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# BINARY OPTION PRICING USING LATTICE METHOD

**Donny Citra Lesmana[1*], Natallia Christina[2], Ravy Ardian Kusuma[3], Siti Salwa Nabila[4]**

[1,2,3,4]*Department of Mathematics, Faculty of Mathematics and Natural Science, IPB University*
*Jalan Meranti, Kampus IPB Dramaga, Bogor, 16680, Indonesia*

*Corresponding author's e-mail: * **donnylesmana@apps.ipb.ac.id***

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*ABSTRACT*

*One way to minimize risks due to uncertainty in stock price movements is by using derivative products, one of which is an option. Binary options, a type of exotic option, provide a fixed payout if certain conditions are met at maturity, but are difficult to solve analytically. In this study, we utilize binomial and trinomial lattice methods, specifically the Cox-Ross-Rubinstein Binomial, Hull-White Trinomial, and Kamrad-Ritchken Trinomial models, to determine the price of binary options. Results indicate that all three methods converge towards the exact solution, demonstrating their applicability for pricing binary options, with the Kamrad-Ritchken Trinomial method showing superior accuracy due to the lowest mean relative error. Additionally, we analyze factors influencing binary option prices, including initial price, strike price, maturity time, volatility, and risk-free interest rate. The study's originality lies in the comparative analysis of these methods under the same market conditions. However, limitations include model assumptions and potential data variability that may affect generalizability. Future research could extend these methods to various stock data or other financial instruments to test robustness. This research provides insights into optimal lattice method selection for practitioners in binary option pricing.*



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## 1. INTRODUCTION

Amidst the uncertainty of stock price movements, derivative instruments are an important foundation for investors to manage risk and optimize potential returns. A derivative instrument is an agreement between two parties to sell or buy a financial asset at a specified date. There are some types of derivative products such as forward contracts, futures contracts, currency swaps and options. Options give the holder the right to buy or sell a financial asset at a certain price (strike price) within an agreed period.

Based on the type of rights, options are divided into call options and put options. Call options are options that give the option holder the right to buy the underlying asset. Meanwhile, a put option is an option that gives the option holder the right to sell an underlying asset [1]. Besides that, based on the payoff structure, options can be divided into vanilla options and exotic options. Vanilla options are European-type or American-type options with a single underlying asset and have a simple payoff calculation [2]. Meanwhile, exotic options are options whose payoff structure not only depends on the price of the underlying asset at the time of exercise, but also depends on the price history of the underlying asset over the life of the option [3]. There are several types of exotic options, including binary options, barrier options, lookback options, chooser options, Asian options, and others. Binary options are the type of option that pays a fixed amount of money if certain conditions are met. These options are categorized into asset-or-nothing and cash-or-nothing.

An asset-or-nothing call option at the maturity time pays out at the price of the underlying asset when the price of the underlying asset exceeds the strike price and does not pay out at all when the price of the underlying asset is below the strike price. Meanwhile, the asset-or-nothing put option at the maturity time does not pay at all when the price of the underlying asset exceeds the strike price and pays the amount of the underlying asset otherwise the price of the underlying asset is below the strike price [4].

Cash-or-nothing call options pay a fixed amount of $Q$ at the maturity time when the price of the underlying asset exceeds the strike price and do not pay at all when the price of the underlying asset is below the strike price. Meanwhile, the cash-or-nothing put option at the maturity time does not pay at all when the price of the underlying asset exceeds the strike price and pays a fixed amount of $Q$ when the price of the underlying asset is below the strike price [4].

Binary options have grown in popularity due to their simplicity in providing a fixed payout if certain conditions are met at maturity. However, pricing these options is complex due to the absence of closed-form analytical solutions, which necessitates the use of numerical methods. Traditional methods like the Black-Scholes model are suitable for simpler options, but they lack flexibility in accurately pricing exotic options, such as binary options, under various market conditions. This creates a significant challenge for financial practitioners in identifying reliable models for pricing binary options, as existing methods may not be applicable or effective.

The model used to calculate the price of the option is the Black Scholes model developed by Fisher Black and Myron Scholes. This model was proposed for pricing European options [5]. The Black Scholes model can be solved analytically to calculate the price of call and put options. However, the model cannot easily be applied to exotic options which in most cases do not have analytical solutions so numerical methods are required to solve them. Binary options are exotic options, it is hard to solve analytically. Therefore, a numerical method is needed in determining the price of binary options. One of the numerical methods that can be used in determining option prices is the binomial and trinomial method. In this study, the calculation of binary option prices was carried out using several methods, namely Binomial Cox-Ross-Rubinstein (CRR), Trinomial Hull-White (HW), and Trinomial Kamrad-Ritchken (KR).

The binomial method is the approach in financial mathematics proposed by Cox, Ross, and Rubinstein in 1979 [5]. The approach involves using discrete random variables that follow a binomial distribution, arranged in the form of a tree to show two possible movements in the price of the underlying asset (which in this case is the stock) over a period of time, as they occur at all points in time during the life of the option. In the binomial method, the stock price is assumed to move up by a factor $u$ and probability $p$, $p \epsilon [0,1]$, and move down by a factor $d$ and probability $1 - p$.

The weakness of this method is its inflexibility in dealing with real situations where stock price movements are not only limited to two possibilities (up or down). Therefore, in 1986, Boyle introduced the trinomial method which considers three possibilities of stock price movements: up, steady, and down [6]. John C. Hull and Alan White then set parameter values to determine option prices in the trinomial method proposed by Boyle [7]. The stock price is expected to move up with factor $u$ and probability $p_u$, remain steady with probability $p_m$, and move down with factor $d$ and probability $p_d$.

