BAREKENG: Journal of Mathematics and Its Applications
March 2025 Volume 19 Issue 1 Page 0617–0628
P-ISSN: 1978-7227 E-ISSN: 2615-3017
https://doi.org/10.30598/barekengvol19iss1pp0617-0628

GEOMETRIC BROWNIAN MOTION WITH JUMP DIFFUSION AND VALUE AT RISK ANALYSIS OF PT BANK NEGARA INDONESIA STOCKS

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Article History:

Received: 9th August 2024
Revised: 22nd November 2024
Accepted: 22nd November 2024
Published: 13th January 2025

Keywords:

Kurtosis;
GBM with Jump Diffusion;
Monte-Carlo;
Peak-Over-Threshold;
VaR.

ABSTRACT

Investments in stocks are made to make a profit, where the higher the expected profit, the greater the risk undertaken. The return on investing in stocks can be influenced by changes in the price of stocks that are difficult to predict, which can lead to uncertainty in the value of the return and the risk of the stock. The application of the Geometric Brownian Motion (GBM) model with Jump Diffusion is crucial for enhancing the accuracy of stock price forecasting and risk analysis by incorporating price jumps resulting from external events within complex market dynamics. The data used in this study are the closing price data of the daily stock of PT Bank Negara Indonesia for the period 1 December 2022 to 31 January 2024, where the stock return data has a kurtosis value greater than 3 (leptokurtic) so that the data indicates a jump. The GBM with Jump Diffusion model was implemented to predict the stock price with a simulation repetition of 1000 times. The analysis shows that the GBM model with Jump Diffusion has an excellent accuracy rate with the smallest MAPE value of 0.86%. The average value of the VaR with Monte Carlo simulation obtained at the reliability levels of 80%, 90%, 95%, and 99% in a row is 0.96%, 1.53, 1.97%, and 2.64%. This result shows that the higher the confidence level used, the greater the risk.

How to cite this article:
A. Zakiah, E. Sulistianingsih and N. Satyahadewi., "GEOMETRIC BROWNIAN MOTION WITH JUMP DIFFUSION AND VALUE AT RISK ANALYSIS OF PT BANK NEGARA INDONESIA STOCKS," *BAREKENG: J. Math. & App.*, vol. 19, iss. 1, pp. 0617-0628, March, 2025.

Journal homepage: *https://ojs3.unpatti.ac.id/index.php/barekeng/*
Journal e-mail: *barekeng.math@yahoo.com; barekeng.journal@mail.unpatti.ac.id*

Research Article · Open Access

1. INTRODUCTION

Investing is an activity that attracts the attention of the public, as it serves as one of the ways to allocate funds into an asset for a certain period to gain profit or returns. Investing in the capital market offers various opportunities for trading long-term instruments, which can be offered by both the government and private enterprises. In the capital market, there are various investment products traded that involve ownership; one of them is stocks. Currently, stocks have become a popular investment choice among millennials, as purchasing stocks no longer requires a large capital [1]. Securities are defined as stocks that serve as evidence of ownership in a limited liability company or corporation, where the owner is referred to as a shareholder [2].

One of the stocks listed on the Indonesia Stock Exchange is the stock of PT Bank Negara Indonesia (Persero) Tbk. The bank is one of the businesses that offers various financial products to the public. Stocks in the financial sector, especially banking, often experience fluctuating stock prices, making them difficult to predict. One technique that can be used to forecast future stock prices is Geometric Brownian Motion (GBM) with Jump Diffusion [2]. The GBM model assumes that stock returns in the past follow a normal distribution. The GBM with the Jump Diffusion model is possible if past stock returns show jumps [3]. When predicting stock prices, the Jump Diffusion approach has smaller modeling errors compared to relying solely on the GBM model [4]. If historical stock return data indicates jumps, then the Jump Diffusion model is used [5]. A jump is a condition where stock return data experiences extreme increases and decreases.

When investing in stocks, understanding the frequently changing stock prices is directly proportional to the magnitude of the risk that may be incurred in an investment. Specifically, this makes risk evaluation important to consider before making an investment decision [6]. Risk is the amount of potential loss that occurs when the average profit deviates significantly from expectations. Risk cannot be avoided but can be managed [7]. One technique for measuring investment-related risk is Value at Risk (VaR). VaR is defined as the measure of the largest potential loss with a certain level of confidence over a specific period [8]. The results of this study are expected to effectively implement the GBM with Jump Diffusion model to project the stock price of PT Bank Negara Indonesia Tbk and assess the maximum investment loss on this stock using VaR estimation with Monte Carlo simulation.

