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STABILITY ANALYSIS OF GAMBLING BEHAVIOR MODEL WITH COGNITIVE BEHAVIORAL THERAPY TREATMENT

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ABSTRACT

Gambling, driven by the desire for quick profits, involves individuals or groups betting money, often resulting in significant financial consequences. Gambling behavior can be influenced by the environment or society. Thus, the dynamics of environmental influences on gambling behavior can be mathematically modeled using differential equations. This study presents a mathematical model of the environmental impact on the dynamics of the SI_1I_2T (Susceptible-Infective1-Infective2-Treatment) population of gamblers undergoing cognitive behavioral therapy (CBT). The model replaces the recovered sub-population with a treatment sub-population, representing individuals receiving CBT, as there is no definitive cure for gambling addiction. It consists four sub-populations: It consists of four sub-populations: (S) individuals susceptible to gambling, (I_1) gamblers who are not yet addicted, (I_2) addicted gamblers, and (T) individuals undergoing treatment but at risk of relapse. Mathematical analysis identifies two equilibrium points: a gambling-free equilibrium and an endemic gambling equilibrium. Furthermore, the results of the stability analysis using the linearization method shows that the balance point has a asymptotically stability characteristic requirement. The basic reproduction number ($\mathcal{R}_0$) was calculate and resulted if $\mathcal{R}_0 < 1$, then the free gambler population equilibrium point is asymptotically stable, and vice versa. Based on the results of the data analysis, the value of $\mathcal{R}_0 = 0.5$. This value is less than 1, so the equilibrium point obtained is the free gambler population and asymptotically stable equilibrium point. This means that the population will be free from gambling behavior. Numerical simulation represents the results of the analysis that has been obtained. Providing cognitive behavioral therapy (CBT) to gamblers in treatment can help reduce the gambler population. The population growth will decrease in such a way that it will eventually lead to a gambling-free population

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1. INTRODUCTION

Gambling is defined as a game involving money as a bet and is regulated as a moral offense under Article 303 bis of the Criminal Code. Several factors contribute to gambling, including a desire to try new things, lack of parental supervision, lack of support from the community, and a lack of positive role models in life [1]. The interaction between individuals and their environment mutually influences each other, making environmental factors significantly impactful. Human behavior can affect the environment, and conversely, the environment can influence human behavior. Gambling occurs along a continuous scale, starting from experimentation, occasional participation, and habitual gambling, and advancing to a compulsive level that results in severe negative effects. The incidence of problem gambling among young individuals tends to grow as they move from adolescence into early adulthood [2]. In addition, research examined gamblers for some older adults as one of the fastest growing group of gamblers across the world [3]. Gambling behavior can cause harm to the individual, including challenges in controlling the amount of money and time spent on gambling. This often results in negative impacts not only for the gambler but also for their family, close relationships, friends, colleagues, and others around them [3].

According to the report from the Indonesian National Police Criminal Investigation Department (Bareskrim Polri), during the period from January to May 2022, a total of 905 gambling cases were uncovered in Indonesia [4]. In this regard, the North Sumatra Regional Police (Polda Sumut) was the most active in handling these cases, with 134 incidents, equivalent to 14.8% of the total gambling cases in the country. East Java Regional Police (Polda Jatim) ranked second with 109 cases, followed by West Nusa Tenggara Regional Police (Polda NTB) with 84 cases, and Central Java with 80 cases. West Kalimantan Regional Police (Polda Kalbar) and Riau Regional Police (Polda Riau) successfully investigated 72 and 56 gambling cases, respectively. Lampung Regional Police (Polda Lampung) dealt with 48 cases, Jakarta Metropolitan Regional Police (Polda Metro Jaya) 40 cases, and West Java 30 cases. Drs. Listyo Sigit Prabowo, M.Si., the Indonesian National Police Chief, ordered the Central Police Headquarters (Mabes Polri) to crackdown on gamblers involved in illegal gambling, both in conventional and online settings [5].

Some research considers problem gambling to be analogous to an infectious disease [3] [6] [7]. The study of infectious disease dynamics involves modeling population growth, disease occurrence, transmission patterns, and social influence factors to develop mathematical models that capture their changing behavior. These methods have also been widely utilized by researchers to explore and address other socially contagious issues, including smoking, alcoholism, substance abuse, gaming addiction, and the spread of information [8]. The influence of the environment on gambling behavior can be mathematically modeled using mathematical modeling approaches [6], [9] - [11]. Tae Sug Do and Young S. Lee propose that young people are introduced to gambling by framing excessive gambling as a socially transmitted disease, with environmental peer contagion represented through mass action terms from epidemiological models [3]. Hiromi Seno, consider focusing on the social nature of online gaming which would be regarded as a factor to reinforce the transition to the addictive gaming [10]. Driss Kada, etc. introduce a PEAR mathematical model to represent the dynamics of a population responding to the spread of e-game addiction, and formulate an optimal control problem for the model in discrete time [7]. The objective of this study is to analyze the dynamics of the gambling population using mathematical models [12] - [15]. The modeling begins with the concept of Susceptible and Infective (SI) and then evolves into the SIR model with a sub-population of those under treatment [4], [14] - [18].

In this research, the recovered sub-population is replaced with a treatment sub-population, representing cognitive behavioral therapy (CBT), as there is no cure for gambling addiction. Previous research conducted by Rozi Wahyudi and colleagues in 2021 [19] divided the population into three groups: those who have never gambled (S), those who gamble but are not addicted (I_1) such as social and problem gamblers, those who are addicted to gambling (I_2) like pathological gamblers, and those who quit gambling but are prone to relapse (R). The variables in that study include the birth rate, the environmental influence on gambling, the death rate, the increase in gambling addicts, the number of gamblers not in jail, the ratio of gamblers caught by the police, the rate of quitting gambling, and the rate of individuals transitioning from quitting gambling to relapse. The model aims to analyze the system's dynamics through stability analysis and the calculation of the basic reproduction number. These parameters were assumed to be accompanied by numerical simulations.

