BAREKENG: Journal of Mathematics and Its Applications
June 2025 Volume 19 Issue 2 Page 0709–0720
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19iss2pp0709-0720

PRICING EMPLOYEE STOCK OPTION USING TRINOMIAL TREE METHOD

Donny Citra Lesmana[1*], Reza Tri Ahmad Ramadhan[2], Siti Nurjanah[3], Vanaya Syahira Dharmawan[4]

[1,2,3,4] *Department of Mathematics, Faculty of Mathematics and Natural Sciences, IPB University*
Jln. Raya Dramaga, Kampus IPB Dramaga, Bogor, 16680, Indonesia

Corresponding author's e-mail: * **donnylesmana@apps.ipb.ac.id**

ABSTRACT

Article History:

Received: 2nd June 2024
Revised: 21st January 2025
Accepted: 1st February 2025
Published: 1st April 2025

Keywords:

Dilution Effect;
Early Exercise;
Employee Forfeiture Rate;
Employee Stock Option;
Trinomial Tree

This study explores the Employee Stock Option (ESO) model proposed by Liao and Lyuu, which provides a robust framework for addressing critical factors such as dilution, early exercise, and employee forfeiture rates. The model is solved using the trinomial tree method, allowing for the consideration of three possible stock price movements: increase, unchanged, or decrease. This approach combines forward and backward calculations to accurately evaluate ESO values by accounting for the complex interactions of these parameters. Dilution effects are modeled by adjusting stock prices based on outstanding shares and strike prices, while early exercise probabilities are addressed using a modified Chi-Square distribution to represent employee behavior. Additionally, the forfeiture rate is dynamically adjusted based on ESO returns and the ratio of stock-to-strike prices. The analysis reveals that ESO price negatively correlates with strike price and forfeiture rate, whereas parameters such as vesting time, maturity date, risk-free rate, volatility, and the number of ESOs granted exhibit positive correlations. This comprehensive methodology demonstrates the practical applicability of the Liao and Lyuu model for real-world ESO valuation. By integrating these critical factors into a unified framework, the study contributes significantly to the literature on financial modeling and provides actionable insights for companies seeking to optimize their ESO programs.

How to cite this article:

D. C. Lesmana, R. T. A. Ramadhan, S. Nurjanah and V. S. Dharmawan., "PRICING EMPLOYEE STOCK OPTION USING TRINOMIAL TREE METHOD," *BAREKENG: J. Math. & App.*, vol. 19, iss. 2, pp. 0709-0720, June, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article • Open Access

1. INTRODUCTION

To compete among competitors, it is necessary for companies to issue policies to enhance the quality of their employees' work. Companies need to take action in retaining their employees. Employees as vital human resources determine the competitiveness of the company. Companies could provide compensation beyond basic salary and bonuses to encourage employees to work to their fullest potential. The compensation provided by companies comes in various forms, such as bonuses, allowances, incentives, and stocks. Employee stock option programs, commonly known as Employee Stock Options (ESOs), are one form of compensation provided by companies [1].

Essentially, stock options represent a contract between an option seller (option writer) and an option buyer (option holder). This contract grants the holder the right to sell or buy stocks at or before a certain date at an agreed-upon price [2]. Options are highly beneficial in managing financial risks as they can be utilized to establish minimum and maximum limits for asset prices. By setting these limit prices, the possibility of asset price fluctuations at specified times can be addressed [3]. Options, based on their function, can be categorized into two types: put options and call options. A call option is a contract that grants the holder the right to buy an asset at a predetermined price, which can be exercised within a pre-agreed-upon period. Conversely, a put option is a type of contract that grants the holder the right to sell an asset at an agreed-upon price within a predetermined timeframe [4].

Employee stock options belong to the type of call options given free of charge by a company to certain employees. ESOs grant the right to purchase a portion of the company's stock at an agreed-upon price at the time the options are granted [5]. Employee, the owner of ESOs could not trade their ESOs to the others. This is what makes ESOs different from other options or common stocks. ESOs also have a vesting period, allowing employees to exercise the option only after the vesting period is over [6]. If an employee leaves the company during the vesting period, the ESOs are canceled. The options will be exercised if they are in the money. ESOs are considered worthless if they are out of the money when an employee leaves the company after the vesting period [7]. Several characteristics of ESOs are early exercises, forfeiture rate, CEO utility maximization, and firm credit risks [8].