2. RESEARCH METHODS

2.1 Stock and Stock Return

Stocks serve as evidence of ownership in a company or Limited Liability Company (LLC) and are considered securities in the capital market, with their holders commonly known as shareholders [1]. Stock return refers to the relative change in the price of a financial asset [2], portraying actual shifts in stock prices. The computation of stock returns involves log returns, which can be expressed as follows:

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{1}$$

where r_t is stock return at time t, S_t is stock price at time t, and S_{t-1} is stock price at time $(t-1)$ [9].

2.2 Kurtosis

Kurtosis represents how peaked or flat a distribution is, usually assumed to be related to its overall spread. It involves identifying stages to determine if there are any significant changes in the data. The kurtosis value indicates the presence of jumps in the data distribution by measuring the sharpness of the peak and the heaviness of the tails. When kurtosis is less than 3 (platykurtic), it suggests there are no significant changes [2]. A kurtosis value of 3 (mesokurtic) indicates no significant deviations, while a kurtosis greater than 3 (leptokurtic) suggests notable changes. Kurtosis estimation can be performed using **Equation (2)** [4]:

$$\gamma = \frac{\frac{1}{n}\sum_{t=1}^{n}(r_t-\mu)^4}{\sigma^4} \tag{2}$$

where γ is kurtosis of stock returns, n is number of stock return data, r_t is stock return at time t, μ is mean stock return, and σ is standard deviation of stock returns.

2.3 Kolmogorov-Smirnov Normality Test

The GBM Model with Jump Diffusion and VaR assumes that stock returns follow a normal distribution. Therefore, it's necessary to conduct a normality test on the stock return data first. This test aims to ascertain whether the stock returns data conform to a normal distribution or not. Several Kolmogorov-Smirnov test procedures can be performed for this purpose [8]:

a. Hypothesis:

   H_0: Data are normally distributed

   H_1: Data are not normally distributed

b. Determining the significance level: α

c. Test Statistic

$$D_{count} = max|S(x) - F_0(x)| \tag{3}$$

   where D_{count} is maximum value for all x of the absolute value $S(x) - F_0(x)$, $S(x)$ is cumulative distribution function of the sample data, and $F_0(x)$ is cumulative distribution function of the normal distribution.

d. Rejection Criteria:

   H_0 is rejected if the value of $D_{count} > D_{(\alpha,n)}$, where $D_{(\alpha,n)}$ represents the critical value obtained from the Kolmogorov-Smirnov table with α indicating the significance level and n represents the number of truncated stock return data. Additionally, H_0 is rejected if the p-value < α.

2.4 Peak Over Threshold

Peaks Over Threshold (POT) is a crucial component in modeling with Jump Diffusion, serving to determine the extent to which stock return data in a sample exhibits significant jumps. The primary function of Peaks Over Threshold (POT) is to identify extreme values that exceed a certain threshold, enabling the analysis of asset price fluctuations. By understanding the patterns and factors influencing these price changes, the analysis becomes a critical aspect in supporting investment decision-making and developing effective risk management strategies. Kurtosis values exceeding 3, which indicate fat tails or leptokurtic phenomena, are considered indicators of significant jumps in the data, signaling the occurrence of extreme events [10]. By utilizing POT to ascertain the quantity of extreme value data in worst-case scenarios, indications of jumps in stock return data can be identified. The computation of the number of data jumps in stock return data is conducted using POT, exclusively with stock return data. The subsequent steps delineate the POT calculation process [11]:

a. Sorting the stock return data from highest to lowest.

b. Determine the quantity of jump data through the following formula:

$$j = 10\% \times n \tag{4}$$

   10% in the context of data analysis, particularly in the Peaks Over Threshold (POT) method, refers to the percentage of the total data used to determine the threshold for extreme values. In other words, using 10% helps ensure that the analysis remains focused on potentially extreme values without sacrificing too much data from the overall dataset. n and j in **Equation (4)** are the number of stock return data and the number of jump return data.

c. Determine the lower threshold value and upper threshold value using **Equation (5)**:

$$u = j + 1 \tag{5}$$

   where u and j in **Equation (5)** are number of truncated stock return data and number of jump return data.

d. Based on step 1, trimming the stock return data using the calculation of the value u in step 3, will serve as a reference to determine the amount of stock return data that should be trimmed at the lower and upper thresholds.

