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MONITORING THE SAUSAGE PRODUCT USING LANEY DEMERIT CHART BASED ANALYTICAL HIERARCHY PROCESS

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ABSTRACT

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Ready-to-eat sausage is a food product that has a limited shelf life. Therefore, regularly monitoring the quality of packaged ready-to-eat sausage products is important to ensure that the products meet the established quality standards. Twelve types of product defects need to be observed in the final checking process to meet the quality standards, namely Wrinkle, Dots, Leaking, Product Stain, Non-standard Form, Poor Print Quality, Vacuum Leaks, Weak Ties, Body Defects, Uneven Length, Broken Node, and Small Stain. This study aims to apply the Laney Demerit Control Chart (LDCC) and Analytical Hierarchy Process-Integrated Statistical Process Control (AHP-ISPC) methods to monitor the quality of packaged ready-to-eat sausage production at XYZ Inc. The data is from the quality testing of ready-to-eat sausage products taken from XYZ Inc. for six months from April 1, 2023, until September 30, 2023. The findings reveal that conventional control charts (u control chart, demerit control chart, and AHP-based demerit control chart) exhibited oversensitivity because it is attributed to the large number of samples produced by the company, prompting the need for a more balanced approach. Implementing the Laney u control chart, Laney demerit control chart, and the AHP-based Laney demerit control charts successfully achieved statistical control in phase I. In contrast, phase II still demonstrated challenges, particularly with the AHP-based Laney Demerit Control Chart detecting the highest number of out-of-control points. This suggests that phase II remains statistically out of control, necessitating further analysis or corrective measures to enhance process stability. Additionally, the process capability analysis indicated that the production process during the specified period lacked capability, as evidenced by a capability index value below one (0.883).

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1. INTRODUCTION

The global food industry, a crucial component of the economy, is experiencing a surge in demand for ready-to-eat products, especially packaged sausages. Ready-to-eat packaged sausages are one of the most popular food products in the market, and their production process requires strict quality control to ensure that the products meet the required standards [1]. XYZ Inc., a major player in the food and beverage sector, specializes in the high production of these famous sausages, emphasizing the need for strict quality control due to their limited shelf life. The production process involves various stages, necessitating consistent and up-to-date quality to ensure consumer confidence and market competitiveness.

The study explores the application of the Laney *u* control chart to recognize the importance of quality control [2], Laney Demerit control chart (LDCC), and Analytical Hierarchy Process-Integrated Statistical Process Control (AHP-ISPC) [3] methods in monitoring ready-to-eat sausage production at XYZ Inc. With 12 types of defects observed, including wrinkles, dots, leaking, product stains, non-standard forms, poor print quality, vacuum leaks, weak ties, body defects, uneven length, broken nodes, and small stains, the study aims to contribute theoretically and practically to the food industry's knowledge.

Highlighting the significance of statistical process control methods, such as LDCC and AHP-ISPC, the study addresses the need for more research on their application to monitor sausage production. The LDCC method, based on a demerit system, uses graphical tools to monitor quality over time, while AHP-ISPC is a decision-making tool to prioritize quality control activities. According to a study by [4] statistical process control (SPC) methods can help improve the quality of food products, including ready-to-eat packaged sausages. One of the SPC methods that can be used is the LDCC, a graphical tool that can be used to monitor the quality of a process over time. Another method that can be used is the AHP-ISPC, which is a decision-making tool that can be used to prioritize quality control activities.

The demerit control chart was introduced by [5] to track the weighted sum of multiple errors in several different error types [6]. The Analytical Hierarchy Process (AHP) is an effective decision-making method to address complex problems involving numerous criteria and alternatives [7]. When evaluating criteria and alternatives using paired comparison evaluation, comparisons are made based on the decision maker's policy by examining the priority of a comparison criterion with other criteria. By employing a hierarchical structure, AHP breaks down the problem into smaller elements, allowing decision-makers to provide relative assessments and weights to each element. AHP's strengths include its ability to handle uncertainty and subjectivity, check decision consistency, and its flexibility in incorporating qualitative and quantitative factors. AHP finds wide applications across various decision-making contexts, such as strategic planning, project selection, resource allocation, and vendor selection. Thus, AHP provides a systematic framework to assist decision-makers in making informed and structured decisions [8].

In conclusion, the study's dual importance lies in advancing theoretical knowledge in the food industry and providing practical insight into the effectiveness of LDCC and AHP-ISPC in improving the quality of ready-to-eat packaged sausages at XYZ Inc. The LDCC method is a graphical tool that can be used to monitor the quality of a process over time. It is based on the demerit system, which assigns a certain number of demerit points to each type of defect that occurs in the process [9]. The demerit points are then plotted on a control chart, which allows the user to monitor the process and detect any changes in the quality of the product.

The quality control procedures within the production process at XYZ Inc., overseen by the quality control division, have thus far relied on a simplistic approach involving utilizing Pareto diagrams and presenting descriptive statistics. The investigation seeks answers to the questions of how to effectively apply the Laney *u* Control Chart, LDCC, and AHP-ISPC methods in monitoring the quality of ready-to-eat sausage production at XYZ Inc. Additionally, the study seeks to conduct a comprehensive capability analysis of the ready-to-eat sausage product at XYZ Inc. These inquiries are motivated by the desire to enhance and diversify existing quality control methodologies, ensuring a more comprehensive and nuanced understanding of the production process and product quality.

2. RESEARCH METHODS

2.1 Demerit Control Chart

When controlling the manufacturing process, many types of defects are often detected. However, not all errors are equally important; Maybe error A is more serious than error B, error C, or vice versa. Therefore, it is necessary to classify many types of defects. A deficiency control chart is a method of classifying defects based on their severity. The classification of disabilities according to severity is as follows [10].

1. Class A (very serious): this type of error is classified as the most severe or cannot even be used. Furthermore, this type of defect can also cause material damage.
2. Class B (critical): this type of fault is classified as a type A malfunction and can lead to reduced service life and increased maintenance costs.
3. Class C (Medium Severe) is classified as a fault that will cause a service failure or cause a no severe problem such as an operational error.
4. Class D (Minor): This type of defect is classified as defects that do not cause damage during use but have minor defects, such as finish, appearance, or quality of work.

