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SOLUTION OF FULLY FUZZY NONLINEAR EQUATION SYSTEMS USING GENETIC ALGORITHM

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ABSTRACT

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A system of nonlinear equations is a collection of several interrelated non-linear equations. Currently, systems of nonlinear equations are used not only on crisp but also on fuzzy numbers. A fuzzy number is an ordered pair function that has a degree of membership [0,1]. Meanwhile, a fully fuzzy system of equations is a system of equations that applies fuzzy number arithmetic operations. The solution of non-linear equation systems is usually complicated to solve analytically, so numerical methods are used as an alternative to solve these problems. In this research, the steps to find the solution of nonlinear fully fuzzy equation systems using genetic algorithms are studied, which in the solution process is based on the theory of evolution and natural selection. The solution steps taken are first converting the fully fuzzy system of equations into a system of crisp equations, next constructing the system of strict equations as a multi-objective optimization problem, and lastly solving the optimization problem using a genetic algorithm which includes initialization, evaluation, selection, crossover, and mutation. As illustrations, several cases of nonlinear fully fuzzy and dual fully fuzzy systems of equations on triangular fuzzy numbers and trapezoidal fuzzy numbers are given. The approximate solutions obtained using genetic algorithms produce solutions that are close to their analytic solutions.

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1. INTRODUCTION

The system of non-linear equations is one part of linear algebra usually used to solve real problems, and it is a collection of several interrelated nonlinear equations. Solving a system of non-linear equations is finding a solution to a system of non-linear equations $f(x)$ such that every equation in the system of equations is 0 [1]. This root value can be found by using analytical methods, but in reality, the solution of this system is usually difficult to solve analytically, so numerical methods are used to find solutions to these problems. Currently, the values of the independent variables in the fully fuzzy equation system can be crisp or fuzzy numbers in practice [2]. A fuzzy number is a generalization of a regular real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1 [3]. Fuzzy numbers are an ordered pair function that has a degree of membership in the interval [0,1] [4]. In general, fuzzy numbers based on the representation of their membership functions are divided into two, namely triangular fuzzy numbers and trapezoidal fuzzy numbers. In addition, fuzzy is also known as a non-linear fully fuzzy equation system. The nonlinear fully fuzzy equation system is an equation system that applies arithmetic operations to fuzzy numbers that are different from arithmetic operations on real numbers.

The solution of nonlinear fully fuzzy systems of equations is usually solved by numerical methods such as Newton's method, gradient method, steepest descent, and others. However, in reality, these methods require differentiability of the objective function. Genetic Algorithm (GA) can be used to solve this problem, as it does not require differentiability in the solution [5]. GA is a search algorithm based on the principles of natural selection and genetics, introduced by J. Holland in the 1970s and inspired by the biological evolution of living things [6]. Some applications of GA in various fields can be found, for example, in [7], [8], and [9]. Simarmata and Mone used GA to solve a problem in object routing shortest tour [7]. In another field, Abyan et al. have proposed the application of GA to solve optimization problems in the design of propellant grain on a rocket [8]. In [9], the authors used GA to estimate maximum-likelihood weighted logistic regression for data on work status in Malang.