In addition, in 1991 Bardia Kamrad and Peter Ritchken developed the Kamrad-Ritchken Trinomial method. The KR Trinomial method is a modification of the ordinary trinomial method by finding the best stretch parameter value based on the smallest error in option pricing [8].

Despite the availability of several numerical methods, including the binomial and trinomial lattice models, there is a lack of consensus on the most accurate and efficient method for binary option pricing. Previous studies have explored the Cox-Ross-Rubinstein (CRR) Binomial, Hull-White (HW) Trinomial, and Kamrad-Ritchken (KR) Trinomial models independently; however, few studies have systematically compared their performance under identical conditions. The absence of a comprehensive comparison leaves a research gap, as financial practitioners and researchers seek clarity on the optimal method for accurately and efficiently pricing binary options.

This study aims to address this gap by conducting a detailed comparison of the CRR Binomial, HW Trinomial, and KR Trinomial methods in pricing binary options. By analysing the convergence and accuracy of these methods under the same market conditions, this research provides valuable insights into their relative performance. Additionally, factors influencing binary option pricing, such as initial price, strike price, maturity time, volatility, and risk-free interest rate, are examined to understand their impact across different models. This approach not only clarifies the comparative strengths and weaknesses of each method but also offers practical guidance for financial practitioners in selecting appropriate models based on specific market conditions.

By filling this gap, our research contributes to the broader field of financial mathematics and enhances the existing literature on option pricing methodologies. The results are expected to assist financial practitioners in making informed decisions regarding model selection for binary options and potentially other exotic derivatives, thus contributing to better risk management and investment strategies in the financial markets.

## 2. RESEARCH METHODS

### 2.1 Data

The data used in this research is the closing price data of PT Mitra Adiperkasa Tbk (MAPI.JK) shares. The data selected is daily stock price data during the period March 14, 2023, to March 14, 2024. The data can be accessed through the Yahoo Finance website (finance.yahoo.com).

The stock data of PT Mitra Adiperkasa Tbk (MAPI.JK) was chosen for this study due to its relevance and representativeness in real-world financial markets, making it an appropriate basis for testing binary option pricing models. PT Mitra Adiperkasa Tbk is a prominent retail company listed on the Indonesia Stock Exchange, and its stock experiences significant trading volume and price fluctuations, which align with the dynamics typically encountered in option pricing scenarios. The company's stock data offers a practical example of a liquid asset, where price movements reflect market responses to various economic conditions, thus providing realistic inputs for binary option pricing models. Furthermore, the high activity in this stock makes it ideal for analyzing volatility and convergence in pricing, which are critical in assessing the performance of different numerical methods. Using PT Mitra Adiperkasa Tbk's data ensures that the findings are applicable to similar stocks in emerging markets, offering insights into the practical utility of these methods in financial decision-making and risk management.

### 2.2 Volatility

Stock price volatility is a fluctuation in stock prices within a certain period of time. The higher the volatility level, the higher the uncertainty of stock returns [9]. Suppose $\sigma$ is the annual volatility value of stock returns and $\Delta t$ is the length of the time subinterval, with the following equation [10]:

$$\sigma = \frac{s}{\sqrt{\Delta t}} \tag{1}$$

where $s$ is the standard deviation of continuously compounding returns, calculated by the formula [10]:

$$s = \sqrt{\frac{1}{k-1}\sum_{i=1}^{k}(u_i - \overline{u})^2}$$

where $u_i = \ln\left(\frac{S_{i+1}}{S_i}\right)$ is the continuously compounding return value on day $i$ and $\overline{u}$ is the average return value, where $S_i$ is the stock price at time $i$.

### 2.3 Binary Option Payoff

The payoff of a binary option depends on the outcome of the price movement of the underlying asset. These binary options can be categorized into asset-or-nothing and cash-or-nothing. The difference between the two types of options lies in what the investor will receive when the option expires. Asset-or-nothing receives in the form of the underlying asset, while cash-or-nothing will receive in the form of cash. Suppose $S_T$ is the stock price at the maturity of the option, $Q$ is the amount of reward that the seller gives to the buyer of the option according to the agreement, and $K$ is the amount of the strike price, then the payoff of the binary option is as follows:

1. Asset-or-nothing call
$$\text{Payoff} = \begin{cases} S_T, & S_T > K \\ 0, & \text{otherwise} \end{cases} \tag{2}$$
2. Asset-or-nothing put
$$\text{Payoff} = \begin{cases} 0, & \text{otherwise} \\ S_T, & S_T \le K \end{cases} \tag{3}$$
3. Cash-or-nothing call
$$\text{Payoff} = \begin{cases} Q, & S_T > K \\ 0, & \text{otherwise} \end{cases} \tag{4}$$
4. Cash-or-nothing put
$$\text{Payoff} = \begin{cases} 0, & \text{otherwise} \\ Q, & S_T \le K \end{cases} \tag{5}$$

### 2.4 Cox Ross Rubinstein Binomial (CRR)

The binomial method is a method used in option pricing that assumes two possible movements in stock prices, namely rising stock prices and falling stock prices [11]. The method was proposed by Cox, Ross, and Rubinstein in 1979 with several assumptions:

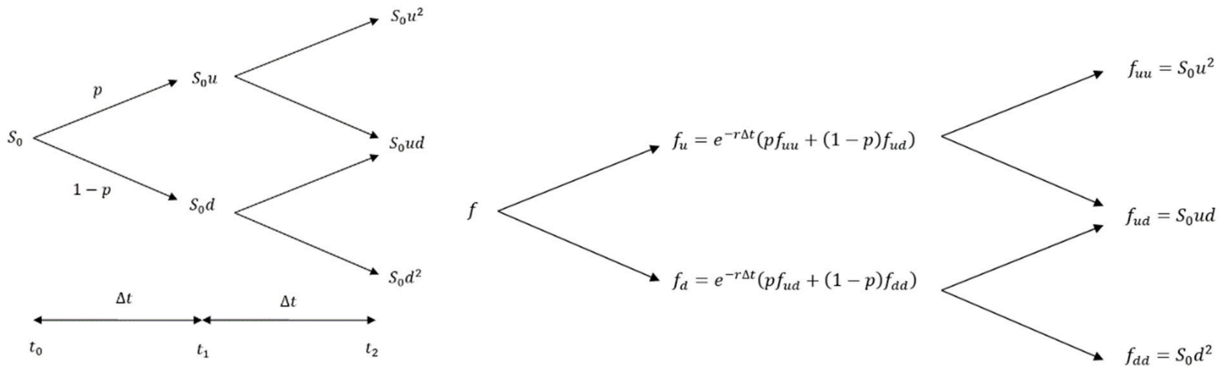
1. At every time period $\Delta t$ the stock price $S$ can increase to $Su$ or decrease to $Sd$ with $0 < d < u < 1$, where $u$ is the upside factor and $d$ is downside factor, which are constant over time.
2. The probability of the price increase is $p$ and price decrease is $1 - p$.
3. The expected stock price with a risk-free interest rate $r$ at time $i + 1$ is [12]:
$$E(S_{i+1}) = S_i e^{r\Delta t}$$

The binomial parameters are determined using these formulas:

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = e^{-\sigma\sqrt{\Delta t}}$$
$$p = \frac{a-d}{u-d}$$
$$a = e^{(r-q)\,t}$$

where $q$ is the dividend yield.

Suppose the stock price at time $t_0$ is $S_0$ and the stock price at maturity $T$ is $S_T$. The interval $[0, T]$ is divided into two subintervals, each of which shows the movement of stock prices up by a factor $u$ and the movement of stock prices down by a factor $d$. The following illustrates the movement of stock prices over two periods using the binomial method:




**(a)** **(b)**

**Figure 1. Two-Period CRR Binomial Method**
**(a) Stock Prices, (b) Option Prices**

As can be seen in **Figure 1** (a), at time $t_1$ the stock price can move up to $S_0u$ or move down to $S_0d$. Then at time $t_2$ the stock price also moves up or down to $S_0u^2$, $S_0ud$, and $S_0d^2$. Furthermore, based on this stock price movement, the option price movement is obtained.

**Figure 1**(b) illustrates the process of calculating option prices at each node in a two-period Cox-Ross-Rubinstein (CRR) binomial tree model, which is a simplified model for predicting stock price changes. In this method, the stock price can either increase by a factor $u$ or decrease by a factor $d$ over each period, representing two possible movements at each step. The diagram shows nodes corresponding to possible stock prices at different points in time, beginning from the initial stock price at time $t_0$.

At the first timestep, the stock price can move up or down based on the model's predefined factors, reaching a new price level. The nodes at time $t_1$ reflect these potential new prices. The process continues to the next time period, $t_2$, with the stock price either increasing or decreasing again, resulting in four possible outcomes for the stock price at this maturity time.

Option prices are determined at each node by working backwards from the final payoffs at maturity. Using the formula

$$f = e^{-r\Delta t}(p_u f_u + p_d f_d) \tag{6}$$

where $r$ is the risk-free interest rate, $\Delta t$ is the time interval, and $p_u$ and $p_d$ are the probabilities of price moving up or down, the model calculates the option's value at each preceding node. This backward induction method continues until reaching the initial node at $t_0$, where the current option price is obtained. **Figure 1**(b) visualizes this process, illustrating how option price calculations flow from maturity back to the starting point in the binomial tree structure.

### 2.5 Hull White Trinomial

The trinomial method discussed in this paper is divided into two models, namely the Hull-White Trinomial method and the Kamrad-Ritchken Trinomial method. In 1988, Hull developed a trinomial method with the probability of a fixed stock price movement of 0.5 [13]. In the Trinomial method, at every time interval $\Delta t$ the stock price can move up by factor $u$, move down by factor $d$ or remain steady. The probability of moving up is $p_u$, moving down is $p_d$, and being steady is $p_m$.

To determine the parameters of this method, the following equations are used [8]:

$$u = e^{\sigma\sqrt{3\Delta t}}$$

$$d = \frac{1}{u}$$

$$p_u = \sqrt{\frac{\Delta t}{12\sigma^2}}(r - \frac{\sigma^2}{2}) + \frac{1}{6}$$

$$p_d = -\sqrt{\frac{\Delta t}{12\sigma^2}}(r - \frac{\sigma^2}{2}) + \frac{1}{6}$$

$$p_m = \frac{2}{3}$$

where $\sigma$ is the volatility and $r$ is the risk-free interest rate.

In trinomial method, at each subinterval, the stock price has a probability of stock prices up, down, and being steady by $p_u$, $p_d$, and $p_m$. The stock movement with the two-period HW Trinomial method is illustrated in Figure 2.

