BAREKENG: Journal of Mathematics and Its Applications
December 2024 Volume 18 Issue 4 Page 2479–2490
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol18iss4pp2479-2490

PROSPECTIVE RESERVE AND FULL PRELIMINARY TERM RESERVE ON ENDOWMENT LAST SURVIVOR LIFE INSURANCE USING CLAYTON COPULA

Hasriati[1], Voundri Nindia Nayunda [2*], Haposan Sirait[3], Ihda Hasbiyati[4]

[1,2,3,4]Mathematics Department, Faculty of Mathematics and Sciences, Universitas Riau
Kampus Bina Widia KM. 12,5, Simpang Baru, Kota Pekanbaru, 28293, Indonesia

*Corresponding author's e-mail: * **voundrinindia.n@gmail.com***

Article History:

Received: 7th, April 2024
Revised: 5th, June 2024
Accepted: 1st,August 2024
Published:14th, October 2024

Keywords:

Clayton Copula;
Endowment Insurance;
Full Preliminary Term Reserve;
Last Survivor;
Prospective Reserve.

ABSTRACT

Combined life insurance is a type of insurance that protects two or more people who are related by family and is divided into two, namely joint-life life insurance and last-survivor life insurance. The last survivor life insurance is a condition of life insurance that will continue if there is at least one of all insurance participants who is still alive and will stop if all insurance participants die. The insurance company has to pay the benefit to the heirs of the insurance participant. When a claim occurs, the insurance company must prepare the reserve fee. The purpose of this research is to determine the amount of premium reserve of endowment last-survivor life insurance using prospective reserve and full preliminary term reserve. Full preliminary term reserve is one of the modified premium reserve calculations from Zillmer Reserve. To determine prospective reserve and full preliminary term reserve using the initial life annuity, single premium, and annual premium. Whereas the initial life annuity is influenced by the combined life and death opportunity of the insurance participants. Furthermore, the combined life and death opportunity of insurance participants will be obtained from Clayton copula and to obtain the parameter of Clayton copula, Rstudio software is used. Based on the result, the value of prospective reserves and full preliminary term reserves has increased every year and prospective reserves produce a greater value than full preliminary term reserves. If the insurance company uses this reserve calculation, the reserve that the company must prepare will increase every year. This is useful for insurance companies in predicting the amount of reserves they must have.

How to cite this article:

Hasriati, V. N. Nayunda, H. Sirait and I. Hasbiyati., "PROSPECTIVE RESERVE AND FULL PRELIMINARY TERM RESERVE ON ENDOWMENT LAST SURVIVOR LIFE INSURANCE USING CLAYTON COPULA," *BAREKENG: J. Math. & App.*, vol. 18, iss. 4, pp. 2479-2490, December, 2024.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

Humans cannot predict what will happen in their lives. Many risks can occur as humans live their lives. One thing that can minimize the impact of these risks is to take part in a life insurance program. Life insurance itself is an insurance program that provides a certain amount of money to the policyholder or heirs upon the policyholder's death or survival specified due date [7]. Endowment life insurance with last survivor status is a type of combined life insurance where premium payments are made until the last death of the insurance participant and payment of the benefit by the company is made after the death of the last insurance participant [3].

When someone has signed an insurance policy, he or she should pay a certain amount of money to the insurance company which is called a premium. The premium will later be used to pay the insurance money to the heirs of the insurance participants if a claim occurs. When a claim occurs by an insurance participant, the insurance company must prepare reserve costs. Reserves have an important role for insurance companies because they can prevent insurance companies from having difficulty paying benefits to heirs or preventing losses. The reserve itself is the amount of money available to the company during the coverage period [9]. One of the reserve calculations that can be used is prospective reserves, the calculation of which is based on the present value of the total expenditure in the future and then reduced by the present value of total income in the future [9].

Insurance companies need to consider the inclusion of their operating costs such as closing costs, renewal fees, and claim fees. In the first year of the policy, quite large costs are required for various purposes such as costs when closing life insurance, including the insurance officer’s provision costs, policy costs, medical examinations, and others [11]. Modification of the premium reserve calculation must be done to avoid losses. One way is using full preliminary term reserve calculations which is a modification of the Zillmer reserve calculation by assuming the policy is valid for one year only so that the premium in the first year is only enough to cover the first year's loading costs [21]. In calculating the reserve value, it is necessary to know the value of the premium and life annuity which is influenced by the life and death opportunity of insurance participants [14]. In determining the combined life opportunity and the combined death opportunity in endowment last-survivor life insurance, Clayton copula is used, which was introduced by Clayton in 1978 [5].

Research regarding the calculation of premiums and premium reserves for endowment last-survivor life insurance has been carried out by various researchers such as the calculation of premiums and benefit reserves using the Gamma-Gompertz Mortality Law for last survivor status [23]. Hasriati et al. [11] in their research, discussed calculating single premium and annual premium for endowment last-survivor life insurance using the Pareto distribution and can know that if participants join in one policy, so they pay less premium. Besides that, the calculation of life insurance premiums can not only be calculated using the Gamma-Gompertz Mortality Law or Pareto distribution but can also use another distribution. Hasriati et. al., [13] in their research used Makeham Law to calculate premiums on endowment life insurance for joint life and last survivor status.

