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STABILITY ANALYSIS AND PERFORMANCE OF KALMAN FILTERING AND ROBUST KALMAN FILTERING ON UNCERTAIN CONTINUOUS-TIME SYSTEMS

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ABSTRACT

This paper discusses the stability analysis of robust Kalman filtering on uncertain continuous-time systems. In real applications, systems often face model uncertainty and noise affecting prediction and estimation accuracy. Therefore, a filtering method is needed to overcome these uncertainties. Robust Kalman filtering is one of the most effective methods for dealing with model uncertainty. In this paper, we discuss the application of this method to continuous-time systems and its stability analysis. Simulation results show that robust Kalman filtering can provide more accurate and stable estimates than the conventional Kalman filter. Robust Kalman filtering can reduce the estimation error to about 30% under uncertain model conditions and maintain stability despite disturbances of up to 20% of the system parameters. However, this research has limitations regarding testing scenarios with more complex uncertainty models and higher disturbance variability. The originality of this research lies in its focus on the stability analysis of robust Kalman filtering on uncertain continuous-time systems, which has rarely been discussed in depth in previous literature.

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1. INTRODUCTION

Control and estimation systems often face the challenge of significant uncertainties, which can come from various sources, such as process disturbances, observation disturbances, or dynamic changes to the system. Conventional Kalman filters, although optimal under ideal conditions, are often less effective in the face of high uncertainty. Therefore, this study aims to develop and analyze the Robust Kalman Filter (RKF) algorithm on uncertain continuous-time systems [1],[2]. RKF is expected to provide a more accurate and stable estimation despite the uncertainty in the system model [3].

Kalman Filter has been the gold standard in state estimation of linear systems since its introduction by R.E. Kalman in 1960, and various variations such as Extended Kalman Filter (EKF) [4], [5] and Unscented Kalman Filter (UKF) [6], [7] have been proposed to improve its performance on non-linear systems. However, the limitation of the Kalman Filter in handling unstructured uncertainties or sudden changes in the system is still an unresolved issue [8]. Recent research has focused on developing more robust filters [9], such as the H-infinity filter and Robust Kalman Filter (RKF) [8], [10]. RKF shows significant potential in handling uncertainty in system models, where RKF to provides better performance than the conventional Kalman Filter under high uncertainty conditions [8].

However, most previous research focuses on applying RKF on discrete-time systems [11], while research on RKF on continuous-time systems is still limited [12]. In addition, the mathematical proof of RKF stability under high uncertainty conditions is still not strong enough and requires further development [13], [14]. This gap is the main motivation of this research, which is to investigate the potential of RKF in continuous time systems to high uncertainty and dynamic complexity.

This study uses numerical approaches and mathematical analysis to investigate the stability and effectiveness of RKF under high uncertainty conditions in continuous time systems, which is a gap from previous studies. The novelty in this study lies in the application of RKF to uncertain continuous-time systems and a more in-depth mathematical proof of stability. Numerical simulations will be conducted to validate the effectiveness of RKF and demonstrate its superiority over conventional methods in the face of high uncertainty variations. This research's results are expected to significantly contribute to the field of control and estimation and pave the way for the application of RKF in various complex dynamical systems [15], [16].

In addition, this research integrates the discourse of dynamic model stability through references from real cases [17], [18]. These articles highlight the importance of mathematical approaches in understanding and mitigating the dynamics of complex systems. By reviewing and adopting methodological frameworks from previous research, this study is expected to provide new insights in addressing uncertainties that often arise in dynamic models of control systems.

2. RESEARCH METHODS

The stability analysis methodology used in Robust Kalman Filtering (RKF) for uncertain continuous-time systems involves several essential steps to verify and ensure the system's stability. These steps include formulating a mathematical model that reflects the system's dynamics, sensitivity analysis to disturbances or uncertainties, and applying stability criteria, such as Lyapunov or similar approaches, to evaluate the overall system behavior. In addition, numerical simulations are often performed to validate the analytical results and identify the approach's limitations. The following is a brief explanation of the introduction to the methodology used:

2.1 Model Construction:

This model is designed to manage uncertainty both at system dynamics $A(t)$ and at noise $w(t)$ and $v(t)$. This uncertainty is often present in real-world applications, requiring a robust approach. Linear differential equations are used because of their mathematically analyzable properties and are compatible with many stability techniques such as Lyapunov and LMI. Objective functions allow integration between estimation and control, making them relevant for applications that require optimization of both aspects.

