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THE COMPARISON OF EXTENDED AND ENSEMBLE KALMAN FILTERS IN MODELING ENVIRONMENTAL POLLUTION INFLUENCES ON ACUTE RESPIRATORY INFECTION DYNAMICS (ISPA)

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ABSTRACT

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Acute Respiratory Infections (ISPA) are a significant health issue. According to the World Health Organization (WHO), ISPA is the leading cause of death among children under five worldwide. ISPA can be caused by environments with high levels of air pollution, particularly in urban areas. Predicting the spread of ISPA is a crucial step in controlling the disease. Since pollution sources are diverse, modeling and prediction can be difficult, which makes advanced methods such as the Kalman Filter (KF) desirable. This study compares two estimation methods, the Extended Kalman Filter (EKF) and the Ensemble Kalman Filter (EnKF), in predicting the spread of ISPA triggered by environmental pollution. Simulation results show that both methods can produce accurate estimations, but EnKF demonstrates superior performance in terms of RMSE compared to EKF. It predicts more accurately for susceptible (X) and infected (Y) populations with EnKF than with EKF. Based on the results of the EnKF for the X and Y populations, the RMSEs are 0.0660 and 0.1114, respectively. EnKF's advantage in handling uncertainty and non-linearity in the model makes it suitable for predicting the spread of ISPA.

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1. INTRODUCTION

Acute respiratory infection (ISPA) is a health problem that attacks the human circulatory system, both the upper and lower respiratory tract. ISPA that attacks the lower circulatory system can cause pneumonia. Symptoms of pneumonia include high fever, chest pain, and difficulty breathing. Pneumonia due to ISPA can increase in individuals with high cardiovascular [1]. Without proper treatment, pneumonia can lead to death [2]. Whenever pollution enters the circulatory system, it disrupts the body's defenses, making it susceptible to ISPA. Environmental pollution causes harmful substances such as NO_2, which can increase cases of pneumonia [3],[4]. Based on a statement from the World Health Organization (WHO), ISPA is the leading cause of death in children under five in the world [5]. ISPA can be triggered by prolonged dry seasons and environmental pollution. The increase in urbanization and industrialization has resulted in worsening ecological pollution in urban areas, so the number of ISPA cases is increasing.

For this reason, predicting the spread of ISPA under environmental pollution is very important in the field of public health. Prediction of the spread of ISPA can help in emergency response planning, such as making decisions about the steps needed to control the spread of the disease. ISPA can be caused by environments with high levels of air pollution, especially in urban areas. It is vital to predict the spread of ISPA in order to control it. To overcome complex problems, a more advanced mathematical approach is required, such as the Kalman Filter (KF) method, which can provide the best estimate of the observed variables [6]. In predicting pollution concentrations, ARIMA and KF show that KF provides prediction results with smaller errors [7]. KF is known for its efficient computational technique in predicting by reducing errors [8]. However, in real-world systems, there are often non-linear dynamics. To solve this issue, the Extended KF (EKF) and Ensemble KF (EnKF) methods, which are advanced versions of KF, are employed as they are better suited for nonlinear systems [9]. This study compares two estimation methods based on the Kalman Filter, the Extended Kalman Filter (EKF) and the Ensemble Kalman Filter (EnKF), in predicting the spread of ISPA triggered by environmental pollution.

Parameter estimates and system states can be updated based on observational data using data assimilation to predict disease spread. The Kalman Filter method is a data assimilation technique used to overcome uncertainty in mathematical modeling and prediction. The Kalman Filter method for nonlinear problems is the Extended Kalman Filter (EKF) method and the Ensemble Kalman Filter (EnKF) method [10],[9]. The initial stage in EKF is to linearize the nonlinear model using the Jacobian matrix [11]. Several previous studies on EKF and EnKF have been conducted. Ritschel et al. 's study applied EKF to the U-loop reactor model, in which the reaction material flows through a curved U-shaped channel. The study discusses the dynamics of methanotroph production in the U-loop reactor with the mathematical model $C_X - C_S - C_O - C_{go}$ each of which is the concentration of biomass, substrate, oxygen in the liquid phase, and oxygen in the gas phase. The EKF approach to the nonlinear equations of the U-loop reactor model shows accurate predictions of biomass and dissolved oxygen concentrations [12]. However, the predictions on substrate concentrations deviate significantly from the actual values. Research on numerical weather prediction (NWP) using EnKF has been conducted. EnKF is used by Environment and Climate Change Canada and the Met Office to initialize predictions for deterministic systems [13]. EnKF's weather prediction using assimilation data variation produced predictions with a global error standard deviation below 77% [13]. The EnKF method can also be applied in motion prediction with the help of robots. Research related to robot motion prediction was conducted by Herlambang et al., including EnKF predicts third finger movement in stroke cases with an accuracy of 92% to 99% [14] and EnKF's application to robotic finger arm motion estimation shows an accuracy of over 98% [15].

