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STOCK PRICE PREDICTION AND SIMULATION USING GEOMETRIC BROWNIAN MOTION-KALMAN FILTER: A COMPARISON BETWEEN KALMAN FILTER ALGORITHMS

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ABSTRACT

Stocks have high-profit potential but also have high risk. Many people have ways to forecast stock prices. The Geometric Brownian Motion (GBM) method forecasts stock prices. The data used in this study are closing stock price data from July 1, 2021 to August 31, 2021 taken from Yahoo! Finance. The stocks used in this research are Bank Rakyat Indonesia (BBRI), Indofood Sukses Makmur (INDF), and Telkom Indonesia (TLKM). A strategy is carried out to improve prediction accuracy by utilising the Kalman Filter (KF). This research will compare the mean absolute percentage error (MAPE) value between GBM-KF, which was manually computed and computed using the Python library. As an example of this research, for BBRI stock, the high GBM MAPE value of 9.02% can be reduced to 3.52% with manually computed GBM-KF and 3.68% with Python library computed GBM-KF. Similarly, INDF and TLKM stocks are showing a significant reduction in MAPE values to deficient levels in some cases. The GBM-KF method employing manual computing may enhance the overall precision of stock price forecasting. Future research may enhance this study by using the GBM-KF model on alternative financial instruments, integrating supplementary market data, or evaluating its efficacy under extreme market conditions.

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1. INTRODUCTION

Investment is the process of buying assets or financial instruments to generate profits or returns in the future. Investments can be made in various forms, including stocks, bonds, property, commodities, and currencies. Investment can be said to be a commitment of a sum of money or other resources made today in the hope of obtaining benefits in the future [1]. Investing aims to bring more significant profits than keeping money in the bank. Investment can also help a person achieve long-term financial goals, such as retirement and children's education. Still, investment also has risks, especially capital loss or not getting the expected return. Along with increasing financial literacy, stock investment is one of the most widely practised forms of investment [2].

Stocks are among the most popular investment options because of their high profit potential. However, stock investment also has a high risk because stock prices can fluctuate dramatically and suddenly depending on external factors that are difficult to predict [3]. Stock price is the value or market price of the stock. Stock prices are calculated based on supply and demand in the stock market, where buyers and sellers meet to buy or sell stocks. External factors such as global economic conditions, government policies, and political and social events can also affect stock prices.

Shareholders benefit from their stock investments through capital gains and dividends. Capital gains are the difference between a purchase and a selling price, while dividends are the distribution of company profits to shareholders. Stock gains can also depend on the company's performance and management decisions. Stock price prediction is crucial for investors and traders as it can help them make the right investment decisions. By predicting future stock prices, investors and stock traders can make better investment decisions and optimise their profits.

Many researchers use different approaches and methods to predict stock prices. One of the standard approaches is using machine learning, such as LSTM and multilayer perceptron. Gao et al. found that both LSTM and GRU models can predict stock prices efficiently; the models do not excel the other [4]. Azizah found that stock prediction by using GBM is better than multilayer perceptron [5]. Researchers use an approach called Geometric Brownian Motion (GBM). GBM is one of the widely used stochastic models to predict stock prices and other financial instruments. This model assumes that stock prices or other financial assets will fluctuate randomly in the short term but tend to rise or fall in the long term.

Many researchers have carried out research on stock price prediction in both mathematics and economics. Much research has shown that the GBM method is suitable for predicting stocks for a short time [6]. Research has simulated stock prices for several companies in Indonesia [7], Australia [8], and Malaysia [9]. The result shows that the actual stock price is the same as the predicted price with the GBM with the MAPE of less than 20%. Other research has examined the prediction of stock index prices with the GBM method [10]. By just applying the sole GBM, the MAPE obtained from the simulation is still considered as high.

The Geometric Brownian Motion (GBM) model, despite its widespread use, has limitations, such as the assumption of constant volatility and the inability to adapt to external market shocks. Such constraints can lead to errors, especially during periods of heightened market volatility or uncertainty. Researchers have investigated the incorporation of adaptive models such as the Kalman Filter to address these difficulties. The Kalman Filter provides recursive updates to model parameters by incorporating new observations, thereby improving prediction accuracy [11].