There have been many studies related to fuzzy and genetic algorithms. Lu & Fang in [10] solved nonlinear optimization problems with fuzzy relation equation constraints using Genetic Algorithms (GA), where this GA is designed to be domain-specific by taking advantage of the structure of the solution set of fuzzy relation equations. In [11], Mashinchi et al. conducted a genetic algorithm approach to solve linear and quadratic fuzzy equations on triangular fuzzy numbers with the form $\tilde{A}\tilde{x} = \tilde{B}$ dan $\tilde{A}\tilde{x}^2 + \tilde{B}\tilde{x} = \tilde{C}$, where $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{x}$ is fuzzy numbers. Kumar et al. [12] solved the system of fully fuzzy linear equations on trapezoidal fuzzy numbers. In 2012, Guchhait et al. made a production inventory model with fuzzy production and demand using fuzzy differential equations with a genetic algorithm approach that is compared with the interval [13]. Research using the Jacobi iteration method on triangular fuzzy numbers to solve the fully fuzzy linear equation system was conducted by Marzuki & Herawati [2]. Gemawati et al. in [14] used QR decomposition in solving the dual fully fuzzy linear equation system on trapezoidal numbers. In [15], the authors solved a nonlinear optimization problem imposed on fuzzy relational equations defined by the Dubois-Prade family of t-norms using a genetic algorithm. In [16], conducted research related to Network Anomaly Detection systems using Genetic Algorithms and Fuzzy Logic. Mangla et al. in [5] studied the numerical solution of nonlinear equation systems using a genetic algorithm. Deswita & Mashadi, in 2019, conducted research on triangular fuzzy number multiplication and its application to fully fuzzy linear systems [17]. Jafarian & Jafari in [18] conducted research with computational methods to solve fully fuzzy matrix equations on triangular fuzzy numbers. The solution of the nonlinear fully fuzzy equation system on triangular fuzzy numbers using the double Newton Rapshon method has been carried out by Anisa et al. [19]. Zakaria et al. in [20] conducted research to solve the nonlinear dual fully fuzzy matrix equations in triangular fuzzy numbers using the Broyden method. By using the same method, Megarani and Zakaria in [21] have proposed a numerical solution for nonlinear fully fuzzy equation system. In the case of singular dual fuzzy nonlinear equation, Moyi et al. have anticipated it by avoiding the point where the Jacobian is singular [22]. Whereas in [23], the authors used the classical conjugate gradient algorithm for solving intuitionistic fuzzy nonlinear equations, especially on triangular fuzzy numbers. While in the form of parameterized fuzzy nonlinear equations, a modified Newton method [24] and Shamanskii Method [25] have been used to solve it.

Based on the literature search above, no research examines the solution of nonlinear fully fuzzy equation systems using metaheuristic methods, especially genetic algorithms. Therefore, in this article, we will discuss the solution of nonlinear fully fuzzy equation systems on triangular fuzzy numbers and trapezoidal fuzzy numbers using genetic algorithms. Here, we organize this paper as follows. Some definitions that we use are described in Section 2. Next, in Section 3, a procedure to find a solution of nonlinear fully fuzzy equation systems using GA and its application are presented. In the last section, we present our conclusions.

2. RESEARCH METHODS

In this section, we refer to some basic theories or definitions that will be used in this study.

2.1 Fuzzy Set

The characteristic function μ_A of a strictly defined set $A \subseteq X$ assigns a value of 0 or 1 to each member in X. This function can be simplified to a function $\mu_{\tilde{A}}$ such that the value assigned to an element of the universe set X is within a certain range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The given value indicates the degree of membership of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function of the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \epsilon X\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \epsilon X$ is called a fuzzy set [12].

2.2 Fuzzy Numbers

$\tilde{A}$ is a fuzzy number if $\mu_{\tilde{A}} : \mathbb{R} \to I = [0,1]$ and it satisfies the following conditions [11].

a) $\mu_{\tilde{A}}(x)$ is upper semicontinuous,

b) There are real numbers a, b, c and d, with $a \le b \le c \le d$. Where,

(i) $\mu_{\tilde{A}}(x) = 0$ outside the interval $[a, d]$,
(ii) $\mu_{\tilde{A}}(x)$ monotonically increasing on $[a, b]$,
(iii) $\mu_{\tilde{A}}(x)$ monotonically decreasing on $[c, d]$,
(iv) $\mu_{\tilde{A}}(x) = 1$, for $b \le x \le c$.

An arbitrary fuzzy number of the form $\tilde{a} = (m - \alpha, m, m + \beta) = (a_m, a_l, a_u)$ with membership function i.e.

$$\mu_{\tilde{a}} = \begin{cases} 1 - \dfrac{m-x}{\alpha}, m - \alpha \le x \le m \\ 1 - \dfrac{x-m}{\beta}, m \le x \le m + \beta \\ 0, \qquad\qquad other \end{cases}$$

is called a triangular fuzzy number. The graphic representation of the membership function of triangular fuzzy numbers is as follows [20].