The weighted number of defects in each category can be calculated using the formula in **Equation (1)** for weight number of defects in class A, **Equation (2)** for weight number of defects in class B, **Equation (3)** for weight number of defects in class C and **Equation (4)** for weight number of defects in class D.

$$w_1 c_1 \tag{1}$$

$$w_2 c_2 \tag{2}$$

$$w_3 c_3 \tag{3}$$

$$w_4 c_4 \tag{4}$$

where c_1, c_2, c_3, c_4 is the number of defects of categories A, B, C and D and w_1, w_2, w_3, w_4 is the weight of the defects in each category. The next step is to calculate the number of defects in each subgroup observation using the formula in **Equation (5)**.

$$D_i = w_1 c_{i1} + w_2 c_{i2} + w_3 c_{i3} + w_4 c_{i4} \tag{5}$$

where $i = 1,2,3, \dots, m$.

To calculate the average value of defects per unit (u_i) for each observation in a subgroup, we must divide the weighted number of defects in each subgrade (D_i) by the number of samples in every observation (n_i). The value is obtained using the formula from **Equation (6)**

$$u_i = \frac{D_i}{n_i} \tag{6}$$

where $i = 1,2,3, \dots, m$.

The values of $\bar{u}_1, \bar{u}_2, \bar{u}_3$, and $\bar{u}_4$ are the average defects per unit for types A, B, C, and D. These values are obtained using the formula in **Equation (7)**, **Equation (8)**, **Equation (9)** , and **Equation (10)** [11].

$$\bar{u}_1 = \frac{\sum_{i=1}^{m} \frac{c_{i1}}{n_i}}{m} \tag{7}$$

$$\bar{u}_2 = \frac{\sum_{i=1}^{m} \frac{c_{i2}}{n_i}}{m} \tag{8}$$

$$\bar{u}_3 = \frac{\sum_{i=1}^{m} \frac{c_{i3}}{n_i}}{m} \tag{9}$$

$$\bar{u}_4 = \frac{\sum_{i=1}^{m} \frac{c_{i4}}{n_i}}{m} \tag{10}$$

The value of the average number of defects per unit for the weighted overall defect category ($\bar{U}$) is calculated using the formula from **Equation (11)**

$$\bar{U} = w_1 \bar{u}_1 + w_2 \bar{u}_2 + w_3 \bar{u}_3 + w_4 \bar{u}_4 \tag{11}$$

Then, the lower control limit, center line, and upper control limit are obtained in **Equation (12)**, **Equation (13),** and **Equation (14)**.

$$LCL_i = \bar{U} - 3\hat{\sigma}_i \tag{12}$$

$$CL \quad = \bar{U} \tag{13}$$

$$UCL_i = \bar{U} + 3\hat{\sigma}_i \tag{14}$$

with the $\hat{\sigma}_i$ obtained using the formula in **Equation (15)**.

$$\hat{\sigma}_i = \sqrt{\frac{{w_1}^2\bar{u}_1 + {w_2}^2\bar{u}_2 + {w_3}^2\bar{u}_3 + {w_4}^2\bar{u}_4}{n_i}} \tag{15}$$

2.2 Analytical Hierarchy Process (AHP)

The analytical hierarchy process is a method for overcoming a complex unstructured problem by organizing/prioritizing it based on the experts' preferences. AHP is based essentially on three basic principles: creating a hierarchy, developing alternative criteria and assessments using pairwise comparison assessments, and calculating the consistency ratio (CR) [12].

The analytical hierarchy process (AHP) method was first developed by Professor Thomas Lorie Saaty of the Wharton School of Business in the early 1970s. It is used to find the ranking of different options or priority order to solve a problem. There are two reasons this AHP method is used as a problem-solving method compared to other methods. The first is the hierarchical structure, which is the consequence of the chosen criteria. Second, consider the validity or consistency, which means having a limited tolerance for inconsistency as the alternative to be selected [13].

When evaluating criteria and alternatives using paired comparison evaluation, comparisons are made based on the decision maker's policy by examining the priority of a comparison criterion with other criteria. The basic value scale for pairwise comparisons is presented in **Table 1** [13].

Table 1. Basic Scale of Pairwise Comparison Values

Basic Scale	Description
1	Both criteria are equally important
3	One criterion is slightly more important than the other
5	One criterion is more important than the other
7	One criterion is clearly more important than the other
9	One criterion is clearly/absolutely more important than the others
2, 4, 6, 8	A value that falls between two adjacent comparisons

If the pairwise comparison matrix has been established, the geometric mean value is calculated based on the relevant experts' assessment. Furthermore, it can be denoted as a pairwise comparison matrix table based on criteria as in **Table 2**.

Table 2. Pairwise Comparison Matrix

K	K_1	K_2	K_3	$\cdots$	K_l
K_1	1	a_{12}	a_{13}	$\cdots$	a_{1l}
K_2	a_{21}	1	a_{23}	$\cdots$	a_{2l}
K_3	a_{31}	a_{32}	1	$\cdots$	a_{3l}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
K_l	a_{l1}	a_{l2}	a_{l3}	$\cdots$	1
Total	N_1	N_2	N_3	$\cdots$	N_l

The geometric mean calculation provides a better averaging approach because it minimizes the differences between one expert's assessment and another in the questionnaire asked by AHP. The geometric mean can be constructed as **Equation (16)** [14].

$$\boldsymbol{G} = \sqrt[r]{\boldsymbol{a_1} \times \boldsymbol{a_2} \times \ldots \times \boldsymbol{a_r}}\ , \tag{16}$$

where $\boldsymbol{G}$ is the geometric mean, $\boldsymbol{a_1}$ is the result of the first inspection, $\boldsymbol{a_2}$ is the result of the second inspection, and $\boldsymbol{r}$ is the number of experts who completed the AHP questionnaire.