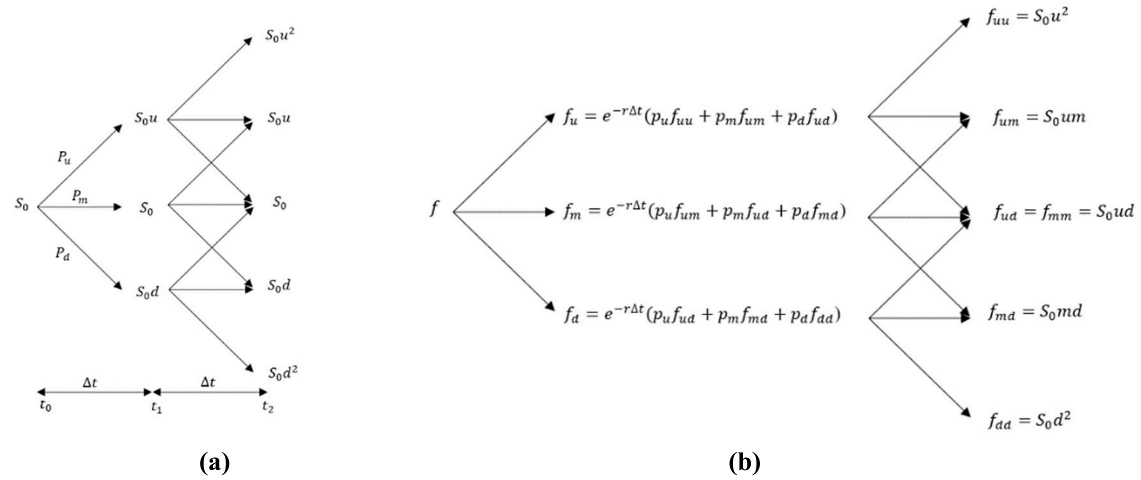



**Figure 2. Two-period HW Trinomial Method**
**(a) Stock Price, (b) Option Price**

From Figure 2(a), at time $t_1$ the stock price moves up to $S_0u$, the stock price remains $S_0$, and the stock price drops to $S_0d$. Whereas at time $t_2$, the stock price can move up, steady, and move down one more time to $S_0u^2$, $S_0u$, $S_0$, $S_0d$, and $S_0d^2$ [5]. Furthermore, from this stock price movement, the option price movement can also be given.

Figure 2(b) demonstrates the calculation of option prices using the Hull-White (HW) Trinomial method in a two-period tree structure. This trinomial model introduces three possible outcomes for the stock price at each step: the price can increase, decrease, or remain steady. This setup better represents potential price movements in real markets, where the price may not only rise or fall but also remain stable for certain periods.

In this figure, each node represents a possible stock price at a given point in time. Starting from an initial stock price at time $t_0$, the model shows three possible price changes in the first timestep, leading to nodes that represent an increase, decrease, or steady price at time $t_1$. From each of these nodes, three further movements are possible, resulting in a total of nine different price outcomes by time $t_2$, the option's maturity date.

To calculate option prices at each node, the model uses a backward induction approach, similar to the binomial method but adjusted for three movement directions. The formula used is [13]

$$f = e^{-r\Delta t}(p_uf_u + p_mf_m + p_df_d) \tag{7}$$

where $r$ is the risk-free rate, $\Delta t$ is the time interval, and $p_u$, $p_m$, and $p_d$ represent the probabilities for the price to move up, stay steady, or move down, respectively. The values at the maturity nodes are determined first based on the final payoff conditions, and these values are then discounted back through the tree structure to calculate the option price at the initial node.

Figure 2(b) visually conveys this process, highlighting how the trinomial method's added flexibility allows for more accurate option pricing in situations with complex market conditions. By incorporating the probability of a steady price, the HW Trinomial model better accommodates varied price behavior and is therefore useful for pricing exotic options like binary options.

### 2.6 Kamrad Ritchken Trinomial

In 1991, Bardia Kamrad and Peter Ritchken made a modification because the ordinary trinomial model was considered to be less effective in option pricing. In the KR Trinomial model, the best stretch parameter value is sought based on the smallest error in option pricing. The stretch parameter denoted by $\lambda$ is a parameter that describes the stretchiness of the tree in the trinomial model [14]. There are three possible movements of the stock price, which are move up by factor $u$ and probability $p_u$, steady with probability $p_m$, and move down by factor $d$ and probability $p_d$.

The parameter values of the Kamrad-Ritchken Trinomial calculated with the following equation [8]:

$$u = e^{\lambda\sigma\sqrt{\Delta t}}$$

$$d = e^{-\lambda\sigma\sqrt{\Delta t}}$$

$$p_u = \frac{1}{2\lambda^2} + \frac{\left(r - \frac{\sigma^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma}$$

$$p_m = 1 - \frac{1}{\lambda^2}$$

$$p_u = \frac{1}{2\lambda^2} - \frac{\left(r - \frac{\sigma^2}{2}\right)\sqrt{\Delta t}}{2\lambda\sigma}$$

In this study, the value of the stretch parameter $\lambda = 1.22474$ is used. Setting $\lambda = 1.22474$ is based on empirical testing and research, showing that this value optimally reduces the pricing error in the model. With this setting, the probability of the stock price staying steady (rather than moving up or down) corresponds to $\frac{1}{3}$, which aligns the model's probability distribution closer to observed market behavior. This distribution enhances the model's convergence toward the actual option price, resulting in smaller mean relative errors when pricing various types of options, especially exotic options like binary options. [15].

### 2.7 Relative Error

Relative error is a comparison of the absolute value of the difference between the predicted value and the exact value [16]. The calculation of the relative error aims to evaluate how accurate a forecast or estimate is. The relative error gives an idea of how much the relative error of the prediction is, making it possible to determine how well a model or method can estimate the desired value. The formula for calculating the relative error value is as follows:

$$\text{Relative Error} = \frac{|\text{predicted value} - \text{exact value}|}{\text{exact value}} \tag{8}$$

where exact value is a solution calculated using the maximum timestep that can be calculated by our personal computer, which uses a Windows operating system with an i5 processor [17].