Okativan et al. [21] in their research discussed the comparison of prospective reserves, Zillmer reserves, full preliminary term reserves, and premium sufficiency reserves on individual life insurance or single status. It was found that in the first year, prospective reserves produced the highest value. In another case, Setianingsih et al. [24] in their research comparing prospective reserve and modification prospective reserve using the Zillmer method for men aged 48 and women aged 69. Moreover, prospective reserve calculations can also be determined on combined status. Hasriati et al. [12] in their research determined prospective reserves on endowment life insurance for joint life status and last survivor status and used Gompertz’s Law to determine the probability of life and death.

Bintoro and Sudding [2] in their research calculated New Jersey and full preliminary term reserve on endowment life insurance for male and female insurance participants using the Indonesian Mortality Table 2019. In other research, Ramadani et al. [22] calculated full preliminary terms using Gompertz Law and without Gompertz Law. Meanwhile, Tarigas et. al., [25] in their research calculated full preliminary term reserve and premium sufficiency reserve for male insurance participants aged 30 years and a coverage period of 25 years.

The distribution function can be used to determine a person's probability of survival and chance of death. Copula is one method that can be used for combined insurance cases. Several researchers have used copulas in actuarial science, such as Erawati and Subhan [8] in their research using GFGM Type II copula in determining term life insurance premiums for last survivor status. Meanwhile, Lestari and Dzakiya [17] in their research compare the calculation of life insurance premiums for joint life status using Frank copula, Clayton copula, Gumbel copula, and without using copula. Clayton copula itself is a type of copula that is quite often used not only in the field of actuarial science but also in other fields of science.

The research and application of the Clayton copula have been widely discussed by several researchers. Among them is the application of the Clayton copula to identify the dependency structure of the Covid-19 outbreak and the average temperature in Jakarta which was discussed by Novianti et al. [19], application of the Clayton copula in the world of investment to determine portfolio-optimization by Darabi and Baghban [6], and estimate the Clayton copula parameters using the Kendall's tau τ correlation [20]. Besides that, the Clayton copula is also one of the considerations in modeling dependence on wind power in Karakas's research [15].

Based on the background that has been developed and the references cited, this research aims to determine premium reserves for last survivor endowment life insurance. In determining premium reserves, the prospective reserve method and full preliminary term reserves are used. The combined probability of life and probability of death are determined by using the Clayton copula.

2. RESEARCH METHODS

This research used a qualitative method based on relevant literature studies and journals. This section will discuss statistical and actuarial theories used to analyze the problems such as the last survivor survival function and Clayton copula.

2.1 Last Survivor Survival Function

Determining the amount of premium for combined life insurance required survival function for combined status [3], which is obtained from the combination of survival function on individual status. A random variable X is said to be a continuous random variable if there is a function $f(x)$ so that the cumulative distribution function can be expressed [26]

$$F(x) = P(\mathrm{X} \leq x) = \int_{-\infty}^{x} f(t)\, dt$$

Survival function of a continuous random variable X is denoted by $S(x)$, i.e

$$S(x) = P(\mathrm{X} > x),$$
$$S(x) = 1 - P(\mathrm{X} \leq x)$$

The relationship between the survival function and distribution function can be obtained as follows.

$$S(x) = 1 - F(x) \tag{1}$$

Suppose that $T(x)$ is a continuous random variable that represents the remaining age of a person who is x years old. The distribution function of variable $T(x)$ is

$$F_{T(x)}(t) = P(T(x) \leq t), \qquad t \geq 0$$

In actuarial science, the distribution function represents someone's probability of death, and the survival function represents someone's probability of life. Distribution function $F_{T(x)}(t)$ represents the probability that someone who aged x will die within a period of t years ($t \geq 0$) and denoted by ${}_tq_x$, i.e.,

$${}_tq_x = P(T(x) \leq t),$$
$$F_{T(x)}(t) = {}_tq_x \tag{2}$$

Survival function of a continuous random variable $T(x)$ is $S_{T(x)}(t)$ which represents the probability that someone aged x years can survive for a period t years ($t \geq 0$), denoted by ${}_tp_x$ i.e.,

$$S_{T(x)}(t) = P(T(x) > t),$$

$$S_{T(x)}(t) = {}_tp_x \tag{3}$$

Based on Equation (1), Equation (2), and Equation (3) can be obtained a relationship between the probability of life ${}_tp_x$ and probability of death ${}_tq_x$ for life insurance who is x years old as follows

$${}_tp_x = 1 - {}_tq_x \tag{4}$$

Next, will be discussed the probability of life and the probability of death for two insurance participants aged x and y years for the last survivor status. Suppose that $T(x)$ is a continuous random variable that represents the remaining age of a person who is x years old and $T(y)$ is a continuous random variable that represents the remaining age of a person who is y years. Continuous random variable last-survivor life insurance denoted by $T(\overline{xy}) = \max[T(x), T(y)]$ and its distributive function [3], [4], namely

$$\begin{aligned} F_{T(\overline{xy})}(t) &= P(\max[T(x), T(y)] \leq t) \\ &= P(T(x) \leq t \text{ and } T(y) \leq t) \\ F_{T(\overline{xy})}(t) &= P(T(x) \leq t) \cdot P(T(y) \leq t) \end{aligned}$$