Early exercise is assumed to occur when the market price of the stock reaches at least a multiple of M (the employee position ratio) from the strike price. Typically, employees holding top management positions will have a higher value of M compared to mid-level management positions. Furthermore, the rules for determining ESO are established using the assumption that options can only be exercised after the vesting period; and options will be exercised if the market price of the stock reaches at least a multiple of M from the strike price [9].

Even though every position has different M value, there is M such that the option price is maximum [10]. Another assumption for early exercise is employees tend to exercise the option exactly when the ESO is vested. After that, the ratio decreases over time and increases again when it is close to maturity date [11].

Most companies prefer to buy existing shares to issue employee stock options. This is because the process of issuing ESOs through new shares is challenging. Executing ESOs can result in an increase in the number of shares outstanding in the market. When the total number of shares in the market increases, the proportion of ownership held by existing shareholders decreases. The dilution effect is the adjustment of stock prices caused by exercised ESOs having a different value from the stock price at that time [8]. Valuing ESOs affected by dilution could use Black-Scholes method [12].

The assumption of a constant forfeiture rate for employees throughout time is highly unrealistic. Adjusting the option value is needed by considering the possibility that employees will leave the company before ESOs vested is lower than after ESOs vested. To estimate the likelihood of employees leaving the company, we can use the Poisson distribution [13]. Another way to make this parameter reliable is to adjust the forfeiture rate based on the value of ESO.

Previous studies have shown how to value ESO using binomial method and include the effect of dilution, early exercise, and forfeiture rate. However, the binomial steps do not accommodate for stocks price to remain unchanged. The trinomial method accommodates this situation and makes the model to be more realistic. Recent studies have shown trinomial method could be used to valuing ESO affected by dilution [14]. Moreover, trinomial methods converge faster than binomial method [15].

Based on the results of previous studies, no studies have developed a trinomial model that includes the effect of dilution, early exercise, and forfeiture rate. In this study, we build upon the ESO model proposed by Liao and Lyuu, which integrates dilution effects, early exercise behavior, and forfeiture rates into a unified framework [11]. Liao and Lyuu's model provides a detailed representation of ESO dynamics, but it was initially developed using a theoretical approach without incorporating the computational advantages of the trinomial method. Unlike Liao and Lyuu's work, which focuses on theoretical parameter exploration, this research applies the trinomial tree method to extend their model. The trinomial approach allows for more realistic stock price movements by considering three potential outcomes in each time step: increase, unchanged, or decrease.

A key distinction in this study is the application of the trinomial method to enhance computational efficiency and model realism. By simulating a range of ESO parameters under various scenarios, this research demonstrates how the trinomial approach converges more rapidly and provides greater accuracy than traditional binomial methods. Additionally, this work introduces dynamic adjustments to forfeiture rates based on stock performance, a feature not fully explored in Liao and Lyuu's original model.

This paper aims to fill a significant gap in ESO valuation by presenting a computationally efficient and realistic framework for pricing ESOs while maintaining consistency with established theoretical foundations. By addressing the practical complexities of ESO valuation, this study offers valuable insights for both academic research and corporate decision-making.

2. RESEARCH METHODS

2.1 Trinomial Method

The research applies the trinomial tree method to evaluate Employee Stock Options (ESOs), building on the framework proposed by Liao and Lyuu. The trinomial tree approach is chosen because it incorporates three potential stock price movements in each time step: upward, unchanged, or downward. This feature improves upon the binomial model, which only considers upward and downward movements, making the trinomial method more realistic for financial modeling. The probabilities of the stock increasing, remaining unchanged, and decreasing are denoted by p_u, p_m, and p_d. The calculation of probabilities using the trinomial method requires stretch coefficients (λ), standard deviation (σ), and the average rate of return (r). The probability parameters formulation in the Kamrad-Ritchken trinomial model is as follows [16]:

$$u = e^{\lambda\sigma\sqrt{\Delta t}} \tag{1}$$

$$d = \frac{1}{u} \tag{2}$$

$$p_u = \frac{1}{2\lambda^2} + \frac{\left(r - \frac{\sigma^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma} \tag{3}$$

$$p_m = 1 - \frac{1}{\lambda^2} \tag{4}$$

$$p_d = 1 - p_u - p_m \tag{5}$$

For enhanced realism, the stretch parameter (λ) is set to $\sqrt{3}$, increasing the likelihood of no price movement compared to standard methods. These probabilities ensure that the trinomial model accurately reflects market dynamics and converges efficiently [9]. Consequently, the following formulas are obtained:

$$u = e^{\lambda\sigma\sqrt{3\Delta t}} \tag{6}$$

$$p_u = \frac{1}{6} + \left(r - \frac{\sigma^2}{2}\right)\sqrt{\frac{\Delta t}{12\sigma^2}} \tag{7}$$

$$p_m = \frac{2}{3} \tag{8}$$

$$p_d = \frac{1}{6} - \left(r - \frac{\sigma^2}{2}\right)\sqrt{\frac{\Delta t}{12\sigma^2}} \tag{9}$$