This research contrasts hand calculations of the GBM-Kalman Filter (GBM-KF) with computations executed via a Python package. Manual calculation promotes transparency in the modelling process. It allows for greater flexibility in tailored modifications, while Python packages such as PyKalman provide efficient and scalable solutions for large-scale or real-time prediction scenarios.

The Kalman filter is capable of adaptively updating the model parameters based on new observations, thereby enhancing the accuracy of stock price predictions [7], [12], [13]. In 2023, Mustika utilised the GBM integrated with the Kalman Filter technique, referred to as GBM-KF. He contrasted the Modification of GBM and ARIMA with the Kalman filter. The results demonstrate that the MAPE form GBM-KF is better than ARIMA-KF. Both outcomes remain highly accurate [14]. Maulidya also examined the alteration of GBM by imposing limitations on the Kalman Filter. The findings indicate that the MAPE of the restricted Kalman Filter is below 10% [15]. Machine learning, GBM, and modified GBM have been employed to forecast stock values, yielding encouraging outcomes. This research compares the use of the Python library with manual computation for predicting three stocks using the GBM-KF method.

2. RESEARCH METHODS

2.1 Data

The data used in this study is closing stock price data, of which three well-known stocks in Indonesia have their respective fields. In this study, price data will be used and processed as research material and used to validate the resulting prediction data. The data are taken from the Yahoo! Finance web portal (https://finance.yahoo.com), which can be presented in the following **Table 1**.

Table 1. Table of Stock Data

No	Name of Stock	Data		Total
		Start	End	
1	Bank Rakyat Indonesia (BBRI)	01/07/2021	31/08/2021	41
2	Indofood Sukses Makmur (INDF)	01/07/2021	31/08/2021	41
3	Telkom Indonesia (TLKM)	01/07/2021	31/08/2021	41

The choice of date has become even more complex due to the challenging predictions during the pandemic, caused by high uncertainty, unpredictable economic fluctuations, and the ever-changing impact of the COVID-19 pandemic on global financial markets.

2.2 Stage of Data Analysis

The steps of data analysis carried out in this study are as follows:

1. Calculation of return value for each stock

   The formula for finding stock price returns is

   $$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \tag{1}$$

   Where R_t is the return of stock on day t, P_t is the closing price of a stock on day t, and P_{t-1} is the price of closing stock on day $t-1$.

2. Normality testing of stock data

   The normality test aims to test whether the dependent and independent variables have a normal distribution in the regression model and whether the residual values have a normal distribution. A good regression model has typical or close-to-normal residual values. The normality test used in this research is Kolmogorov Smirnov, namely with the criteria that if significant Kolmogorov Smirnov < 0.05 then the data is not normal, conversely if significant Kolmogorov Smirnov > 0.05, then the data is normal.

3. Calculation of estimation parameters

   a. Volatility

   Volatility measures changes in a security or market index's price over time. The higher the stock volatility, the more frequently the stock fluctuates, and vice versa if the value is low. It shows that the security price is relatively stable.

   $$\sigma = \frac{s}{\sqrt{t}} \tag{2}$$

   Where σ is volatility, s is standard deviation, and t is the total of time

b. Standard deviation

In the volatility formula, there is a standard deviation which can be calculated with the following formula:

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(R_k - \bar{R})^2} \tag{3}$$

Where s is the standard deviation, n is the sum of return total, $R_k$is the return value of k, and $\bar{R}$ is the average of return.

c. Drift

Drift is a term used to describe a stock's expected annualised rate of return. To find the stock drift as follows:

$$\mu = \frac{\bar{R}}{t} + \frac{\sigma^2}{2} \tag{4}$$

Where μ is drift, and σ^2 is volatility squared.

