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MAPPING EARTHQUAKE MAGNITUDES IN BENGKULU PROVINCE AND SURROUNDING AREAS USING ROBUST ORDINARY KRIGING

Baki Swita[1], Mulia Astuti[2], Fachri Faisal[3*], Aang Nuryaman[4]

[1,2,3]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Bengkulu Jln. W.R. Supratman, Kandang Limun, Bengkulu, 38371, Indonesia

[4]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Lampung Jln. Soemantri Brojonegoro 1, Bandar Lampung, 35141, Indonesia

*Corresponding author's e-mail: * **fachrif@unib.ac.id***

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ABSTRACT

Bengkulu Province, situated in a subduction zone between the Indo-Australian and Eurasian plates, is highly susceptible to significant seismic activity, including major earthquakes in 2000 and 2007 with magnitudes exceeding 7. This research investigates the geographical distribution of earthquake magnitudes in Bengkulu Province and surrounding areas from 2000 to 2023. Understanding these spatial patterns is crucial for enhancing disaster preparedness and risk mitigation strategies in this high-risk region. Previous studies on earthquake distribution in Indonesia have provided valuable insights but often struggle with outliers and data variability, limiting their accuracy. Conventional Ordinary Kriging methods, though widely used, are sensitive to outliers, leading to potential inaccuracies. This study addresses these limitations by applying a robust Ordinary Kriging approach, which effectively mitigates the influence of outliers, thereby improving prediction reliability. The research utilizes earthquake data, including geographical coordinates and recorded magnitudes. It applies both classical and robust experimental semivariograms (Cressie-Hawkins) to model the spatial structure using theoretical variogram models—spherical, exponential, and Gaussian. The best-fit model is determined based on the lowest root mean square error (RMSE), ensuring accurate representation of spatial patterns. The results demonstrate that robust Ordinary Kriging accurately maps the spatial distribution of earthquake magnitudes, revealing clusters of higher magnitude events in specific regions of Bengkulu Province. These findings identify high-risk areas, providing essential data for disaster mitigation and risk management planning. This study significantly contributes to the field of seismology and geostatistics by enhancing the accuracy of magnitude distribution mapping. The resulting maps support local governments, urban planners, and disaster response organizations in developing more effective mitigation strategies, improving infrastructure resilience, and strengthening early warning systems. Ultimately, this research aims to foster safer, more prepared communities in Bengkulu Province and beyond.

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1. INTRODUCTION

Bengkulu Province and the nearby areas experience significant tectonic movements. They are situated in the subduction area of the Indo-Australian and Eurasian plates. The coming together of these two plates leads to the area often encountering earthquakes of different sizes. As one of Indonesia's areas with high earthquake risk, it is crucial to comprehend the spatial arrangement of earthquake magnitudes in Bengkulu Province and the nearby areas [1]. Thorough knowledge of these distribution patterns is beneficial for predicting possible risks and laying the groundwork for disaster preparedness and sustainable development planning.

One of the main challenges in earthquake distribution mapping is the presence of outliers, which often arise due to the wide range of magnitudes recorded in seismic data. Traditional geostatistical methods, such as Ordinary Kriging, are highly sensitive to these outliers, potentially leading to biased predictions and unreliable spatial representations. To address this limitation, this study applies the Robust Ordinary Kriging method, which has been demonstrated to improve prediction accuracy in datasets with extreme values by reducing the influence of outliers on spatial interpolation results [2].

Early research on robust kriging can be traced back to studies on robust variogram estimation methods, particularly the Cressie-Hawkins estimator, which was introduced to improve the reliability of semivariogram modeling in the presence of outliers [2]. Unlike classical variogram estimators that are highly sensitive to extreme values, the Cressie-Hawkins robust estimator assigns lower weights to significant squared differences, reducing their influence on the variogram structure. This approach enhances the robustness of spatial predictions, particularly in datasets with heavy-tailed distributions or measurement errors. The main advantage of robust experimental semivariograms is their ability to provide more stable and resistant estimates, ensuring the spatial dependence structure remains unaffected by extreme values. As a result, robust semivariograms improve the accuracy of kriging predictions, making them particularly valuable in geostatistical applications where outliers are common, such as in earthquake magnitude mapping.

Several researchers have conducted studies on mapping earthquake distribution in Indonesia. Previous studies [3] applied spatial analysis and time series modelling to study the frequency of earthquake events in Bengkulu Province, offering valuable insights into temporal patterns and recurrence intervals of seismic activities. Other research [4] examined the distribution of significant earthquakes based on magnitude and depth in the Mamuju region and surrounding areas, highlighting the correlation between seismic intensity and tectonic settings. Studies [5] mapped the distribution of earthquake occurrences in Papua Province, demonstrating the geographical spread of seismic events and providing a baseline for regional seismic risk assessment. Further investigations [6] explored the use of Ordinary and Robust Kriging methods to gain spatial insights into earthquake strength in Sulawesi, emphasizing the advantages of robust kriging in managing outliers and improving the accuracy of seismic hazard mapping. These studies contribute significantly to our understanding of earthquake distribution across different regions of Indonesia. However, they each exhibit limitations, particularly in managing the outliers prevalent in earthquake data due to the wide range of event magnitudes.