Next, we normalize the pairwise comparison matrix. This matrix normalization can be calculated using the formula in **Equation (17)**.

$$(\boldsymbol{t_i}, \boldsymbol{t_j}) = \frac{a_{ij}}{N_l}, \tag{17}$$

where $\boldsymbol{i}, \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \ldots, \boldsymbol{l}$, $(\boldsymbol{t_i}, \boldsymbol{t_j})$, is the normalization of the pairwise comparison matrix of the *i* th row and *j* th column, $\boldsymbol{a_{ij}}$ is the element of the pairwise comparison matrix of the *i-th* row, and *j* th column, and $\boldsymbol{N_i}$ is the sum of the elements of the column *j-th* of the pairwise comparison matrix. Next, weight the criteria by calculating the average of each row contained in the normalized pairwise comparison matrix using **Equation (18)**.

$$W_c = \frac{\sum_{j=1}^{l} a_{ij}}{l}. \tag{18}$$

Subsequently, the matrix obtained from the normalization of pairwise comparisons can be formally represented and organized into a structured tabular format, exemplified in **Table 3**.

Table 3. Normalized Pairwise Comparison Matrix

K	K_1	K_2	K_3	$\cdots$	K_l	W_{c_i}	W_{s_i}
K_1	t_{11}	t_{12}	t_{13}	$\cdots$	t_{1l}	W_{c_1}	W_{s_1}
K_2	t_{21}	t_{22}	t_{23}	$\cdots$	t_{2l}	W_{c_2}	W_{s_3}
K_3	t_{31}	t_{32}	t_{33}	$\cdots$	t_{3l}	W_{c_3}	W_{s_3}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
K_l	t_{l1}	t_{l2}	t_{l3}	$\cdots$	t_{ll}	W_{c_l}	W_{s_l}

The results of the AHP analysis are valid if they are consistent. The consistency ratio (CR) value must be less than 10% to be said to be consistent. The consistency ratio (CR) value can be calculated using the formula in **Equation (19) [15]**.

$$CR = \frac{CI}{RI}, \tag{19}$$

with RI is the random index value that has been formulated as in **Table 4 [15]**.

Table 4. Random Index Value

Number of Criteria (*l*)	1	2	3	4	5	6	7	8	9	10
Random Index	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The CI value is the consistency index obtained using the formula in **Equation (20)**.

$$CI = \frac{\lambda_{max} - l}{l - 1}, \tag{20}$$

with $\boldsymbol{\lambda_{max}}$ values obtained using the formula in **Equation (21)**.

$$\lambda \frac{\sum_{i=1}^{l} W_{s_i}}{l}_{max}, \tag{21}$$

with the value of $\boldsymbol{W_{s_i}}$ is the result of multiplying the elements in the pairwise comparison matrix by the weight of the criteria. Then divided by the weight of the criteria in accordance with the order of the criteria and $\boldsymbol{l}$ is the number of criteria. The calculation of $\boldsymbol{W_{s_i}}$ is obtained using **Equation (22)**.

$$\begin{pmatrix} 1 & a_{12} & a_{13} & \cdots & a_{1l} \\ a_{21} & 1 & a_{23} & \cdots & a_{2l} \\ a_{31} & a_{32} & 1 & \cdots & a_{3l} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{l1} & a_{l2} & a_{l3} & \cdots & 1 \end{pmatrix} \times \begin{pmatrix} W_{c_1} \\ W_{c_2} \\ W_{c_3} \\ \vdots \\ W_{c_l} \end{pmatrix} = \begin{pmatrix} W_{s_1} \\ W_{s_2} \\ W_{s_3} \\ \vdots \\ W_{s_l} \end{pmatrix}$$

$$\begin{pmatrix} W_{s_1} = \frac{1}{W_{c_1}}[1W_{c_1} + a_{12}W_{c_2} + a_{13}W_{c_3} + \ldots + a_{1l}W_{c_l}] \\ W_{s_2} = \frac{1}{W_{c_2}}[a_{21}W_{c_1} + 1W_{c_2} + a_{23}W_{c_3} + \ldots + a_{2l}W_{c_l}] \\ W_{s_3} = \frac{1}{W_{c_3}}[a_{31}W_{c_1} + a_{32}W_{c_2} + 1W_{c_3} + \ldots + a_{3l}W_{c_l}] \\ \vdots \\ W_{s_l} = \frac{1}{W_{c_l}}[a_{l1}W_{c_1} + a_{l2}W_{c_2} + a_{l3}W_{c_3} + \ldots + 1W_{c_l}] \end{pmatrix} \quad (22)$$

2.3 Laney Demerit Control Chart

Laney Demerit is an extension of the Demerit control chart. It measures the number of defects in each unit by grouping defect types according to their seriousness and having many samples [16]. The weighted number of defects for each subgroup observation is calculated using the formula in **Equation (23).**

$$D_i = w_1 c_{i1} + w_2 c_{i2} + w_3 c_{i3} + w_4 c_{i4}\,, \quad (23)$$

where $\boldsymbol{i = 1, 2, 3, \ldots, m}$. After getting the value of $\boldsymbol{D_i}$, the value will be used to calculate the average value of defects per unit ($\boldsymbol{u_i}$) for each subgroup observation by dividing $\boldsymbol{D_i}$ with the number of samples in each subgroup observation ($\boldsymbol{n_i}$). The value of $\boldsymbol{u_i}$ is obtained by using formula in **Equation (24)**.

$$u_i = \frac{D_i}{n_i}\,, \quad (24)$$

where $\boldsymbol{i = 1, 2, 3, \ldots, m}$. Next, calculate the average defects per unit for classes A, B, C, and D using the formula in **Equation (25)**, **Equation (26)**, **Equation (27)** , and **Equation (28)**.

$$\bar{u}_1 = \frac{\sum_{i=1}^{m} \frac{c_{i1}}{n_i}}{m}\,, \quad (25)$$

$$\bar{u}_2 = \frac{\sum_{i=1}^{m} \frac{c_{i2}}{n_i}}{m}\,, \quad (26)$$

$$\bar{u}_3 = \frac{\sum_{i=1}^{m} \frac{c_{i3}}{n_i}}{m}\,, \quad (27)$$

$$\bar{u}_4 = \frac{\sum_{i=1}^{m} \frac{c_{i4}}{n_i}}{m}\,. \quad (28)$$

After obtaining the average defect per unit value for the classes, the next step is to calculate the average number of defects per unit for the overall weighted defect type ($\boldsymbol{\bar{U}}$) using the formula in **Equation (29)**.

