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THE LOCATING CHROMATIC NUMBER OF $CHAIN(A,4,n)$ GRAPH

Des Welyyanti[1*], Latifa Azhar Abel[2], Lyra Yulianti[3]

[1,2,3]*Department of Mathematics and Data Science, Faculty of Mathematics and Natural Sciences, Universitas Andalas Kampus UNAND Limau Manis, Padang 25163, Indonesia*

*Corresponding author's e-mail: * **wely@sci.unand.ac.id***

ABSTRACT

Let $G=(V,E)$ be a connected graph with a vertex coloring $c: V(G) \to \{1,2,\cdots,k\}$ such that two adjacent vertices have different colors. We denote an ordered partition $\pi = \{S_1, S_2, \cdots, S_k\}$ where S_i is a color class with color-i, consisting of vertices given color i, for $1 \le i \le k$. The color code of a vertex v in G is a k-vector: $c_\pi(v) = (d(v,S_1), d(v,S_2), \cdots, d(v,S_k))$. where $d(v,S_i) = min\{d(v,x) | x \in S_i\}$ is the distance between a vertex v in G and S_i for $1 \le i \le k$. If every two vertices u and v in G have different color codes, $c_\pi(u) \ne c_\pi(v)$, then c is called the locating k-coloring of G. The minimum number of colors k needed in this coloring is defined as the locating chromatic number, denoted by $\chi_L(G)$. This paper determines the locating chromatic number of chain graph and the induction of two graphs A. Graph A is a cyclic graph C_4, which is the identification of P_3, $Chain(A,4,n)$ for $n>2$.

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1. INTRODUCTION

The Locating chromatic numbers of graph, introduced by Chartrand et al. in 2002 [1], are part of graph theory that combines the concepts of vertices coloring and dimension partitions on a graph. Suppose there is a connected graph $G = (V, E)$ and there is a vertex $v \in V(G)$ and a set $S \subset V(G)$. The distance from vertex v to S, denoted $d(v, S)$, is defined as where $d(v, S) = min\{d(v, x) | x \in S\}$ is represents the distance of vertex v to partition S. Let $\pi = \{S_1, S_2, \cdots, S_k\}$ is the partition of $V(G)$ with $S_1, S_2, \cdots, S_k$ classes of π. The representation v of π, denoted by $r(v|\pi)$, is the coordinate$r(v|\pi) = (d(v, S_1), d(v, S_2), \cdots, d(v, S_k))$.

Furthermore, π is called the distinguishing partition of $V(G)$ if $r(u|\pi) \neq r(v|\pi)$ for each of the two different vertices $u, v \in V(G)$. The partition dimension of G, denoted by $pd(G)$ which is the smallest k value so G has a partition distinguishing from k class. *Let* $G = (V, E)$ be a connected graph and $c: V(G) \to \{1, 2, \cdots, k\}$ be a vertex coloring such that two neighboring vertices are colored in different colors. Nominate an ordered partition $\pi = \{S_1, S_2, \cdots, S_k\}$, where S_i the i-color class, i.e. the set of all vertices colored i, for $1 \leq i \leq k$. Next, the color code of a vertex v in G is defined as a k-vector: $c_\Pi(v) = (d(v, S_1), d(v, S_2), \ldots, d(v, S_k))$, where $d(v, S_i) = min\{d(v, x) | x \in S_i\}$ is the distance between a vertex v in G and the i-color class. If every two vertices u and v in G have different color codes, or $c_\pi(u) \neq c_\pi(v)$, then c is called k-locating coloring for graph G. The minimum number of colors k used in k-locating coloring to G is called the locating chromatic number of G, denoted $\mathcal{X}_L(G)$. Other definitions and notations used in this paper are quoted from Diestel [2].

