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MODELING CUSTOMER LIFETIME VALUE WITH MARKOV CHAIN IN THE INSURANCE INDUSTRY

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ABSTRACT

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In the competitive insurance industry, accurately predicting Customer Lifetime Value (CLV) is vital for sustaining long-term profitability and optimizing resource allocation. Traditional static models often fail to capture the dynamic and uncertain nature of customer behavior, which is influenced by factors such as life changes, economic conditions, and evolving product offerings. To address these limitations, this paper proposes an advanced modeling approach that integrates Markov Chains with survival analysis. Markov Chains are well-suited for modeling stochastic processes, where future states depend on current conditions, while survival analysis provides insights into event timing and likelihood for estimating the insurance premium. The proposed model combines these approaches to make a more complete and accurate prediction of CLV. This helps insurers make better decisions and improves the overall performance of their business. We employ the data of customer behavior from the insurance company in Bandung, Indonesia from 1994 to 2020. We found that CLV in the insurance industry is significantly affected by customer behavior.

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1. INTRODUCTION

In the competitive and complex environment of the insurance industry, accurately understanding and predicting customer behavior is crucial for sustaining long-term profitability [1], [2]. Central to this is the concept of Customer Lifetime Value (CLV), which estimates the total expected revenue a company can generate from a customer over the duration of their relationship. CLV provides a forward-looking estimate that incoIDRorates the customer's current value and their likely future behavior, including the duration of the relationship, frequency of engagement, and spending patterns. By offering insights into future customer profitability, CLV enables insurers to optimize marketing efforts by focusing on the most valuable customer segments. Resources can be allocated to marketing strategies that appeal to these customers, with tailored campaigns and targeted promotions designed to foster long-term engagement. Additionally, improving customer retention strategies becomes more efficient, as CLV analysis can reveal which customers are at risk of leaving. Timely interventions, such as personalized offers or premium services, can be deployed to enhance customer satisfaction and loyalty [3], [4].

Traditional approaches to calculating CLV often employ static models that do not fully capture the dynamic and uncertain nature of customer behavior [5]. These models typically fail to consider the various factors influencing customer interactions with insurers, such as changes in life circumstances, economic conditions, and product offerings. Consequently, there is a growing need for a more sophisticated model that can account for the evolving nature of customer relationships [6], [7], [8]. One approach to solve this problem is the application of Markov Chain, which is particularly effective in representing stochastic processes where the future state of a system depends solely on its current state [9], [10], [11]. This makes Markov Chains well-suited for modeling CLV in the insurance industry, where customer transitions between different states (e.g., active, lapsed, reactivated) can be probabilistically determined. However, to enhance the robustness and accuracy of CLV models in the insurance industry, it is essential to recognize the inherent similarities between CLV and survival analysis.

Survival analysis and CLV are conceptually similar in that both are concerned with the timing and likelihood of specific events. In survival analysis, the focus is on the time until an event, for example, to predict engine failures in mechanical components [12], to analyze the tipping point of some process [13], [14], [15], and to assess the failure condition of the ecosystem [16], [17], [18], [19], [20]. Similarly, CLV can be seen as a function of the survival probability of a customer over time, where the survival of the customer represents their continued engagement with the insurer.

By leveraging and combining the principles of survival analysis within a Markov Chain framework, it is possible to develop a more comprehensive and accurate model for predicting CLV in the insurance industry. This integrated approach not only accounts for the likelihood and timing of customer transitions but also provides a more realistic estimation of the long-term value of customers, taking into consideration the various factors that influence their behavior over time.

2. RESEARCH METHODS

2.1 Model Formulation

The fundamental idea behind calculating Customer Lifetime Value (CLV) relies on the net present value generated from customer transactions over their entire relationship with the company. The typical formulation of CLV using net present value is as follows:

$$CLV = \sum_{t=0}^{T} \left(\frac{1}{1+i}\right)^{t} \alpha(t) \tag{1}$$

where i is interest rates, T is the length of transaction time, and $\alpha(t)$ is the profit contributed by the customer at time t.

