BAREKENG: Journal of Mathematics and Its Applications
March 2025 Volume 19 Issue 1 Page 0537–0546
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19iss1pp0537-0546

CLUSTER ANALYSIS OF K-MEANS AND WARD METHOD INFORMING A ROBUST PORTFOLIO: AN EMPIRICAL STUDY OF JAKARTA ISLAMIC INDEX

Zuva Amalina Zain[1], Noor Saif Muhammad Mussafi[2*], Epha Diana Supandi[3]

[1,2,3] *Department of Mathematics, Faculty of Science and Technology, UIN Sunan Kalijaga*
Jln. Timoho No.153, Papringan, Yogyakarta, 55281, Indonesia

Corresponding author's e-mail: * **noor.mussafi@uin-suka.ac.id**

ABSTRACT

Article History:
Received: 12th August 2024
Revised: 25th November 2024
Accepted: 25th November 2024
Published: 13th January 2025

Keywords:
Clustering;
K-Means;
Portfolio Performance;
S-Estimation;
Ward.

Building a portfolio is one method of reducing investment risk. Cluster analysis can shorten the time required to choose companies for a portfolio because it makes it easy to put firms in the same category together. To maintain the best state of the portfolio cluster analysis in the case of data containing outliers, K-means, and ward cluster analysis are employed in conjunction with a robust portfolio strategy. K-means clustering is a popular method for grouping data by assigning observations to clusters based on proximity to the cluster's center meanwhile the Ward method is based on the size of the distance between clusters by minimizing the number of squares. This study seeks to determine the robust portfolio performance comparison outcomes produced by K-Means and Ward clustering utilizing the Sharpe ratio criterion. The Sharpe ratio is one of the most widely used methods to evaluate a portfolio's risk-adjusted performance. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance. Stocks included in the Jakarta Islamic Index 70 (JII70) are used in this research. The results of the formation of a robust portfolio on K-Means clustering produce a return rate of 0.01038627 and risk of 0.1066364, while in the Ward cluster, the portfolio profit rate is obtained at 0.01632749 and the risk is 0.1340073. Based on the Sharpe ratio criteria, in this case, the robust portfolio with the Ward cluster is superior to the K-Means cluster because it produces a higher Sharpe value.

How to cite this article:

Z. A. Zain, N. S. M. Mussafi and E. D. Supandi, "CLUSTER ANALYSIS OF K-MEANS AND WARD METHOD IN FORMING A ROBUST PORTFOLIO: AN EMPIRICAL STUDY OF JAKARTA ISLAMIC INDEX", *BAREKENG: J. Math. & App.*, vol. 19, iss. 1, pp. 0537-0546, March, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

Nowadays, with the rapid advancement of digital technology, everything is easily accessible, even in the corporate sector. The rise in competitiveness is influenced by the development of both new and established businesses that embrace technology. According to [1], technical advancements also give investors or those who make investments the freedom to choose how they want to invest because knowledge about different investment options and methods is widely accessible thanks to the internet and these advancements. Investments are one strategy to get ready for future financial needs [2].

Currently, many investors are interested in investing in Sharia shares. This is shown by the development of the number of sharia stock investors which has increased significantly to 1,650% in the last 5 years. One of the Sharia stock indexes that can be used as a reference for investing is the Jakarta Islamic Index (JII) 70. The performance of Sharia stocks is reflected in the movement of this index.

An investment is a financial commitment made now in order to reap financial rewards later on. Building a portfolio—a grouping of investment assets—is one way to mitigate ambiguous risks in the stock market, which is one of the key elements in making an investing decision [3].

The portfolio model hypothesis was initially put forth by Harry M. Markowitz in 1952. The issue of allocating capital investments to produce significant earnings with minimal risk is covered by this theory [4]. Because the portfolio risk is determined by the weighted average of the risks associated with each asset and the covariance between the assets that comprise the portfolio, the Markowitz portfolio model is also known as the Mean-Variance (MV) portfolio. The MV model in portfolio formation is increasingly developing by modifying the statistical measures used. Group asset selection is one development strategy that should be taken into account when assembling a portfolio. More diversity in the assets in a portfolio will reduce the overall risk and indicate that even the risk of an equally-weighted portfolio decreases with a growing number of assets [5]. The risk in question is the possibility of the same movement when the asset price declines will also occur in other assets in the portfolio. For this reason, in a dataset, there exist conditions where data has different features from other data and some have a perfect linear relationship between some or all variables [6]. This condition leads to the existence of cluster analysis as one of the multivariate statistical techniques that divide objects into many groups, each of which has members that are highly similar to one another inside and without prior knowledge of the group members (clusters), according to [7].