## 3. RESULTS AND DISCUSSION

### 3.1 Volatility of Stock Returns

In this study, volatility is calculated based on daily historical data. From the MAPI stock return data, a standard deviation value of 0.0255 is obtained, and the length of the time interval in one year $(\Delta t)$ is calculated using the number of trading day (252 days). Then, using Equation (1), the annual volatility value of the stock return is obtained as 40.45% per year.

### 3.2 Binary Option Pricing Using CRR Binomial, HW Trinomial, and KR Trinomial

The numerical illustration is given to calculate the option price. In this research, we used these following parameters value:

$S_0 = 1465, K = 1465, Q = 1000, \sigma = 40.45\%, r = 0.06, T = \frac{1}{2}$ (which results in $\Delta t = \frac{1}{2n}$, where $n$ is the number of timesteps). The exact option price is calculated using 10000 timesteps with the final payoffs are given by **Equation (2)** – **Equation (4)**. The results are given in **Table 1**:

**Table 1. Exact Value of Binary Options**

| Options | Methods | | |
|---|---|---|---|
| | CRR Binomial | HW Trinomial | KR Trinomial |
| Asset-or-Nothing Call | 870.2421 | 871.0012 | 872.4415 |
| Asset-or-Nothing Put | 583.4229 | 584.1821 | 585.6172 |
| Cash-or-Nothing Call | 466.6014 | 467.1195 | 468.1006 |
| Cash-or-Nothing Put | 496.6252 | 496.6252 | 497.6069 |

The price of the call and put binary options for several $n$-timesteps are given in **Table 2** to **Table 5**. These tables demonstrate the accuracy and reliability of the model in pricing options.

**Table 2. Option Price and Relative Error for Asset or Nothing Call**

| Step | Option Price | | | Relative Error | | |
|---|---|---|---|---|---|---|
| | CRR | HW | KR | CRR | HW | KR |
| 4 | 601.0764 | 617.3292 | 707.0347 | 0.3093 | 0.2912 | 0.1896 |
| 16 | 734.2336 | 750.8865 | 789.5753 | 0.1563 | 0.1379 | 0.0950 |
| 64 | 804.7943 | 814.0024 | 832.5074 | 0.0752 | 0.0654 | 0.0458 |
| 256 | 840.3873 | 845.0978 | 854.1997 | 0.0343 | 0.0297 | 0.0209 |
| 1024 | 858.1716 | 860.5400 | 865.0598 | 0.0139 | 0.0120 | 0.0085 |
| 4096 | 867.0493 | 868.2352 | 870.4880 | 0.0037 | 0.0032 | 0.0022 |

The results in **Table 2** show that the asset-or-nothing call option price tends to approach the exact value as the number of simulations performed increases. These results also show that the use of the KR method is more optimal than the other two methods.

**Table 3. Option Price and Relative Error for Asset or Nothing Put**

| Step | Option Price | | | Relative Error | | |
|---|---|---|---|---|---|---|
| | CRR | HW | KR | CRR | HW | KR |
| 4 | 331.1667 | 349.8580 | 424.6171 | 0.4324 | 0.4011 | 0.2749 |
| 16 | 451.7736 | 468.5425 | 503.8959 | 0.2256 | 0.1980 | 0.1395 |
| 64 | 519.0666 | 528.2826 | 545.9732 | 0.1103 | 0.0957 | 0.0677 |
| 256 | 553.8362 | 558.5474 | 567.4468 | 0.0507 | 0.0439 | 0.0310 |
| 1024 | 571.4142 | 573.7828 | 578.2519 | 0.0206 | 0.0178 | 0.0126 |
| 4096 | 580.2403 | 581.4262 | 583.6664 | 0.0055 | 0.0047 | 0.0033 |

The results in **Table 3** show that the asset-or-nothing put option price tends to approach the exact value as the number of simulations performed increases. These results also show that the use of the KR method is more optimal than the other two methods.

**Table 4. Option Price and Relative Error for Cash or Nothing Call**

| Step | Option Price | | | Relative Error | | |
|---|---|---|---|---|---|---|
| | CRR | HW | KR | CRR | HW | KR |
| 4 | 289.6267 | 301.5862 | 356.9439 | 0.3793 | 0.3544 | 0.2375 |
| 16 | 375.5049 | 386.8925 | 411.9910 | 0.1952 | 0.1717 | 0.1199 |
| 64 | 422.3634 | 428.6455 | 440.9569 | 0.0948 | 0.0824 | 0.0578 |
| 256 | 446.3298 | 449.5438 | 455.6772 | 0.0434 | 0.0376 | 0.0265 |

| Step | Option Price | | | Relative Error | | |
|---|---|---|---|---|---|---|
| | CRR | HW | KR | CRR | HW | KR |
| 1024 | 458.3868 | 460.0031 | 463.0684 | 0.0176 | 0.0152 | 0.0108 |
| 4096 | 464.4261 | 465.2355 | 466.7683 | 0.0047 | 0.0040 | 0.0028 |

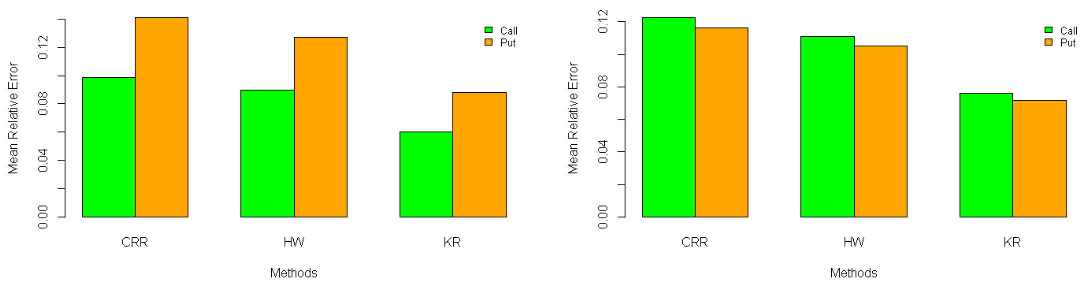
The results in **Table 4** show that the cash-or-nothing call option price is getting closer to the exact value with the increase of the steps performed. These results also show that the use of the KR method is more optimal than the other two methods.