2. RESEARCH METHODS

2.1 Literature Study on Gambling

In this stage, the identification of the problem and literature review is conducted using several books, journals, research papers, or articles obtained from the internet regarding references on the population equilibrium of gambling with cognitive behavioral therapy treatment concerning the growth of the population and increasing gamblers.

2.2 Problem Identification

In the process of problem identification, factors related to the gambling population are considered, limiting the scope of the problem, and making assumptions about each factor. The research begins by studying the dynamics of the gambling population through a review of previous studies. Then, assumptions are formulated to simplify the model, defining the parameters used in the model, which eventually leads to a mathematical model of the influence of the environment on the gambling population. Next, the equilibrium points of the model are determined, where an equilibrium point represents a fixed point where the system does not undergo any change over time. Subsequently, the stability of each equilibrium point in the model is investigated through system linearization.

2.3 Forming the Mathematical Model

After identifying the problem, the relevant factors have been collected. Next, a mathematical model is needed by determining the dependent variables, including the population that has never gambled, the population that gambles but is not addicted, the population addicted to gambling, the population that quit gambling but may relapse, and the population under treatment. In constructing the model, a logistic model is used, assuming that over a certain period, the population will approach an equilibrium point.

2.4 Stability Analysis

After going through the previous steps, this stage involves first finding the equilibrium points. At this stage, the process involves determining the equilibrium points of the model and subsequently analyzing the stability of the model in the vicinity of these points. Once found, local stability analysis is performed to determine the stability around the equilibrium points. The steps in analyzing stability are as follows:

1. Linearization of the system equations;
2. Obtaining the Jacobian matrix;
3. Finding the eigenvalues using direct methods and the Routh-Hurwitz method;
4. Analyzing stability at each equilibrium point.

After analyzing stability at each equilibrium point, solutions to the differential equations will be obtained. From these solutions, it will be determined how the gambling population recovers concerning the group of individuals undergoing cognitive behavioral therapy.

2.5 Numerical Simulation

Based on the model and its equilibrium points obtained, numerical simulations will be conducted using Google Colaboratory. Through this method, a program will be used to check the results of the model's solutions.

3. RESULTS AND DISCUSSION

3.1 Mathematical Model of Gambling Spread

The facts obtained from the process of the gambler population becoming a novice gambler or an acute gambler are as follows:

1. Gambling is often perceived as an enjoyable activity, driven not only by the aspiration to win but also by a strong inherent desire to participate.

2. Gamblers derive satisfaction from modest victories and generally view minor losses as acceptable.
3. In the face of losses, gamblers frequently exhibit a strong determination to continue until they achieve a win.

The assumptions used in forming the model include:

1. The population is described as closed, indicating that it is confined to a specific geographical area.

Gamblers experiencing addiction are provided with treatment through counseling, which includes raising awareness about the risks and negative consequences of gambling.

The formation of the SI_1I_2T model divides the population into four classes, namely the susceptible, gambling population, the non-addicted gambling population (*infected*$_1$), the addicted gambling population (*infected*$_2$), and the gambling population under treatment. Several variables and parameters used in the model of the influence of the environment on gambling behavior are outlined.

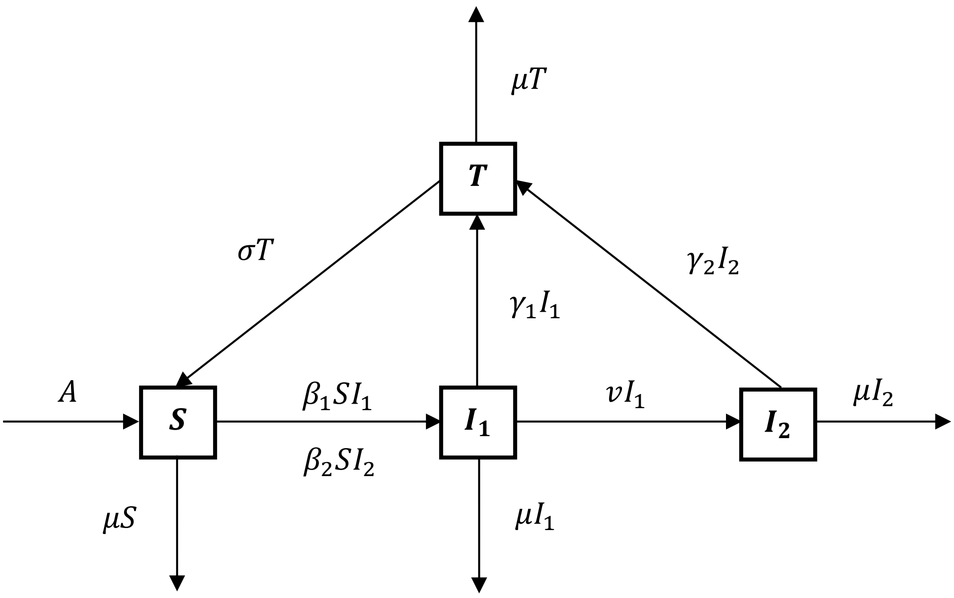

Figure 1. Compartment Diagram

The transitions of these states are as follows:

1. In the population vulnerable to gambling, there is a birth rate (A) within a certain period, which decreases due to natural death. When a susceptible gambling population (S) interact with non-addicted gambling population (I_1) and the addicted gambling population (I_2), can be influenced by the rate of population infected with gambling (β_1) or (β_2).
2. Furthermore, non-addicted gamblers experience an increase due to interaction with both addicted and non-addicted gamblers. However, the non-addicted gamblers decrease because of a personal desire to earn more money quickly, leading them to become addicted gamblers (v) and individuals caught by the police who were then treated with cognitive behavioral therapy (CBT) (γ_1). In addition, the non-addicted gambling population (I_1) is decreasing due to natural deaths (μ).
3. The urgency of an individual's desire to achieve financial gains rapidly (v) increases the growth of the gambling addict population (I_2). Individuals caught by the police who were then treated with cognitive behavioral therapy (CBT) (γ_2) and natural deaths (μ) reducing the growth of the gambling addict population
4. A group of individuals undergoing treatment has increased due to arrests by the police, followed by participation in Cognitive Behavioral Therapy (CBT) sessions, represented by(γ_1) and (γ_2). Besides that, the rate at which vulnerable individuals return to gambling (σ) and natural deaths (μ) reducing the growth of the gambling addict population.

The mathematical model to observe the influence of the environment on gamblers is proposed based on the compartment diagram.

$$\frac{dS}{dt} = A + \sigma T - (\beta_1 I_1 + \beta_2 I_2 + \mu)S, \quad (1)$$

$$\frac{dI_1}{dt} = (\beta_1 S - v - \mu - \gamma_1)I_1 + \beta_2 I_2 S, \quad (2)$$

$$\frac{dI_2}{dt} = vI_1 - (\gamma_2 + \mu)I_2, \quad (3)$$

$$\frac{dT}{dt} = \gamma_1 I_1 + \gamma_2 I_2 - (\mu + \sigma)T. \quad (4)$$

with initial values $S(0) = S_0 > 0, I_1(0) = I_{1_0} \geq 0, I_2(0) = I_{2_0} \geq 0$, dan $T(0) = 0$. Several variables and parameters used in the model of environmental influence on gambling behavior are described in **Table 1** and **Table 2** parameter descriptions in.

Table 1. List of Research Variables

Variable	Description	Conditions
$S(t)$	Groups of individuals who are vulnerable to gambling at times t.	$S(t) \geq 0$
$I_1(t)$	Group of individuals who gamble but are not yet addicted to gambling at the time t.	$I_1(t) \geq 0$
$I_2(t)$	Group of individuals who are addicted to gambling at any time t.	$I_2(t) \geq 0$
$T(t)$	Group of individuals who undergo treatment/treatment at the time t.	$T(t) \geq 0$

Table 2. List of Research Parameters

Parameter	Description	Unit	Condition
A	Birth rate	$\frac{people}{time}$	$A > 0$
μ	Natural death rate	$\frac{1}{time}$	$\mu > 0$
β_1, β_2	The rate of population infected with gambling	$\frac{1}{people \cdot time}$	$\beta_1, \beta_2 > 0$
v	The pace of personal desire to earn more money quickly	$\frac{1}{time}$	$v > 0$
γ_1, γ_2	The rate of individuals caught by the police who were then treated with cognitive behavioral therapy (CBT)	$\frac{1}{time}$	$\gamma_1, \gamma_2 > 0$
σ	The rate at which vulnerable individuals return to gambling	$\frac{1}{time}$	$\sigma > 0$

3.2 Equilibrium Point

There are several steps that can be taken to reach an equilibrium point, including determining a relevant mathematical model for the research problem, expressing the model into mathematical equations by describing the relationships between the involved variables. After that, determining the equilibrium points by setting all equations, which means setting the left-hand side of the system of equations (1 - 4) equal to zero. The following are the results of the obtained two equilibrium points which is $E_0(S_0, 0,0,0)$ representing a gambling-free population, where $S_0^* = \frac{A}{\mu}$. Next, for the equilibrium point with the active gambling population is as follows $E_1(S_1, I_{1_1}, I_{2_1}, T_1)$ representing an existent state where gambling behavior persists, where

$$S_1^* = \frac{\mu^2 + \mu v + \mu\gamma_1 + \mu\gamma_2 + v\gamma_2 + \gamma_1\gamma_2}{\mu\beta_1 + v\beta_2 + \beta_1\gamma_2}, \quad (5)$$

$$I_{1_1}^* = \frac{(C_1)(\mu + \gamma_2)}{\mu C_2}, \quad (6)$$

$$I^*_{2_1} = \frac{vC_1}{\mu C_2}, \tag{7}$$

$$T^*_1 = \frac{(\mu\gamma_1 + v\gamma_2 + \gamma_1\gamma_2)(C_3)}{\mu C_2}. \tag{8}$$

with

$$C_1 = A\mu^2\beta_1 + A\mu\sigma\beta_1 + A\mu v\beta_2 + A\mu\beta_1\gamma_2 + A\sigma v\beta_2 + A\sigma\beta_1\gamma_2 - \mu^4 - \mu^3\sigma - \mu^3 v - \mu^3\gamma_1 - \mu^3\gamma_2 - \mu^2\sigma v - \mu^2\sigma\gamma_1 - \mu^2\sigma\gamma_2 - \mu^2 v\gamma_2 - \mu^2\gamma_1\gamma_2 - \mu\sigma v\gamma_2 - \mu\sigma\gamma_1\gamma_2$$