A significant issue with the Mean-Variance portfolio model is the variance-covariance matrix's minimization of risk and uncertainty in volatile data, and the mean vector's estimate, which prioritizes efforts to optimize the expected return. A robust estimate is applied to the portfolio to minimize this uncertainty and create the best possible robust portfolio. [8] Introduce a robust mean-variance portfolio selection method with preprocessing data using cluster analysis, i.e., Ward and Complete Linkage. The results of their empirical investigation showed that, for all risk aversion values, the portfolio performance generated by the clustering with the Ward algorithm outperforms the portfolio performance produced by the clustering with the complete linkage technique. In the same year, [9] also examined Kamila and Weighted K-Mean clustering methods in selecting a robust mean-variance portfolio. As per this study that refers to the LQ45 index, the portfolios produced using the robust Fast Minimum Covariance Determinant (FMCD) estimate and KAMILA clustering algorithm fared better than those made using the K-Mean. To produce a robust mean-variance portfolio for the Jakarta Islamic Index 30 (JII30) index, [10] employ K-Mean to construct two portfolio groups (the highest return and the lowest return). This research builds upon and modifies the earlier research by applying K-Means and Ward clustering analysis in forming a robust mean-variance portfolio for a larger stock index, i.e., the Jakarta Islamic Index 70 (JII70) index. K-Means and Ward clustering were chosen because they have the advantage of being efficient and easy to use, therefore these methods are the most popular compared to other cluster methods.

Based on the explanation above, the aim of this research is to compare portfolio performance on the JII70 index by combining K-Means and Ward clustering with robust Scale (S) estimation. This is how the rest of the paper is structured. Section 2 covers the study methodologies, section 3 discusses our findings and outcomes, and section 4 wraps up the work.

2. RESEARCH METHODS

The data source of this study merely referred to JII70 as one of the Shariah-compliant indexes in Indonesia. The Financial Services Authority of Indonesia classifies stocks as Shariah-compliant if the issuers are not involved in (1) gambling, (2) conventional financial services, (3) producing, distributing, trading and/or providing products or services that are prohibited by the National Shariah Board-Majelis Ulama Indonesia (MUI) such as alcohol and pork-related products, and (4) trading of risk that contains gharar or uncertainty [11].

Table 1. Stock Selection in K-Means's Cluster

No.	Code	Stocks
1.	AALI	Astra Agro Lestari Tbk.
2.	ACES	Ace Hardware Indonesia Tbk.
3.	ADHI	Adhi Karya (Persero) Tbk.
4.	ADRO	Adaro Energy Tbk.
5.	AGII	Aneka Gas Industri Tbk.
6.	AKRA	AKR Corporindo Tbk.
7.	ANTM	Aneka Tambang Tbk.
8.	BMTR	Global Mediacom Tbk.
9.	BRIS	Bank Syariah Indonesia Tbk.
10.	BRPT	Barito Pacific Tbk.
11.	BSDE	Bumi Serpong Damai Tbk.
12.	BTPS	Bank BTPN Syariah Tbk.
13.	CPIN	Charoen Pokphand Indonesia Tbk
14.	CTRA	Ciputra Development Tbk.
15.	ELSA	Elnusa Tbk.
16.	ERAA	Erajaya Swasembada Tbk.
17.	EXCL	XL Axiata Tbk.
18.	FILM	MD Pictures Tbk.
19.	HEAL	Medikaloka Hermina Tbk.
20.	HRUM	Harum Energy Tbk.
21.	ICBP	Indofood CBP Sukses Makmur Tbk.
22.	INCO	Vale Indonesia Tbk.
23.	INDF	Indofood Sukses Makmur Tbk.
24.	INKP	Indah Kiat Pulp & Paper Tbk.
25.	INTP	Indocement Tunggal Prakarsa Tbk.
26.	ISAT	Indosat Tbk.
27.	ITMG	Indo Tambangraya Megah Tbk.
28.	JPFA	Japfa Comfeed Indonesia Tbk.
29.	KLBF	Kalbe Farma Tbk.
30.	LPPF	Matahari Department Store Tbk.
31.	LSIP	PP London Sumatra Indonesia Tbk.
32.	MAPI	Mitra Adiperkasa Tbk.
33.	MIKA	Mitra Keluarga Karyasehat Tbk.
34.	MNCN	Media Nusantara Citra Tbk.
35.	MYOR	Mayora Indah Tbk.
36.	PGAS	Perusahaan Gas Negara Tbk.
37.	PTBA	Bukit Asam Tbk.
38.	PTPP	PP (Persero) Tbk.
39.	PWON	Pakuwon Jati Tbk.
40.	SCMA	Surya Citra Media Tbk.
41.	SIDO	Industri Jamu dan Farmasi Sido Muncul Tbk.
42.	SMGR	Semen Indonesia (Persero) Tbk.
43.	SMRA	Summarecon Agung Tbk.
44.	TAPG	Triputra Agro Persada Tbk.
45.	TINS	Timah Tbk.
46.	TKIM	Pabrik Kertas Tjiwi Kimia Tbk.
47.	TLKM	Telkom Indonesia (Persero) Tbk.
48.	TPIA	Chandra Asri Petrochemical Tbk.
49.	UNTR	United Tractors Tbk.

No.	Code	Stocks
50.	UNVR	Unilever Indonesia Tbk.
51.	WIKA	Wijaya Karya (Persero) Tbk.

