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A DUAL-CHANNEL SUPPLY CHAIN MODEL OF COMPLEMENTARY PRODUCTS BY CONSIDERING GREEN MANUFACTURING LEVEL

Sabrina Sri Mulyani[1], Ririn Setiyowati [2*], Nughthoh Arfawi Khurdi[3]

[1,2,3]*Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Sebelas Maret*
Jln. Ir. Sutami 36 Kentingan, Surakarta, 57126, Indonesia

Corresponding author's e-mail: * ***ririnsetiyowati@staff.uns.ac.id***

ABSTRACT

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The development of technology and the internet has encouraged industry players to develop their businesses by adding online sales media, which previously only used offline sales media. By adding online sales media, industry players can expand their markets so they can increase profits. This study aims to modify the DCSC model for complementary products by considering the green manufacturing level to maximize the profit of a system consisting of two manufacturers and one retailer.. Based on the model that has been constructed, the optimal solution is determined so that maximum profits are obtained for each model actor in centralized scenarios. After that, the model is applied and sensitivity analysis of its parameters is carried out. Based on the research results, it appears that the policy that is more profitable for each actor in terms of system benefits is the centralized scenario. Analysis of changes in selling price elasticity values and cross price sensitivity of one of the complementary products sold in two channels influences the increase or decrease in profits of each actor and system profits. However, changes in the value of cross-price sensitivity between two complementary products indicate a decrease in the profits of each actor and the profits of the system. The change in the sensitivity value of the green manufacturing level for each product shows an increase in system profits and retailer profits, where when the sensitivity value of the green manufacturing level of product 1 is greater, the profit of manufacturer 1 is also greater, but the profit of manufacturer 2 is smaller. Conversely, when the green manufacturing level sensitivity value of product 2 becomes greater, the profit of manufacturer 2 also becomes greater, but the profit of manufacturer 1 becomes smaller.

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1. INTRODUCTION

One of the mathematical models used in industrial economics is the supply chain (SC) model. Supply chain (SC) is a system that includes a series of processes from manufacturing to selling products to consumers [1],[2]. The rapid development of technology and the internet means that manufacturers who previously only sold products to consumers through retailers now have the opportunity to sell products to consumers directly using online media [3]. The selling products using online media can expand market segments, control product prices and increase profits [4]. Therefore, based on sales media, supply chains are divided into two, namely single-channel supply chain (SCSC) and dual-channel supply chain (DCSC). SCSC only uses one sales media, such as offline or online media. The DCSC uses both types of media [5].

A product in the SC model is grouped into several types. Based on their use in relation to other products, products are divided into substitute and complementary products. Substitute products are products that will be chosen if the desired product is not available [6]), while complementary products are products that complement each other in general use [7]. Applying the supply chain model to complementary products has become an important operation for companies [8]. Complementary products can increase profits if the company succeeds in combining a new product that is related to the main product. Liu [9] also added that the level of complementarity of a complementary product has an impact on sales and pricing strategies.

When purchasing a product, consumers tend to choose to pay more for low-carbon, energy-saving and environmentally friendly products [10]. This awareness certainly has a good impact on research which states that SC has a significant influence on the environment, one of which is the manufacturing process which results in carbon emissions [11].

If carbon emissions in the environment continue to be ignored, future generations will have difficulty obtaining natural resources that are not polluted. To avoid this, green manufacturing is needed. Dheeraj and Vishal [12] define green manufacturing as the application of fast, reliable and energy efficient manufacturing processes and equipment aimed at minimizing waste and increasing productivity. Green manufacturing also has a positive influence on company performance [2]. Lateradded that green manufacturing is carried out throughout the product life cycle, including products that are not sold or returned on the market [13]. Therefore, the main reason for implementing green manufacturing is sustainability in the production of a product [14].

Shan [15] discussed three SC model structures for complementary products by considering the level of green manufacturing which is divided based on sales strategy, namely individual pricing model, pure bundling model, mixed bundling model. the model is in the form of a Single Channel Supply Chain (SCSC). The difference can be seen in product sales from retailers to consumers where retailers sell both products separately in an individual pricing model, retailers sell both products combined in a pure bundling model, and retailers sell both products separately and combined in a mixed bundling model. This research focuses on the structure of the individual pricing model which was developed by adding a direct sales channel to consumers at one of the manufacturers to become the DCSC model. The constructed DCSC model aims to maximize the profits of a system consisting of two manufacturers and one retailer by considering the level of green manufacturing in two scenarios, namely centralized. It is known that the centralized model is the best model as shown by the profits in the centralized scenario being greater than the profits in the decentralized scenario [16], but causing higher investment and operational costs than the centralized scenario [17].

2. RESEARCH METHODS

This research is quantitative research based on the results of a literature review. A literature review was carried out by collecting reference materials in the form of books and journals regarding supply chain models for complementary products, then model development was carried out, the optimal solution was determined from the model obtained, and a sensitivity analysis was carried out to determine the effect of changes in parameter values on the objective function.

2.1 Operational Step

The following are the operational steps used in this research.