Based on Equation (2), we can obtain the probability of death for the last survivor status as

$${}_tq_{\overline{xy}} = {}_tq_x \cdot {}_tq_y$$

Based on Equation (4), then

$$\begin{aligned} 1 - {}_tp_{\overline{xy}} &= (1 - {}_tp_x)(1 - {}_tp_y) \\ 1 - {}_tp_{\overline{xy}} &= 1 - {}_tp_x - {}_tp_y + {}_tp_x \cdot {}_tp_y \\ 1 - {}_tp_{\overline{xy}} &= 1 - ({}_tp_x + {}_tp_y - {}_tp_x \cdot {}_tp_y) \end{aligned}$$

So, the probability of life of last-survivor life insurance and its relationship with the probability of life for joint-life life insurance can be obtained as follows:

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy} \tag{5}$$

2.2 Clayton Copula

One class of copula that is well-known and often used in finance, insurance, and risk management is the Archimedean copula. The Archimedean copula function is expressed as follows:

$$C(u_1, u_2, \dots, u_j) = \phi^{-1}\left(\phi(u_1) + \phi(u_2) + \dots + \phi(u_j)\right) \tag{6}$$

where ϕ is generator function for copula, C is the copula, $u_1, \dots, u_j$ is the random variable, and j is the number of random variables where j is a natural number described in [1], [5], and [18].

Clayton copula was introduced for the first time by Clayton in 1978 [5] and belongs to the Archimedean copula family. The generator function of Clayton copula is described in [1], [5], [15], [18], that is

$$\phi(u) = \frac{1}{\theta}(u^{-\theta} - 1),$$

where θ is the parameter and $\theta > 0$. The inverse of Clayton copula's generator function as follows:

$$\phi^{-1}(u) = (\theta u + 1)^{-\frac{1}{\theta}} \tag{7}$$

By using Equation (6) for $j = 2$ and Equation (7), Clayton copula function can be obtained as follows:

$$\begin{aligned} C(u_1, u_2) &= \phi^{-1}(\phi(u_1) + \phi(u_2)) \\ &= \left(\theta(\phi(u_1) + \phi(u_2)) + 1\right)^{-\frac{1}{\theta}} \\ &= \left(\theta\left(\left(\frac{1}{\theta}(u_1^{-\theta} - 1)\right) + \left(\frac{1}{\theta}(u_2^{-\theta} - 1)\right)\right) + 1\right)^{-\frac{1}{\theta}} \\ &= \left(\theta\left(\frac{1}{\theta}(u_1^{-\theta} + u_2^{-\theta} - 2)\right) + 1\right)^{-\frac{1}{\theta}}, \end{aligned}$$

$$C(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-\frac{1}{\theta}} \tag{8}$$

If u_1 taken as $F_{T(x)}(t)$ and u_2 taken as $F_{T(y)}(t)$, then **Equation (8)** can be written as

$$C\left(F_{T(x)}(t)\,, F_{T(y)}(t)\,\right) = \left(F_{T(x)}(t)^{-\theta} + F_{T(y)}(t)^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{9}$$

Combined survival function using copula for random variables A and B can be expressed as [5], [18]

$$S_{A,B}(a,b) = S_A(a) + S_B(b) - 1 + C\big(1 - S_A(a), 1 - S_B(b)\big) \tag{10}$$

Based on **Equation (10)**, can be obtained combined survival function using copula for continuous variables $T(x)$ and $T(y)$ as follows

$$S_{T(x)T(y)}(t,t) = S_{T(x)}(t) + S_{T(y)}(t) - 1 + C\left(1 - S_{T(x)}(t), 1 - S_{T(y)}(t)\right).$$

By relationship on **Equation (1)** can be written as

$$S_{T(x)T(y)}(t,t) = S_{T(x)}(t) + S_{T(y)}(t) - 1 + C\left(F_{T(x)}(t), F_{T(y)}(t)\right) \tag{11}$$

Then by substituting the **Equation (9)** into **Equation (11)** we obtained a combined survival function Clayton copula namely

$$S_{T(x)T(y)}(t,t) = S_{T(x)}(t) + S_{T(y)}(t) - 1 + \left(F_{T(x)}(t)^{-\theta} + F_{T(y)}(t)^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{12}$$

Function $S_{T(x)T(y)}(t,t)$ represents the combined probability of life for joint life insurance and is denoted by ${}_tp_{xy}$. Then based on the **Equation (2)** and **Equation (3)**, **Equation (12)** can be written as

$${}_tp_{xy} = {}_tp_x + {}_tp_y - 1 + \left({}_tq_x{}^{-\theta} + {}_tq_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{13}$$