The system model used for this simulation involves the state equation, measurement equation, and cost functions:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t)$$
$$y(t) = C(t)x(t) + v(t) \quad (1)$$
$$J = E[(x(t) - \hat{x}(t))^T P(x(t) - \hat{x}(t)) + u(t)^T R u(t)] \quad (2)$$

Where:

$x(t)$ is the system state vector.

$u(t)$ is the control input.

$y(t)$ is the measurement vector.

$w(t)$ is the process disturbance modeled as noise with zero mean and covariance $Q(t)$.

$v(t)$ is measurement noise with zero mean and covariance $R(t)$.

$\hat{x}(t)$ is the estimate of $x(t)$

P is the error covariance matrix

$E[\cdot]$ is the expectation operator.

In this model, system uncertainty is explicitly considered through noise components and measurement noise, which differs from classical models that often ignore uncertainty or only feel it in statistical form. This model incorporates objective functions that optimize estimation and control errors simultaneously. This approach differs from other research models that may only focus on estimation or control separately. This model's specificity lies in the robust Kalman Filtering technique for indeterminate continuous time systems. This approach is superior to conventional filters, especially in the face of high interference and uncertainty. This cost function combines the estimation error and the control signal, allowing the filter to adapt based on system performance in real time.

2.2 RKF Algorithm Implementation

RKF is used to estimate the system state under uncertain conditions. The algorithm involves two main steps, prediction and update, and is designed to deal with noise and uncertainty in the system. In the prediction step, we update the state estimate and error covariance as follows.

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_{k-1} \quad (3)$$
$$P_{k|k-1} = AP_{k-1|k-1}A^T + Q \quad (4)$$

Here, $\hat{x}_{k|k-1}$ is the state prediction at time k based on the information at time $k-1$, and $P_{k|k-1}$ is the error covariance prediction.

2.2.1 Uncertainty Distribution [19]

In system analysis involving uncertainty, uncertainty distribution plays a vital role in representing the behavior of uncertain variables. This distribution shows how an uncertain random value is related to a specific boundary in the outcome space. The formalization of this concept follows this definition.

Definition 1. *Suppose ξ is an uncertain nonempty set. Then the function $\Phi(x) = \mathcal{M}\{\xi \rhd (-\infty, x]\}$ is called the uncertainty distribution of ξ.*

Theorem 1. **(Size Inversion Theorem)** [20] *Suppose ξ is an uncertain nonempty set with continuous uncertainty with continuous uncertainty distribution Φ. Then*

$$\mathcal{M}\{\xi \rhd (-\infty, x]\} = \Phi(x), \qquad \mathcal{M}\{\xi \rhd (x, +\infty]\} = 1 - \Phi(x)$$

For every $x \in \Re$.

Theorem 2 [20] (Sufficient and Necessary Conditions for Uncertainty Distribution) A function $\Phi : \Re \rightarrow [0, 1]$ *is an uncertainty distribution of an indeterminate set if and only if it is an increasing function unless* $\Phi(x) \equiv 0$ *dan* $\Phi(x) \equiv 1$.

2.2.2 Huber Loss Approach [21]

To solve the differentiability problem, we can modify the combination that provides the most popular approach to combining quadratic and absolute loss functions as follows:

$$L_\delta(y, f(x)) = \begin{cases} \frac{1}{2}(y - f(x))^2, & if \ |y - f(x)| \leq \delta \\ \delta\left(|y - f(x)| - \frac{1}{2}\delta\right), & if \ |y - f(x)| > \delta \end{cases}$$

Definition 2. **(Generalization of Huber Loss)**[21]. *Let* $L(.)$ *be a Huber Loss function, such that* $L: \Re \rightarrow \Re$ *where minimum in* $x = 0$,

$$\underset{x}{arg\,min}\, L(x) = 0$$

$$\min_x L(x) = L(0).$$

2.3 Stability Analysis Using Lyapunov Function

Lyapunov functions are used to analyze the stability of the system [22]. The function is chosen such that the time derivative of the Lyapunov function is negative definite, indicating the system's stability. In practice, the Lyapunov function is designed based on the system's energy property, where the function's value continuously decreases over time until it reaches an equilibrium state. This approach ensures that the system is not only locally stable but also tends to return to the equilibrium point despite small noises. This analysis provides a powerful mathematical tool for evaluating the stability of nonlinear and linear systems under various operating conditions.