$$C_2 = \mu^3\beta_1 + \mu^2\sigma\beta_1 + \mu^2 v\beta_1 + \mu^2 v\beta_2 + \mu^2\beta_1\gamma_1 + 2\mu^2\beta_1\gamma_2 + \mu\sigma v\beta_1 + \mu\sigma v\beta_2 + 2\mu\sigma\beta_1\gamma_2 + \mu v^2\beta^2 + 2\mu v\beta_1\gamma_2 + \mu v\beta_2\gamma_1 + \mu v\beta_2\gamma_2 + 2\mu\beta_1\gamma_1\gamma_2 + \mu\beta_1\gamma_2^2 + \sigma v^2\beta_2 + \sigma v\,\beta_2\gamma_2 + \sigma v\beta_2\gamma_2 + \sigma\beta_1\gamma_1^2 + v^2\beta_2\gamma_2 + v\beta_1\gamma_2^2 + v\beta_2\gamma_1\gamma_2 + \beta_1\gamma_1\gamma_2^2$$

$$C_3 = A\mu\beta_1 + Av\beta_2 + A\beta_1\gamma_2 - \mu^3 - \mu^2 v - \mu^2\gamma_1 - \mu^2\gamma_2 - \mu v\gamma_2 - \mu\gamma_1\gamma_2$$

Point $E_1(S_1, I_{1_1}, I_{2_1}, T_1)$ is in an existing state, so it must satisfy the conditions $C_1 > 0$, dan $C_3 > 0$.

3.3 Basic Reproduction Numbers

Theorem 3.1 In general, the conditions that allow for the basic reproduction number are as follows:

1. If $R_0 < 1$, then the number of infected individuals will decrease in each generation, leading to the eradication of the disease.
2. If $R_0 > 1$, then the number of infected individuals will increase in each generation, causing the disease to grow and become an epidemic **[9]**.

The basic reproduction number or the epidemic threshold value can be determined through the Next Generation matrix constructed from the group of infected or exposed individuals. In the following, the basic reproduction number of system (1) will be obtained by the next generation matrix method. Let $E(S, I_1, I_2, T)$, then system (1) can be written as

$$\frac{dx}{dt} = F(x) - V(x),$$

where

$$F = \begin{bmatrix} I_1 S\beta_1 + I_2 S\beta_2 \\ 0 \end{bmatrix}, \tag{9}$$

and

$$V = \begin{bmatrix} \mu I_1 + vI_1 + \gamma_1 I_1 \\ -vI_1 - \mu I_2 + \gamma_2 I_2 \end{bmatrix}. \tag{10}$$

Next, derive the partials of F with respect to I_1 and I_2 and derive the partials of V with respect to I_1 and I_2. The Jacobian matrices of $F(x)$ and $V(x)$ form **Equation (9)** and **Equation (10)** are

$$T = \begin{pmatrix} \frac{\partial F_1}{\partial I_1} & \frac{\partial F_1}{\partial I_2} \\ \frac{\partial F_2}{\partial I_1} & \frac{\partial F_2}{\partial I_2} \end{pmatrix} = \begin{pmatrix} \beta_1 S & \beta_2 S \\ 0 & 0 \end{pmatrix},$$

and

$$U = \begin{pmatrix} \frac{\partial V_1}{\partial I_1} & \frac{\partial V_1}{\partial I_2} \\ \frac{\partial V_2}{\partial I_1} & \frac{\partial V_2}{\partial I_2} \end{pmatrix} = \begin{pmatrix} \mu + v + \gamma_1 & 0 \\ -v & \mu + \gamma_2 \end{pmatrix}.$$

Then, the analysis uses the Next Generation Matrix equation

$$K = TU^{-1},$$

By observing the largest eigenvalue of matrix K, we obtain the basic reproduction number.

$$\mathcal{R}_0 = \frac{\beta_1 S}{(\mu + v + \gamma_1)} + \beta_2 S\left(\frac{v}{(\mu + v + \gamma_2)(\mu + \gamma_2)}\right). \tag{11}$$

3.4 Analysis of Equilibrium Point Stability

After completing the earlier steps, this phase focuses on linearizing the system of equations using a 4×4 Jacobian matrix (J) to assess the stability of a fixed point. The Jacobian matrix (J) is derived from the system equations.

$$\mathrm{J} = \begin{bmatrix} -\beta_1 I_1 - \beta_2 I_2 - \mu & -\beta_1 S & -\beta_2 S & \sigma \\ \beta_1 I_1 + \beta_2 I_2 & \beta_1 S - v - \mu - \gamma_1 & \beta_2 S & 0 \\ 0 & v & -\gamma_2 - \mu & 0 \\ 0 & \gamma_1 & \gamma_2 & -\mu - \sigma \end{bmatrix}. \tag{12}$$

3.4.1 Stability Analysis of Disease – Free Equilibrium Point

Based on the gambling-free population point obtained, the following matrix **Equation (12)** is formed

$$\mathrm{J}(E_0) = \begin{bmatrix} -\beta_1 I_1 - \beta_2 I_2 - \mu & -\beta_1 \frac{A}{\mu} & -\beta_2 \frac{A}{\mu} & \sigma \\ \beta_1 I_1 + \beta_2 I_2 & \beta_1 \frac{A}{\mu} - v - \mu - \gamma_1 & \beta_2 \frac{A}{\mu} & 0 \\ 0 & v & -\gamma_2 - \mu & 0 \\ 0 & \gamma_1 & \gamma_2 & -\mu - \sigma \end{bmatrix},$$

from the Jacobian matrix, by using $|\mathrm{J}(E_0) - \lambda \mathrm{I}| = 0$, then obtained

$$\det\left(\begin{bmatrix} -\mu & -\beta_1 \frac{A}{\mu} & -\beta_2 \frac{A}{\mu} & \sigma \\ 0 & \beta_1 \frac{A}{\mu} - v - \mu - \gamma_1 & \beta_2 \frac{A}{\mu} & 0 \\ 0 & v & -\gamma_2 - \mu & 0 \\ 0 & \gamma_1 & \gamma_2 & -\mu - \sigma \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}\right) = 0,$$

by using the cofactor rule, so that the eigen polynomial is obtained

$$(-\mu - \lambda)\left(\beta_1 \frac{A}{\mu} - v - \mu - \gamma_1 - \lambda\right)(-\gamma_2 - \mu - \lambda)(-\mu - \sigma - \lambda) = 0,$$

thus,

$$\lambda_1 = -\mu, \lambda_2 = \beta_1 \frac{A}{\mu} - v - \mu - \gamma_1, \lambda_3 = -\gamma_2 - \mu, \lambda_4 = -\mu - \sigma$$