The Ordinary Kriging method is widely used in spatial mapping because it considers spatial correlations among data points, leading to more precise predictions. Nevertheless, the data is vulnerable to outliers that may alter the prediction outcomes, leading to a less accurate distribution map. While Ordinary Kriging is widely used in geostatistical mapping due to its ability to model spatial correlations, it assumes that data follows a normal distribution and is highly sensitive to extreme values. In contrast, Robust Kriging introduces a weighting mechanism that reduces the influence of outliers, resulting in more stable and reliable spatial predictions [2]. This advantage is particularly crucial in earthquake magnitude mapping, where extreme values are common due to the natural variability of seismic events. By mitigating the impact of these outliers, Robust Kriging enhances the accuracy and trustworthiness of the generated magnitude distribution maps, making them more suitable for risk assessment and disaster preparedness.

The heightened disaster risk faced by communities in Bengkulu Province and its surroundings emphasizes the urgency of this research. The region has experienced significant historical earthquakes, such as the Bengkulu earthquakes in 2000 and 2007, with magnitudes exceeding 7, highlighting the need for precise maps depicting the distribution of earthquake magnitudes. To address this, the study will utilize daily earthquake data from 2000 to 2023 to generate a distribution map using the Robust Ordinary Kriging method. This approach was chosen because it can produce forecast maps that are less affected by outliers, which is standard in earthquake data. Various stakeholders, including the government, urban planners, and disaster response organizations, can leverage these distribution maps to develop more effective mitigation strategies and policies for addressing earthquake vulnerabilities [7][8]. Therefore, this research is expected to provide a broader and more accurate representation of how earthquake magnitudes are distributed in this region.

The study's findings will add to the knowledge of seismology and geostatistics and have important practical implications. The magnitude distribution maps are valuable for enhancing spatial planning safety and resilience and improving early warning systems. In addition, obtaining a more thorough understanding of these earthquake distribution patterns is expected to help local communities improve their preparedness for possible future earthquakes.

In general, the primary objective of this research is to respond to the pressing demand for improved data on earthquake vulnerability in Bengkulu Province and surrounding areas. By utilizing advanced geostatistical methods, specifically the Robust Ordinary Kriging approach, and analyzing extensive earthquake data from 2000 to 2023, this study aims to address the limitations found in previous research. Prior studies provided valuable insights but often fell short in accurately mapping seismic distributions due to the presence of outliers and varying data quality.

By focusing on this highly seismic region, situated in the subduction zone between the Indo-Australian and Eurasian plates, this research aims to enhance the accuracy and reliability of earthquake magnitude distribution maps. These improved maps are crucial for understanding spatial patterns, identifying high-risk areas, and providing a solid foundation for disaster preparedness strategies.

This study provides a novel contribution to the geostatistical modeling of earthquake magnitudes by integrating Robust Ordinary Kriging to improve the accuracy of magnitude distribution mapping in Bengkulu Province. Unlike previous studies that mainly relied on Ordinary Kriging or other spatial analysis techniques [2]-[5], this research specifically addresses the impact of outliers in seismic data, ensuring a more reliable representation of earthquake risks. Moreover, by utilizing an extensive dataset from 2000 to 2023, this study offers a more comprehensive temporal perspective on seismic activity patterns in the region. These contributions make this study the first to apply Robust Kriging for earthquake magnitude mapping in a highly seismic area of Indonesia, thereby advancing both geostatistical methodologies and disaster risk management strategies.

This enhanced data will significantly contribute to more effective disaster prevention efforts in Indonesia. It will assist local governments, urban planners, and disaster response organizations in developing better mitigation strategies, improving infrastructure resilience, and enhancing early warning systems. Ultimately, this research seeks not only to advance the field of seismology and geostatistics but also to foster safer and more prepared communities in Bengkulu Province and beyond.

2. RESEARCH METHODS

2.1 Data

The data in this study are daily earthquake event data from 2000 to 2023, representing earthquake magnitude occurrences in Bengkulu Province and its surroundings. The secondary data were obtained from the official United States Geological Survey (USGS) website [9]. The population of this study consists of all earthquake events that occurred in Bengkulu Province and its surrounding areas, while the sample includes daily recorded earthquake events from 2000 to 2023 as documented in the USGS database. The research variables include magnitude ($\boldsymbol{Z}$), which measures the strength of the earthquake on a specific scale, latitude ($\boldsymbol{X}$), representing the geographic coordinate indicating the north-south position of the earthquake event, and longitude ($\boldsymbol{Y}$), representing the geographic coordinate indicating the east-west position of the earthquake event.

2.2 Research Steps

The following are the steps used in this study:

1. Data Collection: Gather earthquake data (latitude, longitude, magnitude) from USGS (2000-2023) and clean the data by removing duplicates and handling missing values.

2. Exploratory Data Analysis (EDA): Analyze basic statistics (mean, variance, skewness) and visualize spatial distribution using scatter plots
3. Stationarity Testing: Check time series stationarity using visual plots and statistical tests.
4. Semivariogram Analysis: Compute experimental semivariograms (classical & robust) and fit theoretical models (Spherical, Exponential, Gaussian).
5. Robust Kriging Implementation: Estimate spatial weights using robust semivariogram models and perform spatial interpolation to predict magnitudes in unobserved locations.
6. Model Validation: Compare prediction errors (RMSE) using cross-validation and assess the effectiveness of robust kriging vs. classical kriging.
7. Visualization and Interpretation: Generate contour maps of predicted earthquake magnitudes and identify high-risk zones based on kriging results.