1. Determining the DCSC model assumptions that will be developed from the individual pricing model studied as follows
    a. change the original assumption that both manufacturers sell their products to retailers plus one manufacturer also sells their products directly to consumers,
    b. adding assumptions from previously only using a centralized scenario.
2. Determine the objective function in the form of a system profit function for two manufacturers and one retailer with steps,
    a. determine the number of requests for each product on the sales channels used,
    b. determine total revenue at two manufacturers and one retailer, and
    c. determine total costs at two manufacturers and one retailer.
3. Prove that the constructed DCSC model has a maximum global point with steps,
    a. determine the partial derivative of the model regarding the independent variables,
    b. construct the Hessian matrix,
    c. determine the principal minor determinant of the Hessian matrix, and
    d. the model has a global maximum point if the principal minor determinant of the Hessian matrix is negative definite.
4. Determine the optimal selling price and level of green manufacturing for two manufacturers and one retailer so that maximum profits can be obtained by determining the first partial derivative of the model for the independent variables and the derivative result is equal to zero, then finding a solution for the selling price.
5. Look for appropriate parameter values from previous studies, then substitute them into the model that has been obtained. After that, the optimal solution is determined.
6. Analyze the sensitivity of parameters to the maximum profit of the system based on the results of applying the model with parameter simulation. research method contains explanations in the form of paragraphs about the research design or descriptions of the experimental settings, data sources, data collection techniques, and data analysis conducted by the researcher. This guide will explain writing headings. If your headers exceed one, use the second level of headings as below.

2.2 Model Structure and Assumptions

In this research, the DCSC model consists of manufacturer 1, manufacturer 2, and retailer. It is assumed that manufacturer 1 produces product 1 and manufacturer 2 produces product 2. When combined, product 1 and product 2 will become products that complement each other in general use or are called complementary products. The structure of product sales from manufacturers to consumers via online and offline media is presented in **Figure 1** with D_0 denotes the demand for product 1 on online media at manufacturer 1, D_1 denotes the demand for product 1 at retailers, and D_2 denotes the demand for product 2 at retailers.

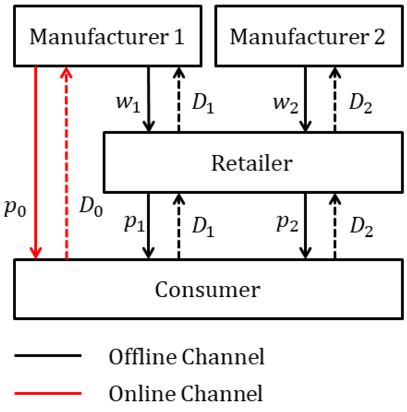

Figure 1. DCSC Model Structure

The following are the assumptions used to construct the model.

1. The interaction between two manufacturers and one retailer follows a centralized scenario.
2. The two manufacturers have different product production costs c_i for $i = 1,2$ and the wholesale price per unit of different products w_i for $i = 1,2$.
3. Complementary products have different values in market demand potential, product price elasticity, and cross-price sensitivity.
4. Retailers sell two complementary products to consumers with an individual pricing model strategy.
5. Consumer demand for a product has a positive linear relationship with the product's green manufacturing level θ_i for $i = 1,2$ which is a continuous variable.
6. Both manufacturers invest additional costs to develop green products of $\eta\theta^2$ where η is the sensitivity of green product development costs. These development costs take the form of a quadratic function because they reflect how initial changes towards green manufacturing can be achieved easily, but subsequent changes are more expensive and difficult to achieve.
7. There is no waiting time for delivery on online sales media.
8. Selling prices p_0, p_1, p_2, w_1, and w_2 are greater than production costs c_1 andc_2.
9. Manufacturer 1 sells products to consumers and retailers respectively for p_0 and $w_1$with$p_0 > w_1$.
10. Retailers sell products offline to consumers at prices p_1 and p_2 where $p_1 > w_1$ and $p_2 > w_2$.
11. Value $\min\{\mu_1, \mu_2\} > \max\{\beta_1, \beta_2\}$.
12. All parameters are positive.

3. RESULTS AND DISCUSSION

3.1 Model Construction

Demand for product 1 consists of demand for product 1 in online media and offline media. Demand in online media is demand for products directly from consumers to manufacturers 1, while demand in offline media is demand for products from consumers to retailers. The basic demand for product 1 in online media is a_0. The unit demand for product 1 in online media decreases as the selling price of product 1 in online media increases p_0, which changes according to the elasticity of the selling price of product 1, which is μ_1, so that $\mu_1 p_0$ is obtained. Changes in consumer preferences in purchasing product 1 in online media are influenced by the selling price of product 1 from retailer p_1 so that the unit demand for product 1 in online media increases along with the cross-price sensitivity of product 1 in both channels α to αp_1. Because product 1 and product 2 are complementary products, the unit demand for product 1 in online media decreases as the selling price of product 2 from retailer p2 increases with the cross-price sensitivity between the two products in the two channels being β_1 to obtain $\beta_1 p_2$. The greater the level of green manufacturing in product 1, namely θ_1, the unit demand for product 1 in online media also increases along with the sensitivity of the level of green manufacturing in product 1 in manufacturing channel 1 in online media γ_0 to $\gamma_0\theta_1$. Obtained demand for product 1 on online media is