**Table 5. Option Price and Relative Error for Cash or Nothing Put**

| Step | Option Price | | | Relative Error | | |
|---|---|---|---|---|---|---|
| | CRR | HW | KR | CRR | HW | KR |
| 4 | 317.1622 | 329.0703 | 386.0237 | 0.3607 | 0.3374 | 0.2242 |
| 16 | 404.5018 | 415.9315 | 441.3860 | 0.1846 | 0.1625 | 0.1130 |
| 64 | 451.7415 | 458.0365 | 470.4349 | 0.0894 | 0.0777 | 0.0546 |
| 256 | 475.8041 | 479.0215 | 485.1765 | 0.0409 | 0.0354 | 0.0250 |
| 1024 | 487.8852 | 489.5024 | 492.5731 | 0.0166 | 0.0143 | 0.0101 |
| 4096 | 493.9305 | 494.7401 | 496.2742 | 0.0044 | 0.0038 | 0.0028 |

The results in **Table 5** show that the cash-or-nothing put option price tends to approach the exact value as the number of simulations performed increases. These results also show that the use of the KR method is more optimal than the other two methods.

The observation shows a significant decrease along with the increase in the number of steps used. The previous tables show that the use of the KR method is more optimal than the other two methods. This is because the option value has the smallest error which causes the value to approach the exact value faster in the KR method. **Figure 3** shows plot of the mean relative errors calculated using the average of the above errors.




**(a)** **(b)**

**Figure 3. Mean Relative Error Value**
**(a) Asset-or-Nothing, (b) Cash-or-Nothing**

### 3.3 Analysis of Factors Affecting the CRR Binomial, HW Trinomial, and KR Trinomial Methods

Option price is influenced by several factors that are important in financial analysis. Some of the main factors that affect option price include volatility, strike price, time to maturity, and risk-free interest rate [18]. These factors have a significant impact on the option price of option price estimates from each method under different market conditions. In this part, the influence of these factors will be explained. **Figure 4** shows the relationship between initial price and option price.

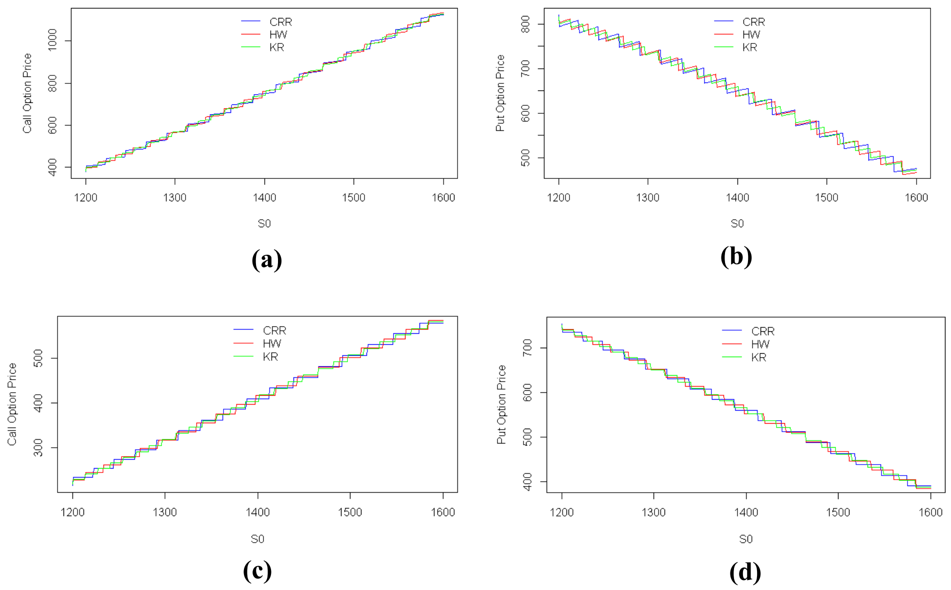



**Figure 4. Relationship between Initial Price Parameters and Option Price**
**(a) Asset-or-Nothing Call, (b) Asset-or-Nothing Put, (c) Cash-or-Nothing Call, (d) Cash-or-Nothing Put**

One of important factors that affect option prices is the initial spot price. This initial price refers to the price of the underlying asset at the time the option is evaluated or purchased. In the lattice model, the initial price is required as the first node of the step. As can be seen in **Figure 5**, there is a positive relationship on call option, while there is a negative relationship on put option.