A fuzzy number $\tilde{a} = (m, n, \alpha, \beta)$ is called a trapezoidal fuzzy number if its membership function is defined by [12]:

$$\mu_{\tilde{a}} = \begin{cases} 1 - \dfrac{m-x}{\alpha}, & m - \alpha \le x \le m; \alpha > 0 \\ 1, & m \le x \le n \\ 1 - \dfrac{x-n}{\beta}, & n \le x \le n + \beta;\ \beta > 0 \\ 0, & other \end{cases}$$

A trapezoidal fuzzy number $\tilde{a} = (m, n, \alpha, \beta)$ is said to be a non-negative trapezoidal fuzzy number $\tilde{a} \ge 0$ if and only if $m - \alpha \ge 0$, and is said to be a zero trapezoidal fuzzy number if and only if $m = 0$,

$n = 0, \alpha = 0$, and $\beta = 0$. Two fuzzy numbers $\tilde{a} = (m, n, \alpha, \beta)$ and $\tilde{b} = (p, q, \gamma, \delta)$ are said to be equal if and only if $m = p, n = q, \alpha = \gamma$, and $\beta = \delta$ [12].

Suppose given two triangular fuzzy numbers $\tilde{a} = (a_m, a_l, a_u)$ and $\tilde{b} = (b_m, b_l, b_u)$ then [18]

a. $\tilde{a} \oplus \tilde{b} = (a_m + b_m, a_l + b_l, a_u + b_u)$,
b. $-\tilde{a} = (-a_m, -a_l, -a_u)$,
c. $\tilde{a} \ominus \tilde{b} = (a_m - b_u, a_l - b_l, a_u - b_m)$
d. Multiplication on fuzzy numbers is denoted by $\hat{*}$, i.e.

$$\tilde{a} \hat{*} \tilde{b} = (c_m, c_l, c_u)$$

where:

$$c_l = a_l \,.\, b_l$$
$$c_m = min(a_m \,.\, b_m, a_m \,.\, b_u, a_u \,.\, b_m, a_u \,.\, b_u)$$
$$c_u = max(a_m \,.\, b_m, a_m \,.\, b_u, a_u \,.\, b_m, a_u \,.\, b_u)$$

If $\tilde{a}$ is any triangular fuzzy number and $\tilde{b}$ is non-negative, then:

$$\tilde{a} \hat{*} \tilde{b} = \begin{cases} (a_m \,.\, b_m, a_l \,.\, b_l, a_u \,.\, b_u), & a_m \geq 0 \\ (a_m \,.\, b_u, a_l \,.\, b_l, a_u \,.\, b_u), & a_m < 0, a_u \geq 0 \\ (a_m \,.\, b_m, a_l \,.\, b_l, a_u \,.\, b_m), & a_m < 0, a_u < 0 \end{cases}$$

Suppose two trapezoidal fuzzy numbers $\tilde{a} = (m, n, \alpha, \beta)$ and $\tilde{b} = (p, q, \gamma, \delta)$ then [14]

a. $\tilde{a} \oplus \tilde{b} = (m + p, n + p, \alpha + \gamma, \beta + \delta)$.
b. $\tilde{a} \ominus \tilde{b} = (m - p, n - q, \alpha + \delta, \beta + \gamma)$
c. for every trapezoidal fuzzy number $\tilde{a} = (m, n, \alpha, \beta)$, there exists a trapezoidal fuzzy number $\tilde{b} = (m, n, -\beta, -\alpha)$ such that $\tilde{a} \ominus \tilde{b} = (0,0,0,0)$
d. $\lambda \hat{*} \tilde{a} = \lambda \hat{*} (m, n, \alpha, \beta) = \begin{cases} (\lambda m, \lambda n, \lambda\alpha, \lambda\beta) & \lambda \geq 0 \\ (\lambda m, \lambda n, -\lambda\beta, -\lambda\alpha,) & \lambda < 0 \end{cases}$.
e. Multiplication $\tilde{a} \hat{*} \tilde{b}$

$$\tilde{a} \hat{*} \tilde{b} = \begin{cases} (mp, nq, m\gamma + p\alpha, n\delta + q\beta), & \tilde{a} > 0 \text{ and } \tilde{b} > 0 \\ (mp, nq, \alpha p - m\delta, \beta q - n\gamma), & \tilde{a} < 0 \text{ and } \tilde{b} > 0 \\ (mp, nq, -m\gamma - \alpha p, -n\delta - \beta q), & \tilde{a} < 0 \text{ and } \tilde{b} < 0 \end{cases}$$

