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SUSCEPTIBLE VACCINATED INFECTED RECOVERED MODEL WITH THE EXCLUSIVE BREASTFEEDING AND ITS APPLICATION TO PNEUMONIA DATA IN INDONESIA

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ABSTRACT

The spread of infectious diseases can occur directly or indirectly. Pneumonia is an infectious respiratory tract disease. Indonesia is among the top 10 countries in the world concerning deaths caused by pneumonia. The spread of infectious diseases can be prevented through vaccination and exclusive breastfeeding, which play a role in providing body immunity. This study aims to formulate an SVIR model with exclusive breastfeeding, apply it to pneumonia in Indonesia, and determine its spread pattern and interpretation regarding the target of free pneumonia by 2030. The methods used were literature and applied studies. Through literature studies, the characteristics of infectious diseases were identified, assumptions and parameters of the model were added, and relationships between variables were determined. The applied method was to estimate the parameters and initial values of the model based on annual data on pneumonia disease in Indonesia. The formulated model is a system of first-order nonlinear differential equations. The model is applied to pneumonia based on annual data from 2013 to 2022 in Indonesia, and its solution is determined using the fourth-order Runge-Kutta method. Based on the model solution and 2021-2022 data, a MAPE value of 15% is obtained, indicating that the model is sufficiently accurate in explaining the spread of pneumonia in Indonesia. The spread pattern of pneumonia in Indonesia from 2013 to 2030 indicates a downward. However, as of 2030, there are still 67,261 individuals infected, indicating that the target of pneumonia-free Indonesia has not been achieved. Simulation shows that with exclusive breastfeeding rate value $\sigma = 0.438852$ and Hib vaccination rate $\alpha = 0.25$ it is estimated that the target of free pneumonia in Indonesia in 2030 will be achieved. The free target can also be achieved by increasing the exclusive breastfeeding rate to 73.9% and the Hib vaccination rate to 0.22.

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1. INTRODUCTION

Infectious diseases are diseases that can be transmitted (moved from one person to another) either directly or through intermediaries [1]. Infectious diseases can be caused by pathogenic microorganisms such as viruses, bacteria, fungi, or parasites. Transmission can happen through the air, saliva, food or drinks, and direct contact with infected individuals. Preventive measures against the spread of infectious diseases include exclusive breastfeeding and vaccination. Vaccines stimulate the body's immune system to recognise infectious diseases, thus reducing the risk of transmission. Exclusive breastfeeding can prevent the spread of infectious diseases. Exclusive breastfeeding is the provision of breast milk without giving the baby any other food or drink other than breast milk for the first 6 months of life. Breast milk provides protection to infants through the various immune components it contains. Breast milk is often referred to as white blood because it contains cells that are important in the destruction and protection of the body against bacteria, germs, viruses and fungi [2]. According to the World Health Organization (WHO) [3], exclusive breastfeeding is one of the best ways to provide adequate nutrition to infants and is a source of protein, fat, carbohydrates, vitamins, minerals and other nutrients that infants need. Exclusive breastfeeding can also reduce the risk of respiratory tract infections, diarrhoea, and ear infections. These preventive actions are carried out to achieve the target of eradicating infectious diseases by 2030 as part of an effort to fulfil the Sustainable Development Goals (SDGs) in 2030 [4].

The spread of infectious diseases can be represented by a mathematical model, namely the susceptible infected recovered (*SIR*) model. The *SIR* model was first proposed by Kermack and McKendrik [5] and further developed by Hethcote [6] under the assumption of a constant population without migration. Chauhan *et al.* [7] investigated the *SIR* model with the influence of vaccination. In 2008, Liu *et al.* [8] developed the susceptible vaccinated infected recovered (*SVIR*) model considering vaccination programs. Individuals who have received the vaccine are categorised as group *vaccinated* (*V*) individuals. In their model, the vaccination target is assumed to be individuals in the susceptible group with a constant population and no migration. Furthermore, Islam [9] formulated the *SVIR* model with the assumptions of death due to disease, non-constant population, and migration factors. Aryani and Widyaningsih [10] examined and applied the *SVIR* model with the assumptions of a non-constant population, no migration factor, vaccine effectiveness period, vaccine failure, and individual group *V* having temporary immunity. In 2022, Wei et al. [11] examined the SVIR model with the assumption of pathogen evolution and the fact that individual V groups have the opportunity to infect. In addition, Widyaningsih *et al.* [12] have also developed an *SVIR* model into an *SVIRS* model by considering the possibility of recovered individuals being infected again. In 2023, Widyaningsih *et al.* [13] formulated the *SVITR* model with relapse and drug resistance. The model assumes that the population is not constant, there is no migration, there is death due to disease, and there is treatment.