$$\bar{U} = w_1\bar{u}_1 + w_2\bar{u}_2 + w_3\bar{u}_3 + w_4\bar{u}_4 \quad (29)$$

Then, after calculating all the values needed to get the three limits on the Laney demerit control chart, the center line, lower control limit, and upper control limit values are obtained using the formula in **Equation (30)**, **Equation (31)** and **Equation (32)**.

$$LCL_i = \bar{U} - 3\hat{\sigma}_i \times \sigma_Z\,, \quad (30)$$

$$CL = \bar{U}\,, \quad (31)$$

$$UCL_i = \bar{U} + 3\hat{\sigma}_i \times \sigma_Z\,, \quad (32)$$

with the $\boldsymbol{\hat{\sigma}_i}$ is obtained using the formula in **Equation (33)**.

$$\hat{\sigma}_i = \sqrt{\frac{{w_1}^2\bar{u}_1+{w_2}^2\bar{u}_2+{w_3}^2\bar{u}_3+{w_4}^2\bar{u}_4}{n_i}}. \tag{33}$$

Next, the $\boldsymbol{\sigma_Z}$ value is the standardized value of the number of defects $(\boldsymbol{u_i})$. Then, the first step is to convert the value of $\boldsymbol{u_i}$ into a *z*-score using **Equation (34)**.

$$Z_i = \frac{(u_i-\bar{U})}{\sqrt{\frac{\bar{U}}{n_i}}}. \tag{34}$$

The next step is to plot the *z* in an individual chart using **Equation (35)** and **Equation (36)**.

$$R_i = |Z_i - Z_{i-1}|\,, \tag{35}$$

$$\overline{MR} = \frac{1}{m-1}\sum_{i=2}^{m} R_i\,. \tag{36}$$

The $\boldsymbol{\sigma_Z}$ value is obtained using **Equation (37)**

$$\sigma_Z = \frac{\overline{MR}}{1.128}. \tag{37}$$

3. RESULTS AND DISCUSSION

This chapter will describe the results of the analysis and discussion conducted in this study on the defect inspection data for ready-to-eat sausage production at XYZ Inc. The analysis steps carried out in this study include analyzing the characteristics of the data, making the u control chart, making the demerit control chart, AHP calculations, making the AHP-based demerit control chart, making the Laney *u* control chart, making the Laney demerit control chart, making the AHP-based Laney demerit control chart, comparison results of the control chart and process capability analysis.

The data source used in this study is secondary data from the quality control division, namely defect inspection data for the production of ready-to-eat packaged sausages at XYZ Inc. in 2023. There are 86 data in subgroups, which indicate daily data from April 1, 2023, until September 30, 2023. The data are separated into two phases. Phase I consists of determining observations in control starting from April 1, 2023, until July 31, 2023. Phase II checks whether shifts are from August 1, 2023, until September 30, 2023.

The variable used in this research is the inspection of defects in the ready-to-eat production process, which has 12 types of defects. The defects will be classified into four different defect classes based on their severity, so it is necessary to weigh each defect class. The weighting given to each defect class with XYZ Inc. policy (based on expert) is shown in **Table 4** below.

Table 4. Research Variable

Class	Description
A	First class (very serious) consisting of leaking, dots, vacuum leaks defect types.
B	Second class (serious) consisting of body defects, weak ties, and product stains defect types.
C	Third class (moderately serious) consisting of uneven length, non-standard form, and poor print quality defect types.
D	Fourth class (minor) consisting small stain, wrinkle, and broken node defect types.

The data structure used in this investigation is presented in **Table 5**.

Table 5. Research Data Structure

Sub-group	Sample	A	B	C	D
1	n_1	C_{11}	C_{12}	C_{13}	C_{14}
2	n_2	C_{21}	C_{22}	C_{23}	C_{24}
3	n_3	C_{31}	C_{32}	C_{33}	C_{34}
⋮	⋮	⋮	⋮	⋮	⋮

Sub-group	Sample	A	B	C	D
i	n_i	C_{i1}	C_{i2}	C_{i3}	C_{i4}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	n_m	C_{m1}	C_{m2}	C_{m3}	C_{m4}

Information:
n_i : Number of samples in the *i*-th subgroup
C_{ia} : Number of class A defects (very serious) in the *I-th* subgroup
C_{ib} : Number of class B defects (serious) in the *i*-th subgroup
C_{ic} : Number of class C defects (moderately serious) in the *i*-th subgroup
C_{id} : Number of class D defects (Almost Serious) in the *i*-th subgroup

3.1 Demerit Control Chart

Continue to the Demerit Control Chart, which provides a concise visual representation of demerit counts over time, allowing quick identification of variations and potential issues in product quality. Its efficiency lies in enabling timely corrective actions, making it an invaluable asset to maintain and improve the overall quality of production. It is used to measure the number of defects in each unit by grouping defect types according to their seriousness and to have a large number of samples. The types of defects found in the ready-to-eat sausage product at XYZ Inc. are grouped into four defect classes with weights shown in **Table 6**.

Table 6. Research Variable

Class	Type of Defect	Weighted Value
A	Leaking, dots, and vacuum leaks	0.70
B	Body defects, weak ties, and product stains	0.15
C	Uneven length, non-standard form, and poor print quality	0.10
D	Small stain, wrinkle, and broken node	0.05

Based on the weighting shown in **Table 6** above, derived from discussions between the author and the relevant parties, the result of the Demerit Control Chart is as follows.