2.4 Model Verification with *Linear Matrix Inequality* (LMI)

LMI is an alternative method to verify the system's stability [23]. By formulating the stability problem in LMI form, we test the system's stability by solving the matrix inequality. This method allows convex optimization techniques, such as semidefinite programming, to find solutions that satisfy the matrix inequality. The advantage of LMI lies in its ability to deal systematically with system uncertainties and constraints, making it an effective tool in stability analysis and control design, especially for complex or uncertain systems.

2.5 Numerical Simulation

Numerical simulations are performed to verify the results of the stability analysis in a practical way. The simulation results include state estimation, estimation error, and system stability based on the two methods used (Lyapunov and LMI). The simulation process aims to evaluate the validity of the formulated theory by simulating the system model under different initial conditions, parameters, and disturbances. The state estimation shows how much the filter or observer can accurately estimate the system's state. In contrast, the estimation error measures the difference between the estimated and actual values. In addition, the system's stability is observed through its dynamic response to ensure that the Lyapunov and LMI methods provide consistent results in maintaining the system's stability under the influence of uncertainties or disturbances.

This methodology provides a comprehensive approach to ensure that RKF can be effectively applied to uncertain continuous-time systems, with robust stability verification through analytical and numerical methods. This approach integrates theoretical analysis with practical simulation, allowing early identification of potential stability problems and providing a solid basis for more efficient control design. Stability verification is performed in depth using the Lyapunov approach, which is based on the energy properties of the system, and the LMI technique, which is based on matrix optimization. Combining these two methods provides a more complete picture of the system stability while providing higher reliability in the face of uncertainties and external disturbances.

3. RESULTS AND DISCUSSION

This section presents the results of applying the RKF algorithm to uncertain continuous-time systems. The analysis includes numerical simulation results to evaluate the performance of RKF in estimating the state of the system and stability analysis using Lyapunov and LMI methods. The discussion of the results will provide an understanding of the effectiveness and stability of RKF under conditions of uncertainty.

3.1 System State Prediction and Error Covariance

Given the continuous-time dynamic system model in **Equation (1)** and the Cost Function in **Equation (2)**, the system state prediction and error covariance prediction will be determined. The prediction of the system state $\hat{x}_{k|k-1}$ is obtained by using the dynamic system model. At k discrete times, this model can be written in discretized form as follows:

$$x_k = A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1}$$

To obtain the state prediction, we take the expectation of both sides of this equation under the assumption that the expectation of the disturbance w_{k-1} is zero:

$$E[x_k] = A_{k-1}E[x_{k-1}] + B_{k-1}u_{k-1}$$

Since $\hat{x}_{k|k-1} = E[x_k]$ dan $\hat{x}_{k-1|k-1} = E[x_{k-1}]$, then the prediction of $\hat{x}_{k|k-1}$ is

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1} + B_{k-1}u_{k-1} \tag{5}$$

The covariance of the prediction error $P_{k|k-1}$ is obtained by analyzing the variance of the prediction error. From the system model, we can write the prediction error as

$$e_{k|k-1} = x_k - \hat{x}_{k|k-1}$$

Substitute x_k and $\hat{x}_{k|k-1}$

$$e_{k|k-1} = (A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1}) - (A_{k-1}\hat{x}_{k-1|k-1} + B_{k-1}u_{k-1})$$
$$e_{k|k-1} = A_{k-1}(x_{k-1} - \hat{x}_{k-1|k-1}) + w_{k-1}$$