Pneumonia is an acute respiratory tract infection affecting lung tissues (alveoli) (Data and Information Center of the Indonesian Ministry of Health [14]). Pneumonia can be caused by pathogens such as viruses, bacteria, and fungi. Transmission can occur through the air or contaminated objects via droplets (sputum spray) when coughing or sneezing. Symptoms may include productive cough, fever, shortness of breath, and headache. According to the *World Health Organization* (*WHO*) [15], pneumonia contributes to nearly one million deaths annually, with a total of 20,084 deaths occurring in Indonesia in 2017. This total ranks Indonesia seventh among countries with the highest number of deaths due to pneumonia. This is in line with basic health research data (Riskesdas) in 2018 [16], which showed that the prevalence of pneumonia in Indonesia based on the diagnosis of health workers was 2%. In 2021, according to the Ministry of Health of the Republic of Indonesia [14], the prevalence rate of pneumonia among under-fives in Indonesia was 3.55%. This means that 3 or 4 out of 100 children under five are affected by pneumonia. Pneumonia can be prevented through *Hib* (*Haemophilus influenza type b*) vaccination. The *Hib* vaccine is administered to children aged 1-1.5 years. The *Hib* vaccine provides a relatively high level of protection. The Centers for Disease Control and Prevention (CDC) [17] records show that the efficacy of the *Hib* vaccine against pneumonia is 95%. In addition, even after being infected with the correct treatment and care, most of the pneumonia can be managed to recover within one to two weeks.

In this article, we formulate the *SVIR* model with exclusive breastfeeding on the spread of infectious diseases. Then, we apply the model to data on pneumonia in Indonesia. The effect of exclusive breastfeeding on the spread of pneumonia in Indonesia should be determined by 2030 so we can estimate whether the target of Indonesia-free pneumonia by 2030 can be achieved or not. For this reason, then the spread pattern of pneumonia in Indonesia and its interpretation regarding the target of free pneumonia by 2030 is determined.

2. RESEARCH METHODS

The literature and applied studies were the research techniques employed in this study. The steps of this research are identifying the characteristics of infectious diseases, reviewing previous studies, and adding assumptions and parameters to the basic SIR model. Based on these steps, an *SVIR* model with the influence of exclusive breastfeeding was obtained. The model was applied to the spread of pneumonia in Indonesia using secondary data, including annual data from the Ministry of Health of the Republic of Indonesia [14] and *World Bank Data* [18] from 2013–2022. The model parameters were estimated using data spanning from 2013 to 2020. The resulting *SVIR* model formulation with the effect of exclusive breastfeeding on the spread of pneumonia in Indonesia was obtained. The model is given an initial value that refers to the 2013 data and is solved using the fourth-order Runge-Kutta method. Based on the model solution and 2021-2022 data, the *mean absolute percentage error* (*MAPE*) value was determined to measure the model's accuracy. The solution of the model is shown through a scatter plot, and its distribution pattern and interpretation of the pneumonia-free target in Indonesia are determined. The steps of this research are shown in Figure 1.