2.5 Parameter Estimation

2.5.1 Average Return

The average stock return over n (number of truncated stock return data) can be estimated using the mean return, formulated as follows [12]:

$$\mu = \frac{1}{n}\sum_{t=1}^{n} r_t \tag{6}$$

where μ is mean stock return, n is number of stock return data, t is time, and r_t is stock return at time t.

2.5.2 Standard Deviation of Return

The standard deviation can be defined as the amount of fluctuation in the price of stock returns. The formula for the mean return of stock can also be used to obtain the standard value of the equity return deviation. The standard deviation of stock returns is formulated as follows [12]:

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \mu)^2 \tag{7}$$

where σ is standard deviation of stock returns, n is number of stock return data, t is time, r_t is stock return at time t, and μ is mean stock return.

2.5.3 Jump Intensity

Parameter estimation for Jump Diffusion on jump intensity can utilize data indicating jumps or jump data through **Equation (8)** [11]:

$$\lambda = \frac{1}{j}\sum_{i=1}^{j} r_i \tag{8}$$

where λ is intensity of jump returns, j is number of jump return data, and r_i is jump return at time i.

2.5.4 Mean Jump Size

The jump size in the Jump Diffusion model refers to the magnitude of price change that occurs due to significant price jumps. It is measured as the difference between the asset price before and after the jump, thus providing insights into the impact of extreme events on asset value and the associated risks. The following formula can be used with jump difference data to estimate parameters in Jump Diffusion for the mean [11]:

$$\beta = \frac{1}{s}\sum_{p=1}^{s} r_p \tag{9}$$

where β is mean jump size, s is number of jump size return data, and r_p is jump size return at time p.

2.5.5 Standard Deviation of Jump Size

The standard deviation using jump size data for parameter estimation in Jump Diffusion [11] can be calculated with **Equation (10)**:

$$\delta = \sqrt{\frac{1}{s-1}\sum_{p=1}^{s}\left(r_p - \beta\right)^2} \tag{10}$$

where δ is a standard deviation of jump size, s is number of jump size return data, r_p is jump size return at time p, and β is mean jump size.

2.6 Brownian Motion

Brownian Motion is a continuous-time stochastic process commonly used to depict the random movement of a variable [13]. The stochastic process $\{X(t), t \geq 0\}$ is a standard Brownian Motion if it satisfies the following characteristics [8]:

a. The change in W_t during the period Δt is $\Delta W_t = \varepsilon\sqrt{\Delta t}$, where ε is a random number with a standard normal distribution, having a mean of 0 and a variance of 1. At time $t = 0$, $\Delta W_t = 0$, the standard deviation of $\Delta W_t = \sqrt{\Delta t}$, and its variance is Δt.

b. Each movement within the time interval $0 \leq s < t \leq T$, $W_t - W_s$ is normally distributed, with a mean of 0 and a variance of σ_{t-s}^2.

c. Within any interval $0 \leq s < t < u < v \leq T$, the changes in $W_t - W_s$ and $W_u - W_v$ are independent or not dependent on past states.

Meanwhile, a stochastic process called Brownian Motion with a drift term μ^*, the drift parameter describes the average return rate of an asset over time. In the GBM model, drift indicates the potential expected return. The drift parameter illustrates the average return rate of an asset over time. Variance σ^2, a stochastic process $\{X_t, t \geq 0\}$ is called Geometric Brownian Motion with ε is a standard normally distributed random number, and $W_t = \varepsilon\sqrt{\Delta t}$ is given as follows:

$$B_t = \mu_t^* + W_t \tag{11}$$

where μ_t^* is the drift parameter with $\mu^* = \mu - \frac{1}{2}\sigma^2$, and B_t is the Brownian Motion at time t.

2.7 Stochastic Differential Equation Method for Jump Diffusion

A stochastic process is a collection of random variables indexed by time $\{W_t, t \in T\}$, where t denotes time and X_t represents the process occurring at time t [2]. The set of indices of a stochastic process is called set T. A stochastic process is called a continuous-time stochastic process and is represented in the form $\{X_t, t \geq 1\}$, if the set T is a time interval $t \in [0, \infty)$ [14]. Meanwhile, if the set T is a countable set $t \in [1, T]$, then the stochastic process used is a discrete-time stochastic process expressed in the form $\{X_t, t = 1,2, \ldots\}$.