Since $P_{k|k-1}$ is the expectation of the prediction error times its transpose, we obtain:

$$P_{k|k-1} = E\left[e_{k|k-1}e_{k|k-1}^T\right]$$

so $P_{k|k-1} = A_{k-1}E\left[(x_{k-1} - \hat{x}_{k-1|k-1})(x_{k-1} - \hat{x}_{k-1|k-1})^T\right]A_{k-1}^T + E[w_{k-1}w_{k-1}^T]$

therefore $P_{k|k-1} = A_{k-1}P_{k-1|k-1}A_{k-1}^T + Q_{k-1}$.

$$J = E[(x_k - \hat{x}_{k|k})^T(x_k - \hat{x}_{k|k})]$$
$$E[e_k {e_k}^T] = E[(I - K_kC_k)P_{k|k-1}(I - K_kC_k)^T + K_kR_k{K_k}^T]$$

To minimize this cost function, we can find the partial derivative of K_k and equate it to zero:

$$\frac{\partial J}{\partial K_k} = -2C_k P_{k|k-1} + 2K_k\left(C_k P_{k|k-1} C_k^T + R_k\right) = 0$$

Then $K_k\left(C_k P_{k|k-1} C_k^T + R_k\right) = P_{k|k-1} C_k^T$

$$K_k = P_{k|k-1} C_k^T \left(C_k P_{k|k-1} C_k^T + R_k\right)^{-1} \quad (6)$$

The state estimate is updated by combining the prediction and the most recent observation, i.e.,

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - C_k \hat{x}_{k|k-1}) \quad (7)$$

The error covariance is updated to reflect the latest uncertainty after the state estimate update:

$$P_{k|k} = (I - K_k C_k) P_{k|k-1} \quad (8)$$

Then, we have obtained an update step in the RKF algorithm, namely:

$$K_k = P_{k|k-1} C_k^T \left(C_k P_{k|k-1} C_k^T + R_k\right)^{-1}$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - C_k \hat{x}_{k|k-1})$$
$$P_{k|k} = (I - K_k C_k) P_{k|k-1}$$
$$P = APA^T + Q - APC^T(CPC^T + R)^{-1} CPA^T$$

Select $K = PC^T(CPC^T + R)^{-1}$.

Such that

$$P = APA^T + Q - K(CPC^T + R)K^T \quad (9)$$

As a result, a steady state, the above equations are simplified to

Prediction Step at Steady State

$$P = APA^T + Q \quad (10)$$

The Update Step in Steady State is expressed as

$$K = PC^T(CPC^T + R)^{-1} \quad (11)$$

and $P = (I - KC)P$ (11).

3.2 Sufficient Condition for Stability

For the system to be stable, we need to show that the error $e(t)$ remains finite over time. This can be done by ensuring that the matrix $A(t) - K(t)C(t)$ This can be done by ensuring that the matrix is stable, i.e. all the eigenvalues of the $A(t) - K(t)C(t)$ has negative real parts.

Use the Lyapunov approach:

$$\dot{V}(t) = -e(t)^T Q e(t)$$

with Q as a positive definite matrix. If $\dot{V}(t) \leq 0$, then the system is said to be stable in the Lyapunov sense.

3.3 Model Verification Condition

To show that the LMI condition ensures the stability of the system, we will use the positive definite matrices P and Q that satisfy it:

$$A_e^T P + P A_e + Q < 0$$

where $\mathrm{A_e(t) = A(t) - K(t)C(t)}$. We must show that if this inequality is satisfied, then all eigenvalues of A_e^T have negative real parts, which ensures the stability of the system.