The data was derived from the JII70 daily stock closing price data that was retrieved on March 4, 2024, and was gathered from the www.finance.yahoo.com website for two years between December 1, 2021, and December 1, 2023. During that time frame, as can be seen in **Table 1**, the JII70 index contains 51 stocks that frequently feature in four times assessments. Normally, the assessments are conducted in May 2022, November 2022, May 2023, and November 2023.

Government security rates have long been utilized as risk-free rates by academics and practitioners, while opinions on whether to use short-term or long-term rates have differed **[12]**. The Bank Indonesia rate at the time of data collection, which is 4.75% annually and can be found on the internet at www.bi.go.id, was utilized as the risk-free rate return in this study. The general procedures that served as the foundation for this study are as follows.

a. Determine the return, risk, and expected return of each stock based on the closing price. Return is obtained by using the formula for the profit level of each stock **[13]**, which is as follows.

$$R_i = ln\frac{P_t+D_t}{P_{t-1}} \tag{1}$$

where P_t is the current closing price of the stock, P_{t-1} is the closing price of the stock at time t-1, D_t is the dividend that is taken to be 0, and R_i is the single asset return. The closing price risk of a stock is obtained by solving the following standard deviation formula **[14]**.

$$\sigma = \sqrt{\frac{\sum_{t=1}^{n}(R_t-\overline{R_t})^2}{n-1}} \tag{2}$$

Perform outlier test, multicollinearity test, and normality test to fulfill the assumption test of cluster analysis. To detect outliers, the concept of Mahalanobis distance is used as follows **[15]**.

$$d_i = (x_i - \overline{x})^t s^{-1}(x_i - \overline{x}) > \chi^2_{p,(1-\alpha)} \tag{3}$$

where d_i is the square of the i-th observation distance, x_i is the Value of the i-th observation, $\overline{x}$ is the mean vector of observations, and s^{-1} is the inverse of the sample variance-covariance matrix.

b. Grouping stocks into clusters using the K-Means and Ward methods. Due to its faster performance than the hierarchical method, K-Means clustering is the alternate cluster method of choice for large data sets. The K-Means technique generates cluster values at random, and the resulting value is known as the centroid **[16]**, or cluster center. On the other hand, one of the hierarchical approaches, the Ward method, has the benefit of being able to maximize the inter-cluster distance while minimizing the variance of the points inside a cluster **[17]**. Determining the number of optimal clusters using the Silhouette Index formula (Gud), namely:

$$SI_i = \frac{b_i-a_i}{max\{a_i,b_i\}} \tag{4}$$

where SI_i is the Silhouette Index of the i-th data, $a_i$is the average distance of the i-th data to all data in one cluster, and $b_i$is the average distance of the i-th data to all data from other clusters.

c. Calculate and select stocks in each cluster using the Sharpe ratio **[18]**.

$$SR = \frac{E(R_p)-R_f}{\sigma} \tag{5}$$

where SR is the Sharpe ratio, $E(R_p)$is the average return of the portfolio, R_f is the average risk-freereturn, and $\sigma$is the standard deviation of the portfolio.

d. Form an optimal portfolio using the concept of mean-variance portfolio and Mean-Variance robust portfolio on each cluster formed. Robust portfolio model parameters are estimated using S-estimation. According to **[10]**, resolving the following optimization problem will yield the S-estimation for the mean vector and covariance matrix.

$$\text{Min}|\Sigma|$$

$$\text{s.t.}\frac{1}{n}\sum_{i=1}^{n}\rho\left(\frac{d_i}{\hat{\sigma}}\right) = \delta \tag{6}$$

where ρ is the loss function, δ is a constant, and d_i is Mahalanobis distance as stated in **Equation (3)**.

e. Using the idea of the Sharpe ratio computation in **Equation (5)**, compare the performance of the best portfolio out of all the portfolios.

3. RESULTS AND DISCUSSION

The return and risk must be calculated first. **Figure 1** illustrates the return movement visualization of 51 stocks registered in JII70 for the period December 1, 2021 - December 1, 2023. It is evident that the 51 stocks under observation exhibit variations. The potential for abrupt increases or decreases in stock prices is indicated by stock movements. It may be concluded that all of the aforementioned stocks carry a risk of erratic stock movements because of the sporadic nature of their changes. Additionally, for 51 stocks included in the JII70, return and risk characteristics are utilized to create K-Means and Ward clusters.

