Modified Exponential Approach Method

Step 1 Formulate the given transshipment problem into a transportation model.

Decision variable:

x_{11} = quantity of vaccines distributed from PT IGM to CV XYZ,
x_{12} = quantity of vaccines distributed from PT IGM to CV PTK,
x_{13} = quantity of vaccines distributed from PT IGM to Ketapang,
x_{14} = quantity of vaccines distributed from PT IGM to Singkawang,
x_{15} = quantity of vaccines distributed from PT IGM to Sintang,
x_{16} = quantity of vaccines distributed from PT IGM to Bengkayang,
x_{17} = quantity of vaccines distributed from PT IGM to Kubu Raya,
x_{21} = quantity of vaccines distributed from CV XYZ to CV XYZ,
x_{22} = quantity of vaccines distributed from CV XYZ to PTK,
x_{23} = quantity of vaccines distributed from CV XYZ to Ketapang,
x_{24} = quantity of vaccines distributed from CV XYZ to Singkawang,
x_{25} = quantity of vaccines distributed from CV XYZ to Sintang,
x_{26} = quantity of vaccines distributed from CV XYZ to Bengkayang,
x_{27} = quantity of vaccines distributed from CV XYZ to Kubu Raya.

Objective Function:

Based on **Table 1,** the distribution cost is the total of the product of the distribution from the source to the destination multiplied by the number of vaccines distributed from the source to the destination.

Minimize $Z = 5{,}000x_{11} + 6{,}000x_{12} + 340{,}000x_{13} + 90{,}000x_{14} + 172{,}000x_{15} + 112{,}000x_{16} + 12{,}000x_{17} + 15{,}000x_{22} + 40{,}000x_{23} + 55{,}000x_{24} + 45{,}000x_{25} + 69{,}000x_{26} + 20{,}000x_{27}$

Constraints on the source:

$x_{11} + x_{12} + \cdots + x_{17} \leq 670{,}000$ (Vaccine Capacity at PT. IGM)

$x_{21} + x_{22} + \cdots + x_{27} \leq 670{,}000$ (Vaccine Capacity at CV. XYZ)

Constraints on the destination:

$x_{11} + x_{21} \geq 670{,}000$ (Vaccine demand in CV. XYZ)

$x_{12} + x_{22} \geq 151{,}000$ (Vaccine demand in Pontianak)

$x_{13} + x_{23} \geq 40{,}000$ (Vaccine demand in Ketapang)

$x_{14} + x_{24} \geq 55{,}000$ (Vaccine demand in Singkawang)

$x_{15} + x_{25} \geq 45{,}000$ (Vaccine demand in Sintang)

$x_{16} + x_{26} \geq 69{,}000$ (Vaccine demand in Bengkayang)

$x_{17} + x_{27} \geq 310{,}000$ (Vaccine demand in Kubu Raya)

Non-negative constraints:
$x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17} \geq 0$

The model is then organized into the Transportation Table in Table 2.

Table 1. Transportation Problem model

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	5,000	6,000	340,000	90,000	172,000	112,000	12,000	670,000
CV. XYZ	0	15,000	50,000	30,000	40,000	35,000	20,000	670,000
Demand	670,000	151,000	40,000	55,000	45,000	69,000	310,000	1,340,000

In Table 2, we can see that $\sum_{j=1}^{n} b_j = \sum_{j=1}^{n} a_i$. Therefore, the process continues to step 3.
Step 3. Subtract the minimum entry c_{ij} in each row from every entry c_{ij} in the transportation table.

Table 2. Reduction of Row Entries

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0	1,000	335,000	85,000	167,000	107,000	7,000	670,000
CV. XYZ	0	15,000	50,000	30,000	40,000	35,000	20,000	670,000
Demand	670,000	151,000	40,000	55,000	45,000	69,000	310,000	1,340,000

Step 4 Subtract the minimum entry c_{ij} in each column from every entry c_{ij} in the transportation table.

Table 4. Reduction on the Column

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0	0	285,000	55,000	127,000	72,000	0	670,000
CV. XYZ	0	14,000	0	0	0	0	13,000	670,000
Demand	670,000	151,000	40,000	55,000	45,000	69,000	310,000	1,340,000

Iteration 21

Step 5 Check if each row and column have a zero element. From Table 4, we can see that each row and column already have a zero element, therefore the process continues to Step 6.
Step 6 Check if for each column b_j is less than or equal to a_i, Next, check if each row a_i is less than or equal to b_j in the column by looking at the row where the reduced c_{ij} entry is zero.

$$670{,}000 < 670{,}000+670{,}000$$
$$151{,}000 < 670{,}000$$
$$40{,}000 < 670{,}000$$
$$55{,}000 < 670{,}000$$

$$\begin{array}{lcl} 45{,}000 & < & 670{,}000 \\ 69{,}000 & < & 670{,}000 \\ 310{,}000 & < & 670{,}000 \\ 670{,}000 & < & 670{,}000 + 151{,}000 + 310{,}000 \\ 670{,}000 & < & 670{,}000 + 40{,}000 + 55{,}000 + 45{,}000 + 69{,}000 \end{array}$$

Since all these requirements are met, the work proceeds to step 9.