4. Modelling with Geometric Brownian Motion (GBM)
Geometric Brownian Motion (GBM) is a stochastic process that models stock price movements. GBM is the result of a combination of Wiener and exponential processes, so it can be used to model stock price movements with characteristics as a stochastic process that follows a normal distribution and has an exponential upward trend [16] . GBM process is used in stock price prediction by assuming that the movement of stock prices over a certain period is a random process described by equation [17]:

$$dS = \mu S dt + \sigma S dW \tag{5}$$

Where dS is a stock price change, dt is the time interval between observations, and dW is the change in the Wiener process. The above model can also be written as follows:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{6}$$

If the equation is written in terms of long-time intervals between consecutive values, the following equation is obtained:

$$\frac{dS(t)}{S(t)} = d\big(lnS(t)\big) = lnS(t) - lnS(t-1) = \ln\left(\frac{S(t)}{S(t-1)}\right) \tag{7}$$

$$\ln\left(\frac{S(t)}{S(t-1)}\right) = \mu dt = \sigma dW(t) \tag{8}$$

Equation (8) is derived from the use of Ito's lemma, which is important for modelling stochastic processes. This equation serves as the foundation for forecasting stock price fluctuations by integrating both deterministic and stochastic elements. For each function $G(S,t)$ from S and t where X satisfies the following stochastic differential equation:

$$dX = adt + bdW(t) \tag{9}$$

For some constants, a, b, and $dW(t)$ are the Brownian motions. Then, the Ito formula itself is defined as follows:

$$dG = \left(\frac{\partial G}{\partial S}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}b^2\right)dt + \frac{\partial G}{\partial S}dW \tag{10}$$

Then, to determine $G(t,S) = \ln\big(S(t)\big)$ in order to satisfy the GBM form, the equation is derived and entered into the Ito formula obtained:

$$d(\ln\big(S(t)\big)) = \left[\left(\frac{1}{S(t)}\right)\mu S(t) + \frac{1}{2}\left(-\frac{1}{S(t)^2}\right)\sigma^2 S(t)^2\right]dt + \left(\frac{1}{S(t)}\right)\sigma S(t)dW(t)$$

$$d(\ln\big(S(t)\big)) = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW(t)$$

$$\frac{S(t)}{S(t-1)} = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW(t) \tag{11}$$

If simplified, we will get a stock price estimation model in GBM as follows:

$$S_{t+1} = S_t \cdot e^{\left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma W t} \tag{12}$$

5. Prediction and Correction of the GBM-KF Model

The GBM process is combined with a Kalman filter to update stock price estimates based on recent stock price data and produce accurate stock price predictions [14]. Kalman Filter (KF) is an algorithm used to update the estimate of a variable that is not directly measured based on directly measured observation data [18]. KF is widely used in various applications, including stock price prediction. Here is the KF algorithm [13].

Table 2. Kalman Filter Algorithm

System Model and Measurement Model	
$x_{k+1} = A_k x_k + B_k u_k + G_k w_k$ $z_k = H_k x_k + v_k$ $x_0 \sim N(\bar{x}_0, P_{x_0})$ $w_k \sim N(0, Q_k)$ $v_k \sim N(0, R_k)$	
Initialisation	
$\hat{x}_0 = \bar{x}_0$ $p_0 = p_{x_0}$	
Prediction Stage	
Covariance Error	: $P_{k+1}^- = A_k P_k A_k^T + G_k Q_k G_k^T$
Estimation	: $\hat{x}_{k+1}^- = A_k x_k + B_k u_k$
Correction Stage	
Covariance Error	: $P_{k+1} = [(P_{k+1}^-)^{-1} + H_{k+1}^T R_{k+1}^{-1} H_{k+1}]^{-1}$
Estimation	: $\hat{x}_{k+1} = \bar{x}_{k+1} + P_{k+1} H_{k+1}^T R_{k+1}^{-1} (z_{k+1} - H_{k+1}\bar{x}_{k+1})$
If using Kalman Gain:	$K_{k+1} = P_{k+1}^- H_{k+1}^T (H_{k+1} P_{k+1}^- H_{k+1}^T + R_{k+1})^{-1}$
Covariance Error :	$P_{k+1} = [I - K_{k+1} H_{k+1}] P_{k+1}^-$
Estimation :	$\hat{x}_{k+1} = \hat{x}_{k+1}^- + K_{k+1}(z_{k+1} - H_{k+1}\hat{x}_{k+1}^-)$