The Markov Chain model was integrated into the CLV calculation to account for the probability of a customer transitioning to another state or purchasing a different insurance product. For a stochastic system to be classified as a Markov Chain, it must satisfy the Markov property. Mathematically, the Markov property states that [21]

$$P_{ij} = P\{X_{t+1} = j | X_0 = i_0 \ldots, X_{t-1} = i_{t-1}, X_t = i\} = P\{X_{t+1} = j | X_t = i\} \tag{2}$$

for all time points t and all states $i_0, \ldots, i_{t-1}, i, j$. The CLV based on the Markov Chain model offers a significant advantage due to its flexibility in capturing a wide range of customer behavior. Additionally, as a probability model, the Markov Chain can account for the uncertainty that arises during the transaction period. Customer behavior, or the probability of a customer transitioning from one state to another, can be effectively represented in a transition probability matrix of the customer. In this model, the transition probability matrix of the customer is constant throughout the transaction period.

Assume every time t, from t_0 to T, there is a reward vector R that records all transaction between the company and the customer

$$\vec{R} = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix} = \begin{pmatrix} NC_1 - M_1 \\ NC_2 - M_2 \\ \vdots \\ NC_n - M_n \end{pmatrix} \tag{3}$$

where $\vec{R}$ is the reward vector, $NC_1, NC_2, \ldots, NC_n$ are the customer net contributions, $M_1, M_2, \cdots M_n$ are the present value of marketing costs, and n is the number of states. The first line in the vector reward $\vec{R}$ associated with the first state of the customer, the second line associated with the second state of the customers, and the n-th line is associated with the n-th state of the customer. By incoIDRorating the transition probability matrix of the customer and reward vector into the CLV formulation in Equation (1), we get

$$CLV_T = \vec{R} + (vC)^1\vec{R} + (vC)^2\vec{R} + \cdots + (vC)^T\vec{R} = \textstyle\sum_{t=0}^{T}(vC)^t\vec{R} \tag{4}$$

where $v = \frac{1}{1+i}$ is the discount factor, $C_{n\,x\,n} = \begin{pmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{pmatrix}$ is the transition probability matrix of the customer, and $\vec{R}$ is the reward vector.

In the insurance industry, the net contributions NC from the customer depend on their expected survival time. This can be calculated through fully discrete n-year terms of insurance gross premium that is paid by the customer [22], [23], [24]. Define gross future loss random variable:

$$L_0^g = \text{PV of benefit outgo} + \text{PV of expenses} - \text{PV of gross premium income.} \tag{5}$$

Under the equivalence principle, premiums are determined such that, on average, the insurer does not gain or lose financially from issuing the policy. That means that $E[L_0^g] = 0$, which is expected present value of benefit outgo + expected present value of expenses – expected present value of gross premium income = 0. For fully discrete n-year term insurance with the death benefit b and premium Z, the expected future loss random variable can be written as

$$E[L_0^g] = bA_{x^1:\overline{n|}} - Z\ddot{a}_{x:\overline{n|}} + \text{PV of expenses} \tag{6}$$

$$A_{x^1:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1}\ {}_kp_x q_{x+k} \tag{7}$$

$$\ddot{a}_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^{k+1}\ {}_kp_x \tag{8}$$

where ${}_kp_x$ and ${}_kq_x$ are probability of (x) survives to age $x+k$ and probability of (x) dies before age $x+k$ respectively. Benefit premium under equivalence principle

$$Z = \frac{bA_{x^1:\overline{n|}} + \text{PV of expenses}}{\ddot{a}_{x:\overline{n|}}} \tag{9}$$

In order to maximize the profit, insurance companies divide the net contribution NC into two assets: risk-free assets and risky assets (Figure 1). The key difference between these two asset types is the amount of return. Risk-free assets offer a smaller but stable return, characterized by low variance, while risky assets provide a higher but more volatile return, characterized by high variance. **Table 1** summarizes the rule of NC allocation that is used to calculate CLV. This rule is obtained from a case study in one of the private life insurance companies in Bandung, Indonesia.