Another study that discussed the application of EKF in disease transmission was conducted by Sebbagh et al. This study focused on the transmission of COVID-19 in Algeria. The spread of COVID-19 with the Susceptible Infection Recovery Death ($SIRD$) mathematical model is predicted using EKF [16]. The results indicate that the application of EKF provides good estimation results from February 25, 2020, to February 13, 20201, as indicated by the small Root Mean Square error (RMSE). EKF has also been applied to the health sector by Giamberardino et al. The application of EKF to the HIV/AIDS virus spread model is used to predict the infected population [17]. The study used a model with five compartments, namely $S_1, S_2,\ I,\ P,\ A$. S_1 is a vulnerable population that underestimates the risk of HIV/AIDS, S_2 is a vulnerable population that has prevention, I is an infected population, P is an HIV-infected population, and A is an AIDS-infected population. According to the results, the EKF method predicts a decrease in the infected population if combined with informative campaigns. This study can be used to suppress the epidemic in 2030.

As opposed to EKF, EnKF does not have linear stages. The EnKF method uses an ensemble of samples to approach the probabilistic distribution of the estimated variables [18]. The application of EnKF in the case of the spread of SI disease was carried out by Mitchell et all. In this study, the results of the status and parameter estimates in the epidemic model using EnKF were given The results showed that the mean square error in the S and I populations were 7.89×10^5 and 2.79×10^3, respectively [19]. The EnKF method was applied to the case of the spread of COVID-19 using an ensemble of 200 [20]. The COVID-19 spread model used is the $S\_I\_R\_D$ model considering the effect of lockdown. The accuracy level of EnKF in the study uses Relative Mean Absolute Error (RMAE). The results show that the RMAE value is less than 1% for the Recover and Decrease populations. EnKF has also been used to predict the spread of dengue fever. The spread of dengue fever using the $S_h\_I_h\_R_h\_S_v\_S_v$ model involving human and mosquito populations. The error difference in the dengue fever transmission model with the EnKF method was 0.6859% [21].

To control ISPA, it is necessary to predict the model of the spread of ISPA under environmental pollution. The model for the spread of ISPA uses the model in Kumari et al.'s research. [22]. The model of ISPA under environmental pollution is X (Population not yet affected by pollution)-P (Population affected by pollution)-Y (Population infected with ISPA). Previous studies have discussed the dynamic analysis of the ISPA $X - P - Y$ model using the theory of ordinary differential equations, dynamic systems, and basic reproduction numbers. The dynamic system model in the $X - P - Y$, model is an essential tool for understanding this pattern, but its nonlinear nature and data uncertainty require a more sophisticated prediction approach. Methods such as EKF and EnKF can be applied to overcome uncertainty in the data and model. This study aims to compare the performance of EKF and EnKF in predicting the dynamics of ISPA under environmental pollution. Therefore, Root Mean Square Error (RMSE) will be used to measure the accuracy of the predictions produced by each method. This study predicts the growth rate of each population $X - P - Y$ in the ISPA model under environmental pollution using MATLAB. The results of this study are expected to contribute to choosing a more appropriate method for handling and preventing the spread of ISPA.

2. RESEARCH METHODS

The research method for predicting the spread of ISPA under environmental pollution using the EKF and EnKF methods is given as follows.

2.1 Mathematical Model

This stage is the identification stage of the model of the spread of ISPA under environmental pollution. The model used in this study consists of three populations, namely $X - P - Y$ [22]. X is a population that has not been affected by environmental pollution, P is a population that has been affected by pollution and is susceptible to ISPA, and Y is a population affected by ISPA.