3. RESULTS AND DISCUSSION

Consider the general form of a nonlinear fully fuzzy equation system as follows

$$\begin{cases} (\tilde{a}_{11} \hat{*} \mathbf{x}_1) \oplus (\tilde{a}_{12} \hat{*} \mathbf{x}_2) \oplus \dots \oplus (\tilde{a}_{1n} \hat{*} \mathbf{x}_n) \oplus (\tilde{c}_{11} \hat{*} \mathbf{x}_1^2) \oplus (\tilde{c}_{12} \hat{*} \mathbf{x}_2^2) \oplus \dots \oplus (\tilde{c}_{1n} \hat{*} \mathbf{x}_n^2) \oplus \dots \oplus (\tilde{e}_{11} \hat{*} \mathbf{x}_1^n) \oplus \\ \quad (\tilde{e}_{12} \hat{*} \mathbf{x}_2^n) \oplus \dots \oplus (\tilde{e}_{1n} \hat{*} \mathbf{x}_n^n) = \tilde{b}_1 \\ \vdots \\ (\tilde{a}_{n1} \hat{*} \mathbf{x}_1) \oplus (\tilde{a}_{n2} \hat{*} \mathbf{x}_2) \oplus \dots \oplus (\tilde{a}_{nn} \hat{*} \mathbf{x}_n) \oplus (\tilde{c}_{n1} \hat{*} \mathbf{x}_1^2) \oplus (\tilde{c}_{n2} \hat{*} \mathbf{x}_2^2) \oplus \dots \oplus (\tilde{c}_{nn} \hat{*} \mathbf{x}_n^2) \oplus \dots \oplus (\tilde{e}_{n1} \hat{*} \mathbf{x}_1^n) \oplus \\ \quad (\tilde{e}_{n2} \hat{*} \mathbf{x}_2^n) \oplus \dots \oplus (\tilde{e}_{nn} \hat{*} \mathbf{x}_n^n) = \tilde{b}_n \end{cases} \quad (1)$$

where $\tilde{a}_{ij}, \tilde{c}_{ij}$, and $\tilde{e}_{ij}$ for $1 \leq i, j \leq n$ are arbitrary fuzzy numbers, while $\tilde{b}_{ij}$ is the right segment, and the unknown element $\mathbf{x}_j$ are nonnegative fuzzy numbers [18]. By using the definition of operation $\oplus$ and $\hat{*}$, we convert **Equation (1)** into the crisp form $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ where $\mathbf{f}$ is a real-valued function in $\mathbb{R}^{2n}$ and $\mathbf{x} = (x_1, x_2, \dots, x_n, x_{n+1}, x_{n+2}, \dots, x_{2n})$. Finding a solution of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ will be considered as an optimization problem

$$z = \min_{\mathbf{x}} \sum_{i=1}^{2n} |f_i(\mathbf{x})|, \quad fitness = \frac{1}{z+1} \quad (2)$$

To solve the problem, we use GA as our method. Generally, GA consists of several stages, namely initialization, evaluation, crossover, mutation, and selection of the best generation. A further explanation of genetic algorithms can be found in [26]. In the next section, we will illustrate some fully fuzzy equation systems that can be solved using GA.

Problem 1. Consider a fully fuzzy equation system as follows [18]

$$\begin{cases}(2,3,5) \circledast \mathbf{x} \oplus (2,4,5) \circledast \mathbf{y} \oplus (1,2,3) \circledast \mathbf{x}^2 \oplus (3,5,6) \circledast \mathbf{y}^2 = (19,140,467)\\ (1,2,3) \circledast \mathbf{x} \oplus (3,4,6) \circledast \mathbf{y} \oplus (3,4,5) \circledast \mathbf{x}^2 \oplus (1,3,4) \circledast \mathbf{y}^2 = (14,136,436)\end{cases}$$

where **x** and **y** are two triangular fuzzy numbers. Given the interval constraints of the values of **x** and **y**, which are between 0 and 10.