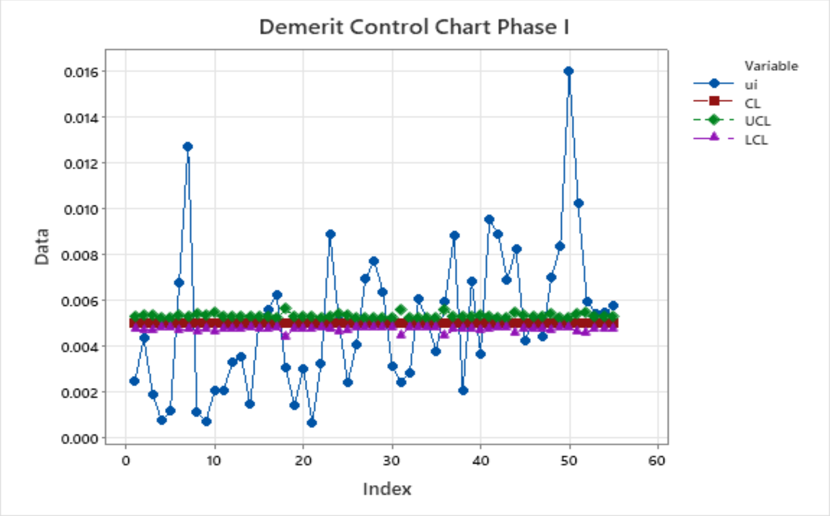

Figure 1. Phase I of the demerit control chart of the demerit control chart

From **Figure 1**, the Demerit Control Chart for Phase I, numerous observations beyond control limits indicate potential oversensitivity in the monitoring process. The weighted values assigned to different classes highlight significant deviations, especially in Class A, that include severe defects such as leaking, dots, and vacuum leaks. This oversensitivity raises concerns about the reliability of the control system, possibly leading to an increased number of false alarms. Therefore, a careful review is necessary to consider potential adjustments to the Demerit Control Chart. This step is crucial as a preparatory measure before proceeding to the Laney Demerit Control Chart method. The goal is to balance sensitivity and precision in capturing meaningful variations while minimizing unnecessary interventions.

3.2 AHP-Based Demerit Control Chart

The Analytical Hierarchy Process (AHP) is employed as a decision-making tool to evaluate the severity of defects. This study involved eight experts in decision-making, using multiple criteria outlined in Table 6, which categorizes the types of defects in the production process. The normalization results of the pairwise comparison matrix are presented in Table 7.

Table 7. The Results of The Normalization of Pairwise Comparison Matrix

	Class A	Class B	Class C	Class D	W_{ci}	W_{si}
Class A	0.730	0.836	0.575	0.546	0.672	4.795
Class B	0.100	0.115	0.310	0.301	0.206	4.200
Class C	0.086	0.025	0.068	0.089	0.067	4.011
Class D	0.084	0.024	0.047	0.063	0.054	4.069

The reliability of the AHP analysis depends on maintaining a CR value below 10%. To determine the CR value, the calculation begins with determining the CI value and subsequently obtaining the λ_{max} value from the CI value. The following section describes the computed values for CI and λ_{max}.

$$\lambda_{max} = \frac{4.795+4.200+4.011+4.069}{4} = 4.269$$

$$CI = \frac{4.269-4}{(4-1)} = 0.089$$

Subsequently, determine the Consistency Ratio (CR). In this study, a random index (RI) value of 0.900 is used due to the inclusion of 4 defect classes. The CR value is then computed based on the formula to assess the degree of consistency in the decision-making process. The results of the CR calculation provide insight into the reliability of the derived weights and the overall consistency of the analytical hierarchy process (AHP) outcomes.

$$CR = \frac{0.089}{0.900} = 0.0996$$

The calculated CR value is 0.0996 or 9.96%, falling below the 10% threshold. It indicates that determining defect weights through the AHP method exhibits consistency. The CR value, being within an acceptable range, signifies the reliability of the Analytical Hierarchy Process (AHP), which results in assigning weights to the various defect classes in the studied production process. After getting the CI, λ_{max}, and CR values, statistical quality control analysis is carried out for phase I.

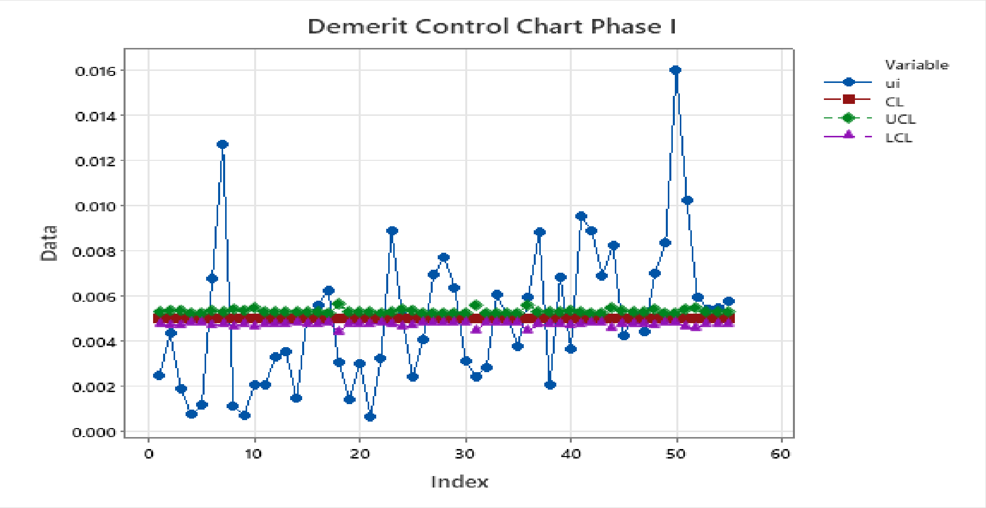

Figure 2. Phase I AHP-based Demerit control charts

The findings in **Figure 2**, derived from analyzing AHP-based Demerit control charts using real data in Phase I, indicate many data points beyond the control limits. This suggests an oversensitivity of all three control charts to the data from the ready-to-eat sausage production at XYZ Inc. This oversensitivity is attributed to the large number of samples the company produces. Consequently, to address this issue, additional analysis is considered necessary, involving the application of the Laney *u* control chart, the Laney demerit control chart, and the AHP-based Laney demerit control charts.

3.3 Laney Demerit Control Chart

To assess statistical control of the production process, Phase I involves statistical quality control, determining which observations fall within acceptable limits. **Figure 3** illustrates the results of the monitoring of ready-to-eat sausage production at XYZ Inc. during Phase I using the Laney Demerit control chart, which presents a standard deviation value of $\sigma_Z = 20.53620815$.