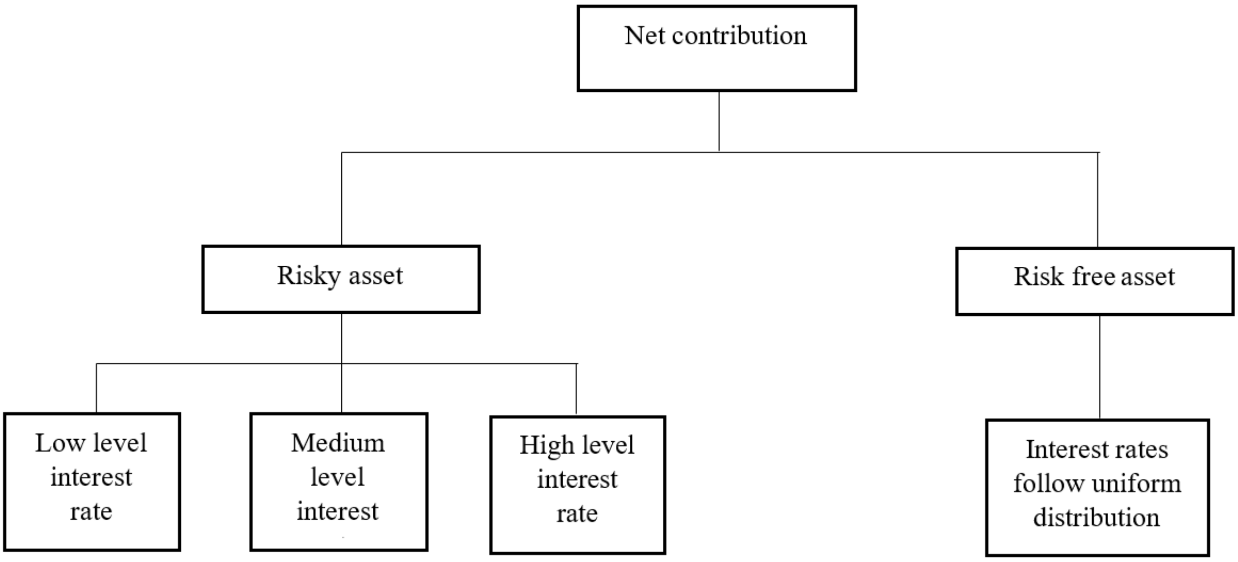


Figure 1. The Schematic Illustration of The Allocation of Net Contribution NC Employed in This Model. NC is Divided into Risky Assets and Risk -Free Assets to Optimize the Company Profit.

The allocation strategy in **Table 1** shifts from conservative (risk-free focus) to aggressive (risky asset focus) over time, often based on the principle that the variance of returns over longer time horizons becomes more tolerable due to the ability to cancel out fluctuations through longer-term growth. The insurance companies typically prefer greater security early in the investment period when they are more risk-averse, shifting to more risk-seeking behavior as the investment horizon lengthens. Moreover, they can tolerate more risk because long-term investments are expected to recover from short-term market volatility.

Table 1. The Proportion of Net Contribution NC Based on the Time of Transaction.

Year	Net Contribution at Risk Free Asset	Net Contribution at Risky Asset
1	100%	0%
2	55%	45%
3	10%	90%
4	10%	90%
5	10%	90%
6	0%	100%
⋮	0%	100%
j	0%	100%

The interest rate on the risk-free asset is assumed to follow a uniform distribution with the value between 6.5% and 7.5% each year. Contrastingly, risky asset interest rates are divided into several levels: low-level interest rates, medium-level interest rates, and high-level interest rates, depending on economic conditions. **Table 2** summarizes the interest rate of a risky asset.