$$D_0 = a_0 - \mu_1 p_0 + \alpha p_1 - \beta_1 p_2 + \gamma_0 \theta_1 \tag{1}$$

The basic demand for product 1 at retailers is a_1. The unit demand for product 1 at the retailer decreases as the selling price of product 1 from retailer p_1 increases, which changes according to the selling price elasticity of product 1 of μ_1 so that $\mu_1 p_1$ is obtained. Changes in consumer preferences in purchasing product 1 at retailers are influenced by the selling price of product 1 from manufacturer 1 p_0 so that the unit demand for product 1 at retailers increases along with the cross price sensitivity of product 1 in both channels α to αp_0. Because product 1 and product 2 are complementary products, the unit demand for product 1 at the retailer decreases as the selling price of product 2 from retailer p_2 increases with the cross-price sensitivity between the two products in the two channels being β_2 so that $\beta_2 p_2$ is obtained. The greater the level of green manufacturing in product 1, namely θ_1, the unit demand for product 1 at retailers also increases along with the sensitivity of the level of green manufacturing of product 1 in the retail channel γ_1 to $\gamma_1\theta_1$. Therefore, the demand for product 1 at retailers is

$$D_1 = a_1 - \mu_1 p_1 + \alpha p_0 - \beta_2 p_2 + \gamma_1 \theta_1 \tag{2}$$

The basic demand for product 2 at retailers is a_2. The unit demand for product 2 at the retailer decreases as the selling price of product 2 from retailer p_2 increases, which changes according to the selling price elasticity of product 2 of μ_2 so that $\mu_2 p_2$ is obtained. Because product 1 and product 2 are complementary products, the unit demand for product 2 at the retailer decreases as the selling price of product 1 from the manufacturer 1 p_0 increases with the cross price sensitivity between the two products in the two channels being β_1 so that $\beta_1 p_0$ is obtained and the unit demand for product 2 at the retailer also decreases as the selling price of product 1 from retailer p_1 increases with the cross price sensitivity between the two products in the two channels being β_2 to obtain $\beta_2 p_1$. The greater the level of green manufacturing in product 2, namely θ_2, the unit demand for product 2 at retailers also increases along with the sensitivity of the level of green manufacturing of product 2 in the retail channel γ_2 to $\gamma_2 \theta_2$. Thus, the demand for product 2 at retailers is

$$D_2 = a_2 - \mu_2 p_2 - \beta_1 p_0 - \beta_2 p_2 + \gamma_2 \theta_2. \tag{3}$$

From the model structure shown in Figure 1, the profit function of manufacturer 1, manufacturer 2 and retailer can be determined. Profits can be obtained from the amount of income minus the amount of expenses. The total income from manufacturer 1 obtained through online sales to consumers is $p_0 D_0$ and sales to retailers is $w_1 D_1$. The total expenditure from manufacturer 1 is obtained from the production costs per product multiplied by the number of requests from manufacturer 1 of $c_1(D_0 + D_1)$ and green product development costs of $\eta_1 \theta_1^2$. Therefore, the profit function of manufacturer 1 can be written as

$$\Pi_{M_1} = (p_0 - c_1)D_0 + (w_1 - c_1)D_1 - \eta_1 \theta_1^2. \tag{4}$$

The total income from manufacturer 2 obtained through sales to retailers is $w_2 D_2$. The total expenditure from manufacturer 2 is obtained from the production costs per product multiplied by the number of requests from manufacturer 2 amounting to $c_2 D_2$ and green product development costs amounting to $\eta_2 \theta_2^2$. Therefore, the profit function of manufacturer 2 can be written as

$$\Pi_{M_2} = (w_2 - c_2)D_2 - \eta_2 \theta_2^2. \tag{5}$$

Total income from retailers obtained through sales to consumers for product 1 is $p_1 D_1$ and for product 2 is $p_2 D_2$. The amount of expenditure from retailers is obtained from the cost of purchasing the product multiplied by the number of requests to manufacturer 1 equal to $w_1 D_1$ and to manufacturer 2 equal to $w_2 D_2$. Therefore, the retailer's profit function can be written as

$$\Pi_R = (p_1 - w_1)D_1 + (p_2 - w_2)D_2. \tag{6}$$

3.2 Optimal Solution

The optimal solution in the centralized scenario is obtained by maximizing system profits. The system profit function is obtained from the sum of **Equation (4)** for manufacturer 1, **Equation (5)** for manufacturer 2, and retailer's profit in **Equation (6)** to obtain

$$\Pi_S(p_0, p_1, p_2, \theta_1, \theta_2) = (p_0 - c_1)D_0 + (p_1 - c_1)D_1 + (p_2 - c_2)D_2 - \eta_1 \theta_1^2 - \eta_2 \theta_2^2. \tag{7}$$