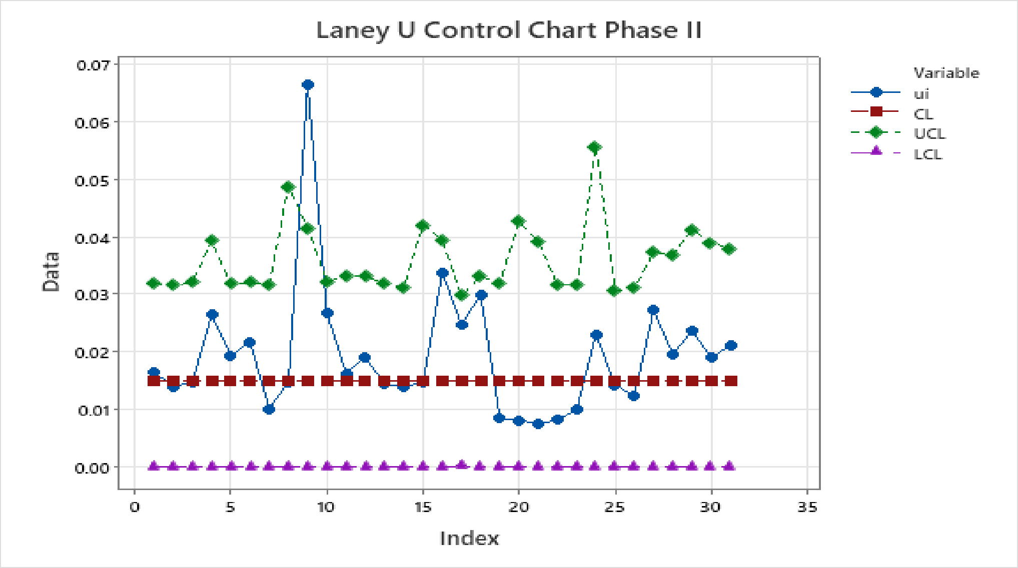

Figure 3. **Laney Demerit control chart phase I**

Using a standard deviation value of $\sigma_Z = 20.53620815$ and a central line set at 0.005033071, the analysis conducted through the Laney Demerit control chart in Phase I reveals the identification of two out-of-control data points. It indicates that XYZ Inc. during this specific period in Phase I lacks statistical control, necessitating corrective actions for the out-of-control data. Addressing these observations involves excluding data points that exhibit the most significant deviation from control limits. The resulting Laney-Demerit control chart for Phase I reflects the successful restoration of statistical control.

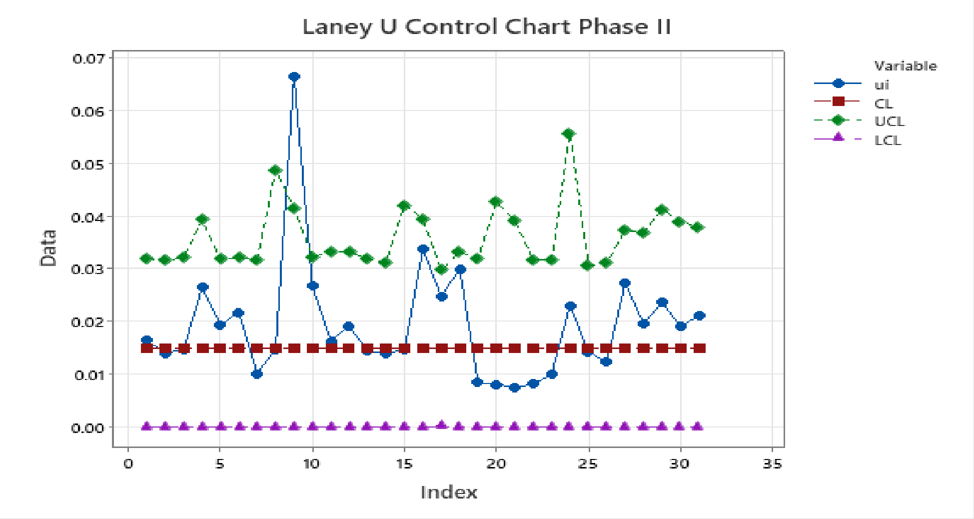

Figure 4. **Laney Demerit control chart phase I in control condition**

The Laney Demerit control chart for Phase I in control is shown in **Figure 4**, having undergone 5 iterations to eliminate out-of-control data, resulting in a standard deviation value of $\sigma_Z = 17.60638298$. These results affirm statistical control in the production of ready-to-eat sausages at XYZ Inc. during the 2023 production period in Phase I. Consequently, the process can confidently advance to Phase II, featuring a central line at 0.004426489.

In the Phase II Laney Demerit control graph, the standard deviation value is $\sigma_Z = 14.49901461$, and the central line remains consistent with that of Phase I at 0.004426489. Phase II analysis is conducted to assess potential process changes between the initial and subsequent phases, while also monitoring the subsequent production processes. This research encompasses Phase II operations that span from August 1, 2023, to September 30, 2023. **Figure 5** encapsulates the results of the monitoring of ready-to-eat sausage production at XYZ Inc. through the Laney Demerit control chart during Phase II.