Table 2. The Interest Rate of Risky Assets, Consists of Equity Funds, Corporate Fixed Income, Sharia Equity Funds, and Money Market Funds.

Risky Asset	Interest rate		
	Low (%)	Medium (%)	High (%)
Equity funds	5%	10%	14%
Corporate fixed income	7%	10%	12%
Sharia equity funds	6%	10%	13%
Money market funds	6%	7%	8%

The algorithm to calculate CLV in the insurance industry with Markov Chain is presented in **Figure 2**. We incoIDRorate the influence of risk-free and risky assets to better capture the real case scenario of fund allocation in the insurance industry.

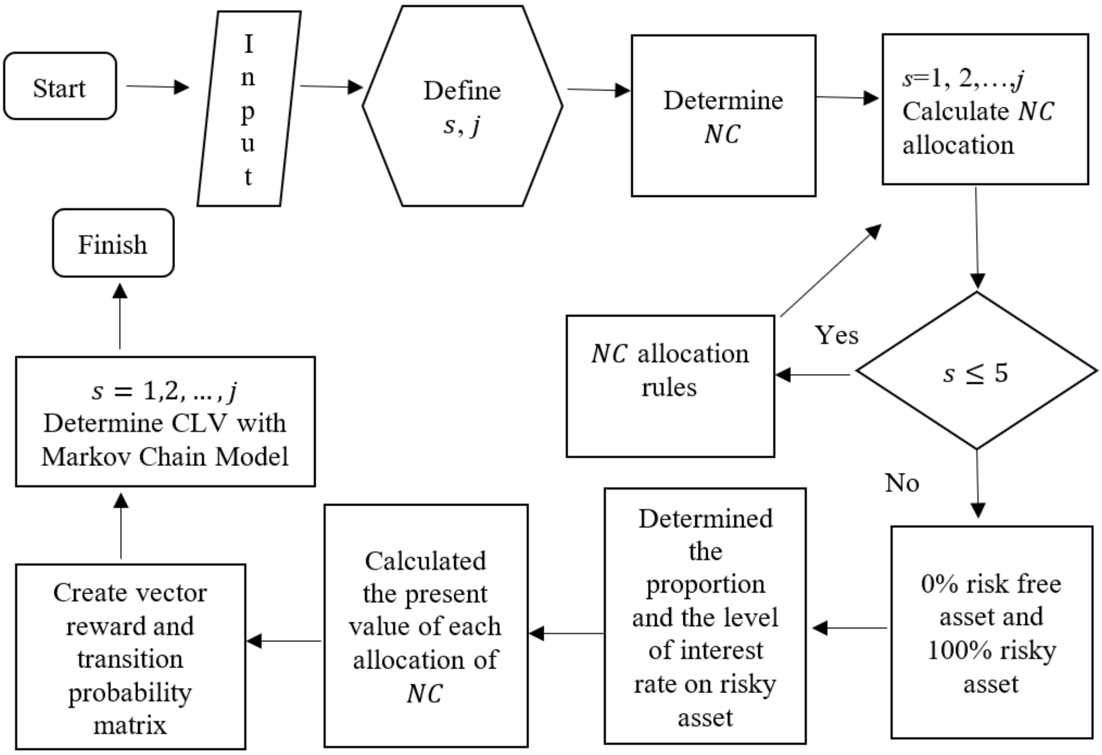


Figure 2. **The Flow Chart to Calculate CLV in The Insurance Industry with Markov Chain.**