Suppose $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{y} = (y_1, y_2, y_3)$ are non-negative triangular numbers. So that the nonlinear fully fuzzy equation system in the equation can be written as follows:

$$\begin{cases}(2,3,5) \circledast (x_1, x_2, x_3) \oplus (2,4,5) \circledast (y_1, y_2, y_3) \oplus (1,2,3) \circledast (x_1^2, x_2^2, x_3^2) \oplus (3,5,6) \circledast (y_1^2, y_2^2, y_3^2) = (19,140,467)\\ (1,2,3) \circledast (x_1, x_2, x_3) \oplus (3,4,6) \circledast (y_1, y_2, y_3) \oplus (3,4,5) \circledast (x_1^2, x_2^2, x_3^2) \oplus (1,3,4) \circledast (y_1^2, y_2^2, y_3^2) = (14,136,436)\end{cases}$$

The nonlinear fully fuzzy equation system is converted into a nonlinear crisp equation system by applying arithmetic operations on triangular fuzzy numbers. So that the new system of equations can be expressed as $\mathbf{f}(\mathbf{x}, \mathbf{y}) = 0$ with $\mathbf{f} = (f_1, f_2, f_3, f_4, f_5, f_6)^t$ and

$$f_1(\mathbf{x}, \mathbf{y}) = 2x_1 + 2y_1 + x_1^2 + 3y_1^2 - 19$$
$$f_2(\mathbf{x}, \mathbf{y}) = 3x_2 + 4y_2 + 2x_2^2 + 5y_2^2 - 140$$
$$f_3(\mathbf{x}, \mathbf{y}) = 5x_3 + 5y_3 + 3x_3^2 + 6y_3^2 - 467$$
$$f_4(\mathbf{x}, \mathbf{y}) = x_1 + 3y_1 + 3x_1^2 + y_1^2 - 14$$
$$f_5(\mathbf{x}, \mathbf{y}) = 2x_2 + 4y_2 + 4x_2^2 + 3y_2^2 - 136$$
$$f_6(\mathbf{x}, \mathbf{y}) = 3x_3 + 6y_3 + 5x_3^2 + 4y_3^2 - 436$$

Therefore, the optimization problem is

$$z = \min_{\mathbf{x}, \mathbf{y}} \sum_{i=1}^{6} |f_i(\mathbf{x}, \mathbf{y})|$$

The solution generated using genetic algorithms is highly dependent on the initial population that is built. The parameters of GA that we use are presented in Table 1. The solutions to the optimization problem when the initial population uses integers and real numbers are presented in Table 2.

Table 1. Parameters of GA of Problem 1.

Parameters	Value	
Interval of **x** and **y**	$0 \le x_i, y_i \le 10, x_i, y_i \in \mathbb{Z}$	$0 \le x_i, y_i \le 10, x_i, y_i \in \mathbb{R}$
Genes (chromosome size)	6	6
Population size (number of chromosomes)	10	100
Generation / number of iterations	1000	200000
Crossover probability	90%	90%
Mutation probability	10%	10%

Table 2. The Solution of Problem 1.

Variable	Initial Population	
	Integer number	Real Number
x_1	1	0.99909276
x_2	4	3.99940729
x_3	6	6.00094986
y_1	2	2.00060058
y_2	4	4.00050735
y_3	7	6.99904013
z	0	0.07074628919
Fitness	1	0.93392805569

Based on the fitness value, it can be concluded that solving with an initial population of integers is better done in this example. This is also because the system of equations, in this case, produces analytic solutions in the form of integers in all its variables; therefore, solving with an initial population of integers produces a better solution. Hence the best solution to Problem 1, namely $\mathbf{x} = (1,4,6)$ and $\mathbf{y} = (2,4,7)$ with its membership degree in the interval $0 \le \mu \le 1$ through the fuzzy triangular membership function as follows:

$$\mu_{\mathbf{x}} = \begin{cases} 1 - \dfrac{4-x}{3}, & 1 \le x \le 4 \\ 1 - \dfrac{x-4}{2}, & 4 \le x \le 6 \\ 0. & other \end{cases}$$

$$\mu_{\mathbf{y}} = \begin{cases} 1 - \dfrac{4-y}{2}, & 2 \le y \le 4 \\ 1 - \dfrac{y-4}{3}, & 4 \le y \le 7 \\ 0, & other \end{cases}$$

The approximate solution of Problem 1 using the genetic algorithm that we obtained is exactly the same or close to the solution obtained in [18]. The triangular fuzzy number representation graph of the solution of Problem 1 for each fuzzy variable **x** and **y** can be seen in Figure 1 and Figure 2, respectively.