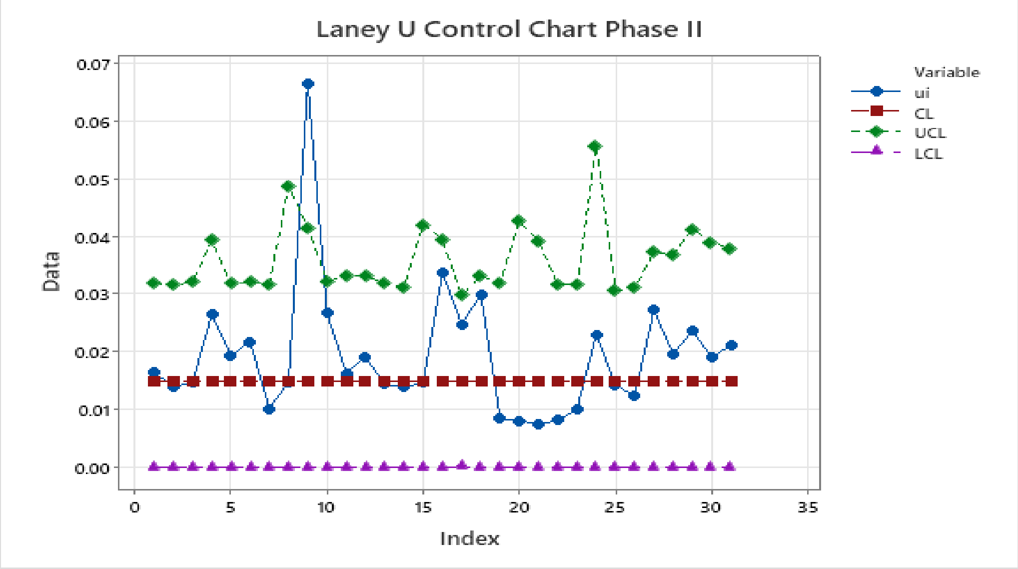

Figure 5. Laney Demerit control chart phase II

Figure 5 presents the Laney Demerit control chart for Phase II, characterized by a standard deviation value of $\sigma_Z = 12.10106$ and a central line set at 0.02759. Upon analysis, six instances of out-of-control data emerge in the fourth, ninth, 17th, 18th, 27th, and 29th observations in Phase II. These analytical findings highlight that the production of ready-to-eat sausages at XYZ Inc. during the 2023 production phase in Phase II lacks statistical control, indicating a discernible process shift between Phase I and II.

3.4 AHP-Based Laney Demerit Control Chart

The Analytical Hierarchy Process (AHP) is used in decision-making to assess the seriousness of defects. In this research, eight experts are involved in decision-making. The multi-criteria used are the types of defects in the production process given in **Table 6**. The rating scale used in this study refers to **Table 3**.

Using the computed values of CI, λ_{max}, and CR, the analysis proceeds to implement statistical quality control in Phase I. This phase involves the examination of the observations to identify those within acceptable limits. **Figure 6** depicts the results of the statistical quality control applied to the production of ready-to-eat sausages at XYZ Inc. The approach integrates the Laney demerit control chart methodology based on AHP, revealing a standard deviation value of $\sigma_Z = 20.5078014$.

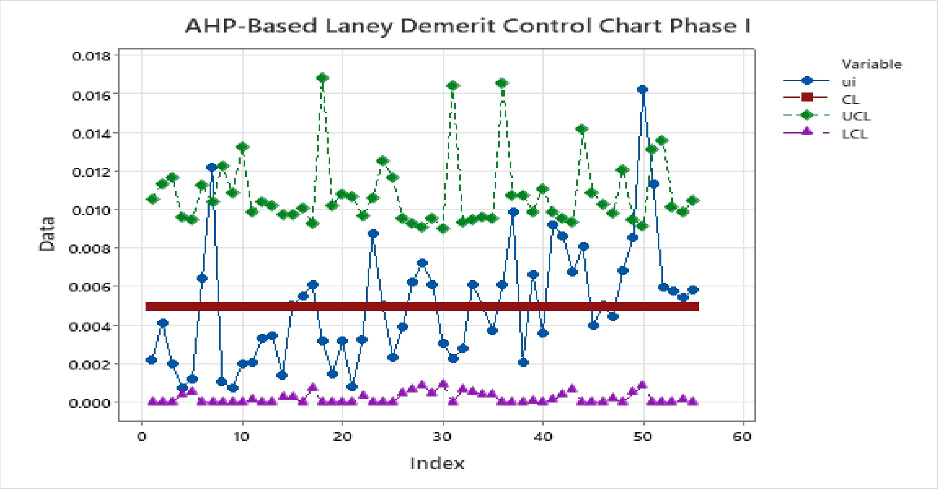

Figure 6. **AHP-based Laney chart phase I**

Conducting statistical quality control on the ready-to-eat sausage production process at XYZ Inc., using the Laney Demerit control chart based on AHP during Phase I with a central line set at 0.04322, revealed two out-of-control instances among the 55 observations. This suggests that the period of this production during Phase I lacks statistical control. Consequently, addressing out-of-control data involves the removal of observations that exhibit the most significant deviation from the control limits. The subsequent results depict the Laney de Merit Control Chart based on AHP for Phase I, demonstrating the successful restoration of statistical control.

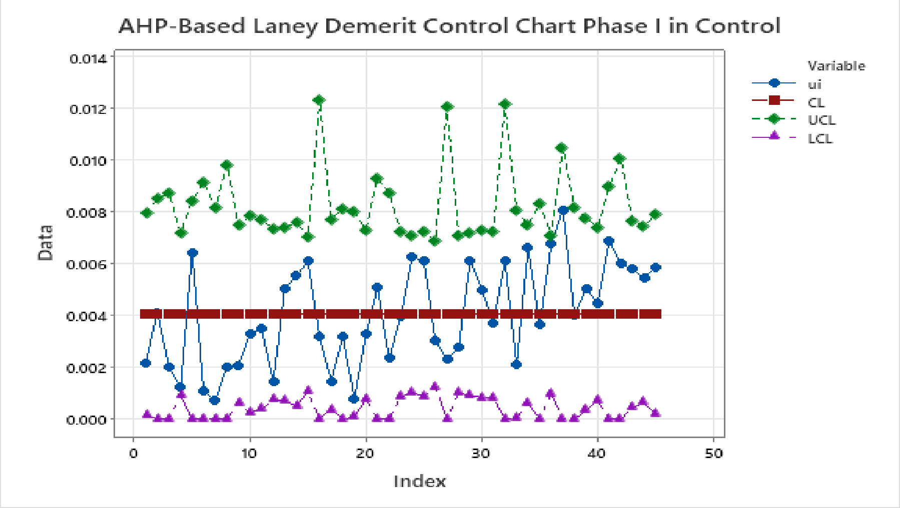

Figure 7. **AHP-based Laney Phase I in control condition**

Figure 7 shows the AHP-based demerit control chart results for Phase I, achieved after 10 iterations to eliminate out-of-control data, with a corresponding standard deviation of $\sigma_Z = 15.78519504$. This outcome signifies that the production of ready-to-eat sausages at XYZ Inc. during this production period in Phase I is now under statistical control. As a result, the process can confidently progress to Phase II, featuring a central line set at 0.00404392.