2.3 The Experimental Semivariogram

Estimating the semivariogram from sample data, such as $Z(s_1)$, $Z(s_2)$,..., where s_1, s_2,... represent the sample locations in a two-dimensional space, is the initial step in implementing theory into practice. It is assumed that those positions were chosen fairly. The common formula for calculating the semivariogram is Matheron's method of moments (MoM) estimator [10][11][12]:

$$\hat{\gamma}(h) = \frac{1}{2|N(h)|}\sum_{i=1}^{N(h)}[Z(s_i+h) - Z(s_i)]^2 \tag{1}$$

where $Z(s_i)$ and $Z(s_i + h)$ represent the observed z values at locations s_i and $s_i + h$, and $N(h)$ indicates the count of paired comparisons at lag h. By varying h, we can generate a sequential series of semivariances that make up the empirical or observed variogram. The manner in which **Equation (1)** is formulated and the implementation relies on whether the data is evenly distributed in one dimension, follows a regular grid, or is scattered irregularly in two dimensions.

2.4 Statistical Distribution

The geostatistical analysis does not need data to adhere to a normal distribution. However, semivariograms consist of a series of variances that may become unreliable in the presence of skewed data and outliers. If the data's distribution is not close to normal and its skewness coefficient exceeds ±1 limits, it may be necessary to consider changing the data. Thus, the data can be altered using a suitable method, such as applying logarithms and analyzing variograms calculated on both original and modified values. The resulting semivariograms significantly differs besides a scaling factor. The responses will be negative in some cases, while they will be affirmative in others[11].

Outliers, such as extremely high or low values outside the main distribution range, can affect the semivariogram. Box plots provide an effective method for detecting outliers. Every outlier must be examined and treated as a possibly incorrect value before deciding whether to include it in the dataset. In the case of contaminated sites, the highest values will be the most significant. If there are only a small number of outliers compared to the entire dataset, eliminating them usually decreases skewness, making it a reasonable strategy. If needed, the values taken out can be put back into the data for kriging. Transformation frequently does not enhance the distribution in the presence of outliers and may exacerbate the situation. Another option is to utilize robust estimators like the ones developed by Cressie-Hawkins, Dowd and Genton [13].

The estimator by Cressie-Hawkins [13][14][15] controls for outliers in the secondary process by taking the fourth root of the squared deviations. It is provided by

$$2\hat{\gamma}(h) = \frac{\left\{\frac{1}{N(h)}\sum_{i=1}^{N(h)}|z(s_i+h) - z(s_i)|^{\frac{1}{2}}\right\}^4}{0{,}457 + \frac{0{,}494}{N(h)} + \frac{0{,}045}{N(h)^2}} \tag{2}$$

The bottom part of **Equation (2)** is an adjustment made assuming that the underlying estimated process has differences that follow a normal distribution across all time intervals.

2.5 Kriging

According to Moraga [16], Kriging is a technique for spatial interpolation that allows for the estimation of values at unsampled places using observed geostatistical data. This technique was first developed in the domain of mining geology and is referred to as the Krige method, in honor of South African mining engineer Danie G. Krige. Assuming we have collected data $Z(s_1)$,..., $Z(s_n)$, our objective is to estimate the value of Z at a specific location $s_0 \in D$. The Ordinary Kriging estimator of $Z(s_0)$ is a linear unbiased estimate,

$$\hat{Z}(s_0) = \sum_{i=1}^{n} \lambda_i Z(s_i) \tag{3}$$

that reduces the average squared prediction error, which is defined as the expected value of

$$E\left[\left(\hat{Z}(s_0) - Z(s_0)\right)^2\right] \tag{4}$$

The Kriging weights are determined based on the estimated spatial structure of the collected data. To generate weights, a variogram model is fitted to the observed data. This model helps us understand how the correlation between observation values varies with the distance between sites. After obtaining the Kriging weights, they are utilized to calculate the predicted values at unsampled sites by applying them to the known data values at the sampled locations. The Kriging weights are determined based on the spatial correlation of the data, taking into consideration the geographical closeness and similarity of the data points. Therefore, places that are seen and have a correlation with the forecast locations, as well as being in close proximity, are assigned a higher weight compared to locations that are uncorrelated and located further away. The weights used in this analysis consider the spatial distribution of all data. Therefore, clustering of observations in oversampled areas are given less weight, as they provide less information compared to individual sites.

Assuming certain conditions, Kriging predictions are the most accurate estimators that are both linear and unbiased. Various forms of Kriging exist, each differing in its underlying assumptions and analytical objectives. Simple Kriging implies that the mean of the random field, $\mu(s)$, is a known value. Ordinary Kriging assumes that the mean, $\mu(s)$, is a constant value that is not known. Universal Kriging is employed when the data has an unknown non-stationary mean structure.