**Figure 5** shows the relationship between strike price and option price.

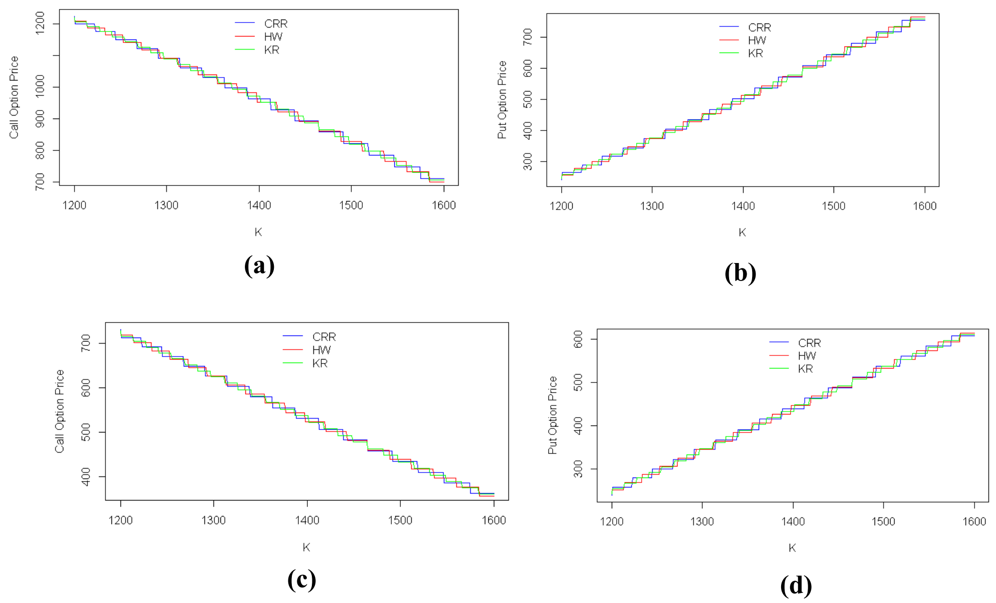
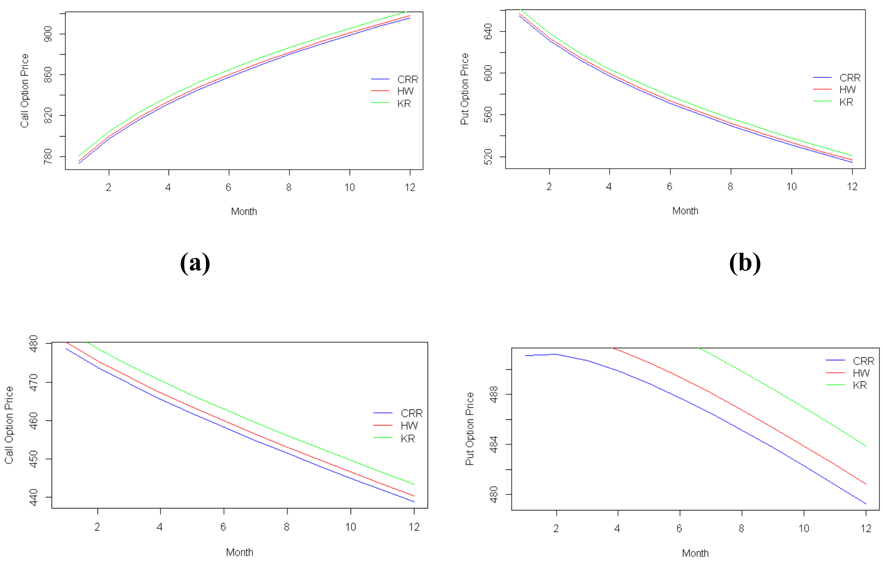



**Figure 5. Relationship between Strike Price Parameters and Option Price**
**(a) Asset-or-Nothing Call, (b) Asset-or-Nothing Put, (c) Cash-or-Nothing Call, (d) Cash-or-Nothing Put**

The strike price is the price agreed upon in the option contract. Meanwhile, the option price is the price paid by the option buyer to the option seller to acquire the right. As can be seen from **Figure 5**, there is a general trend that the call option price decreases as the strike price increases, while the put option price increases as the strike price increases.

**Figure 6** shows the relationship between maturity time and option price.



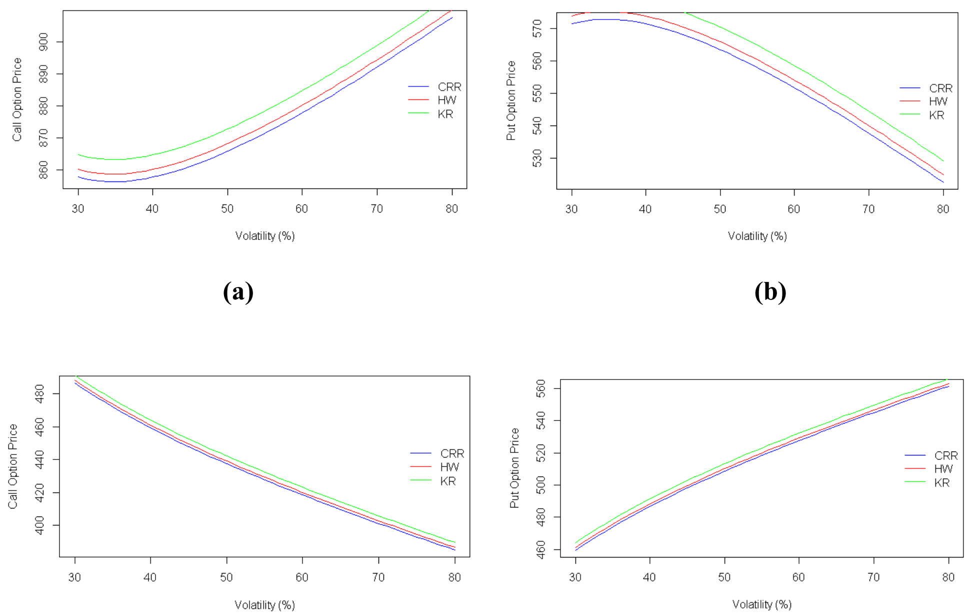

(a) (b)

(c) (d)

**Figure 6. Relationship between Maturity Time Parameters and Option Price**
**(a) Asset-or-Nothing Call, (b) Asset-or-Nothing Put, (c) Cash-or-Nothing Call, (d) Cash-or-Nothing Put**

Maturity time refers to the date on which an option expires. The longer the expiry of the option, the higher the probability that the value of the asset will exceed the exercise price. In this case, delaying the expiry of the option favors the option holder. There is a positive relationship between option duration and option value for asset-or-nothing call options. Meanwhile, there is a downward trend between maturity time and option price for put options and cash-or-nothing call. This relationship can be seen in Figure 6.