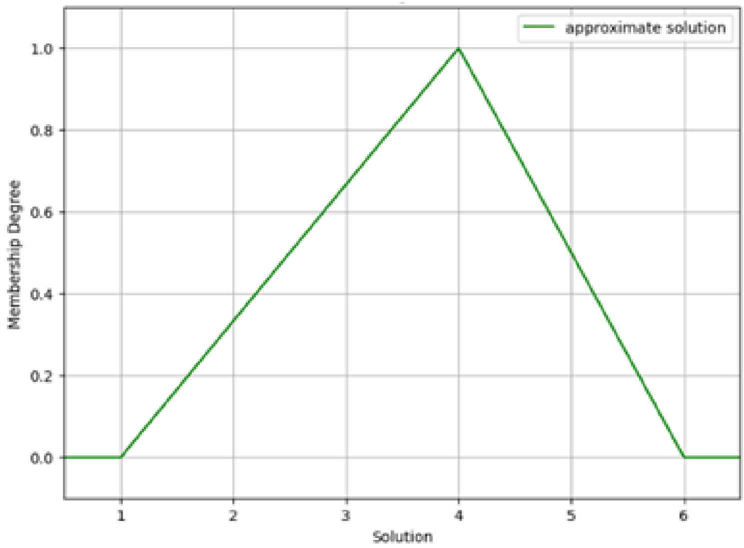

Figure 1. Representation Graph of Triangular Fuzzy Number x for Solution of Problem 1

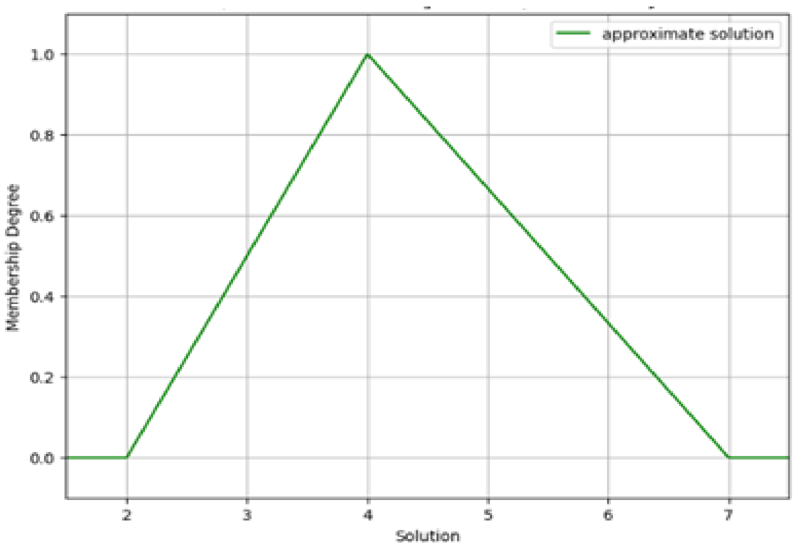

Figure 2. Representation Graph of Triangular Fuzzy Number y for Solution of Problem 1

Problem 2. Consider the following non-linear fully fuzzy equation system

$$\begin{cases}(2,3,1,1) \hat{*}\, \mathbf{x} \oplus (4,6,2,3) \hat{*}\, \mathbf{y} \oplus (3,4,2,3) \hat{*}\, \mathbf{x}^2 \oplus (3,5,1,4) \hat{*}\, \mathbf{y}^2 = (55,331,11,75) \\ (2,4,1,2) \hat{*}\, \mathbf{x} \oplus (5,6,2,3) \hat{*}\, \mathbf{y} \oplus (4,5,1,2) \hat{*}\, \mathbf{x}^2 \oplus (2,3,1,2) \hat{*}\, \mathbf{y}^2 = (53,289,10,51)\end{cases}$$

where **x** and **y** are two trapezoidal fuzzy numbers. Given the interval limit of the values of **x** and **y** which is between 0 and 10.