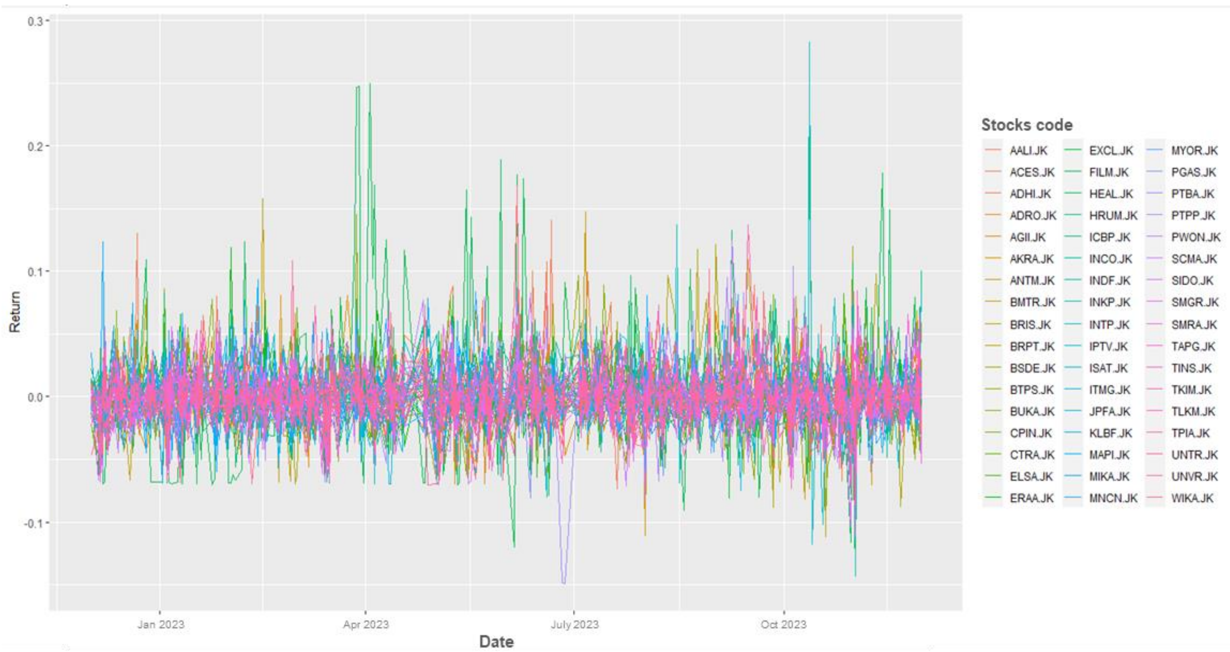

Figure 1. A plot of the Return Movement
Source: RStudio

The outlier test generated by the Mahalanobis distance yields some outliers. The calculation of the Mahalanobis test started with a chi-square value for the freedom degree of $p < 0.05$) so that $\chi^2_{(1;0.05)} = 3.841$ is obtained. Based on the detection of the outlier test using software R, it found three stocks with d_i value above chi-square, i.e., BUKA (4.248855084), FILM (35.879052973), and IPTV (19.734724443). All those three stocks are identified as outlier data. Since this study focuses on a robust portfolio, the outlier data can still be kept for clustering purposes **[10]**.

Figure 2 shows that the optimal number of clusters for 51 stocks is 2 clusters with an average Silhouette coefficient Index value of 0.79. Consequently, **Figure 3** displays the two clusters that resulted from K-Means clustering. There are two clusters in the K-Means clustering method: the first cluster has 29 stocks, while the second cluster has 22 stocks **Figure 3**.