The main puIDRose of CLV calculations is to track how the value of a customer grows or declines over time. This is done by considering the income or profit generated from the customer during different periods. Since money received today is worth more than the same amount received in the future (due to interest rates, inflation, and opportunity cost), future revenues are discounted to reflect their present value. The discount rate can vary based on whether the revenue is considered risk-free or risky. If we assume that the insurance company uses two risky assets, the mathematical formulation to calculate CLV is

$$CLV_0 = \vec{R} \tag{10}$$

$$CLV_1 = CLV_0 + \left(\frac{1}{1+i_{rf}}C\right)^1 \vec{R} \tag{11}$$

$$CLV_2 = CLV_1 + \left(\frac{1}{1+i_{rf}}C\right)^2 (0.55\vec{R}) + \left(\frac{1}{1+i_{rk_1}}C\right)^2 (0.45a\vec{R}) + \left(\frac{1}{1+i_{rk_2}}C\right)^2 (0.45(1-a)\vec{R}) \tag{12}$$

$$CLV_3 = CLV_2 + \left(\frac{1}{1+i_{rf}}C\right)^3 (0.1\vec{R}) + \left(\frac{1}{1+i_{rk_1}}C\right)^3 (0.9a\vec{R}) + \left(\frac{1}{1+i_{rk_2}}C\right)^3 (0.9(1-a)\vec{R}) \tag{13}$$

$$CLV_4 = CLV_3 + \left(\frac{1}{1+i_{rf}}C\right)^4 (0.1\vec{R}) + \left(\frac{1}{1+i_{rk_1}}C\right)^4 (0.9a\vec{R}) + \left(\frac{1}{1+i_{rk_2}}C\right)^4 (0.9(1-a)\vec{R}) \tag{14}$$

$$CLV_5 = CLV_4 + \left(\frac{1}{1+i_{rf}}C\right)^5 (0.1\vec{R}) + \left(\frac{1}{1+i_{rk_1}}C\right)^5 (0.9a\vec{R}) + \left(\frac{1}{1+i_{rk_2}}C\right)^5 (0.9(1-a)\vec{R}) \tag{15}$$

$$CLV_6 = CLV_5 + \left(\frac{1}{1+i_{rk_1}}C\right)^6 (a\vec{R}) + \left(\frac{1}{1+i_{rk_2}}C\right)^6 \left((1-a)\vec{R}\right) \tag{16}$$

$$\vdots$$

$$CLV_j = CLV_{j-1} + \left(\frac{1}{1+i_{rk_1}}C\right)^j (a\vec{R}) + \left(\frac{1}{1+i_{rk_2}}C\right)^j \left((1-a)\vec{R}\right), \text{ for } j > 6 \tag{17}$$

where i_{rf} is the interest rate of risk-free asset, i_{rk_1} is the interest rate of first choice risky asset, i_{rk_2} is the interest rate of second choice risky asset, a is the proportion of risky asset, $\vec{R}$ is the reward vector, and C is the transition probability matrix of the customer.

2.2 Data

We employ data customers from health insurance company in Bandung, Indonesia, from 1994 to 2020. Let's say the name of the customer is Olivia, female, 54 years old. Table 3 summarizes the state of Olivia with '0', '1', '2', indicate not insured, Class 1 low-level insurance contract with the benefit of IDR 500,000,000, and Class 2 high-level insurance contract with the benefit of IDR 1,000,000,000, respectively.

Table 3. The Olivia Data During the Transaction Period with the Insurance Company From 1994 until 2020.

1994	1995	1996	1997	1998	1999	2000	2001	2002
1	1	1	1	1	1	1	1	1
2003	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
1	1	1	1	1	1	1	1	1
2012	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**
1	1	2	2	2	2	2	0	0

Two important factors in calculating CLV using Markov Chain model are determining the transition probability matrix and determining the rewards vector $\vec{R}$. The transition probability matrix from Olivia data is $P_O = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0 & 0.8 & 0.2 \\ 0 & 0 & 1 \end{pmatrix}$. This matrix is obtained by calculating the relative frequency of Olivia states. Each element in the matrix represents the proportion of time Olivia is observed in a specific state relative to the total observations. This calculation involves dividing the frequency of each state by the total occurrences, resulting in a normalized representation of the state distribution. Furthermore, to determine the rewards vector $\vec{R}$, we must record all transactions between the company and Olivia as summarized in Table 4.