Based on model in **Equation (7)**, the Hessian matrix (H) is determined and then the principal minor determinant of the Hessian matrix is obtained as follows

$$|H_{11}| = -2\mu_1,$$
$$|H_{22}| = -4\alpha_2 + 4\mu_1^2,$$
$$|H_{33}| = 8(2\alpha\beta_1\beta_2 + \beta_1^2\mu_1 + \beta_2^2\mu_1 + \alpha^2\mu_2 - \mu_1^2\mu_2),$$
$$|H_{44}| = -8\beta_1\beta_2 P - 4\beta_2^2 J + 4\beta_1^2 K - 4M\mu_2,$$
$$|H_{55}| = -8(\beta_2\gamma_0 - \beta_1\gamma_1)^2\eta_2 + 64O - 2(\gamma_0^2 + \gamma_1^2)\gamma_2^2\mu_1 + 32(\beta_1^2 + \beta_2^2)\eta_1\eta_2\mu_1 + 8\gamma_2^2\eta_1\mu_1^2 + Q$$

with $J = \gamma_0^2 - 4\eta_1\mu_1$, $K = \gamma_1^2 - 4\eta_1\mu_1$, $L = \gamma_2^2 - 4\eta_2\mu_2$, $M = 2\alpha\gamma_0\gamma_1 + 4\alpha_2\eta_1 + \mu_1 N$, $N = \gamma_0^2 + \gamma_1^2 - 4\eta_1\mu_1$, $O = \alpha\beta_1\beta_2\eta_1\eta_2$, $P = \gamma_0\gamma_1 + 4\alpha\eta_1$, and $Q = 8\eta_2\mu_1 N\mu_2 - 4\alpha\gamma_0\gamma_1 L - 8\alpha^2\eta_1 L$.

Based on assumptions 11 and 12, we obtain $\beta_1 < \sqrt{-\frac{\alpha^2\mu_2}{\mu_1} + \mu_1\mu_2}$ so that $H_{11} < 0, H_{22} > 0, H_{33} < 0, H_{44} > 0$, and $H_{55} < 0$. Thus, the system profit function is a strict concave function so that it is guaranteed to have a single optimal solution. The optimal solution of **Equation (7)** is obtained from the first partial derivative of **Equation (7)** for each independent variable and is written

$$\frac{\partial \Pi_s}{\partial p_0} = 0, \frac{\partial \Pi_s}{\partial p_1} = 0, \frac{\partial \Pi_s}{\partial p_2} = 0, \frac{\partial \Pi_s}{\partial \theta_1} = 0, \frac{\partial \Pi_s}{\partial \theta_2} = 0. \tag{8}$$

The solution of system in **Equation (8)** obtains optimal values

$$\begin{aligned}
p_0^{(c*)} = \frac{1}{W}\Big(& c_2\beta_2\gamma_0\gamma_1\gamma_2^2 + a_0\gamma_1^2\gamma_2^2 - c_2\beta_1\gamma_1^2\gamma_2^2 + 4c_2\alpha\beta_2\gamma_2^2\eta_1 - 4a_2\beta_2\gamma_0\gamma_1\eta_2 + 4\alpha_2\beta_1\gamma_1^2\eta_2 \\
& - 16a_2\alpha\beta_2\eta_1\eta_1 - 16a_0\beta_2^2\eta_1\eta_2 - 4a_0\gamma_2^2\eta_1\mu_1 + 4c_2\beta_1\gamma_2^2\eta_1\mu_1 - 16a_2\beta_1\eta_1\eta_2\mu_1 \\
& - 4a_0\eta_2K\mu_2 + a_1\big(-\gamma_0\gamma_2L + 4\eta_1(-\alpha\gamma_2^2 + 4\beta_1\beta_2\eta_2 + 4\alpha\eta_2\mu_2)\big) \\
& + c_1\big(4\beta_1^2\gamma_1^2\eta_2 - 4\beta_1\beta_2\gamma_1(3\gamma_0 + \gamma_1)\eta_2 + 2\gamma_0^2\gamma_2^2\mu_1 + \gamma_0\gamma_1\gamma_2^2\mu_1 + \gamma_1^2\gamma_2^2\mu_1 \\
& - 16\beta_1^2\eta_1\eta_1\mu_1 - 4\gamma_2^2\eta_1\mu_1^2 + 4\beta_2^2\eta_2(\gamma_0(2\gamma_0 + \gamma_1\ ) - 4\eta_1\mu_1) \\
& - 4\eta_2\mu_1(2\gamma_0^2 + \gamma_0\gamma_1 + K)\mu_2 + 4\alpha_2\eta_1L + \alpha(-32\beta_1\beta_2\eta_1\eta_2 + \gamma_1(3\gamma_0 + \gamma_1)L)\big)\Big),
\end{aligned}$$