Next by substituting **Equation (12)** into **Equation (5)** can be obtained probability of life for last-survivor life insurance using Clayton copula, namely

$${}_tp_{\overline{xy}} = 1 - \left({}_tq_x{}^{-\theta} + {}_tq_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{14}$$

and based on the **Equation (4)** and **Equation (14)** we obtained the probability of death for last-survivor life insurance using Clayton copula, namely

$${}_tq_{\overline{xy}} = \left({}_tq_x{}^{-\theta} + {}_tq_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{15}$$

2.3 Life Annuity and Premium Endowment Last-Survivor Life Insurance

An annuity is a sustainable series of payments in a certain amount within a certain period as long as the insurance participant is still alive [16] and annuity payments that are influenced by the insurance participant's probability of life are called life annuity [9]. In an initial life annuity, the first payment is made at the time of signature of the policy, and the last payment at $n-1$ period. In determining initial life annuity is strongly influenced by the interest rate and discount factor. There is a function called discount factors, that is

$$v = \frac{1}{1+i},$$

where i is the interest. Apart from that, there is a function called the discount rate can be expressed [9]

$$d = 1 - v.$$

The cash value of term initial life annuity last survivor for two insurance participants aged x years and y years with n years coverage period is denoted by $\ddot{a}_{\overline{xy}:\overline{n}|}$, which is influenced by a discount factor v and probability of life of the last survivor's life insurance ${}_tp_{\overline{xy}}$, namely [10]

$$\ddot{a}_{\overline{xy}:\overline{n}|} = v^0\ {}_0p_{\overline{xy}} + v^1\ {}_1p_{\overline{xy}} + v^2\ {}_2p_{\overline{xy}} + \cdots + v^{n-1}\ {}_{n-1}p_{\overline{xy}}$$

$$\ddot{a}_{\overline{xy}:\overline{n}|} = \sum_{t=0}^{n-1} v^t\ {}_tp_{\overline{xy}}. \tag{16}$$

In life insurance, the premium amount depends on the benefit received in the future. A premium is a series of payments with a certain amount made by an insurance participant to the insurance company over a predetermined time. Endowment life insurance is a combination of pure endowment insurance and term life insurance. Therefore, the premium for endowment last-survivor life insurance is the sum of the single premium for pure endowment last-survivor life insurance and the premium for term last-survivor life insurance. The single premium of endowment last-survivor life insurance can be expressed as [12], [13]

$$A_{\overline{xy}:\overline{n}|} = A_{\overline{xy}:\overset{1}{\overline{n}|}} + A_{\overset{1}{\overline{xy}}:\overline{n}|}.$$

It can be rewritten as

$$\begin{aligned} A_{\overline{xy}:\overline{n}|} &= \sum_{t=0}^{n-1} v^{t+1}\ {}_{t|}q_{\overline{xy}} + Rv^n\ {}_np_{\overline{xy}} \\ &= \sum_{t=0}^{n-1} v^{t+1}\left(\ {}_tp_{\overline{xy}} - {}_{t+1}p_{\overline{xy}}\right) + Rv^n\ {}_np_{\overline{xy}} \\ &= v\sum_{t=0}^{n-1} v^t\ {}_tp_{\overline{xy}} - \left(\sum_{t=0}^{n-1} v^t\ {}_tp_{\overline{xy}} + v^n\ {}_np_{\overline{xy}} - 1\right) + v^n\ {}_np_{\overline{xy}} \\ &= v\sum_{t=0}^{n-1} v^t\ {}_tp_{\overline{xy}} - \sum_{t=0}^{n-1} v^t\ {}_tp_{\overline{xy}} + 1 \\ &= 1-(1-v)\sum_{t=0}^{n-1} v^t\ {}_tp_{\overline{xy}}\,, \\ A_{\overline{xy}:\overline{n}|} &= 1 - d\ddot{a}_{\overline{xy}:\overline{n}|}, \end{aligned}$$

where d is the discount rate and $\ddot{a}_{\overline{xy}:\overline{n}|}$ is term initial life annuity last survivor. If R is the benefit that is paid at the end of the policy year, then a single premium for endowment last-survivor life insurance is obtained, namely

$$A_{\overline{xy}:\overline{n}|} = R\left(1 - d\ddot{a}_{\overline{xy}:\overline{n}|}\right) \tag{17}$$

The annual premium is a premium paid at the beginning of each year and the amount can be the same or change every year [9]. The annual premium of endowment last-survivor life insurance with n years coverage period and m times the payment and also the benefit of R are denoted by ${}_mP_{\overline{xy}:\overline{n}|}$ can be expressed as

$${}_mP_{\overline{xy}:\overline{n}|} = R\frac{A_{\overline{xy}:\overline{n}|}}{\ddot{a}_{\overline{xy}:\overline{m}|}} \tag{18}$$

3. RESULTS AND DISCUSSION

3.1 Life Annuity and Premium Endowment Last-Survivor Life Insurance Using Clayton Copula

Term initial life annuity last survivor using Clayton copula can be obtained by substituting Equation (14) into Equation (16), namely

$$\ddot{a}_{\overline{xy}:\overline{n}|} = \sum_{t=0}^{n-1} v^t\left(1-\left({}_tq_x{}^{-\theta} + {}_tq_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right) \tag{19}$$