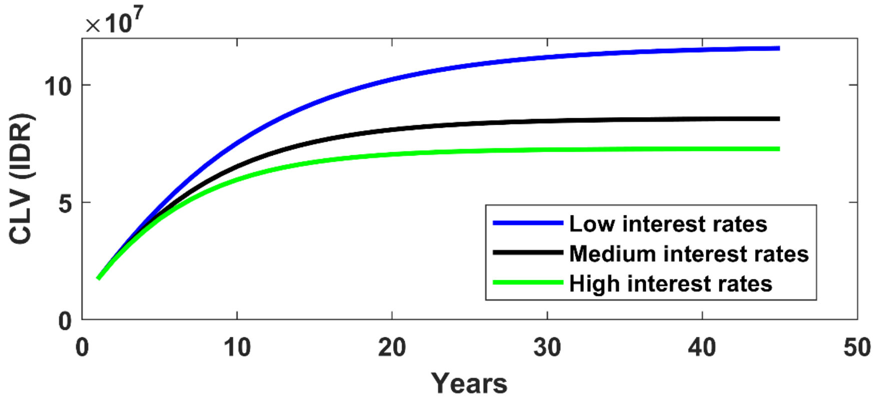
Table 4. The Transaction Data Between Olivia and the Insurance Company, Including Premiums, Marketing Costs, Agency Fees, and Admin Fees.

Income for the Insurance Company	Expenditure for Insurance Company
Premium	Marketing costs IDR 250,000 Agency fees10% of the premium Admin fees IDR 50,000

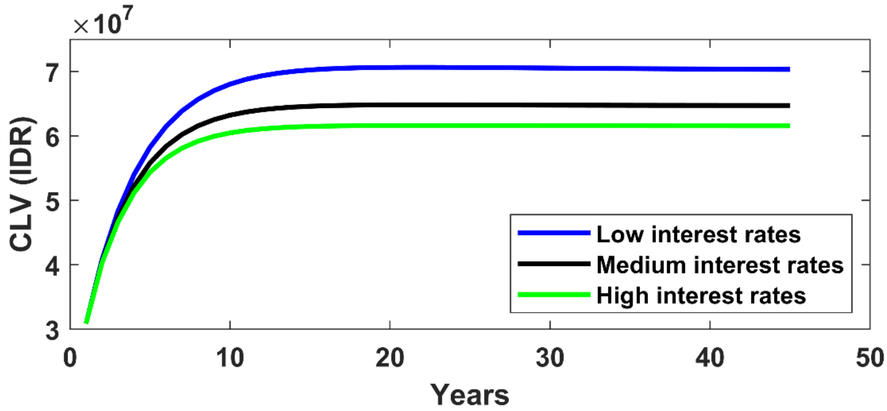
The premium is the main source of income for an insurance company as shown in Table 4. It represents the amount the policyholder pays to the company to cover the insured risk. The premium is calculated based on the class of insurance contract and the benefit. Expenditures for the insurance company include marketing costs, agency fees, and administrative fees. The company incurs IDR 250,000 as a fixed marketing cost for promoting and selling the insurance policy. The insurance company pays 10% of the premium as agency fees to the agents or brokers who help sell the insurance. The company incurs IDR 50,000 in administrative fees, which involve policy processing, customer service, and other administrative tasks associated with issuing and maintaining the insurance policy.