$$\begin{aligned}
p_0^{(c*)} = \frac{1}{W}\Bigg(& -c_2\beta_2\gamma_0^2\gamma_2^2 - a_0\gamma_0\gamma_1\gamma_2^2 + c_2\beta_1\gamma_0\gamma_1\gamma_2^2 - 4a_0\gamma_2^2\eta_1 + 4c_2\alpha\beta_1\gamma_2^2\eta_1 + 4a_2\beta_2\gamma_0^2\eta_2 \\
& - 4a_2\beta_1\gamma_0\gamma_1\eta_2 - 16a_2\alpha\beta_1\eta_1\eta_2 + 16a_0\beta_1\beta_2\eta_1\eta_2 + 4c_2\beta_2\gamma_2^2\eta_1\mu_1 - 16a_2\beta_2\eta_1\eta_2\mu_1 \\
& + 4a_0(\gamma_0\gamma_1 + 4\alpha\eta_1)\eta_2\mu_2 + a_1(-16\beta_1^2\eta_1\eta_2 + JL) \\
& + c_1\big(4\beta_2^2\gamma_0^2\eta_2 - 4\beta_1\beta_2\gamma_0(\gamma_0 + 3\gamma_1)\eta_2 + \gamma_0^2\gamma_2^2\mu_1 + \gamma_0\gamma_1\gamma_2^2\mu_1 + 2\gamma_1^2\gamma_2^2\mu_1 \\
& - 16\beta_2^2\eta_1\eta_2\mu_1 - 4\gamma_2^2\eta_1\mu_1^2 + 4\beta_1^2\eta_2\big(\gamma_1\big(\gamma_{0_{2\gamma_2}}\big) - 4\eta_1\mu_1\big) \\
& - 4\eta_2\mu_1(\gamma_0^2 + \gamma_0\gamma_1 + 2\gamma_1^2 - 4\eta_1\mu_1)\mu_2 + 4\alpha^2\eta_1J \\
& + \alpha(-32\beta_1\beta_2\eta_1\eta_2 + \gamma_0(\gamma_0 + 3\gamma_1\ )L)\big)\Bigg),
\end{aligned}$$

$$\begin{aligned}
p_2^{(c*)} = \frac{1}{X}\Big(& 2\eta_2(2a_2\gamma_0\gamma_1 - a_0\beta_2\gamma_0\gamma_1 + a_0\beta_1\ \gamma_1^2 - 4a_2\alpha^2\eta_1 - 4a_0\alpha\beta_2\eta_1 - a_1\beta_1(\gamma_0\gamma_1 + 4\alpha\eta_1) \\
& - a_2(\gamma_0^2 + \gamma_1^2)\mu_1 - 4a_0\beta_1\eta_1\mu_1 + 4a_2\eta_1\mu_1^2 \\
& + c_1(\gamma_0 + \gamma_1)(\alpha\beta_2\gamma_0 + \alpha\beta_1\gamma_1 + \beta_1\gamma_0\mu_1 + \beta_2\gamma_1\mu_1) + a_1\beta_2J) \\
& + c_2\big(-4\beta_1\beta_2\gamma_0\gamma_1\eta_2 + \gamma_0^2\gamma_2^2\mu_1 + 2\beta_2^2\eta_2J + (2\beta_1^2\eta_2 + \gamma_2^2\mu_1)K - 2\eta_2\mu_1(\gamma_0^2 + K)\mu_2 \\
& + 4\alpha^2\eta_1(\gamma_2^2 - 2\eta_2\mu_2) + 2\alpha\big(-8\beta_1\beta_2\eta_1\eta_2 + \gamma_0\gamma_1(\gamma_2^2 - 2\eta_2\mu_2)\big)\big)\Big),
\end{aligned}$$

$$\begin{aligned}
\theta_1^{(c*)} = \frac{1}{X}\Big(& c_2\alpha\beta_2\gamma_0\gamma_2^2 - a_0\alpha\gamma_1\gamma_2^2 + c_2\alpha\beta_1\gamma_1\gamma_2^2 - 4a_2\alpha\beta_2\gamma_0\eta_2 - a_0\beta_2^2\gamma_0\eta_2 - 4a_2\alpha\beta_1\gamma_1\eta_2 4a_0\beta_1\beta_2\gamma_1\eta_2 \\
& - a_0\gamma_0\gamma_2^2\mu_1 + c_2\beta_1\gamma_0\gamma_2^2\mu_1 + c_2\beta_2\gamma_1\gamma_2^2\mu_1 - 4a_2\beta_1\gamma_0\eta_2\mu_1 - 4a_2\beta_2\gamma_1\eta_2\mu_1 \\
& - a_1(\alpha\gamma_0\gamma_2^2 + 4\beta_1(\beta_2\gamma_0 + \beta_1\gamma_1)\eta_2 + \gamma_1\gamma_2^2\mu_1) + 4a_0\eta_2(\alpha\gamma_1 + \gamma_0\mu_1)\mu_2 \\
& + 4a_1\eta_2(\alpha\gamma_0 + \gamma_1\mu_1)\mu_2 \\
& - c_1(\gamma_0 + \gamma_1)\big(-8\alpha\beta_1\beta_2\eta_2 + \alpha^2L + \mu_1\big(-\gamma_2^2\mu_1 - 4\eta_2(\beta_1^2 + A)\big)\big)\Big),
\end{aligned}$$