The gambling-free population point will be locally asymptotically stable if all the real parts of λ_1, λ_3, and λ_4 are all negative, and for λ_2 this conditional is satisfied when $R_0 < 1$.

$$\begin{aligned} \lambda_2 &= \beta_1 \frac{A}{\mu} - v - \mu - \gamma_1 < 0 \\ &= -\frac{A\beta_2}{\mu}(\mu + v + \gamma_1)\left(\frac{v}{(\mu + v + \gamma_2)(\mu + \gamma_2)}\right) + R_0(\mu + v + \gamma_1) - (v + \mu + \gamma_1) \\ &= -\frac{A\beta_2}{\mu}(\mu + v + \gamma_1)\left(\frac{v}{(\mu + v + \gamma_2)(\mu + \gamma_2)}\right) + (R_0 - 1)(\mu + v + \gamma_1) < 0 \end{aligned}$$

3.4.2 Stability Analysis of $E_1(S_1, I_{1_1}, I_{2_1}, T_1)$

Based on **Equation (12)**, the Jacobian matrix around $E_1(S_1, I_{1_1}, I_{2_1}, T_1)$ is obtained as follows,

$$\mathrm{J}(E_1)=\begin{bmatrix}-\beta_1 I_1-\beta_2 I_2-\mu & -\beta_1 S_1 & -\beta_2 S_1 & \sigma\\ \beta_1 I_1+\beta_2 I_2 & \beta_1 S_1-v-\mu-\gamma_1 & \beta_2 S_1 & 0\\ 0 & v & -\gamma_2-\mu & 0\\ 0 & \gamma_1 & \gamma_2 & -\mu-\sigma\end{bmatrix},$$

from the Jacobian matrix, by using $|\mathrm{J}(E_1)-\lambda\mathrm{I}|=0$, then obtained

$$\det\left[\begin{bmatrix}-\beta_1 I_1-\beta_2 I_2-\mu & -\beta_1 S_1 & -\beta_2 S_1 & \sigma\\ \beta_1 I_1+\beta_2 I_2 & \beta_1 S_1-v-\mu-\gamma_1 & \beta_2 S_1 & 0\\ 0 & v & -\gamma_2-\mu & 0\\ 0 & \gamma_1 & \gamma_2 & -\mu-\sigma\end{bmatrix}-\begin{bmatrix}\lambda & 0 & 0 & 0\\ 0 & \lambda & 0 & 0\\ 0 & 0 & \lambda & 0\\ 0 & 0 & 0 & \lambda\end{bmatrix}\right]=0,$$

$$\begin{vmatrix}-\beta_1 I_1-\beta_2 I_2-\mu-\lambda & -\beta_1 S_1 & -\beta_2 S_1 & \sigma\\ \beta_1 I_1+\beta_2 I_2 & \beta_1 S_1-v-\mu-\gamma_1-\lambda & \beta_2 S_1 & 0\\ 0 & v & -\gamma_2-\mu-\lambda & 0\\ 0 & \gamma_1 & \gamma_2 & -\mu-\sigma-\lambda\end{vmatrix}=0.$$

Following the linearization analysis of the system of equations using the Jacobian matrix, the results were derived from the characteristic equation.

$$\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0.$$

with

$$a_1=\beta_1 I_1+\beta_2 I_2+\beta_1 S_1+4\mu+\sigma+v+\gamma_1+\gamma_2,$$

$$\begin{aligned}a_2=&\,3\mu\beta_1 I_1+\sigma\beta_1 I_1+v\beta_1 I_1+\gamma_1\beta_1 I_1+\gamma_2\beta_1 I_1+3\mu\beta_2 I_2+\sigma\beta_2 I_2+v\beta_2 I_2+\gamma_1\beta_2 I_2+\gamma_2\beta_2 I_2+6\mu^2\\ &+3\mu\sigma+3\mu v+3\mu\gamma_1+3\mu\gamma_2+\sigma v+\sigma\gamma_1+\sigma\gamma_2+v\gamma_2+\gamma_1\gamma_2-3\mu\beta_1 S_1-\sigma\beta_1 S_1\\ &-v\beta_2 S_1-\gamma_2\beta_1 S_1,\end{aligned}$$