3. RESULTS AND DISCUSSION

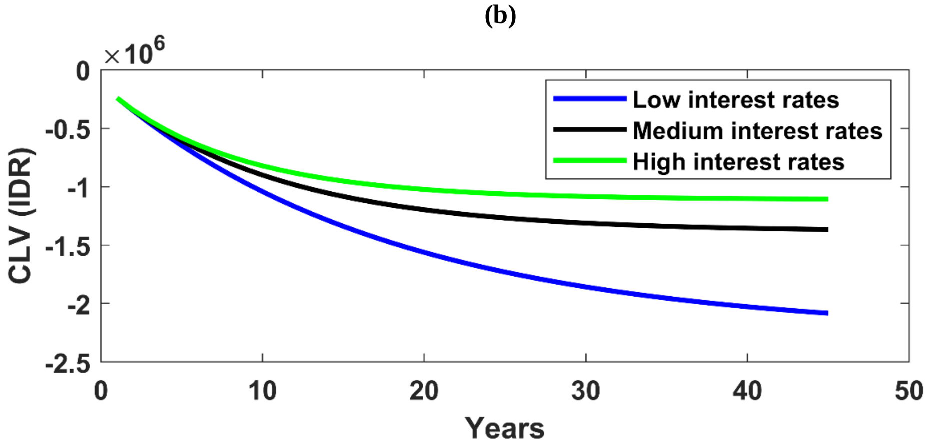
To explain the CLV results using the Markov Chain model, we consider a transaction period of 45 years with an asset allocation of 50% Equity Funds and 50% Corporate Fixed Income. Figure 3(a) and Figure 3(b) illustrate the CLV for three different interest rate levels in the first and second states, where Olivia is classified as a Class 1 and Class 2 customer, respectively. As shown in Figure 3(a) and Figure 3(b), the highest CLV is achieved when the interest rate is low. Figure 3(c) presents the CLV for three interest rate levels in the third state, where Olivia is a former customer. In this scenario, the CLV is negative, indicating a loss for the company, as it incurs continuous marketing costs for Olivia. To minimize this loss, the company could impose a maximum time limit for Olivia's presence in the third state. If Olivia remains in the third state beyond this limit, the company may decide to terminate the relationship. This analysis enables the company to develop effective marketing strategies to maximize Olivia CLV and, consequently, increase overall profitability.



(a)



(b)



(c)

Figure 3. The Olivia CLV Over 45 Years When She Buys (a) Class 1 Low-Level Insurance Contract, (b) Class 2 High-Level Insurance Contract, and (c) Olivia Becomes Former Customer.

Table 5. **The Olivia CLV for the First State in the Last Five Years**

Year	CLV First State		
	Low Interest Rate	Medium Interest Rate	High Interest Rate
41	IDR 115,153,035	IDR 85,643,965	IDR 72,738,024
42	IDR 115,301,568	IDR 85,672,948	IDR 72,748,056
43	IDR 115,434,626	IDR 85,697,875	IDR 72,756,482
44	IDR 115,553,800	IDR 85,719,310	IDR 72,763,559
45	IDR 115,660,519	IDR 85,737,737	IDR 72,769,500

Table 6. **The Olivia CLV for the Second State in the Last Five Years**

Year	CLV Second State		
	Low Interest Rate	Medium Interest Rate	High Interest Rate
41	IDR 70,364,421	IDR 64,846,300	IDR 61,535,930
42	IDR 70,352,865	IDR 64,844,045	IDR 61,535,150
43	IDR 70,341,896	IDR 64,841,990	IDR 61,534,455
44	IDR 70,331,490	IDR 64,840,118	IDR 61,533,837
45	IDR 70,321,620	IDR 64,838,414	IDR 61,533,288

Table 7. **The Olivia CLV for the third state in the last five years.**

Year	CLV Third State		
	Low Interest Rate	Medium Interest Rate	High Interest Rate
41	IDR -2,038,897	IDR -1,359,546	IDR -1,102,732
42	IDR -2,050,595	IDR -1,361,829	IDR -1,103,522
43	IDR -2,061,671	IDR -1,363,904	IDR -1,104,224
44	IDR -2,072,159	IDR -1,365,790	IDR -1,104,846
45	IDR -2,082,091	IDR -1,367,505	IDR -1,105,399