2.6 Model Validation for Determination of Theoretical Semivariogram in Kriging Method

In kriging, cross-validation (CV) is commonly used to assess the accuracy of the variogram model fitted to spatial data. The process involves dividing the data into several subsets or folds, applying the model to the majority of the data, and then predicting the values for the remaining subset. For each data point i, its prediction $\hat{Z}(s_i)$, where s_i is the location of that point, is made using a model trained on all other data points. This process ensures that the model is validated on data not used in its calibration, providing a more reliable measure of its performance. To quantify how well the model predicts the spatial variable, the Root Mean Square Error (RMSE) is often employed. RMSE measures the deviation between the predicted and actual values, calculated using the following formula [17][18] :

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Z(s_i) - \hat{Z}(s_i)\right)^2} \tag{5}$$

where:

n : the total number of observations (data points).

$Z(s_i)$: the true value at the location s_i.

$\hat{Z}(s_i)$: the predicted value at the location s_i obtained from kriging.

By minimizing the RMSE, one can assess how well the kriging model fits the spatial data and refine the semivariogram to improve prediction accuracy.

3. RESULTS AND DISCUSSION

3.1 Descriptive Analysis and Stationary Test

The secondary data was sourced from the official United States Geological Survey (USGS) website [9], and the following are the results of descriptive statistics of data on Earthquake Magnitude in Bengkulu Province and its surroundings, as shown in Table 1.

Table 1. Descriptive Statistics

Magnitude			
Mean	4.672010582	Kurtosis	6.8658378
Standard Error	0.011162201	Skewness	1.7339773
Median	4.6	Range	4.9
Mode	4.4	Minimum	3.5
Standard Deviation	0.485266983	Maximum	8.4
Sample Variance	0.235484044	Count (n)	1890

***Data source**: USGS*[9]

Based on Table 1 above, the distribution of earthquake magnitude data in Bengkulu Province and its surroundings tends to be positively skewed, with the majority of earthquakes having smaller magnitudes, but some earthquakes with much larger magnitudes indicate significant outliers. This is evident from the high kurtosis and positive skewness values. The data also shows that while most earthquakes are relatively moderate, there are large earthquake events that need to be considered in disaster risk analysis and mitigation in this region.

Next, a visual stationary test is performed on the Magnitude data, here a time series plot of the Magnitude column will be made to see if there are significant patterns, trends, or fluctuations over time. Because this data does not have a time column, the assumption is made that the data is sorted chronologically. The following visualization of the Magnitude data can be seen in Figure 1.

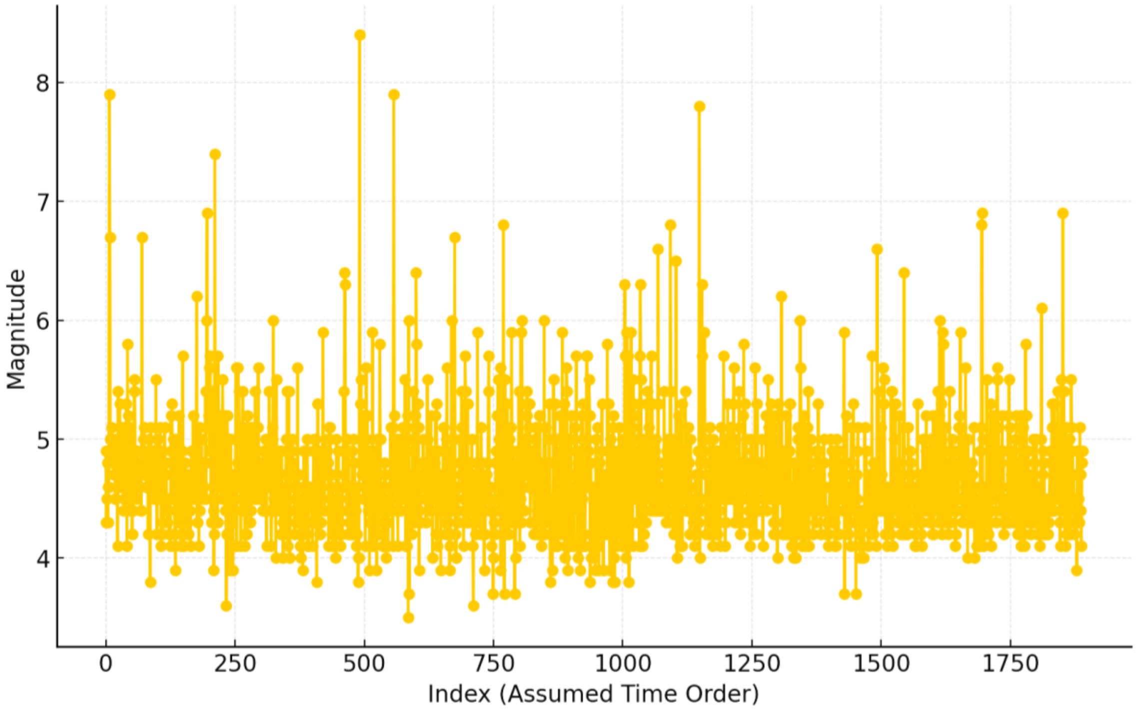

Figure 1. Time Series Plot of Magnitude Data

Based on the visualization of the "Time Series Plot of Magnitude" in Figure 1, it can be observed that the Magnitude data is distributed fairly consistently over time. The plot shows Magnitude values from various locations chronologically, which are assumed to be sorted by time.