Suppose $\mathbf{x} = (x_1, x_2, x_3, x_4)$ and $\mathbf{y} = (y_1, y_2, y_3, y_4)$ are non-negative triangular numbers. So that the nonlinear fully fuzzy equation system in the equation can be written as follows:

$$\begin{cases}(2,3,1,1) \hat{*} (x_1, x_2, x_3, x_4) \oplus (4,6,2,3) \hat{*} (y_1, y_2, y_3, y_4) \oplus (3,4,2,3) \hat{*} (x_1^2, x_2^2, x_3^2, x_4^2) \oplus (3,5,1,4) \hat{*} (y_1^2, y_2^2, y_3^2, y_4^2) \\ \qquad = (55,331,11,75) \\ (2,4,1,2) \hat{*} (x_1, x_2, x_3, x_4) \oplus (5,6,2,3) \hat{*} (y_1, y_2, y_3, y_4) \oplus (4,5,1,2) \hat{*} (x_1^2, x_2^2, x_3^2, x_4^2) \oplus (2,3,1,2) \hat{*} (y_1^2, y_2^2, y_3^2, y_4^2) \\ \qquad = (53,289,10,51)\end{cases}$$

The nonlinear fully fuzzy equation system is converted into a nonlinear crisp equation system by applying arithmetic operations on trapezoidal fuzzy numbers. So that the new system of equations can be expressed as $\mathbf{f}(\mathbf{x}, \mathbf{y}) = 0$ with $\mathbf{f} = (f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8)^t$ and

$$\begin{aligned} f_1(\mathbf{x}, \mathbf{y}) &= 2x_1 + 4y_1 + 3x_1^2 + 3y_1^2 - 55 \\ f_2(\mathbf{x}, \mathbf{y}) &= 3x_2 + 6y_2 + 4x_2^2 + 5y_2^2 - 331 \\ f_3(\mathbf{x}, \mathbf{y}) &= x_3 + 2y_3 + 2x_3^2 + y_3^2 - 11 \\ f_4(\mathbf{x}, \mathbf{y}) &= x_4 + 3y_4 + 3x_4^2 + 4y_4^2 - 75 \\ f_5(\mathbf{x}, \mathbf{y}) &= 2x_1 + 5y_1 + 4x_1^2 + 2y_1^2 - 53 \\ f_6(\mathbf{x}, \mathbf{y}) &= 4x_2 + 6y_2 + 5x_2^2 + 3y_2^2 - 289 \\ f_7(\mathbf{x}, \mathbf{y}) &= x_3 + 2y_3 + x_3^2 + y_3^2 - 10 \\ f_8(\mathbf{x}, \mathbf{y}) &= 2x_4 + 3y_4 + 2x_4^2 + 2y_4^2 - 51 \end{aligned}$$

Therefore, the optimization problem is

$$z = \min_{\mathbf{x}, \mathbf{y}} \sum_{i=1}^{8} |f_i(\mathbf{x}, \mathbf{y})|$$

The parameters of GA that we used in Problem 2 can be seen in Table 3.

Table 3. Parameters of GA of Problem 2.

Parameters	Value
Interval of **x** and **y**	$0 \le x_i, y_i \le 10, x_i, y_i \in \mathbb{Z}$
Genes (chromosome size)	8
Population size (number of chromosomes)	10
Generation/number of iterations	1000
Crossover probability	90%
Mutation probability	10%

GA gives the solutions to the optimization problem when the initial population uses integer numbers $\mathbf{x} = (x_1, x_2, x_3, x_4) = (2,5,1,3)$ and $\mathbf{y} = (y_1, y_2, y_3, y_4) = (3,6,2,3)$ with $z = 0$ and Fitness $= 1$**.** Based on z and the fitness value, it can be concluded that the solution is an exact solution. Furthermore, its membership degree in the interval $0 \le \mu \le 1$ through the fuzzy trapezoidal membership function as follows:

$$\mu_{\mathbf{x}} = \begin{cases} 1 - \dfrac{2-x}{1}, & 1 \le x \le 2 \\ 1, & 2 \le x \le 5 \\ 1 - \dfrac{x-5}{3}, & 5 \le x \le 8 \\ 0, & other \end{cases}$$

$$\mu_{\mathbf{y}} = \begin{cases} 1 - \dfrac{3-y}{2}, & 1 \le y \le 3 \\ 1, & 3 \le y \le 6 \\ 1 - \dfrac{y-6}{3}, & 6 \le y \le 9 \\ 0, & other \end{cases}$$

Representation of the trapezoidal number for the solution of Problem 2 can be found in **Figure 3** and **Figure 4**.