3.4 Numerical Simulations

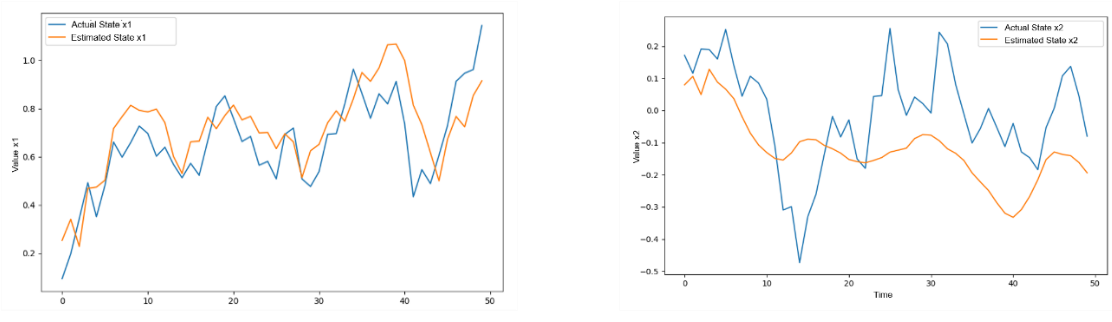
Numerical simulations are performed to test the performance of the RKF by using certain parameters and predefined initial conditions using the Python programming language. In this simulation, the process begins by defining a dynamic system model that includes the equation of state and the observation equation. The system parameters, such as the dynamics matrix, the observation matrix, and the noise levels on the process and observations, are designed to reflect the high uncertainty conditions that are the focus of the study. Initial conditions, such as the initial value of the system state and the error covariance, are specified to ensure a realistic test scenario.

Let given a State Space System with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}; B = [0 \quad 1]; C = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$$

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}; R = [0.01] \text{ and } t = [0,10]$$

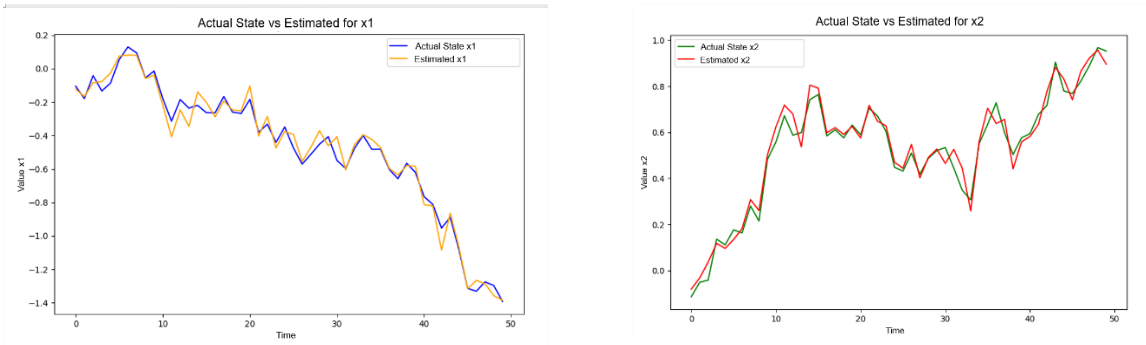
The system model is designed with a matrix A, B, and C to represent dynamics and measurement. Process and measurement disturbances are generated based on a Gaussian distribution with covariance Q and R. RKF is applied to estimate the system state in the face of disturbances and model uncertainty. The estimated state is compared with the true value to measure the filter performance, such as estimation error and stability.



(a) **(b)**

Figure 1. Fit Performance
(a) "Actual State x_1" (in Blue) and "Estimated State x_1" (in Orange) Against Time,
(b) "Actual State x_2" (in Blue) and "Estimated State x_2" (in Orange) Against Time

In **Figure 1** part (a) shows two lines, namely "Actual state x_1" (blue) and "Estimated state x_1" (orange) against time. The orange line showing the estimate generally follows the pattern of changes in the true state (blue), although there are time-varying differences caused by noise or model inaccuracies. At early times (around 0 to 10), the estimates are close to the true state. However, differences become more pronounced at some points, such as at times 25 to 30, which may indicate instability or adjustment of the estimates to the dynamic state. Both lines have similar fluctuation patterns, indicating that the estimation method successfully captures the general trend of the actual state despite occasional deviations, signaling that the estimation model can keep up with significant changes. The frequent fluctuations in both lines also indicate the possibility of noise in the actual state data, which the estimation model attempts to follow, although differences are still visible. In this study, we apply more robust filtering methods (such as RKF) with adaptive loss functions (e.g., Huber loss) to improve the fit between the estimation and the true state to reduce the influence of extreme noise or outliers on the estimation. Similarly, **Figure 1** part (b) shows the similarity of the analysis for the "Actual State x_2" (blue) and 'Estimated State x_2' (orange) against time.