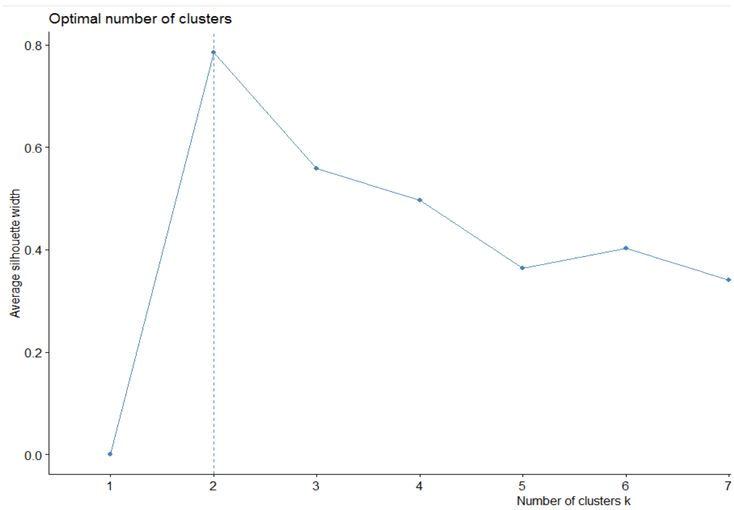

Figure 2. The Plot of the Silhouette Index
Source: RStudio

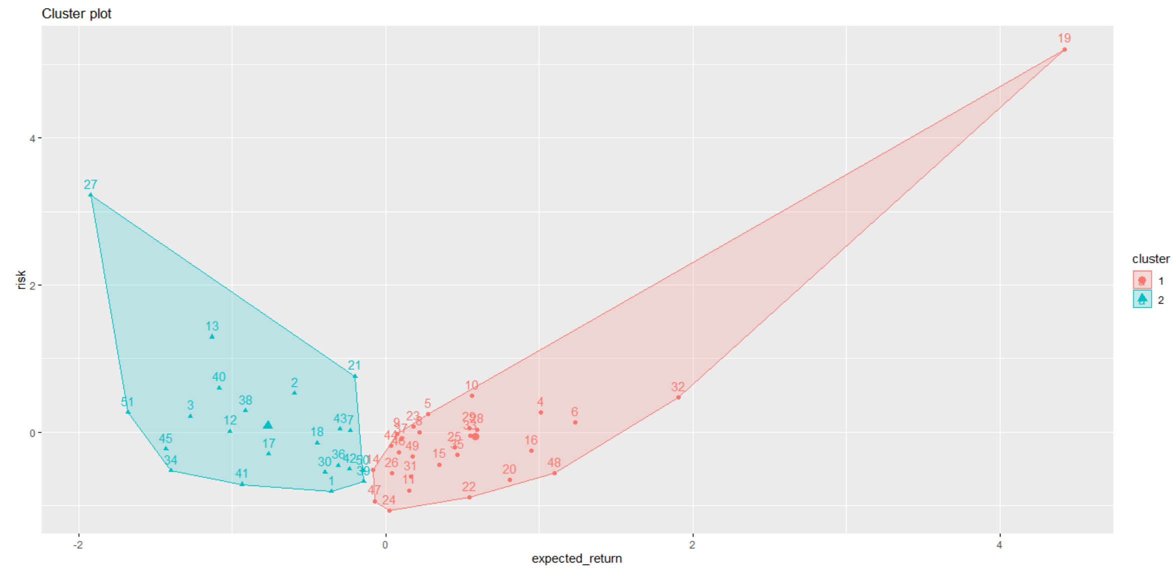

Figure 3. K-Means Clustering
Source: RStudio

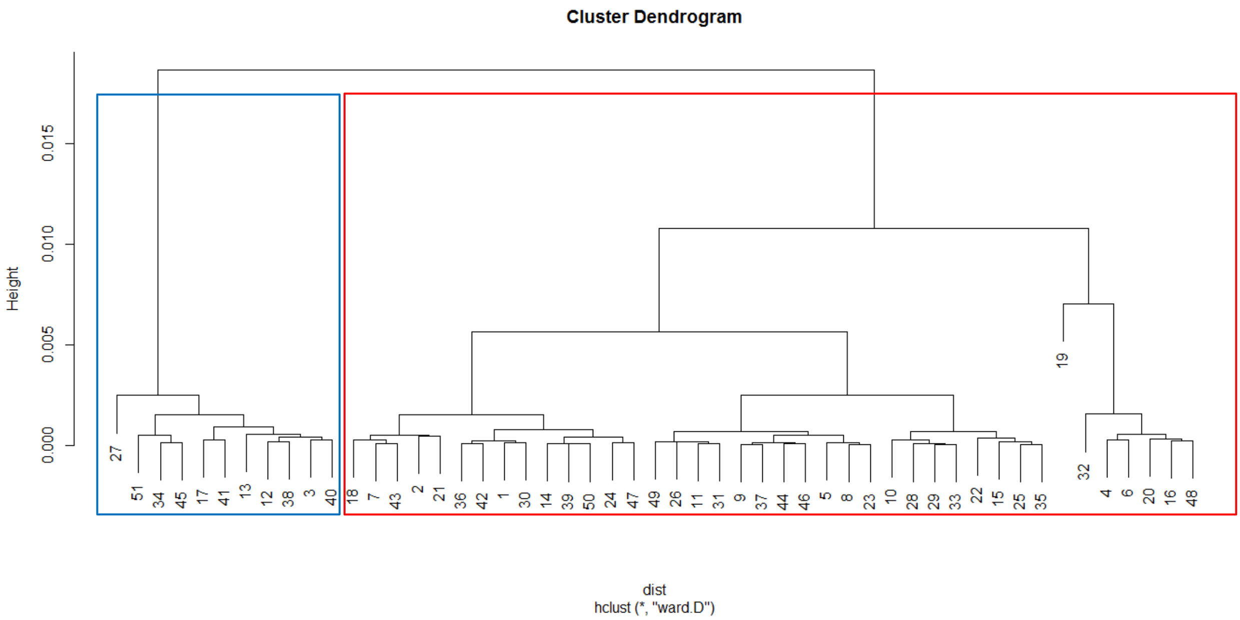

Figure 4. Ward Clustering
Source: RStudio

In addition, **Figure 4** above displays the results of the Ward cluster clustering. It illustrates how the 51 stocks based on Ward's cluster can be divided into two clusters: cluster 1 has 11 stock members (within blue rectangle), while cluster 2 has 40 stocks (within red rectangle). The stocks in the K-Means and Ward clusters are then chosen for the portfolio using the Sharpe ratio (SR) criterion (**Table 2** and **Table 3**).

Using the K-Means method cluster analysis on the JII70 grouping based on **Table 2**, it was discovered that two stocks, FILM and MAPI, with Sharpe ratios of 0.0940134847 and 0.0731702434, were selected as representatives in the first cluster, while TAPG and HRUM stocks, with Sharpe ratio values of -0.0029720408 and -0.0111182151, were selected as representatives in the second cluster. **Table 2** explains the expected return, risk, and Sharpe ratio for each stock using K-Means clustering. FILM shares produce an expected return of 0.00506610, a risk of 0.002779, and a Sharpe ratio of 0.0940134847, MAPI shares produce an expected return of 0.00506610, a risk of 0.002779 and a Sharpe ratio of 0.0940134847 and so on.