In combination with GBM, GBM-KF is a mathematical model that can accurately predict stock price movements [13]. GBM-KF is one of the widely used stock price prediction methods. This method combines the Geometric Brownian Motion model and Kalman Filter to predict future stock prices [15]. In GBM-KF, the GBM model is used to model the stock price and the Kalman Filter is used to update the stock price estimate based on the latest stock price data. Thus, GBM-KF can produce more accurate and reliable stock price predictions [10].

6. Calculation of MAPE value on GBM and GBM-KF models

The model accuracy meter in this research uses Mean Absolute Percentage Error (MAPE), a method widely used to evaluate estimates that consider the influence of the actual value [5]. The MAPE formula can be calculated as follows:

$$MAPE = \frac{\sum\left|\frac{Y_t - F_t}{Y_t}\right|}{n} \times 100\% \tag{13}$$

Where Y_t is the value of testing data at time t, F_t is the value of estimation data at time t, and n is the total of testing data. The smaller the MAPE value, the more accurate the model [9]. Mean absolute percentage error (MAPE) is a measure of the average deviation between a model's predictions and actual values. A 20% MAPE indicates a 20% difference, indicating a 20% average deviation from actual values. A lower MAPE indicates a more accurate prediction, while a 0% MAPE indicates the exact prediction as the actual [19].

3. RESULTS AND DISCUSSION

Daily stock data movement fluctuates or experiences erratic increases and decreases, so it is necessary to calculate the return on the data to facilitate estimation.

Table 3. Stock Return

The t-th return	Return of BBRI	Return of INDF	Return of TLKM
$R_1 = \frac{P_1 - P_0}{P_0}$	0.01261	0.00382	-0.01286
$R_2 = \frac{P_2 - P_1}{P_1},$	-0.02281	-0.01901	-0.00977
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$R_{41} = \frac{P_{41} - P_{40}}{P_{40}}$	-0.00253	-0.05364	0.00000

The normality test is utilised to identify whether a random variable follows a normal distribution. This test uses the Kolmogorov-Smirnov test. The following are the results of the normality test on stock returns:

Table 4. Normality Test

No	Name of Stock	Statistical Test Value	p-Value
1	Bank Rakyat Indonesia (BBRI)	0.121	0.543
2	Indofood Sukses Makmur (INDF)	0.167	0.179
3	Telkom Indonesia (TLKM)	0.113	0.635

From the above results, it can be concluded that the p-value in the normality test, all three, is more significant than α, which is 0.05, so the data can continue to be processed because it is typically distributed.

Before the stock estimation modelling stage, it is necessary to calculate parameter estimates, drift and volatility in the GBM model. Based on the drift and volatility formulas in the previous discussion, the results of the parameter estimation value of the return are as follows:

Table 5. Estimation of Parameters

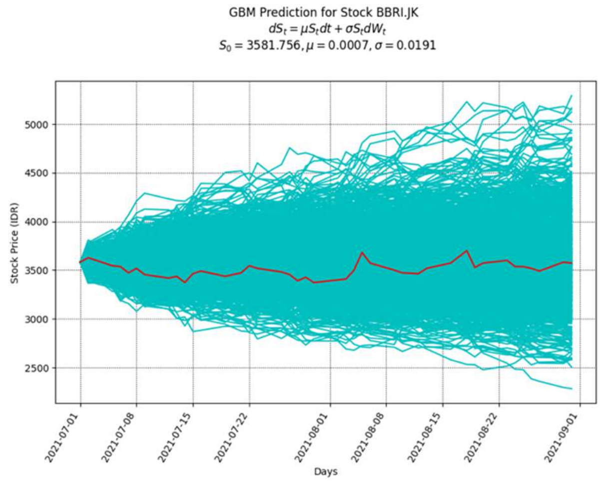
No	Name of Stock	σ	μ
1	Bank Rakyat Indonesia (BBRI)	0.0235	-0.0018
2	Indofood Sukses Makmur (INDF)	0.0157	-0.0012
3	Telkom Indonesia (TLKM)	0.0166	0.0025