Stochastic processes in finance and investment are exemplified by stock price fluctuations. This is due to sudden or uncertain fluctuations that can happen at any time within certain or uncertain time frames [4]. Therefore, the following equation can be used to represent stock price changes [15]:

$$dX_t = \mu X_t dt + \sigma X_t dW_t \tag{12}$$

where X_t is stochastic process at time t, W_t is denotes the standard Wiener process at time t, μ is mean stock return and σ is standard deviation of return stock. Then, for the Jump Diffusion model, the stochastic differential equation follows **Equation (13) [3]**:

$$dX_t = \mu X_t dt + \sigma X_t dW_t + X_t dJ_t \tag{13}$$

where X_t is the stochastic process at time t, W_t denotes the standard Wiener process at time t, μ is the mean stock return, σ is the standard deviation of return stock, J_t is jump process at time t, and t is time.

2.8 The Itô Theorem for Jump Diffusion Models

A mathematician named Kiyoshi Itô discovered Lemma Itô in 1951 [11]. Let X is a Jump Diffusion process, which is defined as the sum of drift, stochastic Brownian motion integral, and Poisson process [2]:

$$X_t = X_{t-1} + \int_{t-1}^{t} \frac{\partial f}{\partial x} \mu dt + \int_{t-1}^{t} \frac{\partial f}{\partial x} \sigma dW_t + \sum_{t=1}^{N_t} (Y_t - 1) \tag{14}$$

Then, for each function $f: [0, T] \times \mathbb{R} \to \mathbb{R}$, the process $G = G(t, X(t))$ can be represented as [16]:

$$G(t, X_t) - G(0, X_0) = \int_0^t \left[\mu \frac{\partial G}{\partial x}(t, X_t) + \frac{\partial G}{\partial s}(t, X_t) + \frac{1}{2}\int_0^t \sigma^2 \frac{\partial^2 G}{\partial x^2}(t, X_t) \right] dt$$
$$+ \int_0^t \frac{\partial G}{\partial x}(t, X_t) \sigma dW_t + \sum_{t=1}^{N_t} (Y_t - 1) \tag{15}$$

The stochastic differential equation with jumps is given by **Equation (12) [17]**, where W_t is denotes the standard Wiener process at time t, J_t is jump process at time t, and defined as:

$$J_t = \sum_{t=1}^{N_t} (Y_t - 1) \tag{16}$$

$$dJ_t = (Y_t - 1) dN_t \tag{17}$$

N_t represents a Poisson process with intensity λ, where W_t, N_t, and $Y_{N(t)}$ are mutually independent. Meanwhile W_t represents Brownian Motion, and the values of μ and σ are parameters of X and t [18]. Based on **Equations (13)** and **Equations (17)**, they are simplified as follows [2]:

$$dX_t = \mu^* X_t dt + \sigma X_t dW_t + X_t dJ(t)J_t \tag{18}$$

when μ is the mean of stock returns, σ is the standard deviation of stock returns, if a jump occurs, B_t is Brownian Motion at time t, N_t is the Poisson process with intensity, and $Y_{N(t)} - 1$ is relative jump size. Based on the Ito process for the jump diffusion model in **Equation (14)**, if there is a function $G = G(t, X_t)$, then the function G will follow **Equation (19) [19]**:

$$dG = \left(\frac{\partial G}{\partial X_t}(\mu - \lambda)X_t + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X_t^2}\sigma^2 X_t^2\right)dt + \frac{\partial G}{\partial X_t}\sigma(X_t)dW_t + \sum_{t=1}^{N_t}(Y_t - 1) \tag{19}$$

Let $G = lnX_t$ be a function, with $\frac{\partial G}{\partial X_t} = \frac{1}{X_t}, \frac{\partial^2 G}{\partial X_t^2} = -\frac{1}{X_t^2}, \frac{\partial G}{\partial t} = 0$, it can be obtained:

$$dG = \left((\mu - \lambda)\frac{1}{X_t}X_t + 0 + \frac{1}{2}\left(-\frac{1}{X_t^2}\right)\sigma^2 X_t^2\right)dt + \frac{1}{X_t}\sigma X_t dW_t + \sum_{t=1}^{N_t}(Y_t - 1)$$

$$dG = \left(\mu - \lambda - \frac{\sigma^2}{2}\right)dt + \sigma dW_t + \sum_{t=1}^{N_t}(Y_t - 1) \tag{20}$$

If there is a change in stock price in the current period compared to the previous period, which is one day with $t_0 < t_1 < t_2 < \cdots < t_n$, then based on the processes in **Equations (15)** and **Equations (20)**, it can be obtained **[2]**:

$$\int_{t-1}^{t} dG = \int_{t-1}^{t}\left(\mu - \lambda - \frac{\sigma^2}{2}\right)dt + \int_{t-1}^{t}\sigma dW_t + \sum_{t=1}^{N_t}(Y_t - 1) \tag{21}$$