In this figure, there is no obvious upward or downward trend, nor is there any significant seasonal or cyclical pattern. Fluctuations in Magnitude values occur around a constant mean, and changes in values appear random with no recurring pattern.

To further assess the data's stationarity, a Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test for Trend Stationarity was performed. The results showed a KPSS statistic of 0.087201 with a p-value of 0.1. Since the p-value is greater than 0.05, we fail to reject the null hypothesis (H_0), suggesting that the data is stationary around a deterministic trend. This further supports the observation from the time series plot that there is no significant trend or pattern in the Magnitude data over time.

This consistency indicates that the variance and average Magnitude values are relatively stable. The pattern seen in the figure does not show significant changes over time, which is one of the early indications of stationarity. Therefore, this pattern can be used in statistical models that require stationary data, such as the Kriging method.

Overall, this “Time Series Plot of Magnitude” provides a visual representation supporting that the Magnitude data is stationary and ready to be used for further geostatistical analysis.

3.2 Experimental Semivariogram Calculation and Theoretical Model Fitting

The calculation results of the two types of experimental semivariograms and the figure were obtained using RStudio software. The following are the experimental semivariograms of the two types of experimental semivariograms as well as the fitting results to the theoretical semivariogram model can be seen in **Table 2** and **Figure 2** below:

Table 2. Experimental Semivariogram and Theoretical Model Fitting

No	Np	Distance	Experimental Semivariogram		Theoretical Model Fitting	
			Classical	Robust	Classical	Robust
1	40398	13.603211	0.207518565	0.152622391	0.211080533	0.154608874
2	86751	32.418266	0.232361529	0.168647037	0.235657306	0.172848125
3	113674	53.395190	0.243058175	0.179527009	0.242490499	0.180615481
4	130337	74.563191	0.243024429	0.181295514	0.242811102	0.181392654
5	144093	95.656740	0.242560222	0.180310667	0.242814164	0.181415011
6	142535	116.64309	0.241403445	0.181756153	0.242814170	0.181415209
7	128821	137.78360	0.253511151	0.189861833	0.242814170	0.181415210
8	121188	159.23519	0.248616571	0.185990147	0.242814170	0.181415210
9	117723	180.28937	0.233760820	0.174824381	0.242814170	0.181415210
10	108369	201.59826	0.232095525	0.176912630	0.242814170	0.181415210
11	98282	222.77428	0.239957520	0.181291234	0.242814170	0.181415210
12	85329	243.99767	0.246053979	0.180244792	0.242814170	0.181415210
13	75803	265.33384	0.225891587	0.168233055	0.242814170	0.181415210
14	71208	286.36729	0.217462926	0.162777098	0.242814170	0.181415210
15	57907	307.28862	0.224496434	0.164187266	0.242814170	0.181415210

The experimental semivariogram in **Table 2** quantifies the spatial dependence of earthquake magnitudes by calculating variance between data pairs at increasing distances. In contrast, the classical method follows the standard calculation but is sensitive to outliers. In contrast, the robust method mitigates their influence for more stable estimates. The theoretical model fitting represents the mathematical function applied to the experimental semivariogram, which is crucial for Kriging-based spatial predictions, with the classical model fitting following standard techniques and the robust model fitting offering improved resistance to data variability. Semivariogram values rise as distance increases, indicating more significant variability between distant points. The robust method yields slightly lower semivariogram values than the classical approach, demonstrating its stability. In contrast, the theoretical model values closely align with experimental ones, validating the effectiveness of the fitting process in capturing spatial variation.

The results of the fitting calculation against the classical theoretical semivariogram and robust semivariogram were carried out using the Gaussian theoretical semivariogram model approach, which successively produced the semivariogram model (Nugget and Gaussian), sill, and range, where sill and range are parameters of the theoretical semivariogram model can be seen in **Table 3.**

Table 3. Theoretical Semivariogram Model Nugget and Gaussian and Parameter Values

Parameter	Theoretical Semivariogram	
	Classical	**Robust**
	Nugget Model	
Sill	0.19918828	0.14720873
Range	0.00000	0.00000
	Gaussian Model	
Sill	0.04362589	0.03420648
Range	24.11244	27.5515

In general, the function for both theoretical Gaussian semivariogram models is as follows:

$$\gamma(h)_{Gau-C} = \begin{cases} 0.19918828 + 0.04362589\left(1 - exp\left(-\dfrac{h^2}{24.11244^2}\right)\right), & h \leq 24.11244 \\ 0.24281417 & , h > 24.11244 \end{cases}$$

and

$$\gamma(h)_{Gau-R} = \begin{cases} 0.14720873 + 0.03420648\left(1 - exp\left(-\dfrac{h^2}{27.5515^2}\right)\right), & h \leq 27.5515 \\ 0.19083462 & , h > 27.5515 \end{cases}$$

Both semivariogram models can later be used in kriging interpolation to determine the mapping of earthquake magnitude distribution in Bengkulu Province and its surroundings. Visually, both the experimental semivariogram model and its theoretical Gaussian semivariogram model can be seen in **Figure 2**.