The implementation of the Laney Demerit control chart based on the Analytical Hierarchy Process (AHP) in Phase II spans from August 1, 2023, to September 30, 2023, with a standard deviation value of $\sigma_Z = 13.44237589$ and maintaining the central line consistent with Phase I, set at 0.00404392 are presented in **Figure 17**.

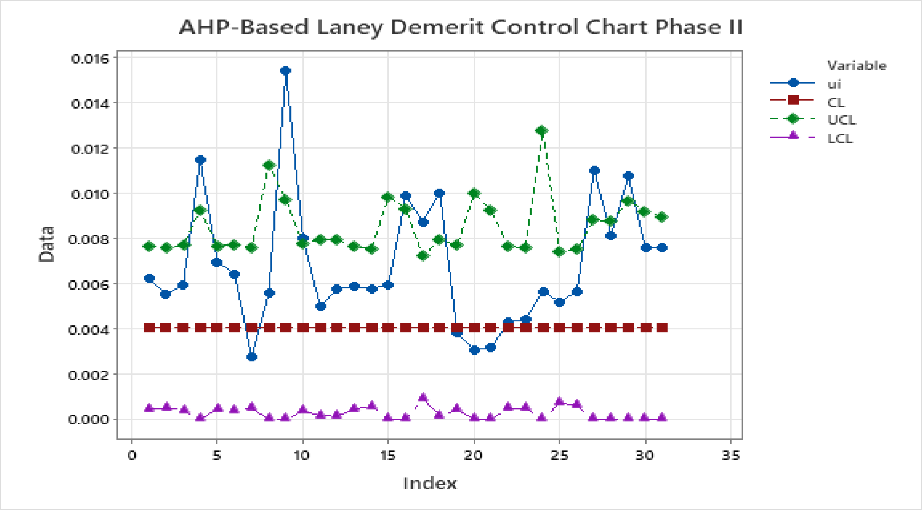

Figure 8. **AHP-based Laney phase II**

Figure 8 shows the Laney Demerit control chart based on AHP for Phase II, characterized by a standard deviation value of $\sigma_Z = 12.10106$ and a central line set at 0.034103. Based on the analysis results, it is evident that 8 data points fall outside the control limits. This observation indicates that the production of ready-to-eat sausages at XYZ Inc. during the production period in Phase II lacks statistical control, highlighting a discernible process shift between Phase I and II.

3.5 Comparison Result of Control Charts

The comparison between the Conventional and Laney is separated into two tables. The comparative findings for Conventional Control Charts and Laney Control Charts are detailed in Table 8.

Table 8. The Comparison Result of Conventional Control Chart

Type of Control Chart	Phase	Total Data Out of Control
Demerit	Phase I	51
AHP-Based Demerit	Phase I	51

Table 8 reveals a notable prevalence of out-of-control data in the Demerit Control Chart and the AHP-based Demerit Control Chart, suggesting their oversensitivity. Laney analysis is employed for these charts to improve control accuracy and mitigate the issue of excessive sensitivity when monitoring ready-to-eat sausage production at XYZ Inc. In the analysis results, the outcomes of the AHP-based Laney Demerit Control Chart exhibit the highest number of out-of-control data points in phase II, with eight observations exceeding control limits. In comparison, the Laney Demerit Control Chart shows six out-of-control observations. This variation suggests that the AHP-based Laney Demerit Control Chart may have a slightly higher sensitivity to certain deviations in the production process at XYZ Inc. during this phase.

3.6 Process Capability

Process capability analysis determines whether ready-to-eat sausage production at XYZ Inc. during this production period is according to the specifications. A process can be considered capable if it is statistically controlled and the products produced are within the specification limits set by the company. This is indicated by the $\hat{P}_{pk}^{\%}$ value is more than one. The process capability analysis begins by calculating the $\hat{p}$ value and the value of $\bar{u} = 0.00404392$. The following are the calculation results of the $\hat{p}$ value.

$$\hat{p} = 1 - e^{-0.00404392}$$

$$= 1 - 0.99596$$

$$= 0.004$$

Next, calculate the value of $\hat{P}_{pk}^{\%}$ and the following are the results of the calculation.

$$\hat{P}_{pk}^{\%} = \left|\frac{-2.649}{3}\right| = 0.883$$

The result of the process capability analysis of the ready-to-eat sausage production at XYZ Inc. during the production period is shown in Table 9.

Table 9. Process Capability Analysis

Index	Score
$\hat{p}$	0.004
$\hat{P}_{pk}^{\%}$	0.883

Table 9 shows that the $\hat{P}_{pk}^{\%}$ value obtained is 0.883, whereas the value is less than one. That result indicates that the ready-to-eat sausage production at XYZ Inc. during this period is incapable.

4. CONCLUSIONS

Based on the analysis and discussion, the following conclusions can be drawn regarding the ready-to-eat sausage production at XYZ Inc.

1. **Statistical Quality Control**: While conventional demerit control charts and AHP-based demerit control charts indicated out-of-control conditions, the Laney demerit control chart, and AHP-based Laney demerit control chart successfully demonstrated statistical control in phase I after necessary adjustments. However, phase II remains out of control, suggesting further investigation and corrective actions to improve process stability.
2. **Process Capability**: The process capability analysis revealed that the ready-to-eat sausage production at XYZ Inc. is currently incapable, with a process capability index of 0.883. This indicates that the process does not consistently produce products within the specified tolerances. The improvements in the production process are necessary to enhance its capability to ensure product quality and meet customer expectations.
3. **Defect Characteristics**: The observed defects in the ready-to-eat sausages primarily relate to sensory attributes, such as off-aromas. This suggests heightened attention should be given to quality control measures related to sensory evaluation and packaging integrity to minimize defect occurrence.

Future research could benefit from implementing of a Fuzzy Analytic Hierarchy Process (AHP) integrated monitoring system to mitigate the subjectivity and potential biases inherent in expert judgments [17].

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