2.9 Geometric Brownian Motion with Jump Diffusion

Assuming the model generated follows a normal distribution based on the Jump Diffusion approach. The Poisson process is used to represent the likelihood of jumps occurring in a short period **[4]**. The continuous stochastic process is Jump Diffusion. When stock prices move rapidly and there are spikes or sharp jumps in previous stock price increases and decreases, the GBM approach known as GBM with Jump Diffusion is used **[12]**. The final stock price model generated using the GBM with Jump Diffusion Model, based on proportions and the Wiener process, is as follows **[2]**:

$$S_t = S_{t-1} exp\left[\left(\mu - \lambda - \frac{\sigma^2}{2}\right) + \sigma Z_{t-1} + N_t\right] \tag{22}$$

where S_t is stock price at time t, S_{t-1} is the stock price at time $(t-1)$, μ is the mean stock return, σ is the standard deviation of stock return, λ is jump intensity, Z_{t-1} is normally distributed generated data at time $(t-1)$, N_t is normally distributed generated data at time t, and t is time.

2.10 Mean Absolute Percentage Error

Mean Absolute Percentage Error (MAPE) is defined as the average absolute percentage of prediction errors. MAPE indicates how large the absolute error rate of the predictions is compared to the actual values. The MAPE value indicates the accuracy of the predictions as given in **Equation (23) [20]**:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{|S_t - \hat{S}_t|}{S_t} \tag{23}$$

where S_t is the actual stock price at time t, $\hat{S}_t$ is the predicted stock price at time t, and n is the number of stock return data.

The prediction results will be better if the obtained MAPE value is smaller. The smallest MAPE value is chosen because it indicates the smallest error. MAPE has a range of accuracy levels that can be used as a measurement for prediction models. This accuracy level can be seen in **Table 1 [20]**.

Table 1. Accuracy Assessment Scale for MAPE

MAPE Value	Prediction Accuracy
$MAPE \leq 10\%$	Prediction accuracy is very good
$10\% < MAPE \leq 20\%$	Prediction accuracy is good
$20\% < MAPE \leq 50\%$	Prediction accuracy is quite good
$MAPE > 50\%$	Prediction accuracy is poor

2.11 VaR in Monte Carlo Simulation

VaR is one of the calculations that can be used to determine the maximum potential losses that may occur on owned assets based on a confidence level, investment amount, and a specific period [8]. Simply put, VaR is used to ascertain the extent of an investor's losses in percentage or monetary terms over the investment period (t) through a confidence level $(1-\alpha)$ [6]. Based on the characteristics of the data that will be generated by the random number generator used to estimate VaR values, the steps in determining VaR through Monte Carlo simulation are as follows [8]:

a. Based on the previous stock price forecasting results, where it is assumed that returns follow a normal distribution with mean (μ) and standard deviation (σ^2), the parameter values for the stock return data are determined.

b. Using n parameters from step (1) to generate random stock return estimates, running simulations on return data to create an empirical distribution, and determining simulated returns.

c. Calculating the largest loss estimate at a confidence level $(1-\alpha)$, represented by R^*, the α quantil value of the empirical return distribution obtained in step (2).

d. Calculating the VaR value at confidence level $(1-\alpha)$ over a period of t days:

$$VaR_{(1-\alpha)} = V_o R^* \sqrt{t} \tag{24}$$

V_o R^* and $\sqrt{t}$ in Equation (24) subsequently are initial investment fund, $R^*$is quantile value of return distribution, and $\sqrt{t}$ is period. The obtained VaR value represents the maximum loss to be incurred when investing.

e. Repeating steps (2) to (4) for m iterations to reflect various possible values of a single asset.

f. Calculating the average based on the VaR values generated in step (5), to stabilizing the VaR value, where the VaR value generated in each simulation differs.

3. RESULTS AND DISCUSSION

3.1 Research Data

The case study was conducted by analyzing the stocks of PT Bank Negara Indonesia Tbk for the period from December 1, 2022, to January 31, 2024, comprising a total of 283 trading days. The data used in this research was sourced from secondary sources accessed through the website https://finance.yahoo.com/. PT Bank Negara Indonesia (Persero) Tbk. is one of the companies offering various financial products to the public. Stocks in the financial sector, especially banking, often experience fluctuating price movements, making them difficult to predict. In this study, software such as Microsoft Excel and R-Studio were utilized for data analysis.