In [3], [4],determined the locating chromatic number for graphs with dominant vertices and fan-tailed amalgamation graphs. Furthermore, in [5] determined the locating chromatic number for the diamond graph Br_n for several values of n. In [6], [7], looked for the characteristics of a tree graph whose locating chromatic number is 3 and proposed an algorithm to determine the upper limit of the locating chromatic number in a tree graph. Regarding this research, Assiyatun et al. in 2020 [8] investigated the upper limit of the locating chromatic number of tree graphs. Another result is the research of Ghanem et al. in 2019 [9], which characterized the locating chromatic number of power of paths and cycle symmetry. In [10], they found that the locating chromatic number of origami graphs $\mathcal{X}_L(O_n)$ is 4 if $n = 3$ and 5 otherwise. As well as Asmiati et al. [11] researched the locating chromatic number of 2 families of barbell graphs and Haryeni and Bakoro in 2022 [12] determined for tree graphs and cycle graphs that have 3 partition dimensions and 4 locating chromatic numbers. Prawinasti et al. [13] showed that the locating chromatic number for split graph of the n cycle graph for $n \geq 3$ is 4 for odd n and 5 for even n.

Azhari et al. in 2020 [14], determined the locating chromatic number of a disconnected graph, which contains a path graph and five double stars graph as its components. The research of the locating chromatic number of the combine of two fan graphs F_n with $n \geq 3$ [15]. Welyyanti et al. [16]-[18] also discuss the locating chromatic number of disconnected graphs, such as the graph $H = P_n \cup C_m$with $n, m \geq 4$ then $X_L(H) = 4$, graph $H = rP_4 \cup kS_{m,n}$ with $r, k \geq 1, m, n \geq 2$ and a graph with 2 components $H = G_1 \cup G_2$, $X_L(G_i) = 3,\ i = 1, 2$. In this research, the locating chromatic number of the $Chain(A, 4, n)$ graph will be determined, for $n > 2$.

2. RESEARCH METHODS

In [1], the upper and lower bounds for the locating chromatic number of connected graphs are determined as follows.

Theorem 1. [1] *For any connected graph* $\boldsymbol{G}$ *of order* $\boldsymbol{n} \geq \boldsymbol{3}$, *it holds that* $\boldsymbol{3} \leq \boldsymbol{\chi_L} \leq \boldsymbol{n}$.

Based on **Theorem 1** above, steps are constructed to determine the locating chromatic number of the Graph $Chain(A, 4, n)$, for $n > 2$ as follows.

a. Determine the lower bound of the locating chromatic number of $Chain(A, 4\ n)$, for $n > 2$, based on **Theorem 1** in [1], If the lower bound does not meet the locating chromatic number requirements, then the addition of vertices colors, thus the requirements of locating chromatic number are filled.

b. The upper bound of the locating chromatic number of the $Chain(A, 4\ n)$, for $n > 2$ will be shown by indicating that the color code of each of vertices of graph $Chain(A, 4\ n)$ must be different.

3. RESULTS AND DISCUSSION

In this section are given the definition of $\boldsymbol{Chain(A, 4\,n)}$ graphs for $\boldsymbol{n > 2}$. Next, we will be determine the locating chromatic number of the graph $\boldsymbol{Chain(A, 4\,n)}$, for $\boldsymbol{n \geq 2}$.

Definition 1. [1] Suppose there are $\boldsymbol{n}$ graphs of cycle graph $\boldsymbol{C_4}$, with $\boldsymbol{n > 2}$. Define the vertices set and edge set of the graph $\boldsymbol{C_4^{(i)}}$, the $\boldsymbol{i}$-cycle graph, as follows.

$$\boldsymbol{V\left(C_4^{z(i)}\right) = \{v_{i,j} | 1 \leq i \leq n, 1 \leq j \leq 4\}}, \tag{1}$$

$$\boldsymbol{E\left(C_4^{(i)}\right) = \{v_{i,k}v_{i,k+1} | 1 \leq i \leq n, 1 \leq k \leq 3\} \cup \{v_{i,1}v_{i,4}\}}. \tag{2}$$

In this paper, the $\boldsymbol{Chain(4, n)}$ graph is constructed by identifying the vertex $\boldsymbol{v_{l,2}}$ with the vertex $\boldsymbol{v_{l+1,1}}$, for $\boldsymbol{1 \leq l \leq n-1}$.