Table 2. Stock Selection in K-Means's Cluster

Code	Expected Return	Risk	Sharpe Ratio
FILM	0.00506610	0.002779	0.0940134847
MAPI	0.002201631	0.000801	0.0731702434
TAPG	0.00006195	0.000526	-0.0029720408
HRUM	-0.0002075	0.000922	-0.0111182151

Table 3. Stock Selection in Ward's Cluster

Code	Expected Return	Risk	Sharpe Ratio
BUKA	-0.0012767	0.001145	-0.0415820604
PTPP	-0.00102604	0.000727	-0.0428947874
FILM	0.00506610	0.002779	0.0940134847
MAPI	0.002201631	0.000801	0.0731702434

Table 3 explains the level of profit, risk, and Sharpe ratio for each stock by Ward's clustering. BUKA shares produce an expected return of -0.0012767, a risk of 0.001145, and a Sharpe ratio of -0.0415820604, PTPP shares produce an expected return of -0.00102604, a risk of 0.000727, and a Sharpe ratio of -0.0428947874 and so on.

The Ward method cluster analysis on the JII70 grouping was found to have two stocks selected as representation in cluster 1 (with a Sharpe ratio of -0.0415820604 and -0.0428947874) and two stocks selected as representation in cluster 2 (with a Sharpe ratio of 0.0940134847 and 0.0731702434). Additionally, the MV portfolio model with robust S estimation is used to discover how to construct the optimal portfolio. This step begins by estimating the mean vector and covariance matrix in **Equation (6)** for the MV robust S portfolio model. Using the robustbase package in R Studio, the CovSest function is implemented to calculate the estimated mean vector and covariance matrix. The K-Means cluster's mean matrix and covariance matrix estimates yielded the following findings.

$$\hat{\mu}_{\kappa m} = \begin{bmatrix} -0.003906 \\ 0.001309 \\ -0.000972 \\ -0.000927 \end{bmatrix} \text{and}$$

$$\hat{\Sigma}_{\kappa m} = \begin{bmatrix} 1.6630e-03 & 3.5423e-05 & -1.33394e-05 & 4.6043-05 \\ 3.5423e-05 & 8.9541e-04 & -1.3458e-05 & -7.9760e-06 \\ -1.3394e-05 & -1.3458e-05 & 4.4996e-04 & 1.6099e-04 \\ 4.6043e-05 & 7.9760e-06 & 1.6099e-04 & 8.1822e-04 \end{bmatrix}$$

After obtaining the estimated mean vector and covariance matrix, the portfolio weights are then determined for various values of risk aversion (γ) based on the K-Means cluster. The following table shows a comparison of robust portfolio performance for the three risk aversions in the K-Means cluster.

Due to **Table 4**, it can be obtained that the optimal portfolio formation with robust S-estimation for the K-Means cluster produces a negative weight (short selling) at risk aversion 1 for FILM and TAPG stocks and risk aversion 10 for FILM stocks (**Table 4**). The optimal portfolio in the K-Means cluster with risk aversion 1 is obtained by allocating funds to FILM shares by -181.84%, MAPI by 263.578%, TAPG by

-1,970%, and HRUM by 20,240%. Likewise, the explanation for portfolio weights at risk aversion 10 and 100 is the same as risk aversion 1.

Table 4. Robust Portfolio Weights with K-Means Cluster Analysis

Model	γ	FILM	MAPI	TAPG	HRUM
Mean-Variance Robust	1	-1.8184858	2.6357815	-0.01970153	0.20240579
	1.5	-1.1700102	1.8410599	0.1338936	0.1950568
	2	-0.8457724	1.4436990	0.2106911	0.1913823
	5	-0.2621444	0.7284495	0.3489267	0.1847682
	10	-0.0676017	0.4900330	0.39500518	0.18256354
	100	0.1074867	0.2754581	0.4364759	0.1805793
	1000	0.1249955	0.2540007	0.4406229	0.1803809

Table 5. Return, Risk, and Sharpe Ratio Portfolio on K-Means Cluster

γ	Return	Risk	Sharpe Ratio
1	0.01038627	0.1066364	0.09617855
1.5	0.006669972	0.07195476	0.09088815
2	0.004811823	0.05486025	0.0853384
5	0.001467155	0.02585305	0.05171609
10	0.00035226	0.01827329	0.01215595
100	-0.00065113	0.01495123	-0.05225464
1000	-0.00075147	0.01491427	-0.05911192

From Table 5, it can be seen that the performance of the Mean-Variance cluster K-Means portfolio that is robust to risk aversion 1 produces a Sharpe ratio of 0.096. For portfolios with $\gamma = 10$ produces a Sharpe Ratio value of 0.012, while at $\gamma = 100$ a Sharpe ratio value of -0.052 is obtained. Thus, it can be concluded that the portfolio performance with risk aversion $\gamma = 1$ is better than $\gamma = 10$ and $\gamma = 100$. After that, a robust portfolio built on Ward's cluster is created. The following are the estimation results of the mean matrix and covariance matrix in the following clusters.