Figure 1. Movement of Stock Closing Prices

In Figure 1, it can be observed that the stock prices fluctuate, with BBNI stock prices experiencing unpredictable increases and decreases. The closing price data of BBNI stock is then divided into in-sample and out-of-sample data. The in-sample data is used to estimate the parameter values of the GBM with Jump

Diffusion model, starting from December 1, 2022, to December 1, 2023, comprising 243 data points. The out-of-sample data is used to validate the model, starting from December 4, 2023, to January 31, 2024, comprising 40 data points.

3.2 Calculating Stock Return In-Sample

After determining the in-sample and out-of-sample data, the next step is to calculate the in-sample stock return data. Stock return is the rate of return obtained based on stock ownership. The stock return value is obtained using **Equation (1)**.

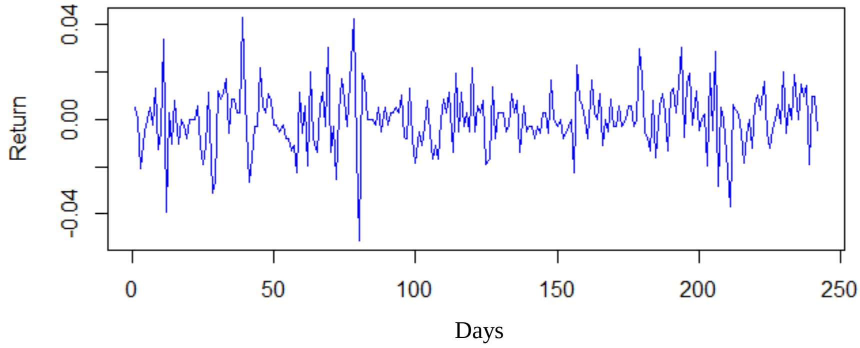

Figure 2. Plot of Stock Return In-Sample

In **Figure 2**, it is evident that the returns obtained can be both positive and negative. Positive returns indicate profit or gain from the investment, meaning that the value of the investment has increased compared to its initial value. Conversely, negative returns indicate a loss, where the value of the investment has decreased. It is important to understand that these fluctuations are a natural part of market dynamics. Investors need to consider the risks and potential rewards when making investment decisions, as well as recognize that inconsistent returns can occur with changing market conditions.

Table 2. Descriptive Statistics

Mean Stock Return	Standard Deviation of Stock Return	Maximum	Minimum	Median	Kurtosis
0.0003	0.0129	0.0431	-0.0513	0.0000	4.67925

In **Table 2**, it is observed that descriptive statistics on in-sample stock returns are provided, where the largest return value in the in-sample stock data is 0.0431 and the smallest return value is -0.0513. The average return obtained is 0.0003. A positive average return value indicates profit in the stock investment. Furthermore, a normality test is conducted on the return data before proceeding to the model formation using the GBM with Jump Diffusion.

3.3 Calculating of Kurtosis and Kolmogorov-Smirnov

The calculation of kurtosis serves as an initial indication of the presence of a jump. Extreme values or jumps are present in the data if the kurtosis value (leptokurtic) is greater than 3. Based on **Table 3**, a kurtosis value of 4.6793 is obtained, indicating that jump data with a kurtosis value greater than 3 is present in the in-sample stock return data. The GBM with Jump Diffusion model assumes that past stock returns follow a normal distribution. The GBM with Jump Diffusion model also assumes that data with stock returns not normally distributed can be used if the resulting kurtosis value is greater than or equal to 3, indicating the presence of jumps in the data **[2]**.

A normality test was conducted based on the Kolmogorov-Smirnov (KS) table with a significance level of $\alpha = 5\%$ or 0,05 using R-Studio software. The analysis results show a p-value of 0.122 is greater than 0.05. The D_{count} value obtained is 0.0760, which is smaller than the $D_{(0.05,242)}$ value of 0.0869. Based on these values, it is concluded that H_0 is accepted, indicating that the in-sample stock return data follows a normal distribution.

3.4 Peak Over Threshold Calculation for Stock Returns In-Sample

After identifying the presence of jumps in the data, the next step is to determine the number of worst-case data with extreme values through Peak Over Threshold (POT) calculation, where the in-sample stock return data indicates the presence of jumps. Based on the 10% quantile value of the in-sample stock return data, the determinants of the upper and lower threshold quantile values are provided in **Table 3**.

Table 3. The Quantile Value of Data Return In-Sample

Quantile	Value
The lower threshold quantile	-0.0142
The upper threshold quantile	0.0162

Based on **Table 3**, the lower threshold quantile value obtained is -0.0142, and the upper threshold quantile value is 0.0162. Thus, it can be concluded that based on the previously sorted data, a total of 48 in-sample stock return data with a 10% threshold have 24 data with values less than -0.0142 and 24 data of values greater than 0.0162, indicating jump data.