Figure 7 shows the relationship between volatility and option price.




(a) (b)

(c) (d)

**Figure 7. Relationship between Volatility Parameters and Option Price**
**(a) Asset-or-Nothing Call, (b) Asset-or-Nothing Put, (c) Cash-or-Nothing Call, (d) Cash-or-Nothing Put**

Volatility, which measures the degree to which the price of the underlying asset fluctuates, has a significant impact on option value. There is a positive correlation between option volatility and option value for asset-or-nothing call and cash-or-nothing put options. Meanwhile, there is a downward trend between volatility and option price for asset-or-nothing put options and cash-or-nothing call options. The relationship between the option prices of the three methods and volatility is shown in Figure 7.

Figure 8 shows the relationship between risk-free interest and option price.

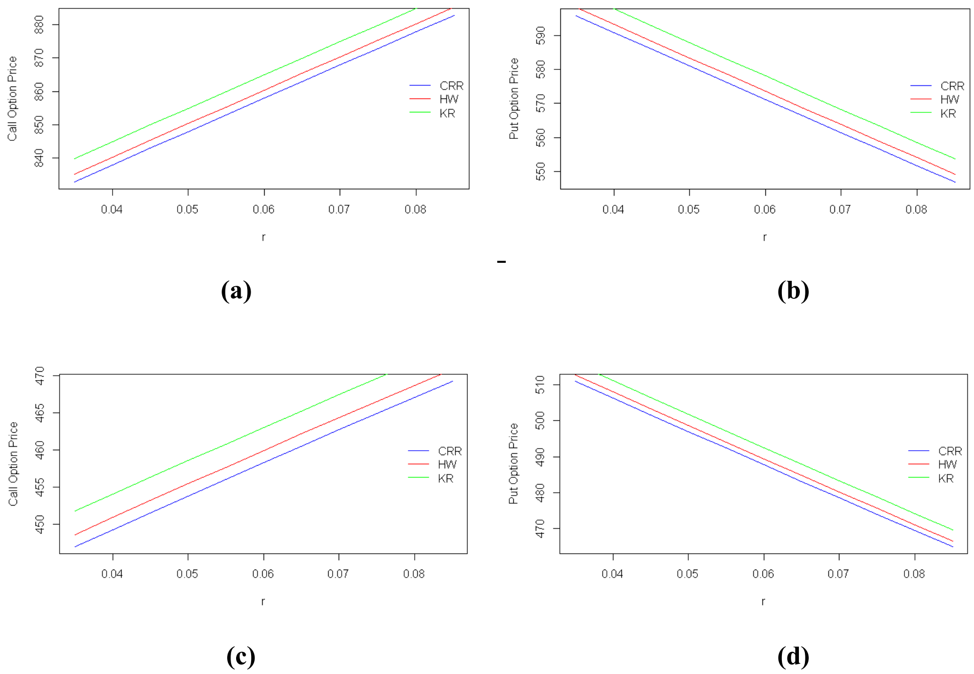



**Figure 8. Relationship between Risk-Free Interest Rate and Option Price**
**(a) Asset-or-Nothing Call, (b) Asset-or-Nothing Put, (c) Cash-or-Nothing Call, (d) Cash-or-Nothing Put**

Another factor that affects option value is the risk-free interest rate. It can be seen from Figure 8 that the option prices of asset-or-nothing and cash-or-nothing call options increase as the risk-free interest rate increases. Meanwhile, the asset-or-nothing and cash-or-nothing option prices of put options decrease as the risk-free interest rate increases. So, it can be concluded that an increase in the interest rate will cause the value of call options to increase and the value of put options to decrease.

## 4. CONCLUSIONS

This study concludes that among the three methods analyzed, the Kamrad-Ritchken (KR) Trinomial method is the most effective for pricing binary options, as it demonstrated the lowest mean relative error compared to the Cox-Ross-Rubinstein (CRR) Binomial and Hull-White (HW) Trinomial methods. The analysis of factors such as initial price, strike price, maturity time, volatility, and risk-free interest rate showed that each method responds differently to market variables, highlighting the importance of selecting an appropriate model based on specific option characteristics.

While this study provides valuable insights into the comparative performance of these numerical methods, it is subject to certain limitations. The analysis is based on stock data from PT Mitra Adiperkasa Tbk, which, while representative, may not capture the full spectrum of market behaviors encountered in other sectors or regions. Additionally, the models rely on certain assumptions, such as constant volatility and risk-free interest rates, which may not hold in real-world market conditions, potentially affecting the accuracy of pricing.

Future studies could expand on this research by testing the CRR, HW, and KR methods under various market conditions, such as during periods of high volatility or economic downturns, to assess their robustness and reliability. Moreover, applying these models to different types of assets, including stocks from diverse sectors or even other financial instruments like commodities, could provide a broader perspective on their applicability. Researchers could also explore modifications to these models to incorporate variable volatility or interest rates, which would more accurately reflect market dynamics and enhance the precision of binary option pricing.

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