$$\hat{\mu}_w = \begin{bmatrix} -0.003028 \\ -0.002607 \\ -0.001774 \\ -0.001760 \end{bmatrix} and \;\; \hat{\Sigma}_w = \begin{bmatrix} 1.0630e-03 & 1.621e-04 & 1.506e-04 & 1.568e-04 \\ 1.621e-04 & 6.389e-04 & 6.733e-05 & 6.075e-05 \\ -1.506e-04 & 6.733e-05 & 1.567e-03 & 3.771e-05 \\ 1.568e-04 & 6.075e-05 & 3.771e-05 & 8.998e-04 \end{bmatrix}$$

The portfolio weights are then established for different levels of risk aversion (γ) based on Ward's cluster after acquiring the estimated mean vector and covariance matrix.

Table 6. Robust Portfolio Weights with Ward Cluster Analysis

Model	γ	BUKA	PTPP	FILM	MAPI
Mean-Variance Robust	1	-1.70528367	-1.49579668	0.03647999	4.16460036
	1.5	-1.08160966	-0.86482156	0.07662069	2.86981052
	2	-0.76977265	-0.54933400	0.09669105	2.22241560
	5	-0.20846604	0.01854361	0.13281768	1.05710475
	10	-0.02136384	0.20783615	0.14485989	0.66866780
	100	0.1470281	0.3781994	0.1556979	0.3190745
	1000	0.1638673	0.3952358	0.1567817	0.2841152

The formation of the optimal portfolio with robust S estimation for the Ward cluster produces a negative weight (short selling) at risk aversion 1 on BUKA and PTPP shares and at risk aversion 10 on BUKA shares (Table 6). For the optimal portfolio in the Ward cluster with risk aversion 1, it is obtained by allocating funds to BUKA shares by -170.528%, PTPP by -149.579%, FILM by 3.64%, and MAPI by 416.460%. Likewise, the explanation for portfolio weights at risk aversion 10 and 100 is the sameas risk aversion 1. Table 7 presents a comparison of robust portfolio performance for the three risk aversions in the Ward cluster.

Based on Table 7, it can be seen that the performance of the Mean-Variance cluster Ward portfolio that is robust to risk aversion 1 produces a Sharpe ratio of 0.120. Portfolios with $\gamma = 10$ produce a Sharpe ratio value of 0.012, while at $\gamma = 100$ a Sharpe ratio value of -0.072 is obtained. Thus, it can be concluded that the portfolio performance with risk aversion $\gamma = 1$ is better than $\gamma = 10$ and $\gamma = 100$. The formation of an optimal portfolio with S-estimation robust in both K-Means and Ward may produce negative weights (see **Table 4** and **Table 6**). The negative values, according to **[10]**, indicate that investors are short-selling or borrowing shares from third parties with the intention of returning them later, presumably after the share price declines.

Table 7. Return, Risk, and Sharpe Ratio Portfolio on Ward Cluster

γ	Return	Risk	Sharpe Ratio
1	0.01632749	0.1340073	0.1208692
1.5	0.01044423	0.09029129	0.1142313
2	0.007502592	0.06870671	0.1073033
5	0.002207652	0.03184579	0.06523673
10	0.00048466	0.022015	0.01419646
100	-0.00114581	0.01760474	-0.07247745
1000	-0.001304658	0.01755505	-0.0817312

Lastly, **Table 8** will compare and display the findings of the best portfolio performance computation on the K-Means and Ward method portfolios. The Sharpe ratio is one of the most commonly utilized measures for assessing a portfolio's risk-adjusted performance. A portfolio's risk-adjusted performance improves as the Sharpe ratio increases.

Table 8. Comparison of Robust Portfolio Performance on K-Means and Ward Clusters

Method	Return	Risk	Sharpe Ratio
K-Means	0.01038627	0.1066364	0.09617855
Ward	0.01632749	0.1340073	0.1208692

The results of the formation of a robust portfolio on k-Means clustering produced a Sharpe ratio of 0.09617855 while the Ward cluster obtained a Sharpe ratio of 0.1208692. In terms of the Sharpe ratio score, it can be inferred that the stock portfolio using the Mean-Variance robust approach based on the Ward cluster performs better than the stock portfolio using the Mean-Variance robust method on the K-Means cluster. This distinction of the Ward cluster is consistent with the findings of the study conducted by **[19]**.

4. CONCLUSIONS

The cluster analysis results of the K-Means and Ward methods on 51 stocks of JII70 were divided in two clusters. The K-Means cluster method divides stocks into two clusters, the first of which has 29 stocks and the second of which has 22 stocks. Similarly, the Ward cluster method divides stocks into two clusters, the first of which has 11 stocks and the second of which has 40 stocks. Four stocks are chosen from each cluster to build the portfolios: FILM, MAPI, TAPG, and HRUM stocks as the K-Means cluster representations and BUKA, PTPP, FILM, and HRUM as the Ward cluster representations.