$$\begin{aligned}
\theta_2^{(c*)} = \frac{1}{X}\Big(& \gamma_2\big(-c_2\beta_2^2\gamma_0^2 - 2a_2\alpha\gamma_0\gamma_1 - a_0\beta_2\gamma_0\gamma_1 + 2c_2\beta_1\beta_2\gamma_0\gamma_1 + a_0\beta_1\gamma_1^2 - c_2\beta_1^2\gamma_1^2 - 4a_2\alpha^2\eta_1 \\
& - 4a_0\alpha\beta_2\eta_1 + 8c_2\alpha\beta_1\beta_2\eta_1 - a_2\gamma_0^2\mu_1 - a_2\gamma_1^2\mu_1 - 4a_0\beta_1\eta_1\mu_1 + 4c_2\beta_1^2\eta_1\mu_1 \\
& + 4c_2\beta_2^2\eta_1\mu_1 + 4a_2\eta_1\mu_1^2 \\
& + c_1(\gamma_0 + \gamma_1)(\alpha\beta_2\gamma_0 + \alpha\beta_1\gamma_1 + \beta_1\gamma_0\mu_1 + \beta_2\gamma_1\mu_1 + a_1(\beta_1(\gamma_0\gamma_1 + 4\alpha\eta_1) + \beta_2J) \\
& + c_2M\mu_2)\big)\Big),
\end{aligned}$$

with

$$\begin{aligned}
W &= 2(-8\beta_1\beta_2\gamma_0\gamma_1\gamma_2 - 32O + \gamma_0^2\gamma_2^2\mu_1 + 4\beta_2^2\eta_2J + (4\beta_1^2\eta_2 + \gamma_2^2\mu_1)K - 4\eta_2\mu_1N\mu_2 + 2\alpha\gamma_0\gamma_1L \\
&\qquad + 4\alpha^2\eta_1L), \\
X &= -8\beta_1\beta_2\gamma_0\gamma_1\gamma_2 - 32\alpha\beta_1\beta_2\eta_1\eta_2 + \gamma_0^2\gamma_2^2\mu_1 + 4\beta_2^2\eta_2J + (4\beta_1^2\eta_2 + \gamma_2^2\mu_1)K - 4\eta_2\mu_1(\gamma_0^2 + K)\mu_2 \\
&\qquad + 2\alpha\gamma_0\gamma_1L + 4\alpha^2\eta_1L.
\end{aligned}$$

Next, the values $w_1^{(c*)}$ and $w_2^{(c*)}$ are obtained by substituting $p_0^{(c*)}, p_1^{(c*)}, p_2^{(c*)}, \theta_1^{(c*)}$ and $\theta_2^{(c*)}$ to **Equation (1)**, **Equation (2)**, and **Equation (3)**. After that, all the optimal values that have been obtained are substituted into the demand function for manufacturer 1 in the online channel, retailer for product 1, and retailer for product 2 respectively as follows

$$D_0^{(c^*)} = a_0 - \mu_1 p_0^{(c^*)} + \alpha p_1^{(c^*)} - \beta_1 p_2^{(c^*)} + \gamma_0 \theta_1^{(c^*)},$$
$$D_1^{(c^*)} = a_1 - \mu_1 p_1^{(c^*)} + \alpha p_0^{(c^*)} - \beta_2 p_2^{(c^*)} + \gamma_1 \theta_1^{(c^*)},$$
$$D_2^{(c^*)} = a_2 - \mu_2 p_2^{(c^*)} - \beta_1 p_0^{(c^*)} - \beta_2 p_2^{(c^*)} + \gamma_2 \theta_2^{(c^*)}.$$

Then the optimal values obtained are substituted into the profit function of each actor so that the maximum profit function is obtained for manufacturer 1, manufacturer 2, retailer and system respectively as follows

$$\Pi_{M_1}^{(c^*)} = \left(p_0^{(c^*)} - c_1\right) D_0^{(c^*)} + \left(w_1^{(c^*)} - c_1\right) D_1^{(c^*)} - \eta_1 \left(\theta_1^{(c^*)}\right)^2,$$
$$\Pi_{M_2}^{(c^*)} = \left(w_2^{(c^*)} - c_2\right) D_2^{(c^*)} - \eta_2 \left(\theta_2^{(c^*)}\right)^2,$$
$$\Pi_R^{(c^*)} = \left(p_1^{(c^*)} - w_1^{(c^*)}\right) D_1^{(c^*)} + \left(p_2^{(c^*)} - w_2^{(c^*)}\right) D_2^{(c^*)},$$
$$\Pi_S^{(c^*)} = \Pi_{M_1}^{(c^*)} + \Pi_{M_2}^{(c^*)} + \Pi_R^{(c^*)}.$$

3.3 Numerical Simulation

In this section, a numerical simulation is conducted to show the optimal values for the decision, demand and profit variables for each actor. Determination of parameter values was taken from research by Shan [15], Ren [18], and Sharma [19]. The parameter values obtained can be seen in Table 1.