Next, define two graphs $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$ with the set of vertices and edges.

$$\boldsymbol{V(A_1 \cup A_2) = \{x_1, y_2, a_{1,1}, a_{1,2}, a_{1,3}, a_{2,1}, a_{2,2}, a_{2,3}\}}, \tag{3}$$

$$\boldsymbol{E(A_1 \cup A_2) = \{x_1a_{1,1}, x_1a_{1,2}, x_1a_{1,3}, y_2a_{2,1}, y_2a_{2,2}, y_2a_{2,3}\}}. \tag{4}$$

The $\boldsymbol{Chain(A, 4, n)}$ graph is defined by identifying the $\boldsymbol{y_1}$ with $\boldsymbol{v_{1,1}}$, and the $\boldsymbol{x_2}$ with $\boldsymbol{v_{n,3}}$. The $\boldsymbol{v_{1,1}}$ and $\boldsymbol{v_{n,3}}$ are two vertices in the $\boldsymbol{Chain(4, n)}$ graph, for $\boldsymbol{n \geq 2}$. The $\boldsymbol{Chain(A, 4, n)}$ graph for $\boldsymbol{n \geq 2}$ can be see in **Figure 1**.

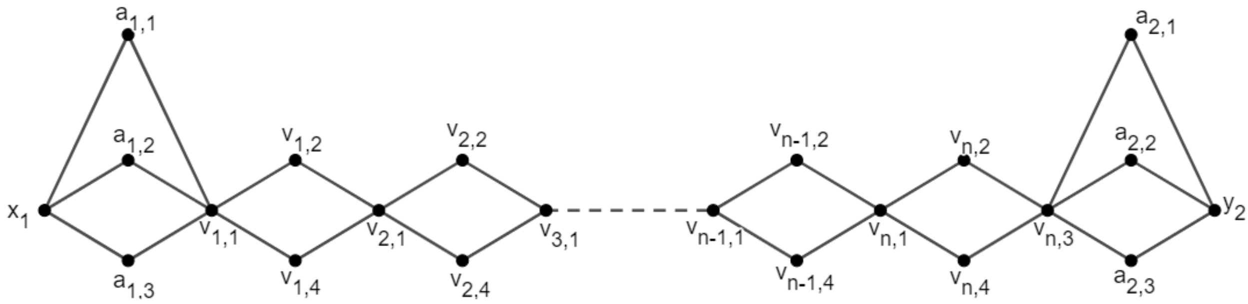

Figure 1. $\boldsymbol{Chain(A, 4, n)}$ for $\boldsymbol{n > 2}$.

Next, **Theorem 2** determined the chromatic number of the Location of the graph $\boldsymbol{Chain(A, 4, 3)}$.

Theorem 2. *Let* $\boldsymbol{chain(A, 4, 3)}$ *be a chain of three cycle graph* $\boldsymbol{C_4}$ *with the A graph is* $\boldsymbol{C_4}$ *identified with* $\boldsymbol{P_3}$*. Then* $\boldsymbol{\mathcal{X}_L(Chain(A, 4, 3)) = 5}$.

Proof.

a. We will determine the lower bound of the locating chromatic number of the $\boldsymbol{Chain(A, 4, 3)}$ graph. Without loss of generality, we assign four colors for every vertex $\boldsymbol{Chain(A, 4, 3)}$. As a result, we have at least two vertices with same color code, such as $\boldsymbol{x_1 = v_{1,1} = (0, 1, 1, 1)}$. Therefore, $\boldsymbol{\mathcal{X}_L(Chain(A, 4, 3)) \geq 5}$.