Based on the above calculations, the estimated value of the volatility and drift parameters of the return of each stock is obtained. After knowing the parameter estimates, the value will be modelled for the value of shares for the next period. From the general equation of the GBM model, the following model is obtained:

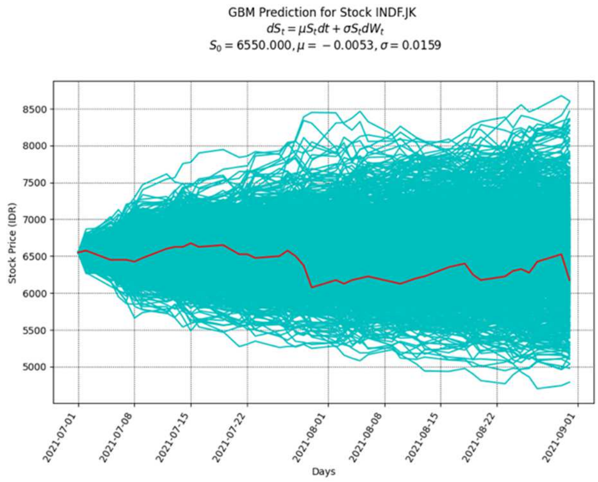
Table 6. Estimation Parameter

No	Name of Stock	GBM Model (S_{t+1})
1	Bank Rakyat Indonesia (BBRI)	$S_t \cdot e^{-0.002076125dt+0,0235Wt}$
2	Indofood Sukses Makmur (INDF)	$S_t \cdot e^{\left(-0,0012-\frac{1}{2}(0,0157)^2\right)dt+0,0157Wt}$
3	Telkom Indonesia (TLKM)	$S_t \cdot e^{0.00236222dt+0,0166Wt}$

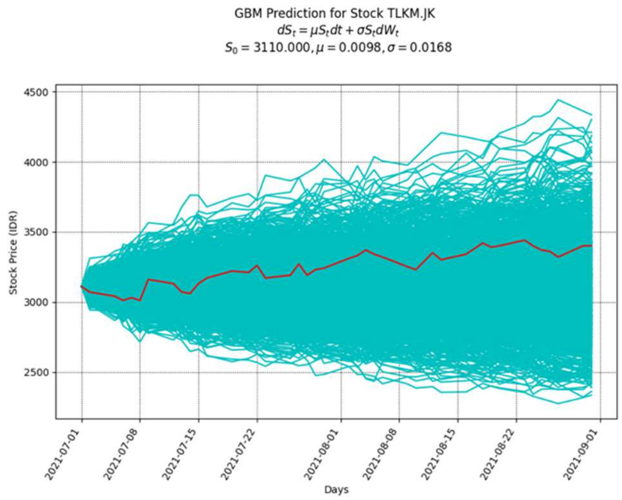
Here are the results of the GBM plot with 1000 iterations:



(a)



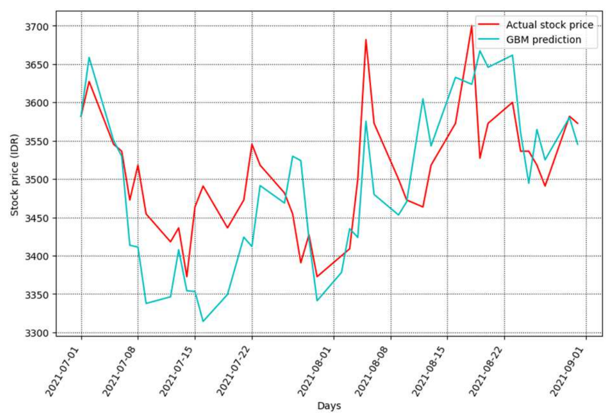
(b)