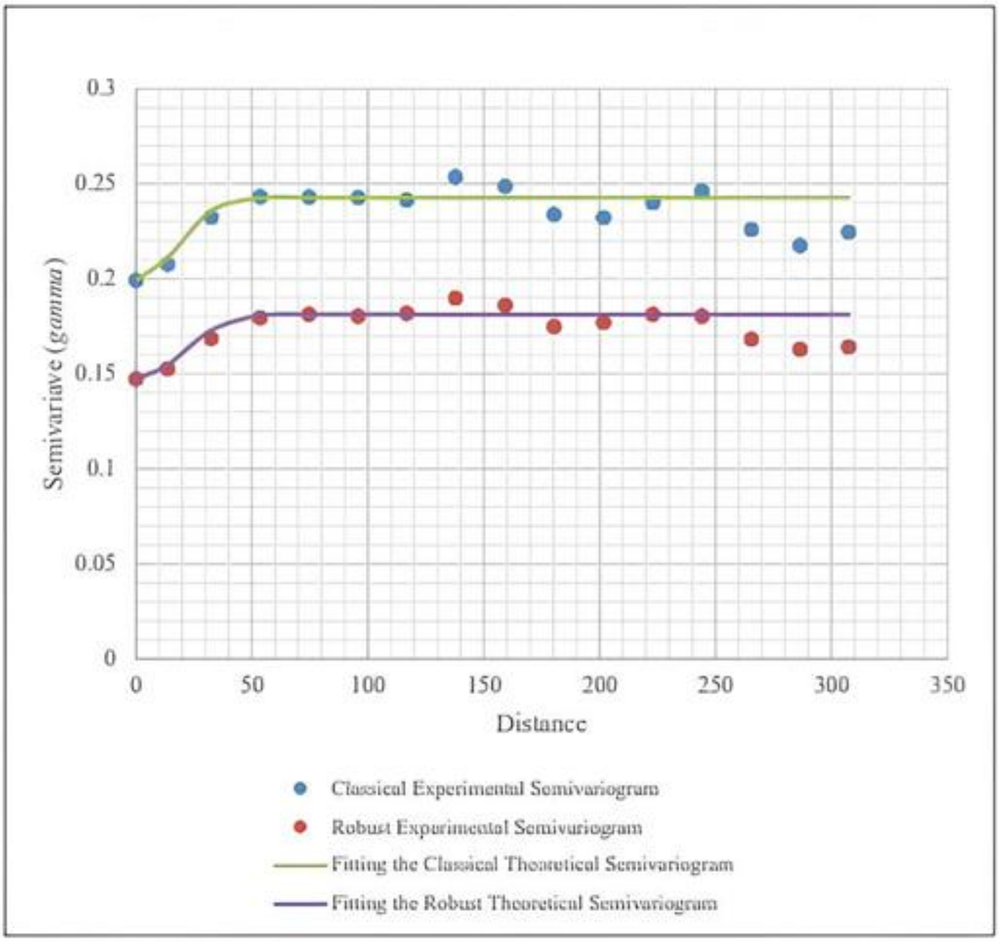

Figure 2. Classical & Robust Experimental Semivariogram and Fitting Results to Theoretical Semivariogram

Figure 2 compares classical and robust semivariograms, illustrating both their experimental values and fitted theoretical models. The classical experimental semivariogram (blue dots) fluctuates more, especially at more considerable distances, while the robust experimental semivariogram (red dots) appears smoother, indicating reduced sensitivity to outliers. Theoretical model fitting is represented by the green (classical) and purple (robust) curves, showing that the robust model maintains a more stable trend. The robust semivariogram reaches a slightly lower sill, suggesting better resistance to extreme values, while its lower nugget effect indicates reduced short-range noise. These differences imply that robust semivariograms provide a more reliable spatial dependence structure, making them preferable for kriging predictions by minimizing the influence of outliers and improving interpolation accuracy.

3.3 Model Validation for Determining Theoretical Semivariogram

After using Equation (5) in the context of classical and robust semivariograms, the RMSE values for the three theoretical semivariogram models are obtained, namely:

Table 4. RMSE Values for Three Theoretical Semivariogram Models

Semivariogram Model	RMSE	
	Classical	Robust
Spherical	0.4752728	0.4761317
Exponential	0.4781665	0.4782262
Gaussian	0.4752569	0.4733682

Based on Table 4 above, it can be seen that the RMSE values of the three theoretical semivariogram models are not significantly different. All six theoretical semivariogram models can be used to predict the magnitude at the location point of 1890 data and make contour maps of earthquake magnitude distribution in Bengkulu Province and its surroundings using ordinary kriging. However, researchers are more likely to choose the Gaussian theoretical semivariogram model because its robust RMSE value is the smallest among the others: 0.4733682.

3.4 Contour Map of Earthquake Magnitude Distribution in Bengkulu Province and its Surroundings using Robust Ordinary Kriging

Before creating the contour map of earthquake magnitude distribution in Bengkulu Province and its surroundings using robust ordinary kriging, a magnitude prediction was first conducted at 1890 data points. The results of the predictions can be seen in Figure 3.