Step 9 Assigning the exponent penalties (number of zero entries c_{ij} from each row i and column j), denoted as by 0_n, to cells with zero cost entries c_{ij}, excluding the c_{ij} whose exponent penalty is to be determined. Repeat Step 9 for all zero cost entries c_{ij} in the table. Then proceed to Step 10.

Table 5. Assigning the Exponent Penalty

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0_3	0_2	285,000	55,000	127,000	72,000	0_2	670,000
CV. XYZ	0_5	14,000	0_4	0_4	0_4	0_4	13,000	670,000
Demand	670,000	151,000	40,000	55,000	45,000	69,000	310,000	1,340,000

Step 10 Allocate the cell values with the maximum possible amount from the supply row and demand column, denoted as x_{ij}, with $x_{ij} = \min\,(a_i,\ b_j)$, considering the following allocation priorities. Based on Step 9, there are the same penalty exponents 0_2 at c_{12}, c_{17}. Thus, the selection is based on the cell with the smallest reduced entry c_{ij} , denoted as (i, j). With $c'_{12} = 151{,}000$, and $c'_{17} = 310{,}000$. Therefore, it is allocated at the entry $x_{12} = \min\,(670{,}000; 151{,}000) = 151{,}000$.

Table 6. Allocation at Exponent Penalty

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0_3	0_2	285,000	55,000	127,000	72,000	0_2	519,000
		151,000						
CV. XYZ	0_5	14,000	0_4	0_4	0_4	0_4	13,000	670,000
Demand	670,000	0	40,000	55,000	45,000	69,000	310,000	1,340,000

Step 11 In the selected cell, $a_1 = 519{,}000$ and $b_2 = 0$, then find the element 0 in row i and allocate $x_{ij} = a_i$. Thus $x_{17} = \min\,(519{,}000;\ 310{,}000) = 310{,}000$ dan $x_{11} = \min\,(209{,}000;\ 670{,}000) = 209{,}000$

Table 7. Checking on a_i and b_j That Have been Allocated

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0_3	0_2	285,000	55,000	127,000	72,000	0_2	0
	209,000	151,000					310,000	
CV. XYZ	0_5	14,000	0_4	0_4	0_4	0_4	13,000	670,000
Demand	461,000	0	40,000	55,000	45,000	69,000	0	1,340,000

And the process continues to step 12.

Step 12 Since the condition $a_i = 0$ for each i and $b_j = 0$ for each j is not yet fulfilled, then proceed to the next iteration, starting from Step 5.

Iteration 2

Step 5 Check whether each row and column have zero elements? In Table 7, we can see that the rows and columns have been fulfilled, so proceed to the next step.

Step 6 Check again if for each column b_j is less than or equal to a_i, Next, check if each row a_i is less than or equal to b_j in the column by looking at the row where the reduced c_{ij} entry is zero.

$$461{,}000 < 670{,}000$$
$$40{,}000 < 670{,}000$$
$$55{,}000 < 670{,}000$$
$$45{,}000 < 670{,}000$$
$$69{,}000 < 670{,}000$$

$$670{,}000 < 461{,}000 + 40{,}000 + 55{,}000 + 45{,}000 + 69{,}000$$

Since all these requirements are met, the work proceeds to step 9.

Iteration 3

Step 9 Assigning the exponent penalties (number of zero entries c_{ij} from each row i and column j), denoted as by 0_n, to cells with zero cost entries c_{ij}, excluding the c_{ij} whose exponent penalty is to be determined. Repeat Step 9 for all zero cost entries c_{ij} in the table. Then proceed to Step 10.

Table 8. Determination of Exponent Penalty

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0_3	0_2	285,000	55,000	127,000	72,000	0_2	0
	209,000	151,000					310,000	
CV. XYZ	0_4	14,000	0_4	0_4	0_4	0_4	13,000	670,000
Demand	461,000	0	40,000	55,000	45,000	69,000	0	1,340,000

Step 10 in Table 8, there are no exponential penalties of 0_1 or 0_2, so it is chosen based on the cell that has the smallest reduced entry c_{ij} and is named (i, j). $c'_{21} = 0$, $c'_{23} = 50{,}000$, $c'_{24} = 30{,}000$, $c'_{25} = 40{,}000$ and $c'_{26} = 35{,}000$. Therefore, it is allocated to the entry $x_{21} = \min(670{,}000; 461{,}000) = 461{,}000$.