2.2 Discretization of ISPA Model

The mathematical model of the spread of Acute Respiratory Infection (ISPA) under pollution is still continuous, so it is discretized. The implementation of the EKF and EnKF algorithms uses a discrete model. The model of the spread of disease under environmental pollution is discretized using the forward finite difference method. The results of discretizing the ISPA model under environmental pollution are transformed into a nonlinear function form.

2.3 Implementation of EKF and EnKF Methods on the ISPA Model

In the EKF and EnKF methods, there are different initial stages when implemented in the spread of ISPA under the environmental pollution model. The EKF method uses the Jacobian matrix for the linearization process. The disease spread model is then converted into discrete and the EKF Algorithm is applied. While the EnKF method uses an ensemble to predict the spread of disease under environmental pollution first. Then, the EnKF Algorithm is applied.

2.4 ISPA Model Simulation Results

The ISPA spread estimation modeling is simulated using MATLAB software. The simulation results are in the form of growth rates of each $X-P-Y$ population using the EKF and EnKF methods.

3. RESULTS AND DISCUSSION

This section explains the prediction results and analysis of the prediction model for the spread of ISPA under environmental pollution using the Extended Kalman Filter (EKF) and Ensemble Kalman Filter (EnKF) methods.

3.1 Model of the Spread of ISPA under Environmental Pollution

The following is a mathematical model of the spread of ISPA under environmental pollution [22].

$$\begin{aligned}\frac{dX}{dt} &= mA-\theta X-\lambda XY+n\xi Y-\mu X\\ \frac{dP}{dt} &= (1-m)A-\theta X-\lambda(1+\delta\lambda')PY+(1-n)\xi Y-\mu P\\ \frac{dY}{dt} &= \lambda XY+\lambda(1+\delta\lambda^{\wedge\prime})PY-(\xi+\phi+\mu)Y\end{aligned} \tag{1}$$

with a description of each parameter in **Table 1**.

Table 1. Parameter Description of the Models of the Spread of ISPA

Parameters	Description
A	Birth rate
m	Probability of birth of a baby
λ	Spread rate of susceptible population
δ	Scale amount of environmental pollution spread
λ'	Pollution impact parameter
θ	Rate of susceptible individuals being moved to stressed compartment
μ	Natural death rate
ξ	Rate of individuals affected by pollution P becoming infected Y
ϕ	Death rate under environmental pollution
n	The rate of healthy individuals becoming susceptible

3.2 Discretization of ISPA Spread Model under Environmental Pollution

Equation (1) is still in the form of a non-linear equation. While the Kalman filter method is linear. Then discretization is carried out to obtain the form of the equation in a discrete state, obtained

$$X_{k+1}=mA\,\Delta t-\theta X_k\,\Delta t-\lambda X_kY_k\,\Delta t+n\xi Y_k-\mu X_k\,\Delta t+X_k$$
$$P_{k+1}=(1-m)A\Delta t-\theta X_k\Delta t-\lambda(1+\delta\lambda')P_kY_k\Delta t+(1-n)\xi Y_k-\mu P_k\Delta t-P_k$$

$$Y_{k+1} = Y_{k+1} = \lambda X_k Y_k \Delta t + \lambda(1+\delta\lambda')P_k Y_k \Delta t - (\xi + \phi + \mu) Y_k \Delta t + Y_k \tag{2}$$

from **Equation (2)**, linearization is carried out by applying the Jacobian matrix **[10]** and we obtain:

$$\boldsymbol{A} = \left[\frac{\partial f_i}{\partial x_j}(\hat{x}_k, u_k)\right]$$

Then

$$\boldsymbol{A} = \begin{pmatrix} 1 + (-\theta - \lambda I_k - \mu)\Delta t & (-\lambda X_k)\Delta t & (n\xi)\Delta t \\ \theta \Delta t & (-\lambda(1+\delta\lambda')Y_k - \mu)\Delta t + 1 & (-\lambda(1+\delta\lambda')P_k + (1-n)\xi)\Delta t \\ \lambda Y_k & (\lambda(1+\delta\lambda')Y_k)\Delta t & (-\lambda(1+\delta\lambda')P_k - (\xi+\phi+\mu)\Delta t + 1 \end{pmatrix} \tag{3}$$

3.3 EKF Method on the Model of the Spread of ISPA under Environmental Pollution

In the model of the spread of ISPA under environmental pollution, the Jacobian matrix has been obtained in **Equation (3)**. Furthermore, the EKF method is applied to the ISPA spread model.