$$\begin{aligned}a_3=&\,2\mu\sigma\beta_1 I_1+2\mu\gamma_1\beta_1 I_1+2\mu v\beta_1 I_1+2\mu\gamma_2\beta_1 I_1+\sigma v\beta_1 I_1+\sigma\gamma_2\beta_1 I_1+\gamma_2 v\beta_1 I_1+\gamma_1\gamma_2\,\beta_1 I_1+3\mu^2\beta_1 I_1\\ &+3\mu^2\beta_2 I_2+2\mu\sigma\beta_2 I_2+2\mu\gamma_1\beta_2 I_2+2\mu v\beta_2 I_2+2\mu\gamma_2\beta_2 I_2+\sigma v\beta_2 I_2+\sigma\gamma_2\beta_2 I_2\\ &+\gamma_2 v\beta_2 I_2+\gamma_1\gamma_2\,\beta_2 I_2-3\mu^2\beta_1 S_1-2\mu\sigma\beta_1 S_1-2\mu v\beta_2 S_1-2\mu\gamma_2\beta_1 S_1-\sigma v\beta_2 S_1\\ &-\sigma\gamma_2\beta_1 S_1+4\mu^3+3\mu^2\sigma+3\mu^2 v+3\mu^2\gamma_1+3\mu^2\gamma_2+2\mu\sigma v+2\mu\sigma\gamma_1+2\mu\sigma\gamma_2+2\mu v\gamma_2\\ &+2\mu\gamma_1\gamma_2+\sigma v\gamma_2+\sigma\gamma_1\gamma_2,\end{aligned}$$

$$\begin{aligned}a_4=&\,\mu^3\beta_1 I_1+\mu^2\sigma\beta_1 I_1+\mu^2 v\beta_1 I_1+\mu^2\beta_1\gamma_1 I_1+\beta_1\gamma_2\mu^2 I_1+\mu\sigma v\beta_1 I_1+\mu\sigma\beta_1\gamma_2 I_1+\mu v\beta_1\gamma_2 I_1\\ &+\mu\beta_1\gamma_1\gamma_2 I_1+\mu^3\beta_2 I_2+\mu^2\sigma\beta_2 I_2+\mu^2 v\beta_2 I_2+\mu^2\beta_2\gamma_1 I_2+\beta_2\gamma_2\mu^2 I_2+\mu\sigma v\beta_2 I_2\\ &+\mu\sigma\beta_2\gamma_2 I_2+\mu v\beta_2\gamma_2 I_2+\mu\beta_2\gamma_1\gamma_2 I_2-\mu^3\beta_1 S_1-\mu^2\sigma\beta_1 S_1-\mu^2 v\beta_2 S_1-\mu^2\beta_1\gamma_2 S_1\\ &-\mu\sigma v\beta_2 S_1-\mu\sigma\beta_1\gamma_2 S_1+\mu^4+\mu^3\sigma+\mu^3 v+\mu^3\gamma_1+\mu^3\gamma_2+\mu^2\sigma v+\mu^2\sigma\gamma_1+\mu^2\sigma\gamma_2\\ &+\mu^2 v\gamma_2+\mu^2\gamma_1\gamma_2+\mu\sigma v\gamma_2+\mu\sigma\gamma_1\gamma_2.\end{aligned}$$

To determine the eigenvalue with a negative real part from the Jacobian matrix calculation, the Routh-Hurwitz criterion is applied. The criterion satisfies the following conditions.

1. $a_1>0$
2. $a_1a_2-a_0a_3>0$
3. $a_1a_2a_3+a_0a_1a_5-a_1^2a_4-a_0a_3^2>0$
4. $2a_0a_1a_4a_5+a_1a_2a_3a_4+a_1^2a_2a_6-a_0a_1a_3a_6-a_1a_2^2a_5-a_1^2a_4^2-a_0a_3^2a_4+a_0a_2a_3a_5-a_0^2a_5^2-a_0a_1a_2a_7+a_0^2a_3a_7>0$

3.5 Numerical Simulation

The simulation of mathematical model of gambling spread is conducted using Maple18 and Google Collaboratory. Maple18 is used to assign values to each parameter based on the given conditions of the basic reproduction number ($\mathcal{R}_0$) set. Since Maple can only handle up to 3 variables to display simulation plots, the simulation is performed using Google Collaboratory, and Jupyter is used to verify whether the simulation results match between the two platforms. This simulation aims to obtain a geometric representation of the equilibrium points and stability theorem of the SI_1I_2T epidemiological model. In this study, the dynamics of the gambling population are analyzed under two different conditions: when gamblers are absent and when there are gamblers present.

In this condition, the simulation uses initial conditions with the following assumptions: the initial values for the susceptible gambling population $S(0) = 13$, the non-addicted gambling population $I_1(0) = 4$, $I_2(0) = 15$, $T(0) = 7$. Meanwhile, the parameter values provided to create the simulation of the gambling population spread model are presented in **Table 3**.

Table 3. Parameter Value

Description	Parameter	Value		Source
		Simulation 1	Simulation 2	
Birth rate	A	0,5	0,9	Assumed
Natural death rate	μ	0,1	0,1	[20]
The rate of population infected with gambling	β_1, β_2	0,1	0,5	[20]
The pace of personal desire to earn more money quickly	v	0,2	0,5	Assumed
The rate of individuals caught by the police who were then treated with cognitive behavioral therapy (CBT)	γ_1, γ_2	0,9	0,2	[20]
The rate at which vulnerable individuals return to gambling	σ	0,1	0,9	Assumed

Based on the **Table 3**, with Simulation 1, there exist equilibrium point E_0 is obtained as $(5,0,0,0)$. Furthermore, the eigenvalue results indicate that for these equilibrium points with the parameter value, was obtained,

$$(-0.1, \quad -0.2, \quad -0.49, \quad -1.2\,),$$

It can be observed that the eigenvalues are negative, indicating that the equilibrium point with gambling-free population is asymptotically stable. While substituting the parameter values in **Equation (11)** and the basic reproduction number is shown

$$\mathcal{R}_0 = \frac{\beta_1 S}{(\mu + v + \gamma_1)} + \beta_2 S\left(\frac{v}{(\mu + v + \gamma_2)(\mu + \gamma_2)}\right) = 0.5 < 1.$$

This simulation results demonstrated through **Figure 1**.