3.5 Parameter Estimation for GBM with Jump Diffusion

Parameter estimation based on the GBM with Jump Diffusion model is calculated using several parameter values, including the mean stock return, standard deviation of stock return, jump intensity value, mean jump difference value, and standard deviation of jump difference value. These values are calculated respectively using **Equation (8)**, **Equation (9)**, and **Equation (10)**:

Table 4. Parameter Value

Parameter	Value
Jump intensity	0.0002
Mean jump difference	0.0020
Standard deviation of jump difference	0.0049

Based on **Table 2** and **Tabel 4** the stock price prediction model using the GBM with Jump Diffusion method in **Equation (22)** is as follows:

$$S_t = S_{t-1} exp\left[\left(0.0003 - 0.0002 - \frac{(0.0129)^2}{2}\right) + 0.0129 Z_{t-1} + N_t\right]$$

After modeling using the GBM with the Jump Diffusion method, the next step is to predict the stock price for the next 40 days using the obtained model.

3.6 Stock Price Prediction using GBM with Jump Diffusion

After obtaining the estimated parameter values and the GBM with Jump Diffusion model, the next step is to predict the stock price. Predictions are made by repeating the process 1000 times to generate price paths to forecast the stock price and its changes over the next 40 periods, starting from December 4, 2023, to January 31, 2024. Additionally, the predicted stock prices are compared with the actual stock prices or out-of-sample stocks. This comparison is presented in **Table 5**:

Table 5. Comparison of Predicted Results and Actual Stock Price of BBNI

No	Date	Actual	Predicted	No	Date	Actual	Predicted
1	4 Dec 2023	5300	5300	21	4 Jan 2024	5600	5465
2	5 Dec 2023	5300	5292	22	5 Jan 2024	5575	5487
3	6 Dec 2023	5225	5273	23	8 Jan 2024	5575	5518
4	7 Dec 2023	5275	5251	24	9 Jan 2024	5650	5509
5	8 Dec 2023	5075	5044	25	10 Jan 2024	5600	5592
6	11 Dec 2023	5175	5293	26	11 Jan 2024	5600	5513
7	12 Dec 2023	5275	5279	27	12 Jan 2024	5600	5566
8	13 Dec 2023	5175	5272	28	15 Jan 2024	5625	5581
9	14 Dec 2023	5350	5327	29	16 Jan 2024	5600	5557
10	15 Dec 2023	5275	5190	30	17 Jan 2024	5550	5536
11	18 Dec 2023	5200	5183	31	18 Jan 2024	5550	5521

No	Date	Actual	Predicted	No	Date	Actual	Predicted
12	19 Dec 2023	5200	5190	32	19 Jan 2024	5500	5500
13	20 Dec 2023	5200	5193	33	22 Jan 2024	5525	5522
14	21 Dec 2023	5150	5141	34	23 Jan 2024	5425	5506
15	22 Dec 2023	5250	5234	35	24 Jan 2024	5475	5499
16	27 Dec 2023	5275	5268	36	25 Jan 2024	5400	5502
17	28 Dec 2023	5350	5288	37	26 Jan 2024	5425	5532
18	29 Dec 2023	5375	5349	38	27 Jan 2024	5575	5512
19	2 Jan 2024	5375	5372	39	30 Jan 2024	5650	5544
20	3 Jan 2024	5350	5311	40	31 Jan 2024	5750	5675

Based on **Table 5**, the stock prediction results obtained using the GBM with Jump Diffusion method are presented. By comparing these predictions with out-of-sample stock prices or actual data, the analysis shows that the difference between the predicted stock prices and the actual prices is not significant, indicating that the prediction model is quite accurate in reflecting the movements of stock prices in the market.