Suppose an option has a lifespan of $[0, T]$, then it assumed that the current price is $S_{0,0}$. So, at time T, when the stock price rises to $S_{0,0}u$ with a probability of p_u, where $S_{0,0}u$ is the result of multiplying the initial stock price by the upward movement factor. Conversely, the stock price falls at the time T to $S_{0,0}d$ with a probability of p_d, where $S_{0,0}d$ is the result of multiplying the initial stock price by the downward movement factor. Meanwhile, when the stock price remains constant at time T with a probability of p_m, it will have the same value as the initial stock price [17]. The following Figure 1 is an illustration of a one-step Trinomial tree:

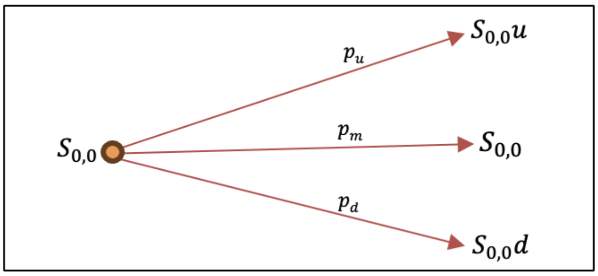

Figure 1. Illustration One-Step Trinomial Tree

The stock price is determined by dividing the time interval into several sub-intervals. After *i*-th period, there will be $j = 2i + 1$ possible stock price outcomes due to price fluctuations. The initial stock price is denoted by $S_{0,0}$. The stock price in the *i*-th period with state *j* is defined as follows:

$$S_{i,j} = S_{0,0}u^i d^j \tag{10}$$

As an illustration, a three-steps trinomial tree is depicted in Figure 2:

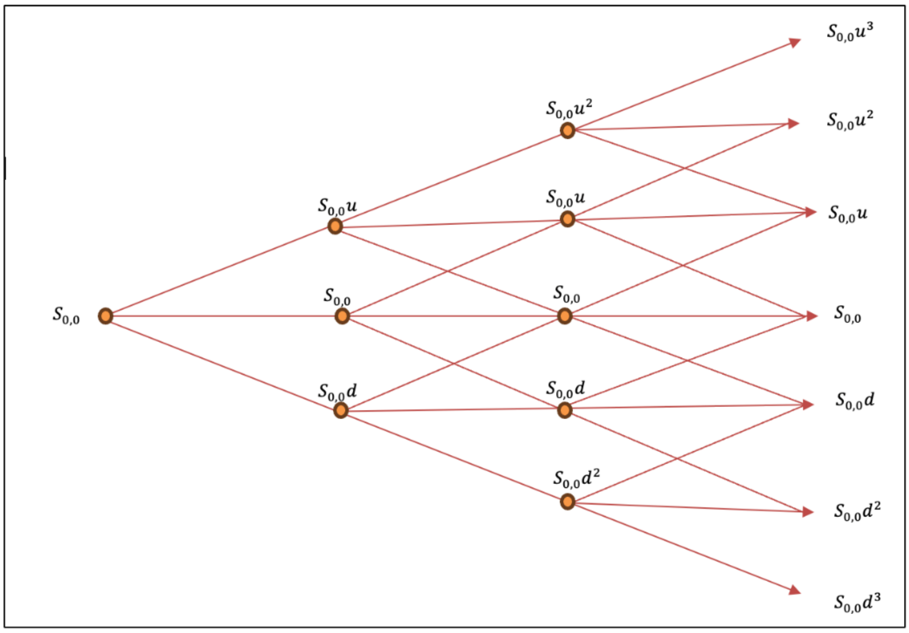

Figure 2. Illustration Three-Steps Trinomial Tree

Figure 2 shows the mechanism of trinomial tree. The initial stock price is $S_{0,0}$. After one step, the stock price has three possibilities. The stock price will change to $S_{0,0}u$, $S_{0,0}d$, or remain the same. From each point, the next step is also the same. The price will increase, decrease, or remain the same. Hence, there will be three points after one time-step and five points after two time-step. The rest will follow the previous mechanism. After n steps, there will be $2n+1$ points.