ISPA spread by environmental pollution follows the following system model:

$$x_{k+1} = f(x_k, u_k) + w_k \tag{4}$$

the system model in the model of the spread of ISPA under environmental pollution is

$$z_k = H x_k + v_k \tag{5}$$

v_k is noise in the measurement following a normal distribution with $v_k \sim N(0, Q_k)$.

The system model and measurements in the ISPA model have been obtained, and then the initialization stage is carried out $\hat{x}_0 = \bar{x}_0$ and $P_0 = P_{x_0}$ as follows.

$$\hat{x}_0 = \begin{pmatrix} 2864 \\ 1.11380 \\ 9528 \end{pmatrix}$$

and

$$P_0 = \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{pmatrix}$$

with noise covariance

$$Q = \begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{pmatrix}$$

and

$$R = 0.0001$$

After obtaining **Equation (4)**, **Equation (5)**, and initiating, the next steps in the EKF method are the prediction and correction stages, which are given as follows **[23]**.

Prediction Step

$$\hat{x}_k^- = f(\hat{x}_k, u_k)$$
$$\hat{P}_{k+1}^- = \boldsymbol{A} P_k \boldsymbol{A}^{\boldsymbol{T}} + Q_K$$

then, the EKF prediction stage in the ISPA spread model under environmental pollution is

$$\hat{x}_{k+1}^- = \begin{pmatrix} 1 + (-\theta - \lambda I_k - \mu)\Delta t & (-\lambda X_k)\Delta t & (n\xi)\Delta t \\ \theta \Delta t & (-\lambda(1+\delta\lambda')Y_k - \mu)\Delta t + 1 & (-\lambda(1+\delta\lambda')P_k + (1-n)\xi)\Delta t \\ \lambda Y_k & (\lambda(1+\delta\lambda')Y_k)\Delta t & (-\lambda(1+\delta\lambda')P_k - (\xi+\phi+\mu)\Delta t + 1 \end{pmatrix} \hat{x}_k + w_k$$

$$\hat{P}_{k+1}^{-} = \begin{pmatrix} 1+(-\theta-\lambda I_k-\mu)\Delta t & (-\lambda X_k)\Delta t & (n\xi)\Delta t \\ \theta\Delta t & (-\lambda(1+\delta\lambda')Y_k-\mu)\Delta t+1 & (-\lambda(1+\delta\lambda')P_k+(1-n)\xi)\Delta t \\ \lambda Y_k & (\lambda(1+\delta\lambda')Y_k)\Delta t & (-\lambda(1+\delta\lambda')P_k-(\xi+\phi+\mu)\Delta t+1 \end{pmatrix}$$

$$P_k \begin{pmatrix} 1+(-\theta-\lambda I_k-\mu)\Delta t & (-\lambda X_k)\Delta t & (n\xi)\Delta t \\ \theta\Delta t & (-\lambda(1+\delta\lambda')Y_k-\mu)\Delta t+1 & (-\lambda(1+\delta\lambda')P_k+(1-n)\xi)\Delta t \\ \lambda Y_k & (\lambda(1+\delta\lambda')Y_k)\Delta t & (-\lambda(1+\delta\lambda')P_k-(\xi+\phi+\mu)\Delta t+1 \end{pmatrix}^T + Q_k \quad (6)$$

Correction Step [11]

The EKF Correction Stage in the model of the spread of ISPA due to environmental pollution is

Kalman Gain EKF

$$K_k = P_k^- H^T (H P_k^- H^T + R_k)^-$$

EKF Correction Estimation

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$$

EKF Correction Error Covariance

$$P_k = (I - K_k H_k)\, P_k^- \quad (7)$$

3.4 EnKF Method on the Model of Spread of ISPA under Environmental Pollution

The initialization stage in the EnKF method is carried out by generating several ensembles $N_\in$ and averaging from several ensembles $N_\in$

$$\hat{x}_{0,i} = 1/\, N_\in \sum_{i=1}^{N_\epsilon} x_{0,i}$$

Prediction Stage

EnKF Prediction Estimation

$$\hat{x}_{k,i}^- = f\left(\hat{x}_{k-1}^-, \hat{u}_{k-1,i}^-\right) + w_{k,i} \quad (8)$$