b. We will determine the upper bound of the locating chromatic number of the $\boldsymbol{Chain(A, 4, 3)}$ graph. Suppose $\mathbf{c}$ the locating coloring on the $\mathbf{Chain(A, 4, 3)}$ graph uses 5-coloring with the partition of $\boldsymbol{V(Chain(A, 4, 3))}$ with $\boldsymbol{S_1, S_2, \cdots, S_k}$ classes of $\boldsymbol{\pi}$ are $\boldsymbol{S_1 = \{a_{1,1}, v_{2,2}, a_{2,1}\}}$, $\boldsymbol{S_2 = \{x_1, v_{1,2}, v_{3,2}, a_{2,2}\}}$, $\boldsymbol{S_3 = \{v_{1,1}, v_{2,1}, v_{3,1}, v_{4,1}, y_2\}}$, $\boldsymbol{S_4 = \{a_{1,2}\}}$ **and** $\boldsymbol{S_5 = \{a_{1,3}, v_{1,4}, v_{2,4}, v_{3,4}, a_{2,3}\}}$, seen in **Figure 2.**

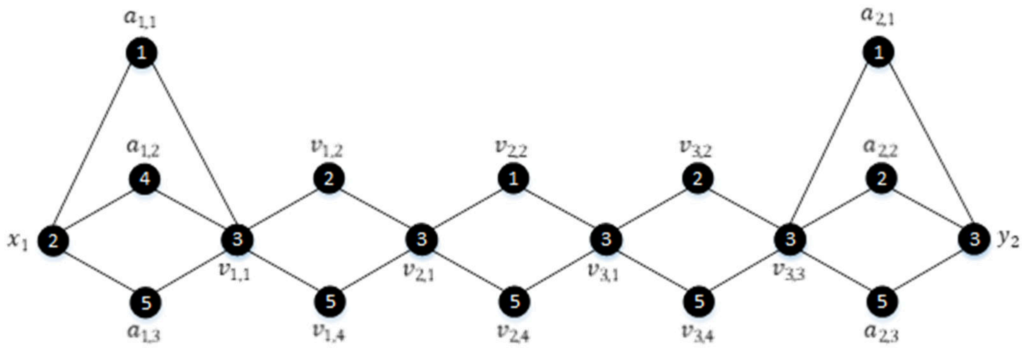

Figure 2. 5-Locating Coloring of $Chain(A,4,3)$ Graph

The distance a vertex and the $\boldsymbol{k}$-color class ($\boldsymbol{d(v,S_k)}$) in $\boldsymbol{Chain(A,4,3)}$ graph seen in Table 1:

Table 1. The Distance a Vertex and the k-color Class $Chain(A,4,3)$ Graph

$d(v,S_k)$	S_1	S_2	S_3	S_4	S_5
$a_{1,1}$	0	1	1	2	2
$v_{2,2}$	0	2	1	4	2
$a_{2,1}$	0	2	1	8	2
x_1	1	0	2	1	1
$v_{1,2}$	2	0	1	2	2
$v_{3,2}$	2	0	1	6	2
$a_{2,2}$	2	0	1	8	2
$a_{1,3}$	2	1	1	2	0
$v_{1,1}$	1	1	0	1	1
$v_{2,1}$	1	1	0	3	1
$v_{3,1}$	1	1	0	5	1
$v_{4,1}$	1	1	0	7	1
y_2	1	1	0	9	1
$a_{1,2}$	2	1	1	0	2
$a_{2,3}$	2	2	1	8	0
$v_{1,4}$	2	2	1	2	0
$v_{3,4}$	2	2	1	6	0
$v_{2,4}$	2	2	1	4	0