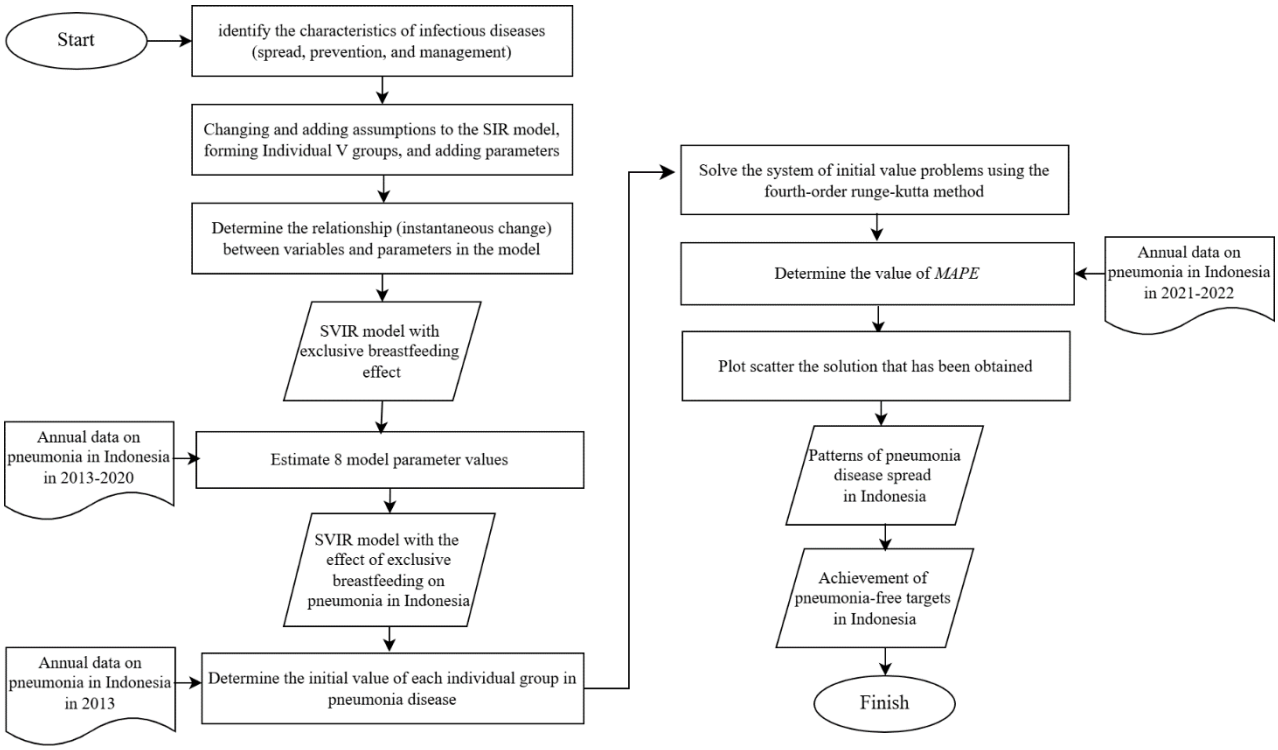

Figure 1. Research Flow Chart

3. RESULTS AND DISCUSSION

3.1 *SVIR* Model with Exclusive Breastfeeding Influence.

The spread of infectious diseases can be represented by the *SIR* model. The *SIR* model was studied by Hethcote [6] in 1998, assuming a constant population (N) without migration. In his model, Hethcote divides the population into three groups: susceptible *(S)*, infected *(I)*, and recovered (R). Notation $S(t)$ represents the number of individuals who are healthy but susceptible to the disease, that is, who are not yet infected at time t. $I(t)$ represents the number of infected individuals who are infectious and able to spread the disease by contact with susceptible individuals at time t. $R(t)$ represents the number of individuals who have been infected and recovered from the disease and gained immunity at time t. The relationship between groups of variables is written as

$$N = S(t) + I(t) + R(t) \tag{1}$$

The Hethcote *SIR* model [6] was used as the basis for this study. In this study, the population is assumed to be inconstant without migration, so the birth rate is not equal to the mortality. If θ denotes the birth rate and N is the population size, then the number of births is θN. Each individual born is assumed to be healthy but susceptible to disease. As a result, the number of individuals in group S increases by θN. In the spread of infectious diseases, death can occur. There are two types of death: natural death and death due to disease. Natural death occurs in individuals in each group. If μ states the natural death rate, the number of individuals in each group decreases by μS, μI, and μR, respectively. Death due to disease can occur if the group I individuals fail to receive treatment. If η denotes the disease mortality rate, the number of groups I individuals who die of disease is ηI. As a result, the number of groups I individuals decrease by ηI.

Vaccination is the administration of vaccines to increase one's immunity to infectious diseases. Liu *et al*. have developed the *SIR* model from Hetchote [6] by considering vaccination. Similar to Liu *et al.* [8], in this study, vaccinated individuals are categorised as vaccinated (V) individuals. The vaccination target is individual S. If α expresses the vaccination rate, the number of individuals who get the vaccine is αS. As a result, group S individuals decrease by αS and group V individuals increase by αS. Individual V can experience natural death, so the number of group V individuals decrease by μV.