Correction Stage

In the correction stage, because it involves ensembles, several ensembles are generated on the measurement data $z_{k,i} = z_k + v_{k,i}$, with

Kalman gain EnKF

$$K_k = P_k^- H^T (H P_k^- H^T + R_k)^{-1}$$

EnKF correction estimation

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_{k,i}^-)$$

Correction error covariance EnKF

$$P_k = (I - K_k H_k)\, P_k^- \quad (9)$$

3.5 Simulation Results of EKF and EnKF on the Model of the Spread of ISPA under Environmental Pollution

The application of the model of the spread of ISPA due to environmental pollution with the $X-P-Y$ model with the EKF and EnKF methods was simulated against the rate of each population. The initial values in the $X-P-Y$ model and the parameters of the ISPA spread model are given in Table 2 [22]. The results of the two EKF and EnKF methods were compared using RMSE to see which method was better in predicting the model of the spread of ISPA due to environmental pollution.

Table 2. Values in the X-P-Y Model and the Parameters of the ISPA Spread Model

Symbol	Value	Unit
X	2864	individuals
P	1.11380	individuals
Y	9528	individuals
A	200	per unit time
m	0.8	per unit time
λ	0.00002	per unit time
δ	0.3	per unit time
λ'	0.1	per unit time
θ	0.004	per unit time
μ	0.035	per unit time
ξ	0.012	per unit time
ϕ	0.01	per unit time
n	0.7	per unit time

The initial values and parameters in Table 1 and Table 2 are simulated at each population growth rate $X - P - Y$. The rate of spread of ISPA due to environmental pollution in the population X using the EKF and EnKF methods is shown in Figure 1. Prediction of the spread of ISPA within a time span of 180 days. At the beginning of time, the susceptible population increases due to the initial dynamics of the system, then decreases sharply until around day 40, reflecting the high infection rate in the population. After that, the susceptible population slowly decreases at a decreasing rate until it reaches a stabilization near day 100, where the system begins to enter a state of dynamic equilibrium. This occurs because the number of susceptible individuals decreases, reducing the chance of infection. By using EKF and generating N = 200 ensembles in the EnKF method, it shows that both methods are equally close to the actual value quite well.

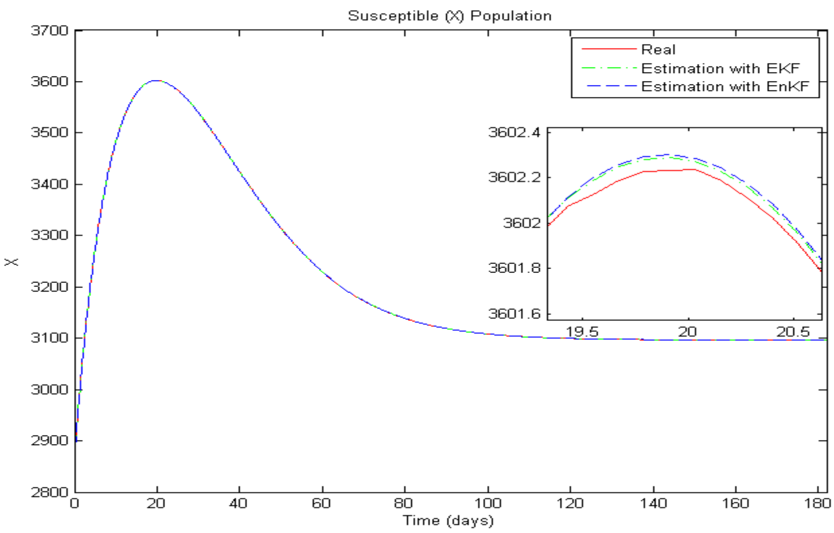

Figure 1. Prediction of the Spread of ISPA Under Environmental Pollution in Population X with EKF and EnKF