Table 1. Parameter Values Used in the Numerical Simulation of the DCSC Model

Parameter	Value	Parameter	Value
a_0	650 unit	γ_0	0.35 unit/$
a_1	600 unit	γ_1	0.8 unit/$
a_2	1000 unit	γ_2	0.5 unit/$
μ_1	4 unit/$	η_1	0.4
μ_2	6 unit/$	η_2	0.2
α	0.8 unit/$	c_1	80 $/unit
β_1	0.2 unit/$	c_2	20 $/unit
β_2	0.6 unit/$		

The parameter values in Table 1 are substituted for the optimal solution in the centralized scenario. The optimal solution for the DCSC model in the centralized scenario is presented in Table 2.

Table 2. Optimal Solution of the DCSC Model

Decision Variable	Optimal Solution
w_1	107.01
w_2	59.80
p_0	139.21
p_1	131.44
p_2	84.23
θ_1	77.35
θ_2	80.28
Optimal Demand	
D_0	208.54
D_1	196.94
D_2	428.07
Maximum Profit	
Π_{M_1}	15273.79
Π_{M_2}	15748.21
Π_R	15268.99
Π_S	46291.00

Based on Table 2, the selling price of product 1 in online media for each DCSC model is higher than by the selling price of product 1 from retailer in the centralized scenario. However, the total demand of manufacturer 1 is lower than the total demand of manufacturer 2. In addition, the profit of manufacturer 2 in the centralized scenario is higher than the profit of manufacturer 1. The system profit in centralized scenario is obtained $46291.

3.4 Sensitivity Analysis

In this section, a sensitivity analysis of parameters related to the demand function is done, such as selling price elasticity (μ_1 and μ_2), cross-price sensitivity of product 1 in online and offline media (α), cross-price sensitivity of product 1 and product 2 (β_1 and β_2) , and the sensitivity of demand to the level of green manufacturing (γ_0, γ_1, and γ_2). The purpose of this analysis is to determine the effect of changes in parameter values on each actor's profits.

a. Analysis of Selling Price Elasticity

The influence of changes in the selling price elasticity value of product 1 in online and offline channels on the profits of each actor as well as system profits is shown in Table 3.

Table 3. The Influence of μ_1 on the Profits of Each Actor and System

μ_1	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
3.1	35490.60	13890.17	18471.64	67852.41
3.2	32187.27	14184.72	17957.39	64329.38
3.3	29232.74	14449.51	17503.57	61185.82
3.4	26579.22	14688.81	17099.98	58368.02
3.5	24187.30	14906.12	16738.53	55831.95

Based on Table 3, it can be seen that in the centralized scenarios, an increase in the value of μ_1 causes an increase in profits for manufacturer 2. However, profits for manufacturer 1, retailers, and the system decrease. The largest decrease occurred in manufacturer 1 with an average difference of $2260.66 in the centralized scenario. Furthermore, the effect of changes in the selling price elasticity value of product 2 at retailers on the profits of each actor as well as system profits is shown in Table 4.

Table 4. The influence of μ_2 on the profits of each actor and system

μ_2	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
8.1	16018.05	9551.74	12535.80	38105.59
8.2	16047.01	9342.73	12439.55	37829.29
8.3	16075.24	9139.29	12345.76	37560.29
8.4	16102.74	8941.22	12254.34	37298.30
8.5	16129.57	8748.31	12165.20	37043.07

Based on Table 4, it can be seen that in the centralized scenarios, an increase in the value of μ_2 causes an increase in profits for manufacturer 1. However, profits for manufacturer 2, retailers, and the system decrease. The largest decrease occurred in manufacturer 2 with an average difference of $160.69 in the centralized scenario.

b. Analysis of Cross Price Sensitivity

The effect of changes in the cross-price sensitivity value of product 1 in online and offline media on the profits of each actor as well as system profits is shown in Table 5.

Table 5. The Influence of α on the Profits of Each Actor and System

α	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.1	7776.34	16538.20	14131.34	38445.87
0.2	8581.55	16444.79	14277.21	39303.55
0.3	9457.64	16345.97	14431.81	40235.41
0.4	10411.27	16241.18	14596.06	41248.51
0.5	11449.93	16129.82	14771.07	42350.82

Based on Table 5, it can be seen that in the centralized scenarios, an increase in the value α causes a decrease in profits for manufacturer 2. However, profits for manufacturer 1, retailers, and systems have increased. The largest increase occurred in manufacturer 1 with an average difference of $734.72 in the centralized scenario. Furthermore, the effect of changes in the cross-price sensitivity value of product 1 in online media and product 2 in retailers on the profits of each actor as well as system profits is shown in Table 6.