Individual V may experience vaccine failure. Vaccine failure is when the vaccine administered does not protect against an infectious disease as expected. This can occur due to errors in vaccine administration procedures, inappropriate immunity, and so on. Aryani and Widyaningsih [10] have examined the *SVIR* model by considering vaccine failure. Similar to Aryani and Widyaningsih [10], this study considers vaccine failure. If δ represents the vaccine failure rate, the number of individuals who experience vaccine failure is $\delta \frac{VI}{N}$. As a result, group V individuals decrease by $\delta \frac{VI}{N}$, and group I individuals increase by $\delta \frac{VI}{N}$. Thus, the instantaneous change in the number of individuals in the Hethcote *SIR* model [6] and V is formulated as

$$\begin{aligned}
\frac{dS}{dt} &= \theta N - \alpha S - \beta \frac{SI}{N} - \mu S \\
\frac{dV}{dt} &= \alpha S - \delta \frac{VI}{N} - \mu V \\
\frac{dI}{dt} &= \delta \frac{VI}{N} + \beta \frac{SI}{N} - \eta I - \gamma I - \mu I \\
\frac{dR}{dt} &= \gamma I - \mu R
\end{aligned} \tag{2}$$

The *United Nations International Children's Emergency Fund* (*UNICEF*) [19] states that exclusive breastfeeding is the first vaccine for infants, providing protection from disease and death. Therefore, in this study, newborns who are exclusively breastfed are assumed to have vaccine-equivalent immunity and are categorised as group V individuals. If σ denotes the rate of exclusive breastfeeding, then the number of newborns who are exclusively breastfed is $\sigma\theta N$. As a result, group V individuals increase by $\sigma\theta N$ and group S individuals decrease by $\sigma\theta N$. Thus, the instantaneous change in the number of S and V individuals in the system of **Equation (2)** change into

$$\begin{aligned}
\frac{dS}{dt} &= (1-\sigma)\theta N - \alpha S - \beta \frac{SI}{N} - \mu S \\
\frac{dV}{dt} &= \alpha S - \delta \frac{VI}{N} - \mu V + \sigma\theta N
\end{aligned} \tag{3}$$

Based on the system of **Equation (3)**, the *SVIR* model with the effect of exclusive breastfeeding is formulated as follows

$$\begin{aligned}
\frac{dS}{dt} &= (1-\sigma)\theta N - \alpha S - \beta \frac{SI}{N} - \mu S \\
\frac{dV}{dt} &= \alpha S - \delta \frac{VI}{N} - \mu V + \sigma\theta N \\
\frac{dI}{dt} &= \delta \frac{VI}{N} + \beta \frac{SI}{N} - \eta I - \gamma I - \mu I
\end{aligned} \tag{4}$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

with $S(0) \geq 0, V(0) \geq 0, I(0) > 0, R(0) \geq 0$ and parameters $\theta, \alpha, \beta, \delta, \sigma, \eta, \gamma, \mu > 0$. These parameters represent the birth rate, vaccination rate, contact rate, vaccine failure rate, disease death rate, cure rate, and natural death rate, respectively. Whereas σ is the rate of exclusive breastfeeding that has an effect in reducing the number of susceptible individuals and increasing the number of individuals who get the vaccine. **Equation (4)** is a system of first-order nonlinear differential equations with dependent variable t and independent variables S, V, I, R. Furthermore, the population dynamics of the $SVIR$ model with exclusive breastfeeding effect is shown in **Figure 2**.