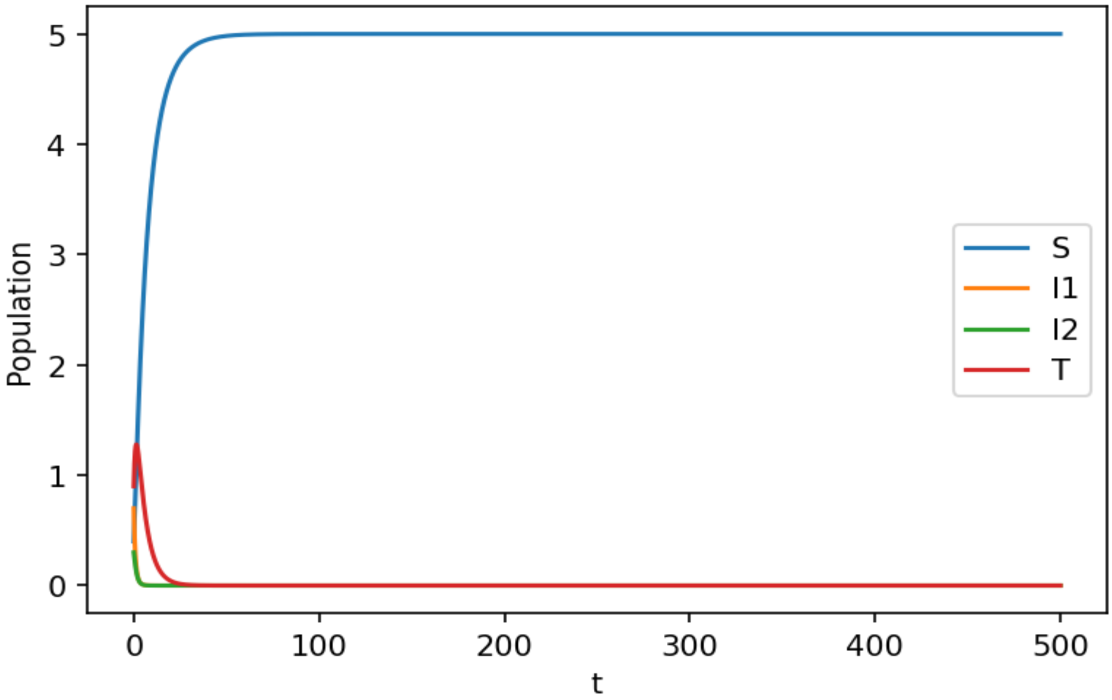

Figure 1. Simulation Graph of Equilibrium Point with the Gambling Free Population (E0)

Figure 1 present graphs of gambling free population. The growth rate of the vulnerable gambling population shows a significant increase, indicating that the population reaches a stable state to values 5. Next, for the equilibrium points of non-addicted gamblers, addicted gamblers, and the gamblers under treatment, the population decreases until it reaches a point where the simulation generates constant graphs that converge to the value of 0. This is because there are no addicted gamblers present nor the gamblers undergoing treatment.

Based on **Table 3**, with Simulation 2, there exist two equilibrium point, $E_0(9,0,0,0)$ and E_1 is obtained as $(0.6,2.625,4.375,1.4)$. Furthermore, the eigenvalue results indicate that for E_0unstable, while E_1 with the parameter value, was obtained,

$$(-3.1788, \quad -0.1, \quad -1.321, \quad -0.8),$$

It can be observed that the eigenvalues are negative, indicating that the equilibrium point with the active gambling population (E_1) is asymptotically stable. While substituting the parameter values in **Equation (11)** and the basic reproduction number is shows

$$\mathcal{R}_0 = \frac{\beta_1 S}{(\mu + v + \gamma_1)} + \beta_2 S\left(\frac{v}{(\mu + v + \gamma_2)(\mu + \gamma_2)}\right) = 15 > 1.$$

The results of numerical solutions for simulating the spread gaming behavior with cognitive behavioral therapy (CBT) are shown in graphic plots **Figure 2**.