The color code of a vertex in $Chain(A,4,3)$ graph are $c_\Pi(a_{1,1}) = (0,1,1,2,2)$, $c_\Pi(v_{2,2}) = (0,2,1,4,2)$, $c_\Pi(a_{2,1}) = (0,2,1,8,2)$, $c_\Pi(x_1) = (1,0,2,1,1)$, $c_\Pi(v_{1,2}) = (2,0,1,2,2)$, $c_\Pi(v_{3,2}) = (2,0,1,6,2)$, $c_\Pi(a_{2,2}) = (2,0,1,8,2)$, $c_\Pi(a_{1,3}) = (2,1,1,2,0)$, $c_\Pi(v_{1,1}) = (1,1,0,1,1)$, $c_\Pi(v_{2,1}) = (1,1,0,3,1)$, $c_\Pi(v_{3,1}) = (1,1,0,5,1)$, $c_\Pi(v_{4,1}) = (1,1,0,7,1)$, $c_\Pi(y_2) = (1,1,0,9,1)$, $c_\Pi(a_{1,2}) = (2,1,1,0,2)$, $c_\Pi(a_{2,3}) = (2,2,1,8,0)$, $c_\Pi(v_{1,4}) = (2,2,1,2,0)$, $c_\Pi(v_{2,4}) = (2,2,1,4,0)$, $c_\Pi(v_{3,4}) = (2,2,1,6,0)$.

Because every vertex on the graph has a different color code, then the locating chromatic number from Graph $\mathcal{X}_L(Chain(A,4,3)) = 5$. ∎

In Theorem 2 we have locating chromatic number of $Chain(A,4,n) = 5$ with $n = 3$, so, in Theorem 3 we will determine locating chromatic number $Chain(A,4,n)$ with $n = 4$.

Theorem 3. *Let $\boldsymbol{chain(A,4,4)}$ be a chain of three cycle graph $\boldsymbol{C_4}$ with the A graph is $\boldsymbol{C_4}$ identified with $\boldsymbol{P_3}$. Then $\boldsymbol{\mathcal{X}_L(Chain(A,4,4)) = 5}$.*

Proof. We will find the lower bound of the locating chromatic number of the $\boldsymbol{Chain(A,4,4)}$. Without loss of generality, we assign of different four colors for every vertex $\boldsymbol{Chain(A,4,4)}$, then that 4-coloring vertices in $\boldsymbol{Chain(A,4,4)}$, have at least two vertices with the same color code, such as $\boldsymbol{a_{1,1} = a_{1,3} = (1,1,0,2)}$.

Next, we will find the upper bound of the locating chromatic number of the $\boldsymbol{Chain(A,4,4)}$. Let the $\boldsymbol{Chain(A,4,4)}$ have 5-coloring for every vertices, then the partition of $\boldsymbol{V(\mathrm{Chain}(A,4,4)}$ are $\boldsymbol{S_1}$ $\{\boldsymbol{a_{1,1}, v_{2,2}, v_{2,2}, a_{2,1}}\}$, $\boldsymbol{S_2 = \{x_1, v_{1,2}, v_{3,2}, a_{2,2}\}}$, $\boldsymbol{S_3 = \{v_{1,1}, v_{2,1}, v_{3,1}, v_{4,1}, v_{4,3} y_2\}}$, $\boldsymbol{S_4 = \{a_{1,2}\}}$ and $\boldsymbol{S_5 = \{a_{1,3}, v_{1,4}, v_{2,4}, v_{3,4}, v_{4,4}, a_{2,3}\}}$,