The rate of spread of ISPA due to environmental pollution in the population P affected by environmental pollution using the EKF and EnKF methods is shown in Figure 2. From time $t = 0$ to $t = 180$, the population affected by environmental pollution continues to increase due to the lack of responsive handling. The exposed population (P) in the ISPA model increases to around ($t = 100$) and becomes constant. This happens because the dynamic system reaches equilibrium, the rate of individuals entering the susceptible group (X) is equal to the rate of individuals leaving the group (P) to the infected group (Y). This stability reflects that the spread of ISPA has approached the dynamic equilibrium point, in other words, interactions between populations no longer produce significant changes in the exposed group. Predictions using EKF and EnKF show good accuracy where small differences between EKF and EnKF predictions are seen in the transition phase towards equilibrium, indicating the effectiveness of EnKF in capturing tiny changes in system dynamics because of its flexibility in handling nonlinear dynamics.

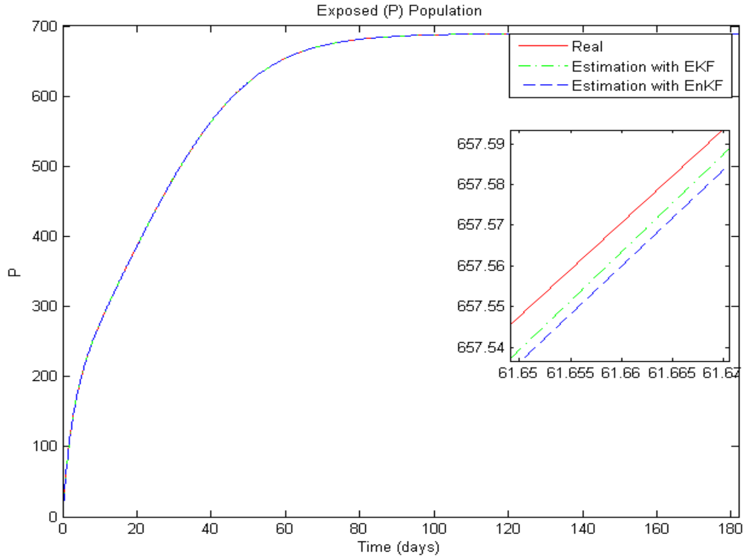

Figure 2. Prediction of the Spread of ISPA under Environmental Pollution in Population P with EKF and EnKF

The rate of spread in the population infected with ISPA under environmental pollution Y using the EKF and EnKF methods is shown in **Figure 3**. Y is very high at the beginning of time ($t = 0$), because the outbreak has just begun but it soon decreases significantly. This decrease continues until about ($t = 100$), after which the infected population stabilizes, indicating effective control of the spread of the disease. The growth rate of the infected population is influenced by several factors, namely the rate of individuals affected by pollution P becoming infected $Y(\xi)$ and the Scale amount of environmental pollution spread (δ). Predictions using the EKF and EnKF show good agreement with the actual data, with EnKF giving more stable results than EKF.

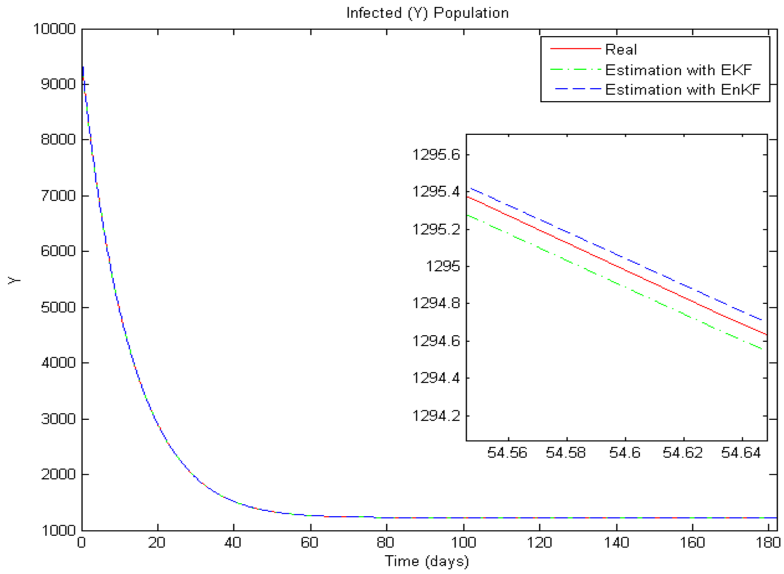

Figure 3. Prediction of the Spread of ISPA under Environmental Pollution in Population Y with EKF and EnKF