The single premium for endowment last-survivor life insurance with n years coverage period and the benefit of R using Clayton copula can be obtained by substituting Equation (19) into Equation (17) as follows:

$$A_{\overline{xy}:\overline{n}|} = R\left(1 - d\sum_{t=0}^{n-1} v^t\left(1-\left({}_tq_x{}^{-\theta} + {}_tq_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right) \tag{20}$$

Based on **Equation (19)** and by substituting the **Equation (20)** into **Equation (18)** we can obtain the annual premium of endowment last-survivor life insurance with n years coverage period and m times the payment and also the benefit of R using Clayton copula as follows:

$$ {}_{m}P_{\overline{xy}:\overline{n}|} = \frac{R\left(1 - d\sum_{t=0}^{n-1} v^t \left(1 - \left({}_{t}q_x{}^{-\theta} + {}_{t}q_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right)}{\sum_{t=0}^{m-1} v^t \left(1 - \left({}_{t}q_x{}^{-\theta} + {}_{t}q_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)} \tag{21} $$

3.2 Prospective Reserve on Endowment Last-Survivor Life Insurance Using Clayton Copula

In endowment last-survivor life insurance, the calculation of prospective reserve can be expressed in three cases [9], [12]

(i) Both insurance participants alive until the end of the policy year
(ii) Insurance participant aged x years died and insurance participant y years alive
(iii) Insurance participant aged x years alive and insurance participant y years died.

In this research will be discussed if both insurance participants live until the end of the year policy. Prospective reserve on endowment last survivor life if both insurance participants live until the end of the year policy can be expressed [9]

$$ {}_{t}^{m}V_{\overline{xy}:\overline{n}|} = A_{\overline{x+t,y+t}:\overline{n-t}|} - {}_{m}P_{\overline{xy}:\overline{n}|}\ddot{a}_{\overline{x+t,y+t}:\overline{m-t}|} \tag{22} $$

Based on **Equation (19)**, **Equation (20)**, and **Equation (21)** for insurance participants aged $(x+t)$ and $(y+t)$ years with $(n-t)$ years coverage period and m times the payment premium can be obtained prospective reserve of endowment last-survivor life insurance in **Equation (22)** using Clayton Copula as

$$ {}_{t}^{m}V_{\overline{xy}:\overline{n}|} = R\left(1 - d\sum_{k=0}^{(n-t)-1} v^k \left(1 - \left({}_{k}q_{x+t}{}^{-\theta} + {}_{k}q_{y+t}{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right) $$
$$ - \left(\frac{R\left(1 - d\sum_{k=0}^{n-1} v^k \left(1 - \left({}_{k}q_x{}^{-\theta} + {}_{k}q_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right)}{\sum_{k=0}^{m-1} v^k \left(1 - \left({}_{k}q_x{}^{-\theta} + {}_{k}q_y{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)}\right) $$
$$ \left(\sum_{k=0}^{(m-t)-1} v^k \left(1 - \left({}_{k}q_{x+t}{}^{-\theta} + {}_{k}q_{y+t}{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right) $$

3.3 Full Preliminary Term Reserve on Endowment Last-Survivor Life Insurance Using Clayton Copula

Full preliminary reserves are a modification of Zillmer's reserve calculation. The premium in the first year is only enough to cover the first year's loading costs, or in other words, the reserve in the first year is zero [21]. As previously mentioned, the full preliminary reserve is a modification of the Zillmer reserve by assuming $t = 1$ the value of the Zillmer reserve is zero. This is done to avoid a negative reserve value [10]. Zillmer reserve for individual life insurance or single status can be expressed [21]

$$ {}_{t}^{m}V_{x:\overline{n}|}^{(z)} = A_{x+t:\overline{n-t}|} - \left({}_{m}P_{x:\overline{n}|} + \frac{\alpha}{\ddot{a}_{x:\overline{m}|}}\right)\ddot{a}_{x+t:\overline{m-t}|}, $$

then, Zillmer reserves on endowment last-survivor life insurance if both insurance participants live until the end of the policy year is

$$ {}_{t}^{m}V_{\overline{xy}:\overline{n}|}^{(z)} = A_{\overline{x+t,y+t}:\overline{n-t}|} - \left({}_{m}P_{\overline{xy}:\overline{n}|} + \frac{\alpha}{\ddot{a}_{\overline{xy}:\overline{m}|}} \right) \ddot{a}_{\overline{x+t,y+t}:\overline{m-t}|} \tag{23} $$

For $t = 1$, in **Equation (23)** it is assumed to be zero, ${}_{1}^{m}V_{\overline{xy}:\overline{n}|}^{(z)} = 0$, then can be obtained α is

$$ 0 = A_{\overline{x+1,y+1}:\overline{n-1}|} - \left({}_{m}P_{\overline{xy}:\overline{n}|} + \frac{\alpha}{\ddot{a}_{\overline{xy}:\overline{m}|}} \right) \ddot{a}_{\overline{x+t,y+t}:\overline{m-1}|}, $$

$$ \alpha = \left(\frac{A_{\overline{x+1,y+1}:\overline{n-1}|}}{\ddot{a}_{\overline{x+t,y+t}:\overline{m-1}|}} - {}_{m}P_{\overline{xy}:\overline{n}|} \right) \ddot{a}_{\overline{xy}:\overline{m}|}, $$

$$ \alpha = \left({}_{m-1}P_{\overline{xy}:\overline{n-1}|} - {}_{m}P_{\overline{xy}:\overline{n}|} \right) \ddot{a}_{\overline{xy}:\overline{m}|} \tag{24} $$