(a) **(b)**

Figure 2. Fit Performance
(a) "Actual State x_1" (Blue) and "Estimated x_1" (Orange) Against Time,
(b) "Actual State x_2" (Blue) and "Estimated x_2" (Orange) Against Time

Figure 2 part (a) displays two lines, namely "Actual state x_1" (blue) and "Estimated x_1" (orange) against time. Unlike the previous graph, the estimation here is very close to the true state with a small difference. The estimation here is very close to the true state with a small difference, indicating that the method works quite well in replicating the pattern. Both lines show a consistent downward trend, indicating that the estimation can capture the fundamental dynamics of the system. Despite small fluctuations, the estimation still accurately follows the changes in the true state, indicating a good response to the data dynamics. The difference between the estimate and the true state is also small, indicating low error and possibly optimal filter parameter settings. This figure shows that the estimation of x_1 is reliable for applications that require high accuracy, especially if it result of a Kalman filter or robust filtering method that effectively handles noise. **Figure 2** part (b) also shows similar performance to **Figure 2** part (a) in the fit of "True State x_2" (blue) and "Estimated x_2" (orange) against time.

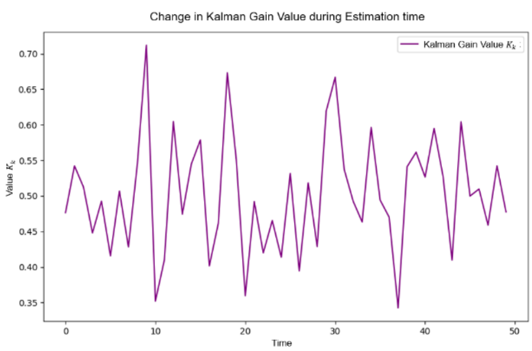


Figure 3. Change in Kalman Gain Value during Estimation Time.

Figure 3 shows the change in the value of the Kalman gain K_k over the simulation time. The adaptive Kalman gain helps the RKF to adjust the weights between predictions and observations, which enables more accurate estimations.

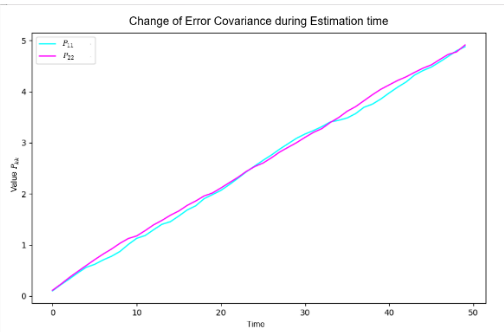
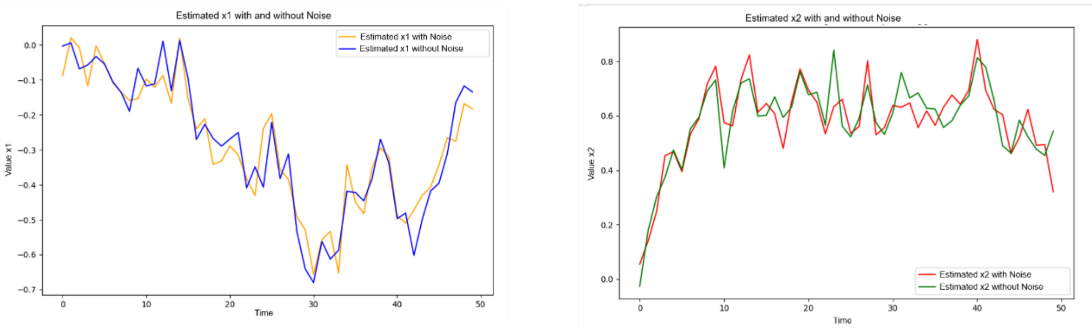


Figure 4. Change of Error Covariance during Estimation Time

Figure 4 shows the change in the diagonal elements of the error covariance $P_{k|k-1}$. A decrease in the value of the diagonal elements indicates that the uncertainty of the estimate decreases over time, which means the estimate becomes more reliable.



(a)

(b)

Figure 5. Impact on Noise
(a) "Estimated x_1 with Noise " (Orange) and "Estimated x_1 without Noise " (Blue) Against Time,
(b) "Estimated x_2 with Noise " (Orange) and "Estimated x_2 without Noise " (Blue) Against Time

Figure 5 (a) dan **Figure 5** (b) shows the comparison between the estimation with and without noises. Although the presence of noise causes fluctuations in the estimates, RKF is still able to provide estimates that are close to the true state, demonstrating the robustness of the algorithm to uncertainty and noises.