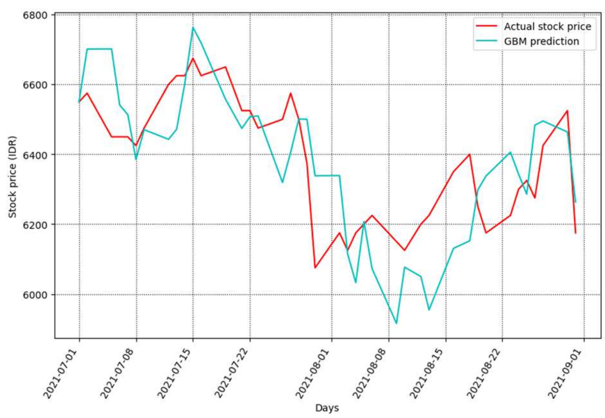
(c)

Figure 1. GBM Simulation for (a) BBRI, (b) INDF, and (c) TLKM

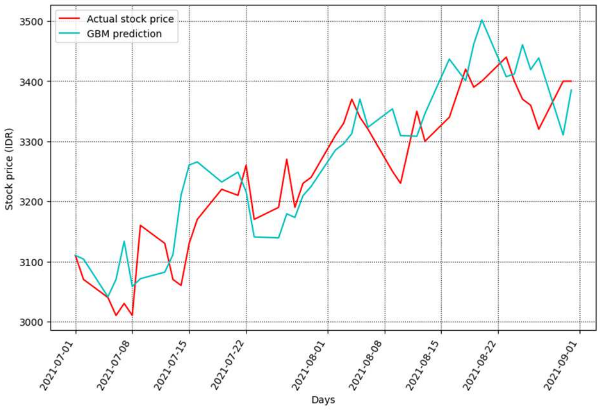
These trajectories occur for several reasons that have been explained previously. These many trajectories take one trajectory with the smallest MAPE value of all existing trajectories. Here is one of the trajectory images generated by GBM:



(a)



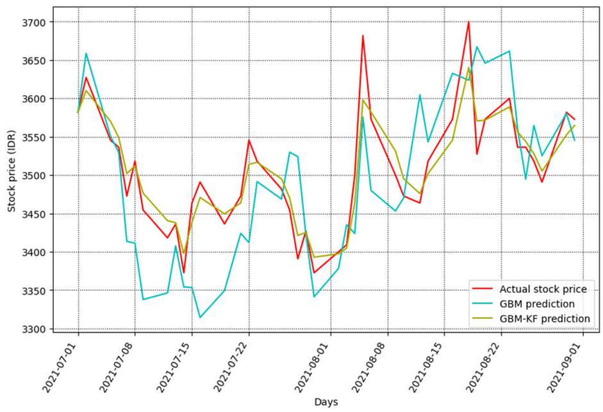
(b)



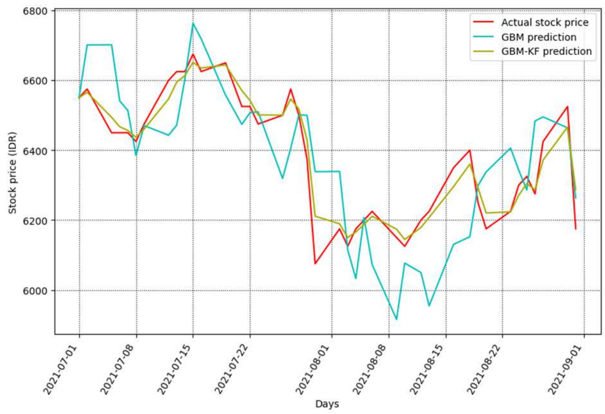
(c)

Figure 2. Comparison of actual and predicted stock prices results for 1000 iterations of (a) BBRI, (b) INDF, and (c) TLKM