2.2 Dilution Effect

The dilution effect is modeled by adjusting stock prices when ESOs are exercised. The adjustment accounts for the increased number of shares outstanding and reflects this change in the valuation. The post-dilution stock price is calculated as a weighted average based on the number of existing shares (ω), the number of ESOs granted (θ), the strike price (K), and the stock price at time i and state j $(S_{i,j})$. If it is assumed that all Employee Stock Options (ESOs) are executed at a certain point or not executed at all, then the post-dilution stock price is the weighted average of ω shares outstanding with $S_{i,j}$ price and θ shares resulted from ESO with K price [9].

$$\text{The stock price after affected by the dilution effect } = \frac{\omega S_{i,j} + \theta K}{\omega + \theta} \tag{11}$$

2.3 Early Exercise Effect

The exercise rate of Employee Stock Options (ESOs) cannot be assumed to be the same at all points in time. Exercise rates are high in the early periods after the vesting time (s) then it will decrease and increase again as the maturity date (T) approaches. Early exercise is modeled using a modified Chi-Square distribution, which captures the likelihood of employees exercising their options before maturity. This distribution accounts for the time elapsed since vesting and the relationship between stock price and strike price.

2.4 Employee Forfeiture Effect

Employee forfeiture could be influenced by Employee Stock Options (ESOs). When ESOs are still in the vesting period, employees are less likely to leave the company because the benefits of ESOs can only be realized after the vesting period. Let q represent the employee forfeiture rate in all periods. When the options are in the money, employees are more likely to exercise their ESOs. Therefore, the employee forfeiture rate tends to increase to $(q+R_n)\Delta t$. However, when the option returns are less than or equal to zero, the options are not exercised. This is leading to a decrease in the employee forfeiture rate to $(q-R_n)\Delta t$. Adjustments in the employee forfeiture rate also occur during the vesting period. The employee forfeiture rate is negatively correlated with the ratio of stock price to exercise price, $S_{i,j}/K$. Therefore, the adjustment of the employee forfeiture rate during the vesting period is $Kq\Delta t/S_{i,j}$ [11].

2.5 The Modified Employee Stock Option

The modified Employee Stock Option (ESO) model considers dilution, the influence of early exercise rates, and the effect of employee forfeiture [11]. The parameters in this model are the number of shares outstanding before ESO exercised (ω), the number of ESOs granted (θ), the strike price (K), the time interval for each step (Δt), the vesting period (s), the length of periods after ESOs vested (n), the number of periods from ESOs vested to maturity date (h), maturity date (T), the employee forfeiture rate (q), and the exercise ratio (R_n).

To obtain the price of ESO, the first step is to determine the value of the option in each node. For each node after the ESO vested, the option value is

$$C_{m,j} = \max\left(\frac{S_{m,j}\omega + K\theta}{\omega+\theta} - K, 0\right) \tag{12}$$

for $m = s+n, n = 1, 2, \dots, h$, and $j = 0,1,2, \dots ,2n$.

Recall that there are h different times to exercise option. Option exercised time is $m\Delta t$. For each exercise time, find the option value to determine the final price for ESO. Hence, there are h different option prices based on the time they are exercised. The current price of ESO at each node is calculated using the implementation of trinomial approach. The value of ESO is calculated based on four conditions.

1. If ESO is vested and $S_{i,j} > K$, then:

$$C_{i,j} = [1 - (q + R_n)\Delta t]e^{-r\Delta t}[p_u C_{i+1,j} + p_m C_{i+1,j+1} + p_d C_{i+1,j+2}] \quad (13)$$

2. If ESO is vested and $S_{i,j} < K$, then:

$$C_{i,j} = [1 - (q - R_n)\Delta t]e^{-r\Delta t}[p_u C_{i+1,j} + p_m C_{i+1,j+1} + p_d C_{i+1,j+2}] \quad (14)$$

3. If ESO is vesting and $\frac{K}{S_{i,j}} q\Delta t < 1$, then:

$$C_{i,j} = \left[1 - \frac{K}{S_{i,j}} q\Delta t\right] e^{-r\Delta t}[p_u C_{i+1,j} + p_m C_{i+1,j+1} + p_d C_{i+1,j+2}] \quad (15)$$