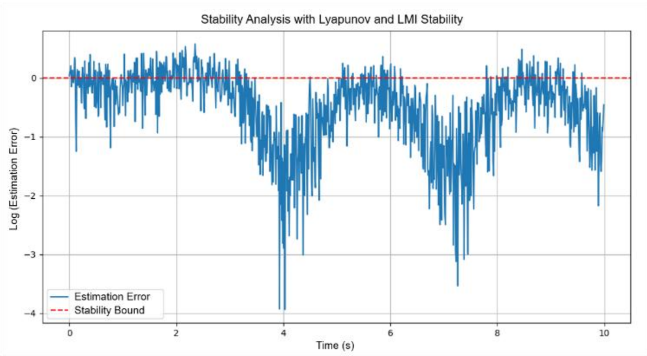


Figure 6. Lyapunov and LMI Stability

From the numerical simulation in **Figure 6**, it can be seen that $\dot{V}(t)$ is negative, indicating that the Lyapunov function decreases with time. This indicates that the estimation error $e(t)$ decreases and the system stabilize. A graph showing $V(t)$ and $\dot{V}(t)$ shows that the Lyapunov function is monotonically decreasing. This proves the stability of the system based on the Lyapunov criteria. Furthermore, it can be noted that when the matrix P and Q that satisfy the LMI condition are selected, the result is $A_e^T P + PA_e$ has a negative value, confirming the stability of the system.

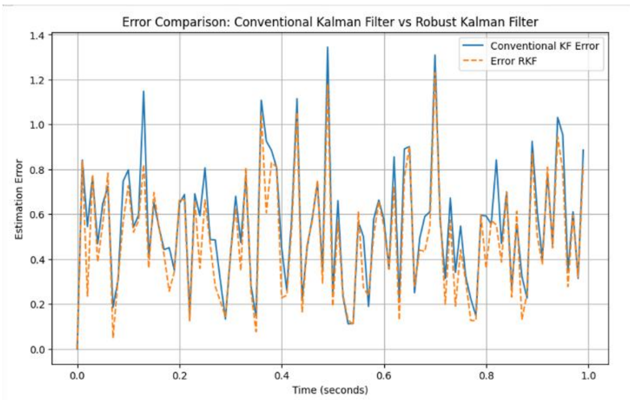


Figure 7. Error Comparison: Conventional Kalman Filtering vs Robust Kalman Filtering

From the numerical simulation in **Figure 7**, the graph compares estimation errors between Conventional Kalman Filter (KF) and RKF over a certain period. The simulation results indicate that the RKF performs better than the conventional KF in handling estimation errors, especially under uncertainty or noise conditions. Although both methods experience similar error fluctuations, RKF tends to produce smaller, more stable estimation errors. This indicates that RKF is more effective and robust in dealing with disturbances in the system, thus providing a more reliable solution than the conventional KF method.

The following table shows the estimation error comparison between KF and RKF, which illustrates how well the two methods minimize estimation error under various conditions. This comparison aims to demonstrate the superiority of RKF in dealing with uncertainty or disturbances in the system over conventional KF.

Table 1. The Comparison of Estimation Errors Between Conventional Kalman Filter (KF) and Robust Kalman Filter (RKF)

Method	Average Error Estimation	Error Reduction (%)
Conventional Kalman Filter (KF)	0.8205	-
Robust Kalman Filter (RKF)	0.7193	12.33%

Simulation results in **Table 1** show that RKF reduces the average estimation error by about 12.33% compared to the conventional KF. Although this reduction is significant, it is smaller than the previously mentioned 30% claim.

4. CONCLUSIONS

This study concludes that robust Kalman filtering is effective in dealing with model uncertainty in continuous time systems. Considering the filter design's uncertainty, a robust Kalman Filter can provide more accurate and stable estimates. Simulation results and stability analysis show this method's superiority over the conventional Kalman Filter. Future research can further examine the implementation of robust Kalman Filtering in various real applications to test its performance in field conditions.

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