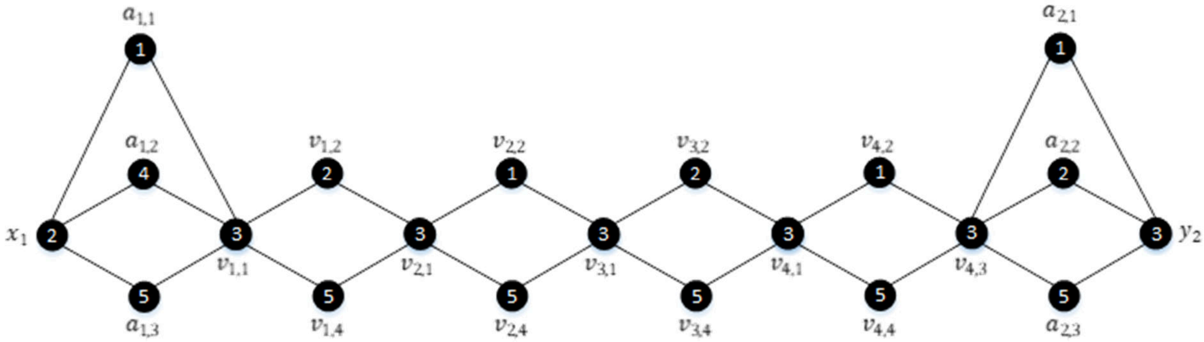

Figure 2 The 5-Locating Coloring of $Chain(A,4,4)$ *Graph*

The distance a vertex and the k-color class ($\boldsymbol{d(v,S_k)}$) in $\boldsymbol{Chain(A,4,4)}$ graph seen in **Table 2**:

Table 2. The Distance a Vertex and The $\boldsymbol{k}$-color Class $\boldsymbol{Chain(A,4,4)}$ **Graph**

$\boldsymbol{d(v,S_k)}$	$\boldsymbol{S_1}$	$\boldsymbol{S_2}$	$\boldsymbol{S_3}$	$\boldsymbol{S_4}$	$\boldsymbol{S_5}$
$a_{1,1}$	0	1	1	2	2
$v_{2,2}$	0	2	1	4	2
$v_{4,2}$	0	2	1	8	2
$a_{2,1}$	0	2	1	10	2
x_1	1	0	2	1	1
$a_{2,2}$	1	0	1	10	2
$v_{1,2}$	2	0	1	2	2
$v_{3,2}$	2	0	1	6	2
$v_{1,1}$	1	1	0	1	1
$v_{2,1}$	1	1	0	3	1
$v_{3,1}$	1	1	0	5	1
$v_{4,1}$	1	1	0	7	1
$v_{4,3}$	1	1	0	9	1
y_2	1	1	0	11	1
$a_{1,2}$	2	1	1	0	2
$a_{1,3}$	2	1	1	2	0
$v_{1,4}$	2	2	1	2	0
$v_{2,4}$	2	2	1	4	0
$v_{3,4}$	2	2	1	6	0
$v_{4,4}$	2	2	1	8	0
$a_{2,3}$	2	2	1	10	0

According to **Table 2**, the color code of a vertices in $Chain(A,4,4)$ graph is $c_\pi(a_{1,1}) = (0,1,1,2,2)$ $c_\pi(v_{2,2}) = (0,2,1,4,2)$, $c_\pi(a_{1,2}) = (2,1,1,0,2)$, $c_\pi(v_{4,4}) = (2,2,1,8,0)$, $c_\pi(v_{2,1}) = (1,1,0,3,1)$, $c_\pi(v_{4,2}) = (0,2,1,8,2)$, $c_\pi(a_{1,3}) = (2,1,1,2,0)$, $c_\pi(a_{2,3}) = (2,2,1,10,0)$, $c_\pi(v_{3,1}) = (1,1,0,5,1)$, $c_\pi(a_{2,1}) = (0,2,1,10,2)$, $c_\pi(v_{1,4}) = (2,2,1,2,0)$, $c_\pi(x_1) = (1,0,2,1,1)$, $c_\pi(v_{4,1}) = (1,1,0,7,1)$, $c_\pi(v_{1,2}) = (2,0,1,2,2)$, $c_\pi(v_{2,4}) = (2,2,1,4,0)$, $c_\pi(a_{2,2}) = (1,0,1,10,2)$, $c_\pi(v_{4,3}) = (1,1,0,9,1)$, $c_\pi(v_{3,2}) = (2,0,1,6,2)$, $c_\pi(v_{3,4}) = (2,2,1,6,0)$, $c_\pi(v_{1,1}) = (1,1,0,1,1)$, $c_\pi(y_2) = (1,1,0,11,1)$.