Next, by substituting the α in **Equation (24)** into **Equation (23)**, can be obtained full preliminary term on endowment last-survivor life insurance which is denoted by ${}_{t}^{m}V_{\overline{xy}:\overline{n}|}^{(F)}$ as follows [10]:

$$ {}_{t}^{m}V_{\overline{xy}:\overline{n}|}^{(F)} = A_{\overline{x+t,y+t}:\overline{n-t}|} - \left({}_{m}P_{\overline{xy}:\overline{n}|} + \frac{\left({}_{m-1}P_{\overline{xy}:\overline{n-1}|} - {}_{m}P_{\overline{xy}:\overline{n}|} \right) \ddot{a}_{\overline{xy}:\overline{m}|}}{\ddot{a}_{\overline{xy}:\overline{m}|}} \right) \ddot{a}_{\overline{x+t,y+t}:\overline{m-t}|} $$

$$ {}_{t}^{m}V_{\overline{xy}:\overline{n}|}^{(F)} = A_{\overline{x+t,y+t}:\overline{n-t}|} - {}_{m-1}P_{\overline{xy}:\overline{n-1}|} \cdot \ddot{a}_{\overline{x+t,y+t}:\overline{m-t}|} \tag{25} $$

Based on **Equation (19)**, **Equation (20)**, and **Equation (21)** for insurance participants aged $(x + t)$ and $(y + t)$ years with $(n - t)$ years coverage period and m times the payment premium can be obtained full preliminary term reserve of endowment last-survivor life insurance in **Equation (25)** using Clayton copula as

$$ {}_{t}^{m}V_{\overline{xy}:\overline{n}|}^{(F)} = R\left(1 - d \sum_{k=0}^{(n-t)-1} v^k \left(1 - \left({}_{k}q_{x+t}{}^{-\theta} + {}_{k}q_{y+t}{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right) $$
$$ - \left(\frac{R\left(1 - d\sum_{k=0}^{(n-1)-1} v^k \left(1 - \left({}_{k}q_{x}{}^{-\theta} + {}_{k}q_{y}{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right)}{\sum_{k=0}^{(n-1)-1} v^k \left(1 - \left({}_{k}q_{x}{}^{-\theta} + {}_{k}q_{y}{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)} \right) $$
$$ \left(\sum_{k=0}^{(m-t)-1} v^k \left(1 - \left({}_{k}q_{x+t}{}^{-\theta} + {}_{k}q_{y+t}{}^{-\theta} - 1\right)^{-\frac{1}{\theta}}\right)\right) $$

As an example, A husband and wife who are 35 years old and 33 years old respectively take part in the last-survivor endowment life insurance program with a coverage period of 20 years. If the benefit received by the heirs is Rp150,000,000, the applicable interest rate is 5%, and the premium payment period is 15 years, then determine the following cases if both insurance participants live until the end of the policy year:

(i) Prospective reserve
(ii) Full preliminary term reserve

Using the package VineCopula in the Rstudio software, the combined probability of death for insurance participants aged 35 years and 33 years based on the Indonesian Mortality Table IV (2019) obtained a parameter value is $\theta = 28$.

Table 1. Prospective and Full Preliminary Term Reserve of Last-Survivor Endowment Life Insurance

t^{th} year	Prospective Reserve		Full Preliminary Term Reserve	
1	Rp	5.461.129	Rp	0
2	Rp	11.192.328	Rp	6.009.257

t^{th} year	Prospective Reserve		Full Preliminary Term Reserve	
3	Rp	17.206.570	Rp	12.315.382
4	Rp	23.518.985	Rp	18.934.244
5	Rp	30.144.329	Rp	25.881.336
6	Rp	37.099.505	Rp	33.174.387
7	Rp	44.401.294	Rp	40.831.014
8	Rp	52.066.658	Rp	48.869.040
9	Rp	60.115.799	Rp	57.309.658
10	Rp	68.567.630	Rp	66.172.754
11	Rp	77.445.138	Rp	75.482.445
12	Rp	86.771.398	Rp	85.262.971
13	Rp	96.570.916	Rp	95.540.090
14	Rp	106.870.181	Rp	106.341.658

The comparison of prospective reserves and the full preliminary term of the last survivor endowment life insurance in **Table 1** can be illustrated in **Figure 1** to see their differences.