4. If ESO is vesting and $\frac{K}{S_{i,j}} q\Delta t \geq 1$, then:

$$C_{i,j} = 0 \quad (16)$$

Let C_n be ESO value when the option is exercised at $(s + n)\Delta t$. To value the final price for ESO, the price of each C_n will be taken to find the weighted average. R_n is used to define weight of C_n. The formula of weighted average for C is

$$C = \sum_{n=1}^{h} R_n C_n \quad (17)$$

3. RESULTS AND DISCUSSION

Chi Squared distribution with parameter $k = 4$ is used for the initial simulation based on its practical suitability for modeling employee behavior. This choice reflects the assumption that employees are more likely to exercise their options when the stock price reaches at least four times the strike price. The parameter captures the tendency of employees in higher management levels to exercise their options at a higher threshold compared to those in mid-level positions. Additionally, previous research has demonstrated that aligns well with observed behaviors in similar financial models, ensuring consistency and reliability in the simulation results. Other parameters are listed in Table 1. With these parameters, the ESO value is Rp22.52. Another valuation of ESO price in this section is using parameters value in the Table 1, unless stated otherwise. The data used in this study is derived from a combination of theoretical assumptions and values commonly applied in financial modeling literature. The parameters listed in Table 1 were selected to provide a representative baseline for analyzing Employee Stock Options (ESOs). Each value was chosen based on practical relevance and consistency with prior studies, ensuring the results are both robust and comparable. These parameter values provide a realistic foundation for analyzing ESO pricing and ensure the results are applicable to a broad range of corporate contexts.

Table 1. Initial Parameters

Parameter	Value
Shares outstanding (ω)	200000000
ESOs granted (θ)	5000000
Initial stock price (S_0)	Rp100
Strike price (K)	Rp100
Risk-free rate (r)	6%
Volatility (σ)	25%
Forfeiture rate (q)	5%
Time interval (Δt)	0.25 year
Vesting time (s)	5 years
Maturity date (T)	10 years

Table 2 shows the effect of the time interval used for pricing ESOs. As an illustration, $\Delta t = 0.25$ year (3 months) means employees are allowed to exercise the option every three months. The more exercise time employees could do every year, the more valuable ESOs price is. If the transaction could happen at any time ($\Delta t \to 0$), the option price would converge. This can be demonstrated by looking at the graph in **Figure 3**. **Figure 3** shows that the option price will gradually increase until converging to a certain point.

Table 2. ESO Price with Different Value of Δt

Time Interval (Δt) (year)	Price (Rp)
1.000	17.71
0.500	20.89
0.333	21.91
0.250	22.52
0.167	23.18

Furthermore, it is illustrated in **Figure 3**.