Because every vertex on the graph has a different color code, so, upper bound of $\mathcal{X}_L(\boldsymbol{Chain}(A,4,4)) \le 5$. Then, the locating chromatic number of $\boldsymbol{Chain}(A,4,4)$ gra[redacted], $\mathcal{X}_L(\boldsymbol{Chain}(A,4,4)) = 5$.

In **Theorem 2** and **Theorem 3**, then $\boldsymbol{Chain}(A,4,n)$ with $n > 2$ can be generalized, as in **Theorem 4**.

Theorem 4. *If $\boldsymbol{Chain}(A,4,n)$ is n graphs of cycle graph C_4 with $n > 2$ and of the cycle C_4 induced with 2 graphs A i.e., the graph of cycle C_4 identified with P_3, then $\mathcal{X}_L(\boldsymbol{Chain}(A,4,n)) = 5$.*

Proof. First, we determine the lower bound of $\mathcal{X}_L(\boldsymbol{Chain}(A,4,n))$ for $n > 2$. Without loss of generality, we assign of different four colors for every vertex $\mathcal{X}_L(\boldsymbol{Chain}(A,4,2))$, then that 4-coloring vertices in $\boldsymbol{Chain}(A,4,n)$ with $n > 2$ have at least two vertices $a_{1,1}$ **and** $a_{1,2}$ or $a_{1,1}$ **and** $a_{1,3}$ with same color code. Therefore, $\mathcal{X}_L(\boldsymbol{Chain}(A,4,n)) \ge 5$ for $n > 2$. Next, we determine the upper bound of the locating chromatic number of $\boldsymbol{Chain}(A,4,n)$ for $n > 2$.

Define $c : V(\boldsymbol{Chain}(A,4,n)) \to \{1,2,3,4,5\}$ such that:

$c(x_1) = 2$

$c(y_2) = 3$

$$c(v_{i,j}) = \begin{cases} 1, & \text{for } 1 \le i \le n, j = 2 \text{ with } i \text{ even} \\ 2, & \text{for } 1 \le i \le n, j = 2 \text{ with } i \text{ odd} \\ 3, & \text{for } 1 \le i \le n, j = 1, 3 \\ 5, & \text{for } 1 \le i \le n, j = 4 \end{cases}$$

$$c(a_{i,j}) = \begin{cases} 1, & \text{for } i = 1,2 \quad ,j = 1 \\ 2, & \text{for } i = 2 \quad ,j = 2 \\ 4, & \text{for } i = 1 \quad ,j = 2 \\ 5, & \text{for } i = 1,2 \quad ,j = 3. \end{cases}$$

The distance between vertices in $\boldsymbol{Chain}(A,4,n)$ and the 5-color class are:

$$d(x_1, S_k) = \begin{cases} 0 & ,\text{for } k = 2 \\ 1 & ,\text{for } k = 1,4,5 \\ 2 & ,\text{for } k = 3 \end{cases}$$

$$d(y_2, S_k) = \begin{cases} 0 & ,\text{for } k = 3 \\ 1 & ,\text{for } k = 1,2,5 \\ 5 + (n-1)2 & ,\text{for } k = 4, n \ge 3 \end{cases}$$

$$d(v_{i,1}, S_k) = \begin{cases} 0 & ,k = 3 \text{ for } 1 \le i \le n \\ 1 & ,k = 1,2,5 \text{ for } 1 \le i \le n \text{ and } k = 4 \text{ for } i = 1 \\ 1 + (i-1)2 & ,k = 4 \text{ for } 2 \le i \le n \end{cases}$$

$$d(v_{i,2}, S_k) = \begin{cases} 0 & ,k = 1 \text{ for } i \text{ even and } k = 2 \text{ for } i \text{ odd}, 1 \le i \le n \\ 1 & ,k = 3 \text{ for } 1 \le i \le n \\ 2 & ,k = 1 \text{ for } i \text{ odd}, k = 2 \text{ for } i \text{ even and } k = 5, 1 \le i \le n \\ 2 + (i-1)2 & ,k = 4 \text{ for } 1 \le i \le n \end{cases}$$