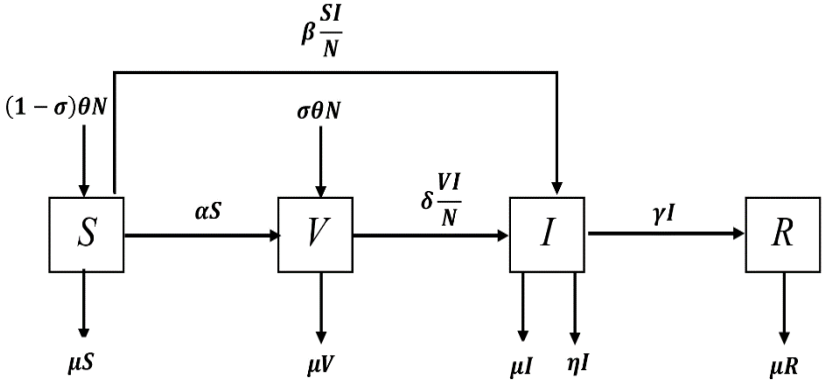

Figure 2. Population Dynamics of the SVIR Model with the Exclusive Breastfeeding

3.2 Application

Equation (4) was applied to the spread of pneumonia in Indonesia based on annual pneumonia data spanning from the Ministry of Health of the Republic of Indonesia **[14]** and World Bank Data **[18]**. The parameter values of the model were estimated using data spanning 2013 to 2020. The vaccination rate is determined by the number of individuals who get the *Hib* vaccine divided by the number of susceptible individuals, the birth rate is determined by the number of births divided by the population, the death rate is determined by the CDR (Crude Death Rate) by sourced World Bank Data **[18]**, the contact rate is determined by the number of new cases that occur, the vaccine failure rate is determined based on information from the Centre for Disease and Prevention (CDC) **[17]**, the disease mortality rate is determined by the number of deaths due to pneumonia divided by the number of infected individuals, the cure rate is determined by the number of recovered individuals divided by the number of infected individuals, and the exclusive breastfeeding rate is determined by the number of exclusively breastfeed babies divided by the number of births that occur. **Table 1** displays estimated parameter values.

Table 1. Eight-Parameter Value Estimation

Parameter	Notation	Estimation value
Birth rate	θ	0.018282
Vaccination rate	α	0.017038
Contact rate	β	1.083444
Vaccine failure rate	δ	0.050000
Exclusive breastfeeding rate	σ	0.438852
Death rate due to disease	η	0.001379
Cure rate	γ	0.998621
Natural death rate	μ	0.007511

Table 1 shows that the estimated exclusive breastfeeding rate is $0.438852 \approx 0.4$, which means that out of 10 babies born, 4 are exclusively breastfed. The contact rate has an estimated value of 1.083444, which means that out of 1,000,000 infected individuals can transmit the disease to 1,083,444 other individuals. The parameter values in **Table 1** were then substituted into **Equation (4)**. Thus, we obtained the $SVIR$ model with the effect of exclusive breastfeeding on the spread of pneumonia in Indonesia, namely

$$\begin{aligned}
\frac{dS}{dt} &= 0{,}010259N - 1{,}083444\frac{SI}{N} - 0{,}017038S \\
\frac{dV}{dt} &= 0{,}017038S - 0{,}050000\frac{VI}{N} - 0{,}007511V + 0{,}008022N \\
\frac{dI}{dt} &= 0{,}050000\frac{VI}{N} + 1{,}083444\frac{SI}{N} - 0{,}007511I \\
\frac{dR}{dt} &= 0{,}998621I - 0{,}007511R
\end{aligned} \tag{5}$$

Equation (5) is a system of first-order nonlinear differential equations. **Equation (3)** is given an initial value that refers to pneumonia disease data in 2013 sourced from the Data and Information Center of the Indonesian Ministry of Health **[14]**, namely

$$S(0) = 247058159, V(0) = 22847, I(0) = 571547, R(0) = 564773 \tag{6}$$

The fourth-order Runge-Kutta method is used to solve a system of initial value problems in **Equation (5)** and **Equation (6)**. Based on the solution of the model and data for 2021-2022, the *MAPE* value is 15%. According to Lewis' criteria **[20]**, if the *MAPE* value is between 10%-20%, **Equation (5)** that explains the spread of pneumonia in Indonesia is accurate. The number of individuals in groups $S, V, I,$ and R from 2013 to 2030 is explained by the model solution, as seen by the scatter plots in **Figure 3** and **Figure 4**.