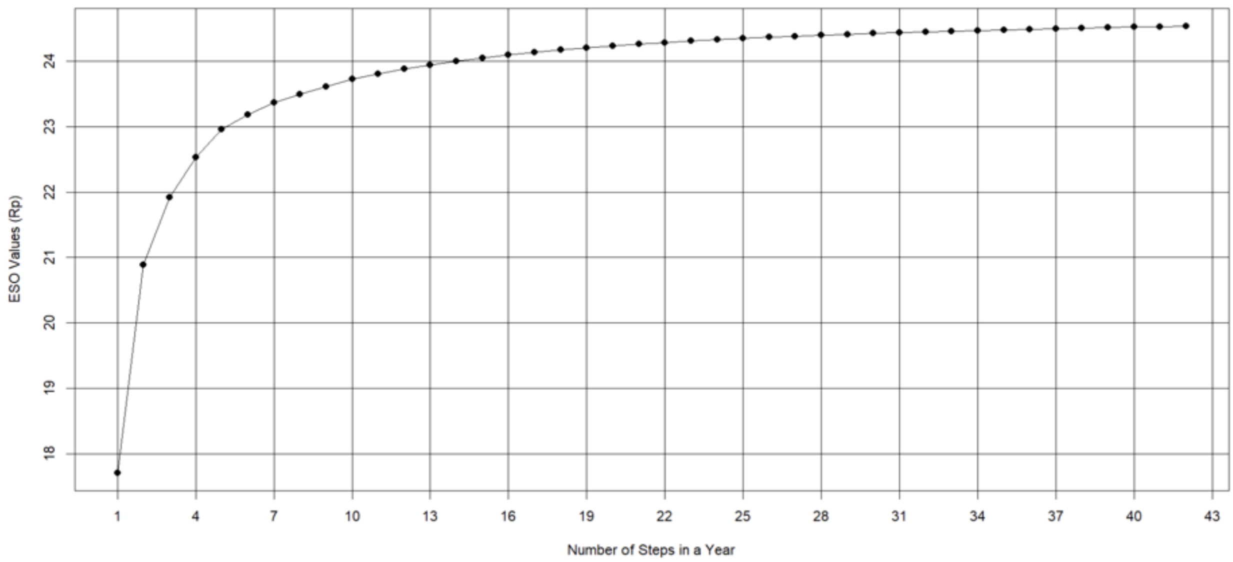

Figure 3. A Graph Illustrating the Relationship Between the Number of Steps and the Price of ESO

Strike price and the numbers of ESOs granted determine how much dilatation affects the stock price. **Table 3** shows that strike price and price of ESO have a reciprocal relationship. The relationship between the number of ESOs granted and the price of ESO are also reciprocal.

Table 3. ESO Price with Different Value of Strike Price

Strike price (K) (Rp)	Price (Rp)
96	24.15
97	23.74
98	23.33
99	22.93
100	22.53
101	22.25
102	21.88
103	21.50
104	21.13
105	20.76

The effect of the number of ESOs granted to the option price is listed in **Table 4**. As shown in **Table 4**, the increase in the number of options granted reduces the price value of ESO. But the change in the number of ESO granted does not affect significantly to the option price.

Table 4. ESO Price with Different Number of ESOs Granted

Number of ESOs granted (θ)	Price (Rp)
1000000	22.97
2000000	22.86
3000000	22.75
4000000	22.63
5000000	22.53
6000000	22.42
7000000	22.30
8000000	22.20
9000000	22.09
10000000	21.99

In line with the strike price, the forfeiture rate affects ESOs' price negatively. **Table 5** shows that an increase in the forfeiture rate decreases the value of ESO. The more employees forfeit from the company in a year, the less the value of ESO to account.

As an illustration, let company A and B have the same characteristic ESOs except for forfeiture rate. Company A need Rp30 for each option because the forfeiture rate is around 1%. Company B, with a forfeiture rate of 10%, needs around Rp16 for each option. This scheme generates a tradeoff that companies with lower forfeiture rate should account higher option value.

Table 5. ESO Price with Different Forfeiture Rate

Forfeiture rate (q) (%)	Price (Rp)
1	29.69
2	27.70
3	25.85
4	24.13
5	22.53
6	21.03
7	19.63
8	18.33
9	17.12
10	15.99

The company could accommodate ESOs with different vesting times. The shorter the vesting time, the less option price is for longer vesting time option. This effect is shown in **Table 6**. The company could grant lower value ESOs with a short vesting time. However, the employees' odds of resigning are higher when their ESOs are exercised.

Consider there are two ESOs with vesting time 3 year and 5 years. In the 4th year, companies with ESOs vesting time 3 year tend to have higher forfeiture rate compared to ESOs with 5 year vesting time.

Table 6. ESO Price with Different Vesting Time

Vesting time (year)	Price (Rp)
1	18.34
2	19.51
3	20.55
4	21.55
5	22.53
6	23.48
7	24.39
8	25.21
9	25.68

The common maturity date used by a company is 10 years. The company could make this maturity date shorter or longer based on demand. **Table 7** shows longer maturity date leads to higher prices for ESOs. However, the differences of ESO price are not significant for longer maturity date.

Table 7. ESO Price with Different Maturity Date

Maturity date (year)	Price (Rp)
1	19.51
2	20.63
3	21.41
4	22.02
5	22.53
6	22.94
7	23.27
8	23.53
9	23.73
10	23.87

Risk free rate used for modeling ESO price has a positive correlation with ESO price. The positive correlation also appears in volatility with ESO price. The increase in the risk-free rate increases the price for ESO. This relationship is shown in **Table 8** and **Table 9**.

Table 8. ESO Price with Different Risk-free Rate

Risk-free rate (%)	Price (Rp)
3	14.93
4	17.44
5	19.99
6	22.53
7	25.01
8	27.41
9	29.71
10	31.89
11	33.93
12	35.85

Table 9 shows that the increase in volatility increases the price of ESO. It is shown that the price gradually increases from Rp19.51 when the volatility 5% to Rp23.87 when the volatility is 50%.