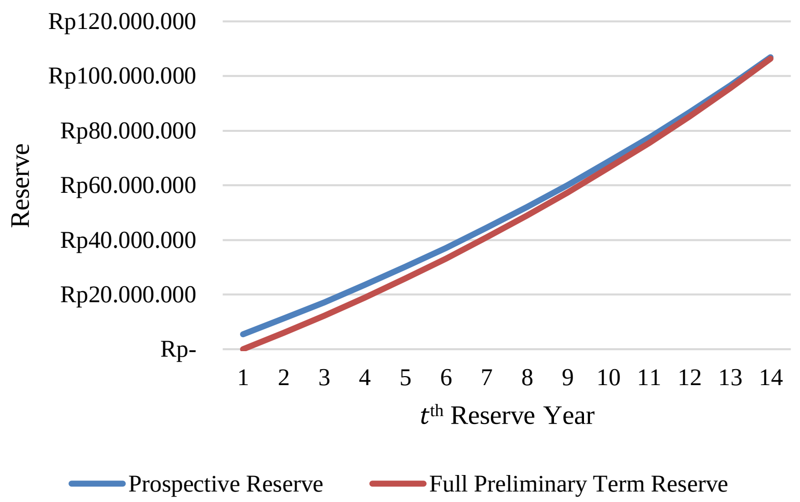

Figure 1. Prospective and Full Preliminary Term Reserve of Last-Survivor Endowment Life Insurance

Table 1 and **Figure 1** show the prospective reserves and full preliminary term of the last survivor endowment life insurance using Clayton copula. The value of prospective reserves and full preliminary term reserves has increased every year and has almost the same rate of increase as seen in **Figure 1**. However, the calculation of prospective reserves has a greater value than full preliminary term reserves for $t = 1$ to $t = 10$, so it can be seen quite a significant difference. As the period increases, prospective reserves and full preliminary term reserves have almost the same value, because in **Figure 1** for $t = 11$ to $t = 14$ it can be seen that the lines overlap. However, calculating for $t = 11$ to $t = 14$, prospective reserves have a slightly larger value than full preliminary term reserves, this is shown in **Table 1**.

Based on **Table 1** and **Figure 1**, by using the Clayton copula in calculating premium reserves, the calculation of prospective reserves produces a greater value than full preliminary term reserves. When compared with other cases, Oktavian et. al., **[21]** in their research assuming a single insurance participant, prospective reserves have a greater value than full preliminary term reserves. This can happen because, in full preliminary term reserves, the reserve value in the first year is assumed to be zero. But in reality, the value of the first year's reserves is not always completely used up to cover operational costs.

As the t value increases the value of the reserves obtained increases. On the other hand, the age of insurance participants is also increasing. Increasing the age of insurance participants can increase the probability of death of the insurance participant. This can happen because as a person's age increases, the risks he or she will face also become greater, such as declining health. This is one of the reasons why the value of t increases, and the premium reserve also increases.

4. CONCLUSIONS

Clayton copula can be used to model the inequalities of probability of life and probability of death for combined life insurance participants and can be used to determine the amount of premium reserves. One of the reserve calculations that is widely used is prospective reserves. Moreover, an alternative method for calculating premium reserves is the use of a full preliminary term reserve. The full preliminary term reserve itself is a modification of the Zillmer reserve by setting the value of the Zillmer reserve in the first year to zero. In this case, the Clayton copula is used to determine the probability of life and death, annuity, premiums, and also reserves for last survivor endowment life insurance. By using $\theta = 28$, it is obtained that prospective reserves and preliminary term reserves increase every year. Also, prospective reserves with the Clayton copula produce a greater value than full preliminary term reserves. So, if the insurance company uses prospective reserve and full preliminary term reserve last-survivor life insurance using Clayton copula, the reserve that the company must prepare will increase every year. This is useful for insurance companies in predicting the amount of reserves they must have.

ACKNOWLEDGMENT

Thank you to the Mathematics Department and the Faculty of Mathematics and Natural Sciences (FMIPA) at Riau University for providing support and suggestions for this research.