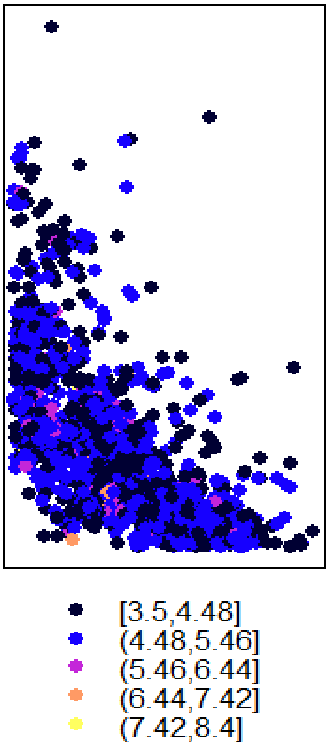

Figure 3. Robust Kriging Prediction for Magnitude

Figure 3 above is the result of robust kriging to predict the distribution of earthquake magnitudes in the Bengkulu Province and its surroundings. Here is a detailed analysis of the image:

1. Color Interpretation:
   a. Color Palette: The image shows the use of a color palette to represent predicted magnitude values. Dark colors (dark blue) indicate lower magnitude values, while light colors (yellow) indicate higher magnitude values.
   b. Color Scale: The color scale is divided into five intervals that indicate the range of predicted magnitude values:
      i. Dark blue: [3.5, 4.48]
      ii. Light blue: [4.48, 5.46]
      iii. Purple: [5.46, 6.44]
      iv. Pink: [6.44, 7.42]
      v. Yellow: [7.42, 8.4]
2. Distribution Pattern:
   a. Spatial Pole: The image shows that the magnitude of the earthquake varies across the mapped area. Areas with lighter colors (pink to yellow) indicate that the predicted earthquake magnitude in those areas is higher.
   b. Distribution of Points: The points in the image are evenly distributed across the study area, indicating that predictions have been made for many locations within the studied region.
3. Robust Kriging:
   a. Robust Kriging Method: The robust kriging method used in this prediction is designed to reduce the influence of outliers or deviating data that may exist in the dataset. This results in predictions that are more stable and resilient to anomalies in the data.
   b. The Influence of Outliers: With the robust kriging method, the predictions generated tend to be more accurate for data that has a non-normal distribution or significant outliers. This is

evident from the more even spread of predicted values, without any extreme predictions that do not align with the overall pattern.

4. Interpretation of the Magnitude Scale:

   a. Magnitude Value: A scale used to indicate the predicted earthquake magnitude ranging from 3.5 to 8.4. These values represent the severity levels of the predicted earthquakes at various locations within the study area.

   b. High-Value Distribution: Areas with high magnitude values (from pink to yellow) need special attention as they indicate regions predicted to have the potential for large earthquakes, which could have significant impacts.

5. Uses of Prediction:

   a. Mitigation Planning: The results of this prediction are very useful for disaster mitigation planning. Areas marked in bright colors (indicating high magnitude) may require special attention regarding building reinforcement, emergency preparedness, and evacuation strategies.

   b. Increased Awareness: Prediction maps like this can also be used to raise awareness among communities in earthquake-prone areas.

Next, after obtaining the magnitude prediction results using robust kriging as shown in **Figure 3**, we will now discuss the contour map of earthquake magnitude distribution in Bengkulu Province and its surroundings using ordinary kriging. The contour results can be seen in **Figure 4**.