The EKF and EnKF methods are compared using Root Means Square Error (RMSE). Root Mean Square Error (RMSE) is a method used to measure the difference between the predicted value and the actual value of a model. RMSE describes how well the model can provide predictions by measuring the root mean square of the difference between the predicted value and the observed value, the following is the RMSE equation [24].

$$RMSE = \sqrt{\frac{\sum_{n=1}^{N}(\hat{r}_n - r_n)^2}{N}}$$

The RMSE of the EKF and EnKF methods in the ISPA spread model is shown in Table 3. The prediction in the EnKF method in the population X has an RMSE value of 0.0648, while the EKF method has a value of 0.0660. The difference between the two methods is 0.0012. The prediction with EnKF is slightly closer to the actual value than EKF. This shows that EnKF is superior in handling the spread of ISPA under environmental pollution in the population X. EnKF can provide more stable and accurate estimates due to the ensemble approach. The prediction using the EnKF method in the population P has an RMSE value of 0.0084, while the EKF method has a value of 0.0080. The difference between the two methods is 0.0004. It can be seen that both methods are equally close to the actual value. The prediction using the EnKF method in the population Y has an RMSE value of 0.1114, while the EKF method has a value of 0.1310. The difference between the two methods is 0.0196. It can be seen that both methods are equally close to the actual value. When compared to EKF, predictions with EnKF are closer to the actual values of the Y population predictions in the ISPA spread model. This shows that EnKF is superior in handling the spread of ISPA due to environmental pollution in the population P.

Table 3. RMSE EKF and EnKF in the ISPA Spread Model Under Environmental Pollution

Population	RMSE EKF	RMSE EnKF
X	0.0660	0.0648
P	0.0080	0.0084
Y	0.1310	0.1114

The criteria used to assess effectiveness in this study is the RMSE value. A lower RMSE indicates that the prediction method accurately represents actual data. The study results showed that the RMSE of the EnKF method was consistently lower than that of EKF. This shows that EnKF is more effective in capturing the pattern of ISPA spread due to environmental pollution. RMSE shows that EnKF is more adaptive in dealing with uncertainty and variance in the data

4. CONCLUSIONS

The study's results showed that in the model of the spread of ISPA in environmental pollution conditions, the Ensemble Kalman Filter (EnKF) is superior to the Extended Kalman Filter (EKF). This is evident from the lower RMSE (Root Mean Square Error) values produced by EnKF compared to EKF for both the susceptible population (X) and the infected population (Y). Meanwhile, in the exposed population (P), the RMSE values between EKF and EnKF did not differ much.

- For the susceptible population (X), EnKF generates an RMSE of 0.0648, while EKF generates an RMSE of 0.0660. Although the difference is small, it shows that EnKF is slightly more accurate in tracking changes in the susceptible population.
- For the population exposed to environmental pollution (P), there is a small difference between the EKF and EnKF, as seen in the RMSE difference of 0.0004. The RMSE in the EKF method is 0.0080, while the RMSE in the EnKF method is 0.0084. The most significant difference can be found in the infected population.
- The most significant difference can be found in the infected population (Y), where EnKF achieves an RMSE of 0.1114, much lower than EKF's RMSE of 0.1310. Therefore, EnKF performs much better under varying pollution conditions for estimating infected population dynamics.

An ensemble framework can be used to accurately estimate how a population's health evolves so that it can be used for predicting both the susceptible and the infected populations. This study aims to improve the accuracy of predicting the spread of ISPA due to environmental pollution by comparing the EKF and EnKF methods. The results show that EnKF is more effective in handling data uncertainty and nonlinearity, providing a more adaptive solution for epidemiological models. Practically, health authorities can use this study to predict the spread of diseases more accurately, supporting decision-making in resource allocation, intervention strategies, and pollution control to improve public health.

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