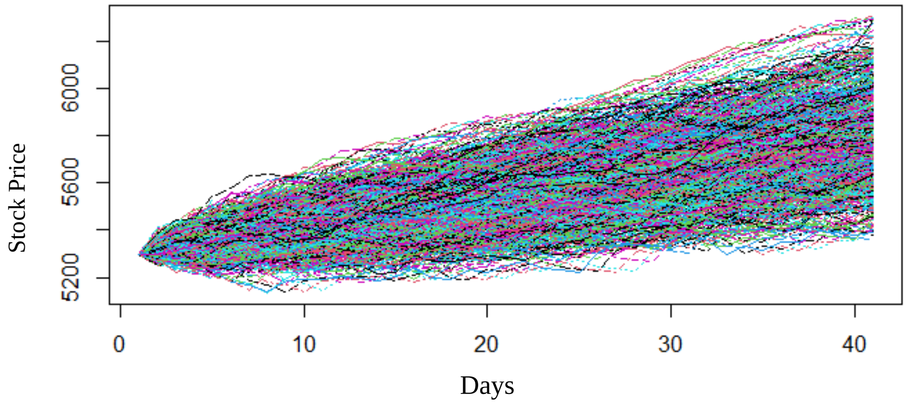

Figure 3. Stock Price Movement Prediction using GBM with Jump Diffusion Model and Monte Carlo Simulation

In **Figure 3**, it can be seen that the prediction results using the GBM with Jump Diffusion model spread randomly and produce predicted stock price paths that differ from one another. This is because there is a stochastic element in the form of a random variable with a standard normal distribution. The calculation of the MAPE value in **Equation (23)** yields the smallest MAPE value of 0.86%. Thus, it can be concluded that the accuracy level of the MAPE value using the GBM with Jump Diffusion model is categorized as very good because the MAPE value is below 10%, which is a common threshold in prediction analysis; a MAPE value below 10% indicates that the model has relatively small errors in estimation, making it reliable for investment decision-making.

The calculation of predicted stock returns is carried out to determine the parameters used in measuring the Value at Risk (VaR). Before proceeding to the calculation of predicted stock returns, a Kolmogorov-Smirnov normality test is conducted on the predicted stock returns with a significance level of $\alpha = 5\%$ or 0.05. The obtained p-value is 0.0628 which is greater than 0.05. The D_{count} which is smaller than the $D_{(0.05,39)}$ value of 0.2100. Based on these results, it is concluded that H_0 is accepted, indicating that the predicted BBNI stock data follows a normal distribution.

3.7 Monte Carlo Simulation for VaR Calculation

After conducting the Kolmogorov-Smirnov normality test, estimation of predicted stock returns and parameter calculations were performed. Value at Risk (VaR) calculation with Monte Carlo simulation will use the average and standard deviation parameters of predicted stock returns. Since the previous normality test results showed that the predicted stock return data follows a normal distribution, it can proceed to estimate the VaR value. Based on the predicted stock return values, VaR parameters are obtained using **Equation (6)** for the average predicted stock returns and **Equation (7)** for the standard deviation of predicted stock returns as follows:

Table 6. Parameters for VaR of Predicted Stock Return Data

Parameter	Value
Mean predicted stock return	0.0021
Standard deviation of predicted stock return	0.0157

Based on **Table 6**, the average predicted stock return is obtained as 0.0021, and the standard deviation of predicted stock return is 0.0157. After obtaining the parameter values from the predicted stock returns, the VaR calculation using Monte Carlo simulation is performed using **Equation (24)**. The VaR calculation is conducted with confidence levels of 80%, 90%, 95%, and 99% over a one-day, with 1000 repetitions. The VaR calculation results are obtained using R-Studio and Microsoft Excel software and can be seen in **Table 7**.

Table 7. VaR Value

Confidence Level	VaR(%)	VaR(Rp)
80%	0.96	963,362
90%	1.53	1,530,266
95%	1.97	1,974,099
99%	2.64	2,642,933

The VaR value at a confidence level of 90% is higher than that at a confidence level of 80%, as outlined in **Table 7**, indicating that when investors choose a higher confidence level, they anticipate greater potential losses over a specific period. Furthermore, the VaR value also increases at a 95% confidence level compared to that at 90%, suggesting that as the confidence level rises, the risk that investors must accept not only increases but also reflects the need to account for the possibility of larger extreme losses, thereby emphasizing the importance of a deep understanding of the relationship between confidence levels and potential risk in investment decision-making.

4. CONCLUSIONS

The Geometric Brownian Motion (GBM) with Jump Diffusion method applied to the stock prices of PT Bank Negara Indonesia Tbk (BBNI) from December 1, 2022, to January 31, 2024, resulted in the smallest MAPE value, which is 0.86%. The prediction accuracy using the Geometric Brownian Motion (GBM) with the Jump Diffusion method is categorized as very good because the MAPE value is below 10%. The maximum loss estimation using VaR in the Monte Carlo simulation based on confidence levels of 80%, 90%, 95%, and 99%, respectively, amounts to Rp963,362, Rp1,530,266, Rp1,974,099, and Rp2,642,933.

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