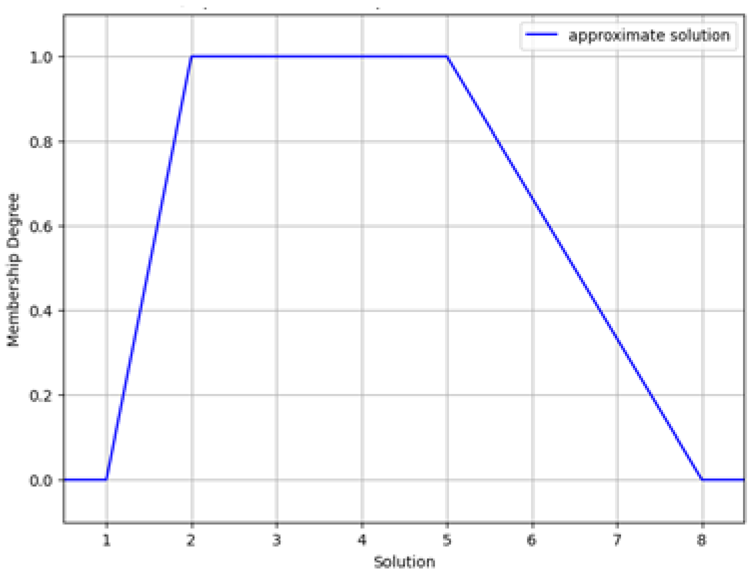

Figure 3. Representation Graph of Trapezoidal Fuzzy Number x for Solution of Problem 2

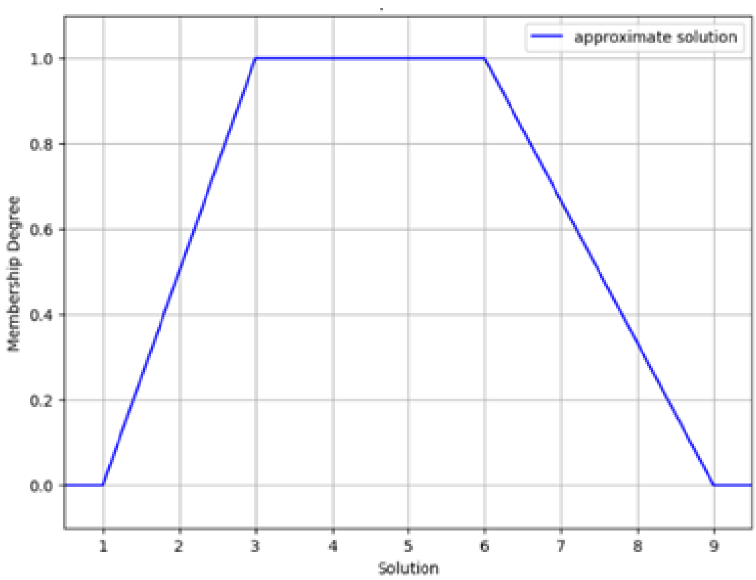

Figure 4. Representation Graph of Trapezoidal Fuzzy Number y for Solution of Problem 2

4. CONCLUSIONS

In this article, we have shown a numerical approach to solving dual fully fuzzy equation systems using genetic algorithms. In our study, the problem was considered an optimization problem and then solved using the genetic algorithm. Based on the illustration of some problems, the solution obtained is highly dependent on the initialization of the population, where if the analytical solution to all variables of the system equations is an integer, then the initial population with integers produces a better solution than the initial population using real numbers. Meanwhile, if the analytic solution of the system of equations is a real number, the initial population with real numbers produces a better solution.

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