Table 9. ESO Price with Different Volatility

Volatility (%)	Price (Rp)
5	19.51
10	20.63
15	21.41
20	22.02
25	22.53
30	22.94
35	23.27
40	23.53
45	23.73
50	23.87

The results of this study highlight the effectiveness of the trinomial method in pricing Employee Stock Options (ESOs) under various conditions. Simulation results show that ESO prices are influenced by multiple factors, with notable trends and correlations observed.

Firstly, the analysis confirms a negative correlation between ESO price and strike price. As the strike price increases, the value of the ESO decreases, reflecting the reduced financial benefit for employees. Similarly, a higher forfeiture rate negatively impacts ESO prices, as the likelihood of employees leaving before exercising their options increases. In contrast, positive correlations are observed with parameters such as volatility, vesting period, and maturity date. Higher volatility leads to a broader range of potential stock price outcomes, increasing the ESO's expected value. Longer vesting periods and maturity dates provide employees with more opportunities to benefit from favorable stock price movements, enhancing the option's value. The dilution effect is effectively modeled by incorporating the impact of additional shares on stock prices. Results indicate that an increased number of ESOs granted slightly reduces individual ESO value due to the adjustment in stock price caused by dilution. However, this effect is mitigated when the number of outstanding shares is significantly larger than the granted options.

Early exercise behavior, modeled using a modified Chi-Square distribution, reveals that employees are more likely to exercise their options shortly after vesting or as the maturity date approaches. This aligns with observed real-world behaviors and underscores the importance of accurately modeling early exercise probabilities.

The study also demonstrates the computational advantages of the trinomial method. Compared to the binomial model, the trinomial approach converges more rapidly and provides more accurate results by accounting for scenarios where stock prices remain unchanged. This feature enhances the model's realism and applicability in practice.

Practically, these findings offer valuable insights for companies designing ESO programs. Optimizing parameters such as strike price and vesting period can balance employee incentives with organizational costs. Additionally, understanding the interplay between forfeiture rates and ESO value can inform retention strategies and financial planning.

Overall, this study validates the applicability of the Liao and Lyuu model [11] when extended with the trinomial method. By addressing key factors such as dilution, early exercise, and forfeiture rates, the research provides a robust framework for ESO valuation and practical recommendations for implementation in real-world scenarios.

4. CONCLUSIONS

This study highlights significant advantages of the proposed method over Liao and Lyuu's original model. By applying the trinomial tree method, this research achieves greater computational efficiency and realism. The trinomial approach accounts for scenarios where stock prices remain unchanged, a feature absent in Liao and Lyuu's binomial-based framework. Additionally, the dynamic adjustment of forfeiture rates based on stock performance provides a more accurate representation of employee behavior, further enhancing the model's practical applicability. This study successfully demonstrates the application of the modified trinomial tree method to the Employee Stock Option (ESO) model proposed by Liao and Lyuu. By incorporating dilution effects, early exercise probabilities, and forfeiture rates, the research provides a robust framework for ESO valuation. The trinomial method's ability to account for three possible stock price movements enhances its realism and computational efficiency compared to the binomial model. ESO price is shown to be negatively correlated with strike price and forfeiture rate, while positively influenced by factors such as volatility, vesting period, and maturity date. This work bridges theoretical advancements and practical applications, offering a reliable tool for companies to optimize ESO programs. Future research could expand the model to include additional factors such as macroeconomic conditions and sector-specific dynamics to further enhance its relevance and applicability.

REFERENCES

[1] N. M. W. Menawati and I. B. P. Astika, “PENGARUH RENTABILITAS DAN LIKUIDITAS PADA JUMLAH OPSI SAHAM DAN DAMPAKNYA PADA NILAI PERUSAHAAN,” *E-Jurnal Akuntansi Universitas Udayana*, vol. 18, no. 3, pp. 1915–1942, 2017.

[2] N. Gusnela and D. Ahmad, “PENENTUAN NILAI OPSI SAHAM KARYAWAN (OSK) DENGAN MEMPERHITUNGKAN EFEK DILUSI MENGGUNAKAN METODE LATTICE TRINOMIAL,” *UNP Journal of Mathematics*, vol. 3, no. 1, pp. 48–52, 2020.