From **Figure 3**(a) and **Figure 3**(b), it is observed that if the transaction period is less than 5 years, the CLV for the second state is higher than for the first state. However, over a longer period, the CLV for the first state suIDRasses that of the second state, even though the second state involves Olivia purchasing a higher-level insurance contract. This occurs because, if Olivia selects a low-level insurance contract, there is a 95% probability that she will continue with this contract and a 5% probability that she will upgrade to a higher-level contract. Conversely, if Olivia chooses a high-level insurance contract, there is an 80% probability that she will renew it and a 20% probability that she will not. As a result, over the long term, the CLV for the first state eventually exceeds that of the second state.

Table 5, **Table 6**, and **Table 7** summarize Olivia CLV over the last five years. CLV is a fundamental concept in customer relationship management (CRM), providing a forward-looking measure of the total financial contribution a customer is expected to bring over their lifetime with the insurance company. It allows insurance companies not only to assess profitability on a customer-by-customer basis but also to develop targeted strategies to maximize the value derived from each customer relationship. By focusing on the CLV of Olivia, the company gains insights into how different customer states affect long-term profitability and, more importantly, how strategic interventions can optimize this value. From **Table 5** and **Table 6**, it is evident that maintaining the relationship with Olivia has resulted in consistent profits, particularly in the first and second states. In customer lifecycle theory, these states often represent high engagement and positive interactions, which contribute to increased customer satisfaction and loyalty. Satisfied and loyal customers are less sensitive to price changes, more likely to make repeat purchases, and are often more receptive to cross-selling and upselling efforts. In this context, Olivia's continued profitability in these states reinforces the importance of nurturing customer relationships to sustain long-term value.

However, **Table 7** highlights a potential challenge if Olivia transitions to the third state, a stage that may signal disengagement or lower interaction levels with the company. This is consistent with customer lifecycle models where customer engagement often fluctuates, requiring companies to implement retention strategies. In this state, Olivia's profitability decreases, which might be a result of reduced purchases, increased service costs, or competitive pressures that make it harder to maintain her loyalty. The costs associated with re-engaging customers in declining states can outweigh the benefits if appropriate actions are not taken swiftly. To prevent a decline in CLV in the third state, the company may need to employ specific marketing strategies tailored to Olivia's needs and behavior. Personalized marketing, for instance, has been shown to increase customer retention by addressing individual preferences and needs. Furthermore, targeted loyalty programs and customer engagement campaigns can enhance the customer experience, reigniting interest and reversing potential profit declines. Companies can also leverage predictive analytics to anticipate customer state transitions and proactively offer solutions or incentives before significant losses occur. Therefore, while Olivia's profitability is sustained in her earlier states, her transition to the third state presents a risk to the company overall profit margins. This highlights the importance of state-based CLV models in guiding marketing and retention strategies, ensuring that insurance companies not only maintain relationships with high-value customers but also effectively manage transitions that could lead to customer churn. Ultimately, understanding and managing CLV dynamics across different customer states allows companies to adopt a more strategic, data-driven approach to optimizing long-term profitability.

4. CONCLUSIONS

Accurately predicting Customer Lifetime Value (CLV) is essential for the insurance industry to sustain profitability and optimize business strategies. Traditional static models fall short in capturing the dynamic nature of customer behavior, necessitating more advanced approaches. The integration of Markov Chains with survival analysis presents a promising solution, offering a robust framework to model the stochastic and evolving interactions between customers and insurers. The robustness of this approach is ensured by its ability to incoIDRorate a wide range of time-dependent factors and variability in customer behavior. By adopting this integrated approach, insurers can achieve more precise and realistic estimations of CLV, leading to better-informed decisions and improved long-term customer value management. Based on customer data from the insurance company in Bandung, Indonesia, we found that the behavior of customers, represented in the transition probability matrix, has a significant influence on the CLV prediction.

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