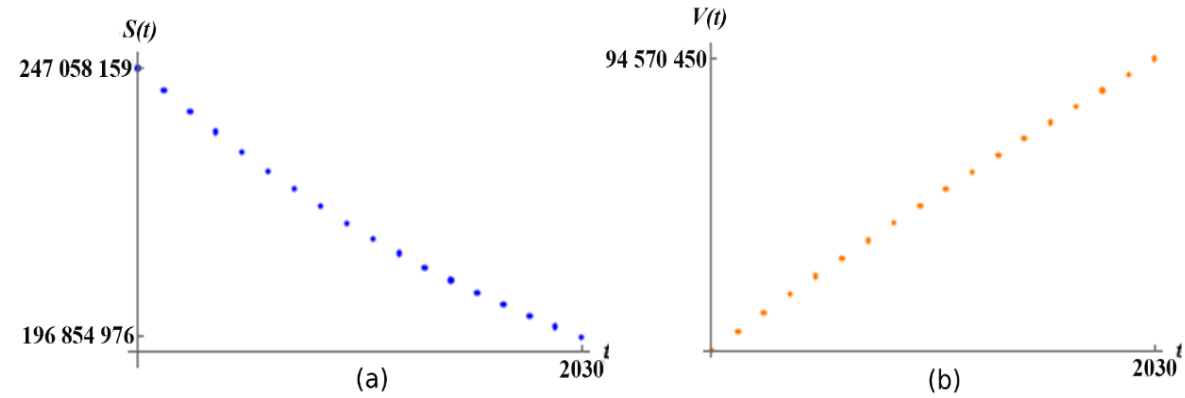

Figure 3. (a) Scatter Plot of the Number of *S* Individuals In 2013-2030, (b) Scatter Plot of the Number of *V* Individuals In 2013-2030

Based on **Figure 3** (a), it can be seen that the number of group *S* individuals decreased annually until 2030, from 247,058,159 individuals to 196,854,976 individuals. This occurs due to exclusive breastfeeding, which results in newborns who are exclusively breastfed being categorised as group *V* individuals. In addition, the vaccination resulted in S individuals who had been vaccinated being categorised as *V* individuals. Therefore, the number of individuals in group S has decreased because they have moved to other groups of individuals or died. From **Figure 3** (b), it can be seen that the number of group *V* individuals keeps increasing until, in 2030, there are 94,570,450 individuals. This happens because of the vaccination program, which results in the group of *S* individuals who have been vaccinated being categorised as group *V* individuals. In addition, group *V* individuals are less likely to contract pneumonia.

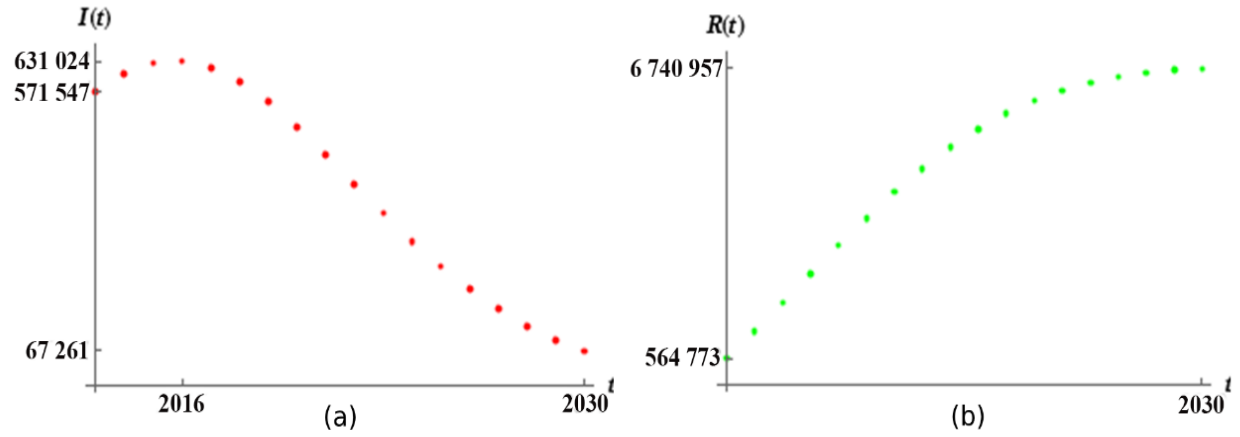

Figure 4. (a) Scatter Plot of the Number of *I* individuals in 2013-2030, (b) Scatter Plot of the Number of *R* Individuals in 2013-2030

Based on **Figure 4** (b), it can be seen that the number of group R individuals keep increasing every year until 2030 from 564,773 individuals to 6,740,957 individuals. This happens because infected individuals can recover and gain immunity so that they become group R individuals. **Figure 4** (a) shows that the number of group I individuals increased until 2017 and gradually decreased in the following year until 2030, when there were still 67,261 individuals. This happens because the vaccination program and the effect of exclusive breastfeeding result in individuals contracting the disease less.