[3] F. J. Fabozzi and P. P. Peterson, *FINANCIAL MANAGEMENT AND ANALYSIS SECOND EDITION*, Second. New Jersey: John wiley & Sons, Inc., 2013.

[4] E. Wahyuni, R. Lestari, and M. Syafwan, “MODEL BLACK-SCHOLES OPSI CALL DAN OPSI PUT TIPE EROPA DENGAN DIVIDEN PADA KEADAAN CONSTANT MARKET,” *Jurnal Matematika UNAND*, vol. 6, no. 2, pp. 43–49, 2017.

[5] S. N. Hayuningrum and R. Artiono, “PEMODELAN MATEMATIKA OPSI SAHAM KARYAWAN MENGGUNAKAN METODE BINOMIAN ENHANCED AMERICAN,” *MATHunesa: Jurnal Ilmiah Matematika*, vol. 11, no. 2, pp. 186–191, 2023.

[6] M. Athar, “EMPLOYEE STOCK OPTION PLANS: A META-ANALYSIS (UNDERSTANDING IMPACT OF ESOPS THROUGH LITERATURE),” *STUDIES IN BUSINESS AND ECONOMICS*, vol. 15, no. 1, pp. 100–114, Apr. 2020, doi: 10.2478/sbe-2020-0009.

[7] E. Chendra, L. Chin, and A. Sukmana, “PRICING EMPLOYEE STOCK OPTIONS (ESOS) WITH RANDOM LATTICE,” *IOP Conf Ser Mater Sci Eng*, vol. 335, p. 012041, Apr. 2018, doi: 10.1088/1757-899X/335/1/012041.

[8] M.-C. Wu, I.-C. Lin, and Y.-T. Huang, “MANAGING EMPLOYEE STOCK OPTION EXPENSE: A FAIR VALUE APPROACH,” *Australian Accounting Review*, vol. 26, no. 79, pp. 429–438, 2016.

[9] J. Hull, *Options, FUTURES AND OTHER DERIVATIVES*, 11th ed. New York: Pearson, 2022.

[10] E. Rahmi, “TRINOMIAL MODEL ON EMPLOYEE STOCK OPTION VALUE,” *IOSR-JM*, vol. 13, no. 5, pp. 23–28, 2017.

[11] F. Y. Liao and Y. D. Lyuu, “AN EXPANDED MODEL FOR THE VALUATION OF EMPLOYEE STOCKS OPTIONS,” *The Journal of Futures Markets*, vol. 29, no. 8, pp. 713–735, 2009.

[12] A. Fitri and Suherman, “PENENTUAN NILSI OPSI SAHAM KARYAWAN (OSK) DENGAN MEMPERHITUNGKAN EFEK DILUSI DENGAN MENGGUNAKAN METODE BLACK-SCHOLES,” *UNPjoMath* , vol. 3, no. 2, pp. 7–11, 2020.

[13] H. Maulana and K. A. Sidarto, “VALUING EMPLOYEE STOCK OPTION (ESO) UNDER EMPLOYEE FORFEITURE RATE,” in The International Confrence on Mathematics, Science, Education and Technology,” Faculty of Mathematics and Sciences, Universitas Negeri Padang, 2015, pp. 71–77.

[14] V. Rosalina, “PEMODELAN MATEMATIKA OPSI SAHAM KARYAWAN MENGGUNAKAN METODE TRINOMIAL YANG MEMPERHITUNGKAN EFEK DILUSI,” *Jurnal Ilmiah Matematika*, vol. 11, no. 2, pp. 192–198, 2023.

[15] K. K. Langat, J. I. Mwaniki, and G. K. Kiprop, “PRICING OPTION USING TRINOMIAL LATTICE METHOD,” *Journal of Finance and Economics*, vol. 7, no. 3, pp. 81–87, 2019.

[16] J. Lee, J.-R. Chang, L.-J. Kao, and C.-F. Lee, *ESSENTIALS OF EXCEL VBA, PYTHON, AND R: FINANCIAL DERIVATIVES, RISK MANAGEMENT AND MACHINE LEARNING*, 2nd ed., vol. 2. 2023.

[17] R. Meliyani, E. H. Nugrahani, and D. C. Lesmana, “PENENTUAN HARGA OPSI CALL WINDOW RESET MENGGUNAKAN METODE BINOMIAL TREE DAN TRINOMIAL TREE,” *Journal of Mathematics and Its Applications*, vol. 11, no. 2, pp. 192–198, 2023.