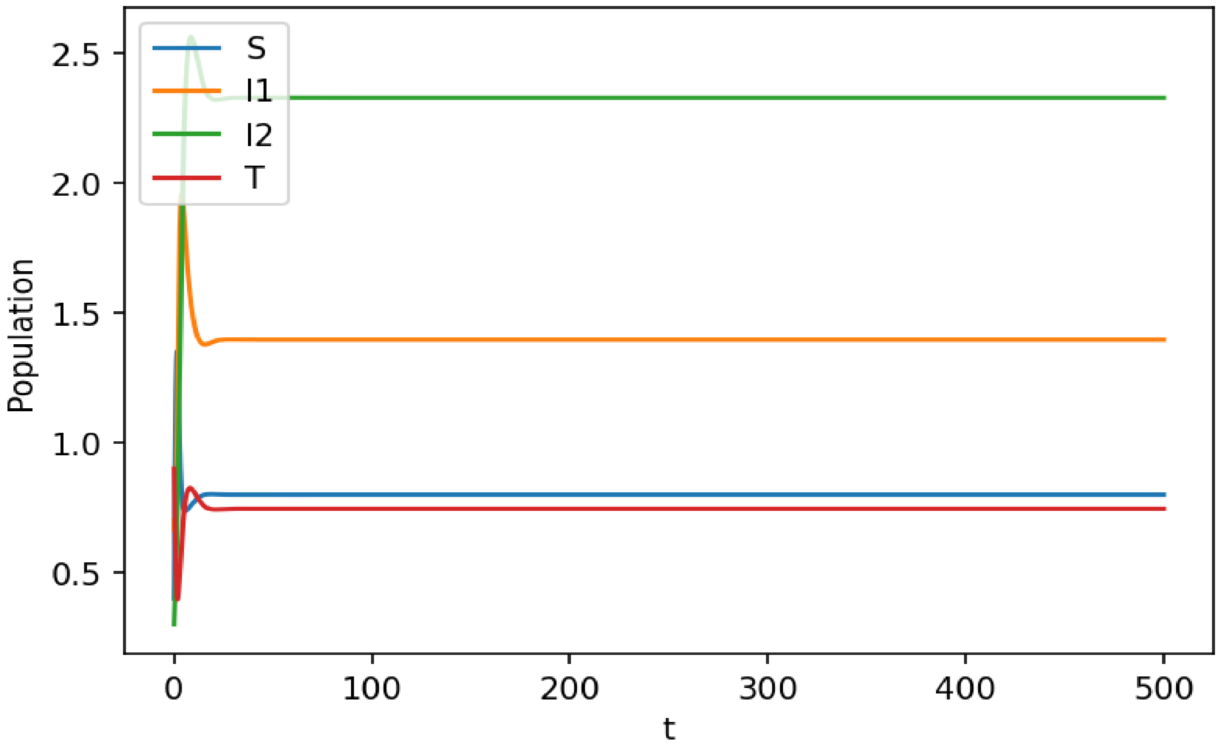

Figure 2. Graph of the Numerical Simulation of the Equilibrium Point of the Endemic Gambler Population

Figure 2 depicts the rates of the vulnerable gambling population, non-addicted gamblers, addicted gamblers, and gamblers under treatment. The growth rate of the vulnerable gambling population shows a significant increase, followed by a continuous decrease after a certain time t, indicating that the population reaches a stable state due to experiencing cycles of increase and decrease. In the population of non-addicted gamblers, there is a decline, but it is possible that some non-addicted gamblers remain. Although the population consistently decreases, it stabilizes within a certain time period 2.625. Based on the numerical results, the addicted gambling population shows generates constant graphs that converge to the value of 4.375. Next, for the equilibrium points of the gamblers under treatment, the population decreases until it reaches a point where the infected cell movement remains unchanged or in a balanced state. This decline occurs due to the absence of a transition from addicted gamblers to gamblers under treatment. The population of gamblers under treatment reaches equilibrium at 1.4 at a certain time t.

Based on the results above, it can be concluded that the equilibrium point E_0 is stable asymptotically local while $\mathcal{R}_0 < 1$ and vice versa. Besides that, the equilibrium point E_1 stable asymptotically local if only if

1. $a_1 > 0$
2. $a_1 a_2 - a_0 a_3 > 0$
3. $a_1 a_2 a_3 + a_0 a_1 a_5 - a_1^2 a_4 - a_0 a_3^2 > 0$
4. $2a_0 a_1 a_4 a_5 + a_1 a_2 a_3 a_4 + a_1^2 a_2 a_6 - a_0 a_1 a_3 a_6 - a_1 a_2^2 a_5 - a_1^2 a_4^2 - a_0 a_3^2 a_4 + a_0 a_2 a_3 a_5 - a_0^2 a_5^2 - a_0 a_1 a_2 a_7 + a_0^2 a_3 a_7 > 0$

Providing cognitive behavioral therapy (CBT) to gamblers in treatment can reduce the growth of the gambler population. The population growth will decrease in such a way that it will eventually lead to a Gambling-Free Population.

4. CONCLUSIONS

The conclusion of this study is as follows:

1. The model has identified two main equilibrium (fixed) points: E_0 (representing a gambling-free population) and E_1 (representing an existent state where gambling behavior persists).
2. For $E_0\ (A/\mu, 0,0,0)$ (free population): This point is stable asymptotically local and exist if all variables have non-negative values, meaning there's no members of the population engage in gambling.
3. For $E_1\left(S_1, I_{1_1}, I_{2_1}, T_1\right)$ (existent state) with

$$S_1^* = \frac{\mu^2 + \mu v + \mu\gamma_1 + \mu\gamma_2 + v\gamma_2 + \gamma_1\gamma_2}{\mu\beta_1 + v\beta_2 + \beta_1\gamma_2}$$

$$I_{1_1}^* = \frac{(C_1)(\mu + \gamma_2)}{\mu C_2},$$

$$I_{2_1}^* = \frac{vC_1}{\mu C_2},$$

$$T_1^* = \frac{(\mu\gamma_1 + v\gamma_2 + \gamma_1\gamma_2)(C_3)}{\mu C_2}.$$

This equilibrium is stable asymptotically local if all model variables are positive; otherwise, the model declares this state non-existent, as negative values would indicate unfeasible population states.
4. In the stability analysis, the equilibrium points will be stable if they meet the following conditions:
   a. If all values of the fixed point $E_0\ (A/\mu, 0,0,0)$ exist, they are declared to be existent.
   b. If any variable has a negative value in the fixed point $E_1\left(S_1, I_{1_1}, I_{2_1}, T_1\right)$, it is declared non-existent as it does not meet the fixed point criteria.
5. The basic reproduction number $\mathcal{R}_0 = \frac{\beta_1 S}{(\mu+v+\gamma_1)} + \beta_2 S\left(\frac{v}{(\mu+v+\gamma_2)(\mu+\gamma_2)}\right)$ are obtained, the disease-free equilibrium point is locally asymptotically stable under the condition that $\mathcal{R}_0 < 1$ and vice versa.
6. Performing numerical analysis on the gambling population with cognitive behavioral therapy treatment. Based on the conducted analysis, simulations for E_0 and E_1 are in an existent state and asymptotically stable.

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