Thus, based on **Figure 3** and **Figure 4**, the spread pattern of pneumonia in Indonesia from 2013 to 2030 shows that the number of S and I experienced a downward trend. On the other hand, the number of individuals V and R experienced an upward trend. Then, based on **Figure 4** (a), it appears that there are still 67,261 individuals infected with pneumonia in Indonesia, so it is suspected that the target of a pneumonia-free Indonesia by 2030 will not be achieved.

3.3 Simulation

The results of the application show that the number of I individuals have a downward trend. However, until 2030, there are still infected individuals, so it is suspected that the pneumonia-free target in Indonesia will not be achieved. The decrease in the number of I individuals can be influenced by exclusive breastfeeding and vaccination programs. Therefore, simulations were conducted on these two things to determine achieving the pneumonia-free target in Indonesia by 2030. The first simulation was performed by increasing the value of the exclusive breastfeeding rate parameter (σ) 0.50-0.70 with increment $\Delta\sigma$=0.05, while the other parameters were fixed (see **Table 1**). The simulation is shown in **Table 2**

Table 2. The Number of *I* Individuals Resulting from Simulation of the σ Parameter Value

Parameter	σ					
Year	**0.438852**	**0.50**	**0.55**	**0.6**	**0.65**	**0.70**
2013	571547	571547	571547	571547	571547	571547
2017	618850	613878	609838	605823	601831	597862
2021	449904	435476	424002	412813	401901	391260
2024	281758	265149	252263	239974	228255	217082
2027	148610	134912	124618	115078	106240	98054
2028	115961	103845	94850	86605	79050	72131
2030	67261	58469	52113	46425	41337	36789

Based on **Table 2**, it is known that by increasing the value of σ from 0.438852 to 0.5, the number of individuals I in 2030 has decreased from 67,261 to 58,469 individuals. Similarly, by increasing the value of σ to 0.70, there is a significant decrease (30.472 individuals) in the number of infected individuals, but there are still 36,789 infected individuals in 2030. So, the target of free pneumonia in Indonesia in 2030 is still not expected to be achieved. The second simulation is done by increasing the value of vaccination rate (α)

0.05-0.25 with increment $\Delta\alpha$ = 0.05, while the other parameters are fixed as in **Table 1**. The number of I individuals resulted from the simulation is shown in **Table 3**

Table 3. The Number of I Individuals Resulting from Simulation of the α Parameter Value

Parameter	α					
Year	**0.017038**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**
2013	571547	571547	571547	571547	571547	571547
2017	618850	494845	359893	268019	204090	158692
2021	449904	192857	62803	24376	11014	5665
2024	281758	62433	9417	2128	665	267
2027	148610	15035	986	134	31	10
2028	115961	8833	437	50	11	3
2030	67261	2824	79	7	1	0

Table 3 shows that increasing the value of the vaccination rate parameter (α) can result in a decrease in the number of infected individuals. It can be seen that by increasing the value of the vaccination rate α to 0.1 and 0.2 the number of infected individuals drops to 79 and 1 individuals in 2030. **Table 3** also shows that by increasing the vaccination rate to 0.25 in 2030, it is predicted that there will be no individuals infected with pneumonia. So, with σ = 0,438852 and α = 0,25 it is estimated that the target of free pneumonia in Indonesia 2030 will be achieved.

The third simulations were conducted by increasing the value of the parameter vaccination rate (α) 0.2-0.22 with increment $\Delta\alpha$ = 0.005 and σ = 0.739, with other parameters fixed. The σ parameter value of 0.739 was taken based on Indonesia's exclusive breastfeeding target of 73.97% of infants under 6 months of age in 2023 (Badan Pusat Statistik Indonesia **[21]**). The simulation results are shown in **Figure 5** or **Table 4**.