Table 6. The Influence of β_1 on the Profits of Each Actor and System

β_1	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.1	15619.99	16378.12	15716.01	47714.12
0.2	15167.32	15748.67	15374.40	46290.40
0.3	14735.95	15145.06	15046.21	44927.21
0.4	14324.71	14565.71	14730.59	43621.01
0.5	13932.59	14009.15	14426.77	42368.51

Based on Table 6, it can be seen that in the centralized scenarios, an increase in the value of β_1 causes a decrease in the profits of each actor and system. The largest decrease occurred in manufacturer 2 with an average difference of $473.79 in the centralized scenario. Furthermore, the effect of changes in the cross price sensitivity values of product 1 and product 2 at retailers on the profits of each actor as well as system profits is shown in Table 7.

Table 7. The Influence of β_2 on the Profits of Each Actor and System

β_2	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.1	17034.94	18796.15	17599.53	53430.62
0.2	16628.12	18128.85	17114.76	51871.73
0.3	16239.01	17492.21	16651.20	50382.42
0.4	15866.52	16884.39	16207.44	48958.36
0.5	15509.61	16303.73	15782.23	47595.57

Based on Table 7, it can be seen that in the centralized scenarios, an increase in the value of β_2 causes a decrease in the profits of each actor and system. The largest decrease occurred in manufacturer 2 with an average difference of $498.48 in the centralized scenario.

c. Analysis of Green Manufacturing Level Sensitivity

The effect of changing the sensitivity value of the green manufacturing level of product 1 in the online channel on the profits of each actor as well as the system profits is shown in Table 8.

Table 8. The Influence of γ_0 on the Profits of Each Actor and System

γ_0	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.1	14365.26	15865.24	15108.24	45338.74
0.2	14644.03	15822.70	15209.03	45675.76
0.3	14977.87	15774.84	15317.07	46069.79
0.4	15373.02	15720.85	15434.29	46528.16
0.5	15837.57	15659.71	15563.00	47060.28

Based on Table 8, it can be seen that in the centralized scenarios, an increase in the value of γ_0 causes a decrease in profits for manufacturer 2. However, profits for 1, retailers and the system increase. The largest increase occurred in manufacturer 1 with an average difference of $294.46 in the centralized scenario. Furthermore, the effect of changes in the green manufacturing level sensitivity value of product 1 at retailers on the profits of each actor as well as system profits is shown in Table 9.

Table 9. The Influence of γ_1 on the Profits of Each Actor and System

γ_1	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.1	14053.38	16026.51	14508.13	44588.02

γ_1	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.2	14158.36	16005.43	14565.41	44729.20
0.3	14278.77	15979.18	14640.32	44898.27
0.4	14415.89	15947.31	14734.78	45097.97
0.5	14571.27	15909.22	14851.35	45331.84

Based on **Table 9**, it can be seen that in the centralized scenarios, an increase in the value of γ_1 causes a decrease in profits for manufacturer 2. However, profits for 1, retailers and the system increase. The largest increase occurred in manufacturer 1 with an average difference of \$103.58 in the centralized scenario. Furthermore, the effect of changes in the green manufacturing level sensitivity value of product 1 at retailers on the profits of each actor as well as system profits is shown in **Table 10**.

Table 10. The Influence of γ_2 on the Profits of Each Actor and System

γ_2	Centralized Scenario			
	Π_{M_1}	Π_{M_2}	Π_R	Π_S
0.1	15305.03	15267.19	14544.09	45116.30
0.2	15288.60	15326.59	14641.25	45256.44
0.3	15260.73	15426.13	14807.20	45494.06
0.4	15220.68	15566.51	15048.34	45835.54
0.5	15167.32	15748.67	15374.40	46290.40

Based on **Table 10**, it can be seen that in the centralized scenarios, an increase in the value of γ_2 causes a decrease in profits for manufacturer 1. However, profits for manufacturer 2, retailers and the system increase. The largest decrease occurred in manufacturer 2 with an average difference of \$166.06 in the centralized scenario.

4. CONCLUSIONS

Based on the results of the discussion described in the research, it can be concluded as follows.

1. A dual-channel supply chain model for complementary products was developed by considering the level of green manufacturing. From this model, the maximum profit can be determined in centralized scenarios by proving that the profit function of the model is a strict concave function so that it has a single solution.
2. Based on numerical simulations the profit of manufacturer 2 in the centralized scenario is higher than the profit of manufacturer 1 as well as the total demand of manufacturer 1 is lower than the total demand of manufacturer 2.
3. Based on sensitivity analysis of selling price elasticity parameters and cross price sensitivity, one of the complementary products sold in two channels influences the increase or decrease in profits of each actor and system profits. However, changes in the value of cross-price sensitivity between two complementary products indicate a decrease in the profits of each actor and the profits of the system. The change in the sensitivity value of the green manufacturing level for each product shows an increase in system profits and retailer profits, where when the sensitivity value of the green manufacturing level of product 1 is greater, the profit of manufacturer 1 is also greater, but the profit of manufacturer 2 is smaller. Conversely, when the green manufacturing level sensitivity value of product 2 becomes greater, the profit of manufacturer 2 also becomes greater, but the profit of manufacturer 1 becomes smaller. In further research, it can be developed by adding direct channels from manufacturers to consumers. Apart from that, research can also be developed by changing the sales strategy for complementary products to a pure bundling model or mixed bundling model.

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