REFERENCES

[1] M. A. Boateng, A. Y. Omari-Sasu, R. K. Avuglah, and N. K. Frempong, "A Mixture of Clayton, Gumbel, and Frank Copulas: A Complete Dependence Model", Journal of Probability and Statistics, vol. 2022, pp. 1-7, Aprill 2022
[2] F. F. Bintoro and F. N. F. Sudding, "Comparison of Premium Reserves with New Jersey Methods and Full Preliminary Term on Endowment Insurance", Journal of Actuarial, Finance and Risk Management, vol. 1, no. 2, pp. 1-7, November 2022.
[3] N. L. Bowers, H. U. Geerber, J. C. Hickman, D. A. Jones, and C. J. Nesbitt, Actuarial Mathematics, Schaumburg: The Society of Actuaries, 1997.
[4] S. J. Camilli, I. Duncan, and R. L. London, Models for Quantifying Risk, Sixth Edition, Winsted: ACTEX publications, 2004.
[5] U. Cherubini, E. Luciano dan W. Vecchiato, Copula Methods in Finance, John Wiley & Sons, Hokoben, 2004.
[6] R. Darabi and M. Baghban, "Application of Clayton Copula in Portfolio Optimization and its Comparison with Markowitz Mean-Variance Analysis", Advances in Mathematical Finance and Applications, vol. 3, no. 1, pp. 33-51, February 2018.
[7] D. C. M. Dickson, M. R. Hardy, and H. R. Waters, Actuarial Mathematics for Life Contingent Risk, Cambridge University Press, New York: Cambridge University Press, 2009.
[8] P. Erawati and M. Subhan, "Penentuan Premi Asuransi Jiwa Berjangka Status Last Survivor Menggunakan Model GFGM-Type II Copula", Journal of Mathematics UNP, vo.7, no.2, pp.69-75, 2022.
[9] T. Futami, Matematika Asuransi Jiwa Bagian I, terjemahan. dari seimei hoken sugaku, jokan (92 Revision). Japan: Incorporated Foundation Oriental Life Insurance Cultural Development Center, 1993.
[10] T. Futami, Matematika Asuransi Jiwa Bagian II, terjemahan. dari seimei hoken sugaku, jokan (92 Revision). Japan: Incorporated Foundation Oriental Life Insurance Cultural Development Center, 1994.
[11] Hasriati, and T. P. Nababan, "Private Premium of Endowment Last Survivor and Joint Life Insurance with Pareto Distribution," International Journal of Statistical Distributions and Applications, vol. 5, no. 4, pp. 76–81, December 2019.
[12] Hasriati, T. P. Nababan and Ihda Hasbiyati, "Reserve of Life Insurance Prospective Dwiguna Joint Life and Last Survivor with Gompertz Law," in Proc. 2nd African Int. Conf. on Industrial Engineering and Operations Management Harare, pp. 1815-1824, Dec. 7-10, 2020.
[13] Hasriati, Haposan Sirait, Agung Prabowo, Sukono, and Abdul Talib Bon, "Life Insurance Premiums Dwiguna Joint Life and Last Survivor with Makeham Law", in Proc. 11th Annual Int. Conf. on Industrial Engineering and Operations Management Singapura, pp. 3270-3279, March 7-11, 2021.
[14] Hasriati, Sukono, I. Hasbiyati, T. P. Nababan and A. Kirana, "Zillmer Reserves in Dwiguna Life Insurance with the Cox-Ingersoll-Ross (CIR) Interest Rate," Komputer: J. Ilmiah Ilmu Komputer dan Matematika, vol. 20, no. 1, pp. 01–09, December 2023.
[15] A. M. Karakas, "Using Copulas for Modeling Dependence in Wind Power", Asian Journal of Engineering and Technology, vol.7, pp.33-49, February 2019.
[16] S. G. Kellison, The theory of interest, Homewood: Richard D. Irwin Inc, 1991.
[17] F. Lestari and A. Dzakiya, "Perbandingan Estimasi Premi Asuransi Jiwa Joint Life Menggunakan Asumsi Kebebasan Mortalita dan Metode Copula", Jurnal Matematika Integratif, vol.19, no.2, pp.201-212, 2023.

[18] R. B. Nelsen, An Introduction to Copulas, Second Edition, Springer, New York, 2006.

[19] P. Novianti, S. H. Kartiko, dan D. Rosadi, “Application of Clayton copula to identify dependency structure of Covid-19 outbreak and average tempe- rature in Jakarta Indonesia”, Journal of Physics: Conference Series, 1943 (2021), 1-10.

[20] A. W. Nurcahyani, D. R. S. Saputro, and N. A. Kurdhi, “Korelasi Kendall (τ) untuk Estimasi Parameter Korelasi Kendall (τ) untuk Estimasi Parameter”, Jurusan Pendidikan Matematika FMIPA UNY, pp.53-58, 2016

[21] M. R. Oktavian, D. Devianto, and F. Yanuar, “Kajian Metode Zillmer, Full Preliminary Term, Dan Premium Sufficiency Dalam Menentukan Cadangan Premi Pada Asuransi Jiwa Dwiguna”, Jurnal Matematika UNAND, vol.3, no.4, pp.160-167, 2014.

[22] K. Ramadani, D. Devianto, I. Rahmi HG, “Pengaruh Penggunaan Hukum Mortalitas Gompertz Pada Penentuan Besarnya Asuransi Jiwa Dwiguna Dengan Metode Full Preliminary Term”, Jurnal Matematika UNAND, vol.8, no.1, pp.163-170, May 2019.

[23] Ruhiyat, W. Erliana, K. Lamberto, and E. Ardelia, “Last-Survivor Insurance Premium and Benefit Reserve Calculation using Gamma-Gompertz Mortality Law,” J. Mat. Integr., vol. 18, no. 1, pp. 9–18, May 2022.

[24] D. Setianingsih, S. Rosita, and S. Resti S, “Modification of Prospective Life Insurance Reserve Prospective Calculations Using The Zillmer Method”, Journal of Mathematical Sciences and Optimization, vol.1, no.2, pp.63-71, 2024.

[25] L. A. Tarigas, N. Satyahadewi, and H. Perdana, “Penentuan Cadangan Premi Asuransi Jiwa Dwiguna Menggunakan Metode Full Preliminary Term Dan Premium Sufficiency”, Buletin Ilmiah Math. Stat. dan Terapannya, vol.8, no.3, pp.379-386, 2019.

[26] R. E. Walpole, R. H. Myers, dan S. L. Myers, Probability and Statistics for Engineering and Scientist, Ninth Edition, Prentice Hall, Boston, 2012.