The optimal portfolio in the K-Means cluster with risk aversion 1 is obtained by allocating funds to FILM sharesby -181.84%, MAPI by 263.578%, TAPG by -1,970%, and HRUM by 20,240%. Furthermore, the optimal portfolio in the Ward cluster with risk aversion 1 is obtained by allocating its funds to BUKA shares by - 170.528%, PTPP by -149.579%, FILM by 3.64%, and MAPI by 416.460%. The comparison results indicate that the Ward cluster's MV robust portfolio outperforms the K-Means cluster. This is evident from the Ward cluster's Sharpe ratio value, which is higher than the K-Means cluster despite the small differences.

REFERENCES

[1] R. C. Pajar and A. Pustikaningsih, “Pengaruh motivasi investasi dan pengetahuan investasi terhadap minat investasi di pasar modal pada mahasiswa FE UNY,” *J. Profita Kaji. Ilmu Akunt.*, vol. 5, no. 1, 2017.

[2] D. Tambunan, "Investasi saham di masa pandemi COVID-19," *J. Khatulistiwa Inform.*, vol. 4, no. 2, pp. 117–123, 2020.
[3] I. M. Adnyana, *Manajemen Investasi dan Portofolio*. Jakarta Selatan: Lembaga Penerbitan Universitas Nasional (LPU-UNAS), 2020.
[4] I. Kulali, "Portfolio Optimization Analysis with Markowitz Quadratic Mean-Variance Model," *Eur. J. Bus. Manag.*, vol. 8, no. 7, pp. 73–79, 2016.
[5] Y. Lee, W. C. Kim, and J. H. Kim, "Achieving portfolio diversification for individuals with low financial sustainability," *Sustainability*, vol. 20, no. 17, p. 7073, 2020.
[6] K. Wada, "Outliers in official statistics," *Japanese J. Stat. Data Sci.*, vol. 3, no. 2, pp. 669–691, 2020.
[7] A. Wicaksono, *K-Medians Clustering (Studi Kasus pada Data Jumlah Tenaga Kesehatan Menurut Kabupaten/Kota di Provinsi Jawa Barat Tahun 2015)*. Bandung: Universitas Pendidikan Indonesia., 2017.
[8] L. Gubu and D. Rosadi, "Robust Mean-Variance Portfolio Selection With Ward And Complete Linkage Clustering Algorithm," *Econ. Comput. Econ. Cybern. Stud. Res.*, vol. 54, no. 3, 2020.
[9] L. Gubu and D. Rosadi, "Robust mean–variance portfolio selection using cluster analysis: A comparison between Kamila and weighted k-mean clustering," *Asian Econ. Financ. Rev.*, vol. 10, no. 10, p. 1169, 2020.
[10] E. D. Supandi and Y. Anggara, "Analisis Klaster dalam Pembentukan Portofolio Robust Mean-Variance," *J. Sains Mat. dan Stat.*, vol. 9, no. 1, pp. 37–47, 2023.
[11] N. S. M. Mussafi, Z. Ismail, and N. A. B. Aziz, "The downside deviation quadratic programming for stock portfolio optimisation: an empirical study of shariah and conventional indices in Indonesia," *Afro-Asian J. Financ. Account.*, vol. 14, no. 3, pp. 350–371, 2024.
[12] A. Damodaran, "What is the riskfree rate? A Search for the Basic Building Block," *J. New Financ.*, vol. 1, no. 3, 2020.
[13] N. S. M. Mussafi, "Analisis Risk Asset Portfolio Berbasis Reward To Variability Pada Saham Syariah Di Indonesia Menggunakan Nonlinear Programming," *J. Mat. "MANTIK,"* vol. 3, no. 2, pp. 57–64, 2017, doi: 10.15642/mantik.2017.3.2.57-64.
[14] P. Jorion, *Value at Risk: The New Benchmark for Managing Financial Risk*, 3rd ed. McGraw-Hill, 2007.
[15] H. Ghorbani, "Mahalanobis distance and its application for detecting multivariate outliers," *Facta Univ. Ser. Math. Informatics*, pp. 583–595, 2019.
[16] I. R. Sitepu, "Analisis Cluster terhadap Tingkat Pencemaran Udara pada Sektor Industri di Sumatera Selatan," *J. Penelit. Sains*, vol. 14, no. 3, 2011.
[17] E. K. Tokuda, C. H. Comin, and L. D. F. Costa, "Revisiting agglomerative clustering," *Phys. A Stat. Mech. its Appl.*, vol. 585, no. 126433, 2022.
[18] E. Pratama, A. N. Satyahadewi, N. Sulistianingsih, "Analysis Of Optimal Portfolio Formation On Idx30 Indexed Stock With The Mean Absolute Deviation Method," *BAREKENG J. Ilmu Mat. dan Terap.*, vol. 18, no. 3, pp. 1753–1764, 2024.
[19] L. Ferreira and D. B. Hitchcock, "A comparison of hierarchical methods for clustering functional data," *Commun. Stat. Comput.*, vol. 38, no. 9, pp. 1925–1949, 2009.