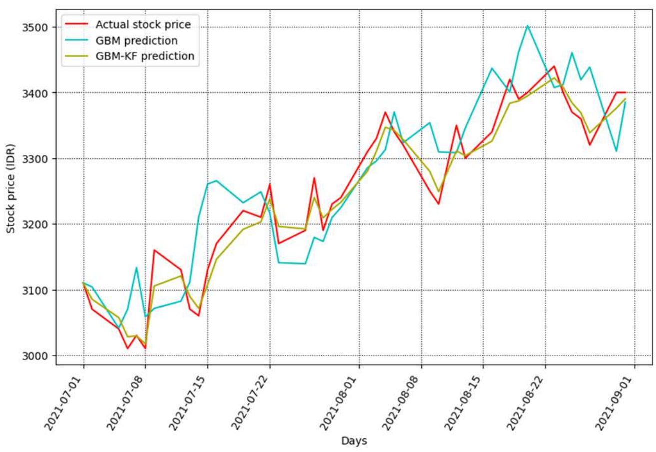
Next, the prediction process will be carried out by using the GBM Kalman Filter model. In this process, several manual stages will first be carried out to calculate the accuracy of the results. After the manual stages are completed, the prediction program will be implemented using Python programming. The automatic running process will provide stock price predictions based on the previously prepared GBM-KF model. Furthermore, to measure the accuracy of the prediction, the predicted data will be evaluated using the Mean Absolute Percentage Error (MAPE) method, which also provides an overview of how close the predicted value is to the actual value. The graph below represents the results of the GBM and manual GBM-KF (manually computed GBM-KF):



(a)



(b)



(c)

Figure 3. Comparison of actual, predicted stock prices using GBM and manual GBM-KF results for (a) BBRI, (b) INDF, and (c) TLKM

Then, we use the GBM method, which is provided by Python (library GBM-KF). The Pykalman library gives the Python library. The comparison of the graphs is as follows:

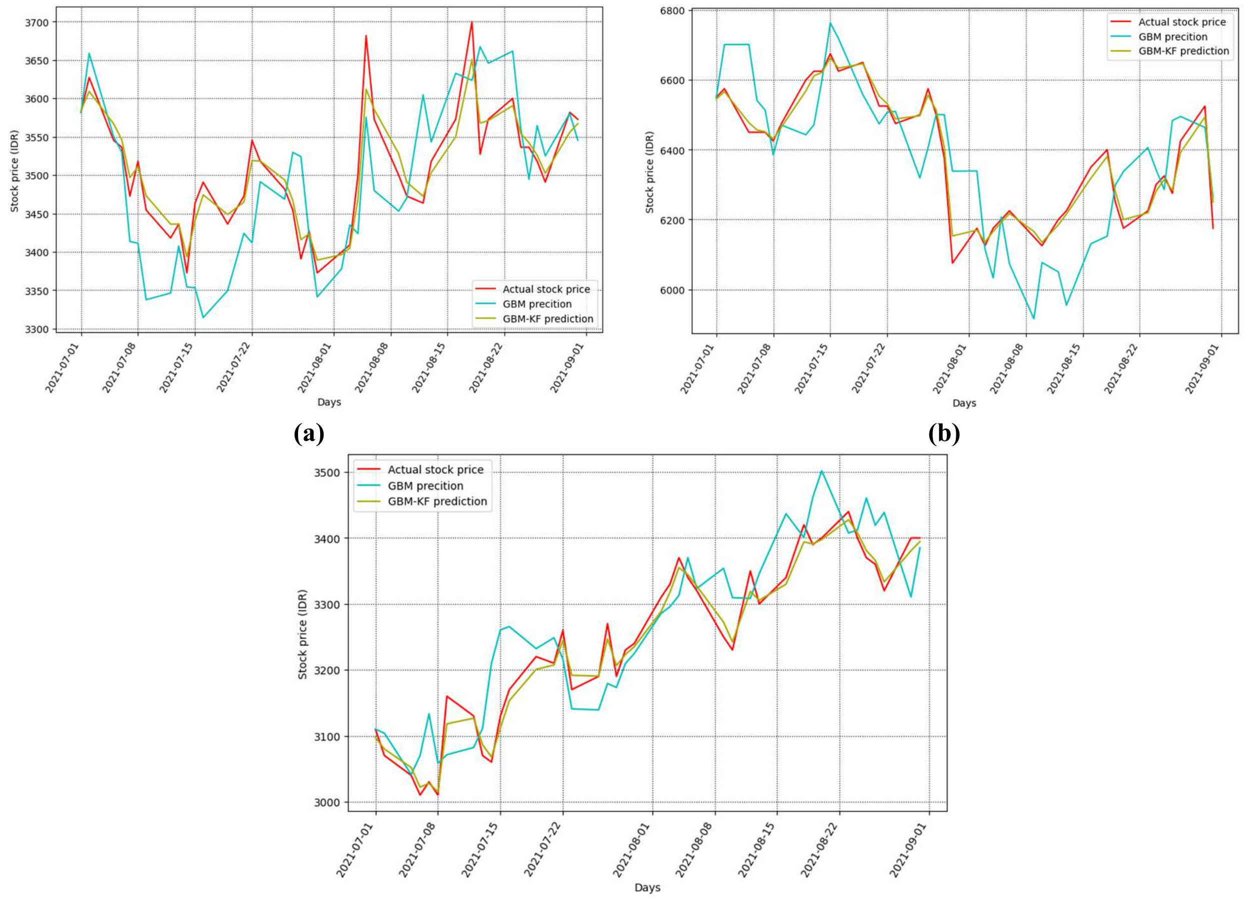


Figure 4. Comparison of actual, predicted stock prices using GBM and library GBM-KF results for (a) BBRI, (b) INDF, and (c) TLKM

The graphs above show a noticeable difference, where the GBM Method results are close to the actual data, while the GBM-KF predictions align more closely with observed stock prices compared to standalone GBM. The next step involves the simulated data results, which are then evaluated for accuracy using the MAPE method. The resulting MAPE results are as follows in per cent.

Table 7. Average MAPE of GBM and GBM-KF

No	Name of Stock	Average MAPE of GBM	Average MAPE of GBM-KF Manual	Average MAPE of GBM-KF Python Library
1	Bank Rakyat Indonesia (BBRI)	9,02%	3,52%	3,68%
2	Indofood Sukses Makmur (INDF)	6,14%	3,04%	3,07%
3	Telkom Indonesia (TLKM)	6,69%	4,40%	4,43%

From Table 7, it can be observed that estimating the results using GBM-KF produces a lower MAPE value compared to GBM. However, according to the MAPE criteria of less than 10%, both are still classified as having excellent accuracy. This shows a significant enhancement in prediction accuracy when applying the Kalman Filter, underscoring its potential to improve the overall accuracy of stock price prediction.

4. CONCLUSIONS

The main objective of this work was to improve the precision of stock price forecasts by combining the Geometric Brownian Motion model with the Kalman Filter technique. The results indicate that GBM-KF markedly enhances prediction accuracy compared to standalone GBM, as shown by decreased MAPE values for all evaluated stocks. The GBM computation indicates MAPE values of 9.02%, 6.14%, and 6.69% for BBRI, INDF, and TLKM, respectively, reflecting a high level of accuracy. A strategy is adopted to enhance prediction accuracy through the utilisation of the Kalman Filter. An example explored in this research is that as for BBRI, the MAPE value has been lowered from 9.02% to 3.52% and 3.68% for manual and library, respectively. The same applies to INDF and TLKM stocks. The application of the Kalman Filter results in a substantial decrease in MAPE values, achieving notably low levels in certain instances. The GBM-KF method employing manual computing may enhance the overall precision of stock price forecasting. Future research may enhance this study by using the GBM-KF model on alternative financial instruments, integrating supplementary market data, or evaluating its efficacy under extreme market conditions.

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