Table 9. Allocation at Exponent Penalty

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0_3	0_2	285,000	55,000	127,000	72,000	0_2	0
	209,000	151,000					310,000	
CV. XYZ	0_4	14,000	0_4	0_4	0_4	0_4	13,000	209,000
	461,000							
Demand	0	0	40,000	55,000	45,000	69,000	0	1,340,000

Step 11 In the selected cell $a_2 = 209{,}000$ and $b_1 = 0$ f, then find the element 0 in row i and allocate $x_{ij} = a_i$. Thus $x_{23} = 40{,}000$, $x_{24} = 55{,}000$, $x_{25} = 45{,}000$, and $x_{26} = 69{,}000$.

Step 12 Since $a_i = 0$ for each i and $b_j = 0$ for each j, then the solution is optimal, and the process continues to Step 13.

Table 10. Checking on a_i and b_j that have been Allocated

Sources	Destinations							Supply
	CV.XYZ	Pontianak	Ketapang	Singkawang	Sintang	Bengkayang	Kubu Raya	
PT. IGM	0_3	0_2	285,000	55,000	127,000	72,000	0_2	0
	209,000	151,000					310,000	
CV. XYZ	0_4	14,000	0_4	0_4	0_4	0_4	13,000	0
	461,000		40,000	55,000	45,000	69,000		
Demand	0	0	0	0	0	0	0	1,340,000

Step 13 Calculate the optimal cost.

Thus, $x_{11} = 209{,}000$, $x_{12} = 151{,}000$, $x_{13} = 0$, $x_{14} = 0$, $x_{15} = 0$, $x_{16} = 0$, $x_{17} = 310{,}000$, $x_{21} = 461{,}000, x_{22} = 0, x_{23} = 40{,}000, x_{24} = 55{,}000$, $x_{25} = 45{,}000$, $x_{26} = 69{,}000$, $x_{27} = 0$. And the transportation cost is

$$\begin{aligned} Z = {} & 5{,}000x_{11} + 6{,}000x_{12} + 340{,}000x_{13} + 90{,}000x_{14} + 172{,}000x_{15} + 112{,}000x_{16} + 12{,}000x_{17} \\ & + 15{,}000x_{22} + 40{,}000x_{23} + 55{,}000x_{24} + 45{,}000x_{25} + 69{,}000x_{26} + 20{,}000x_{27} \\ & = 13{,}536{,}000{,}000 \end{aligned}$$

Based on Table 10, it can be seen that PT IGM distributes vaccines to CV. XYZ, Pontianak and Kuburaya with amounts of 209,000 units, 151,000 units and 310,000 units, respectively. CV. XYZ distributes vaccines to Ketapang, Singkawang, Sintang and Bengkayang with amounts of 40,000 units, 55,000 units, 45,000 units, and 69,000 units, respectively. The results align with PT Indofarma Global Medika's current practices, where company-owned vehicles are used for Pontianak and Kubu Raya. For Ketapang, Singkawang, Sintang and Bengkayang, delivery services are provided by CV. XYZ. As a comparison, using QM for Windows software also resulted in the same vaccine distribution allocation.

The Modified Exponential Approach offers a higher level of optimization compared to other methods, although with greater complexity. The zero Neighbouring [16] [17], Zero Suffix [18] [19], Zero point methods [4] [20] provide quick and reasonably good solutions for simple transportation problems but may be less optimal for more complex cases. The ASM method [8] [9] offers quick and easily implemented solutions but may require further optimization to achieve optimal results. Additionally, this method includes three stages in the transportation method: determining the initial feasible solution, table revision, and the optimality test. The modified Exponential Approach is suitable for cases with many variables and constraints, requiring a more complex approach to optimize allocation.

4. CONCLUSION

The vaccine distribution problem at PT Indofarma Global Medika was solved using the Modified Exponential Approach Method, resulting in the optimal solution: PT IGM distributes vaccines to CV. XYZ, Pontianak and Kuburaya respectively in amounts of 209,000 units, 151,000 units and 310,000 units. CV. XYZ acts as an intermediary for PT. IGM in distributing the vaccines. CV. XYZ distributes vaccines to Ketapang, Singkawang, Sintang and Bengkayang respectively in amount of 40,000 units, 55,000 units, 45,000 units, and 69,000 units. Therefore, the transportation cost is 13,536,000000.

These results are consistent with PT Indofarma Global Medika's current practices, where the company uses its own vehicles for Pontianak and Kubu Raya, and relies on CV. XYZ for deliveries to Ketapang, Singkawang, Sintang and Bengkayang. As a comparison, using QM for Windows software also resulted in the same vaccine distribution allocation.

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