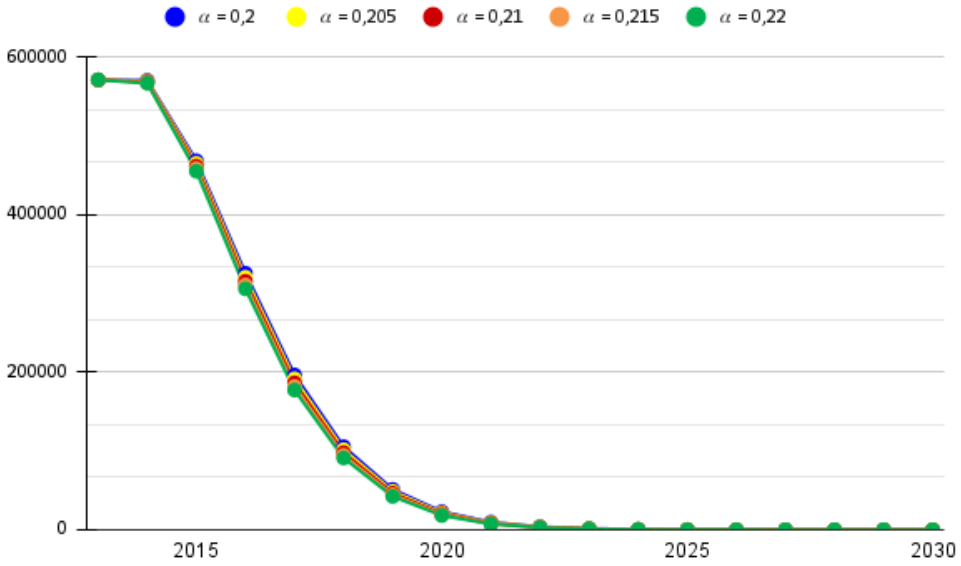

Figure 5. Scatter Plot Simulation of the Number of I Individuals in 2013-2030 with $\sigma = 0.739$ and the Variation of α Value Variation

Table 4. The Number of I Individuals Resulting from Simulation with $\sigma = 0,739$ and the Variation of α Parameter Value

Parameter	α				
Year	**0.2**	**0.205**	**0.21**	**0.215**	**0.22**
2013	571547	571547	571547	571547	571547
2017	196692	191643	186763	182045	177482
2021	9610	8949	8345	7790	7282
2024	525	475	431	392	358
2027	22	19	17	15	14
2028	7	6	6	5	4
2030	1	1	1	1	0

Based on **Figure 5** and **Table 4**, it appears that vaccination and exclusive breastfeeding can reduce the number of infected individuals. By increasing the value of the vaccination rate (α) and exclusive breastfeeding rate (σ), the decline can be faster. An increase σ and α together so that $\alpha = 0{,}22$ and $\sigma = 0.739$, resulting in zero in the number of infected individuals in 2030. Thus, by increasing the exclusive breastfeeding rate value to 73.9% and vaccination rate to 0.22, it is predicted that the target of free pneumonia in Indonesia by 2030 can be achieved.

4. CONCLUSIONS

The $SVIR$ model with the exclusive breastfeeding on the spread of infectious diseases obtained is the system in **Equation (4)**, the system of first-order nonlinear differential equations with the dependent variable t and independent variables *S, V, I, R*. The model is applied to the spread of pneumonia in Indonesia based on annual pneumonia data from 2013 to 2022 to obtain the $SVIR$ model with exclusive breastfeeding on the spread of pneumonia in Indonesia. The *MAPE* value of the model in **Equation (5)** is 15%. This means that the model accurately explains the spread of pneumonia in Indonesia. The solution of the model showed that the spread pattern of pneumonia in Indonesia from 2013 to 2030 experienced a downward trend. However, by 2030, there are still 67.261 infected individuals, so the target of free pneumonia in Indonesia has not been achieved yet. Simulation shows that with $\sigma = 0.438852$ and $\alpha = 0.25$, it is estimated that the target of free pneumonia in Indonesia in 2030 will be achieved. The simulation also shows that by increasing the exclusive breastfeeding rate value to 73.9% and the *Hib* vaccination rate to 0.22, it is predicted that the target of free pneumonia in Indonesia by 2030 can be achieved.

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