$$d(v_{i,4}, S_k) = \begin{cases} 0 & ,k = 5 \text{ for } 1 \le i \le n \\ 1 & ,k = 3 \text{ for } 1 \le i \le n \\ 2 & ,k = 1,2 \text{ for } 1 \le i \le n \\ 2 + (i-1)2 & ,k = 4 \text{ for } 1 \le i \le n \end{cases}$$

$$d(a_{i,2}, S_k) = \begin{cases} 0 & ,k = 2 \text{ for } i = 2 \text{ and } k = 4 \text{ for } i = 1 \\ 1 & ,k = 2 \text{ for } i = 1 \text{ and } k = 3 \text{ for } i = 1,2 \\ 2 & ,k = 1,5 \text{ for } i = 1,2 \\ 2n + 2 & ,k = 4 \text{ for } n \ge 3, i = 2 \end{cases}$$

$$d(a_{i,1}, S_k) = \begin{cases} 0 & ,k = 1 \text{ for } i = 1,2 \\ 1 & ,k = 2 \text{ for } i = 1 \text{ and } k = 3 \text{ for } i = 1,2 \\ 2 & ,k = 2 \text{ for } i = 2, k = 4 \text{ for } i = 1 \text{ and } k = 5 \text{ for } i = 1, 2 \\ 2n + 2 & ,k = 4 \text{ for } n \ge 3, i = 2 \end{cases}$$

$$d(a_{i,3}, S_k) = \begin{cases} 0 & , k = 5 \text{ for } i = 1,2 \\ 1 & , k = 2 \text{ for } i = 1 \text{ and } k = 3 \text{ for } i = 1,2 \\ 2 & , k = 1 \text{ for } i = 1,2, k = 2 \text{ for } i = 2 \text{ and } k = 4 \text{ for } i = 1 \\ 2n+2 & , k = 4 \text{ for } i = 2. \end{cases}$$

So, the color code of a vertices in $Chain(A, 4, n)$ for $n > 2$ are:

$c_\Pi(y_2) = (1, 1, 0, 5 + (2n - 2), 1)$ for $n \geq 3$
$c_\Pi(x_1) = (1, 0, 2, 1, 1)$
$c_\Pi(v_{1,1}) = (1, 1, 0, 1, 1)$
$c_\Pi(v_{i,1}) = (1, 1, 0, 1 + (i - 1)2, 1)$ for $i \geq 2$
$c_\Pi(v_{i,2}) = (0, 2, 1, 2 + (i - 1)2, 2)$ for i even, $1 \leq i \leq n$
$c_\Pi(v_{i,2}) = (2, 0, 1, 2 + (i - 1)2, 2)$ for i odd, $1 \leq i \leq n$
$c_\Pi(v_{i,4}) = (2, 2, 1, 2 + (i - 1)2, 0)$ for $1 \leq i \leq n$
$c_\Pi(a_{1,3}) = (2, 1, 1, 2, 0)$
$c_\Pi(a_{1,2}) = (2, 1, 1, 0, 2)$
$c_\Pi(a_{1,1}) = (0, 1, 1, 2, 2)$
$c_\Pi(a_{2,1}) = (0, 2, 1, 2n + 2, 2)$ for $n \geq 3$
$c_\Pi(a_{2,2}) = (2, 0, 1, 2n + 2, 2)$ for $n \geq 3$
$c_\Pi(a_{2,3}) = (2, 2, 1, 2n + 2, 0)$ for $n \geq 3$.

It can be seen that the color code for each vertex is different, so the upper locating chromatic number of $\boldsymbol{Chain(A, 4, n)}$ is 5. ■

4. CONCLUSION

In this article, the locating chromatic number of the graph $Chain(A, 4, n)$ is 5 for $n > 2$, so $\mathcal{X}_L(Chain(A, 4, n)) = 5$.

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