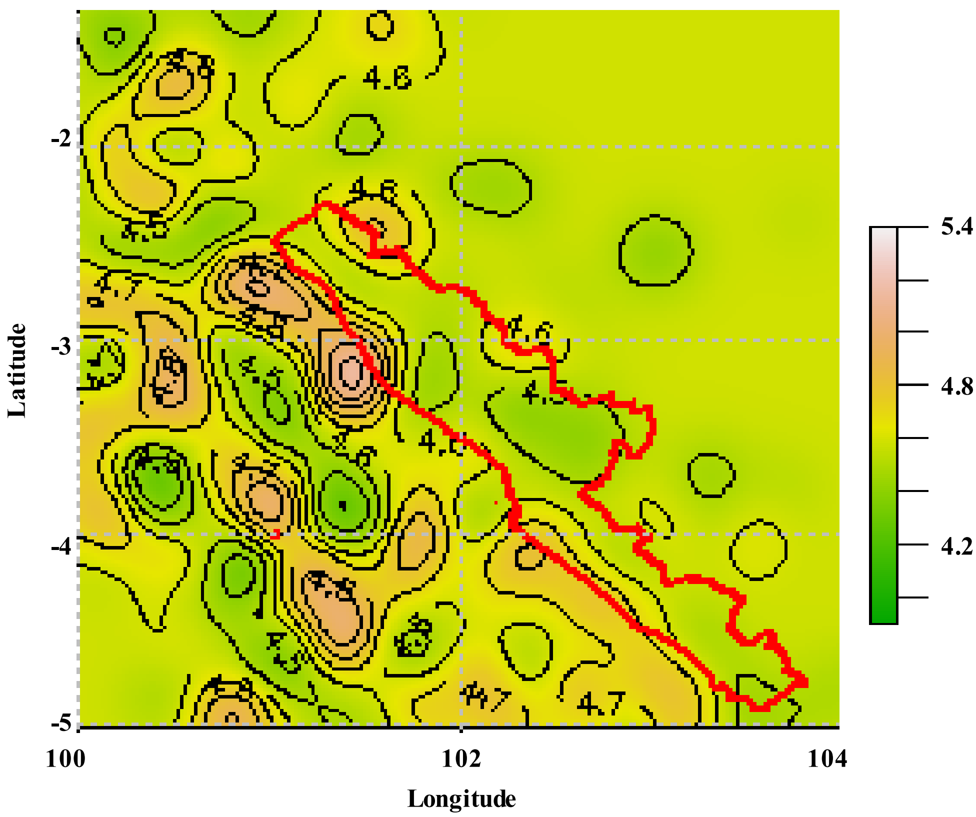

Figure 4. Contour Plot with the Best Robust Semivariogram Model

Figure 4 above illustrates the kriging contour plots result on the distribution of earthquake magnitudes surrounding Bengkulu Province. The result is generated by using the best robust semivarious model: Cressie-Hawkins. This kriging contour map describes the variation of predicted earthquake magnitudes area habituating in the area, and it spans from a longitude of about 100.0 to 104.0 and a latitude of -5.0 to -2.0. This approach presented heterogeneity in term of earthquake magnitudes in each location using the ordinary kriging method, which turns to be robust against the outlier, hence offering more robust estimates in the area where the data are heterogeneity in an extreme form. As illustrated in the map, a closed contour line represents the location with a similar value of magnitude. The closes the contour is, the sharper is the change in magnitude as it indicates the magnitude of local earthquake activity. For example, Bengkulu area presented by a red line, the sharp differencing of the controlling curve shows a significantly varying magnitude in the region. Variation of colour degradation from a bright green to a dark brown shows an idea of earthquake intensity. Green colors indicate the lowest magnitudes approximate 4.0 to 4.2 while yellow coloration is the medium magnitude with 4.4 to 4.9. The dark brown indicated the most significant magnitude with an approximate 5.0 to 5.4. With this concept, the gradual variation in the magnitude of earthquake is shown on the region as light colors indicate a low intensity while darkening indicates the most intensity due to active tectonic.

This map additionally shows the marked differences which exist across the west coast of Sumatra, specifically in Bengkulu, where more frequent earthquake events are represented by closer contour lines and darker colours. The fading of this color shows us the magnitude and the spatial distribution of earthquake sizes which is an important piece in understanding where earthquakes are more likely to strike! The darker red colour on the map represents the highest risk of high impact, and may be considered in disaster mitigation efforts or policy planning in areas that are vulnerable to seismic influences.

The study aims to answer important research questions such as the distribution patterns of earthquake magnitudes and modelling techniques in Bengkulu Province and its surrounding area. Descriptive Statistics of the Dataset The descriptive statistics (**Table 1**), used as clarification and validation here, show a positively−sketched nature of distribution of earthquake magnitudes with a mean close to 4.67 according to analyses conducted in different parts of this research work. The vast majority of recorded earthquakes have intermediate magnitudes, but there are significant outliers with high kurtosis and skewness values that illustrate unusually frequent occurrence of the extreme events. Furthermore, the magnitude appears stationary in time as evidenced by the time series plot (**Figure 1**) showing that the distribution is nearly consistent over time without dominant trends or patterns.

The contour map of the distribution of earthquake magnitudes was estimated on this robust semivariogram model (specifically with the Cressie-Hawkins model, shown in **Figure 4**). Stable estimates for areas defined by large premiums or penetrations, observed in both risk maps with and without aggregation (**Figure 3**) indicates how effective this was at addressing outliers where robust ordinary kriging was applied. Evaluating the aggregation or fusion using prediction and contour mapping results has shown that robust kriging significantly improves predictive efficiency, especially in non-normal distributions.

Descriptive analysis (**Table 1**) indicates that most earthquake magnitudes are concentrated in the lower range. However, the presence of numerous outliers—values significantly higher than the mean—has important implications for disaster risk assessment. Skewness and kurtosis metrics further highlight the impact of these extreme events, emphasizing their importance in seismic risk planning.

To ensure that the dataset meets the assumptions required for geostatistical modeling, a stationarity test was conducted. The visual stationarity test (**Figure 1**) confirms that the magnitude data remains stable over time. This stability is crucial for models such as Kriging, which rely on stationary data to produce reliable spatial predictions.

Further geostatistical analysis was carried out using robust semivariogram fitting (**Table 3**), which verifies the stationarity of the data and provides essential parameters for spatial interpolation. To assess the accuracy of different theoretical semivariogram models, their root mean square error (RMSE) values were compared (**Table 4**). The results indicate that the Gaussian model yields the lowest RMSE, suggesting its suitability for accurate magnitude prediction.

The effectiveness of the selected geostatistical model is further evaluated through a scatter plot (**Figure 5**), which compares observed earthquake magnitudes (x-axis) with their predicted values (y-axis) obtained using Robust Ordinary Kriging. Each blue dot represents an individual data point, illustrating the relationship between actual and estimated magnitudes. The trend line (dotted) and the R^2 value of 0.6196 indicate a moderate correlation, meaning that the model captures some variability in earthquake magnitudes but has room for improvement. Clusters of predicted values are more consistent in the mid-range magnitudes (5.0–6.5), whereas for higher magnitudes (>7.0), predictions exhibit greater variability, suggesting challenges in accurately estimating extreme earthquake magnitudes.

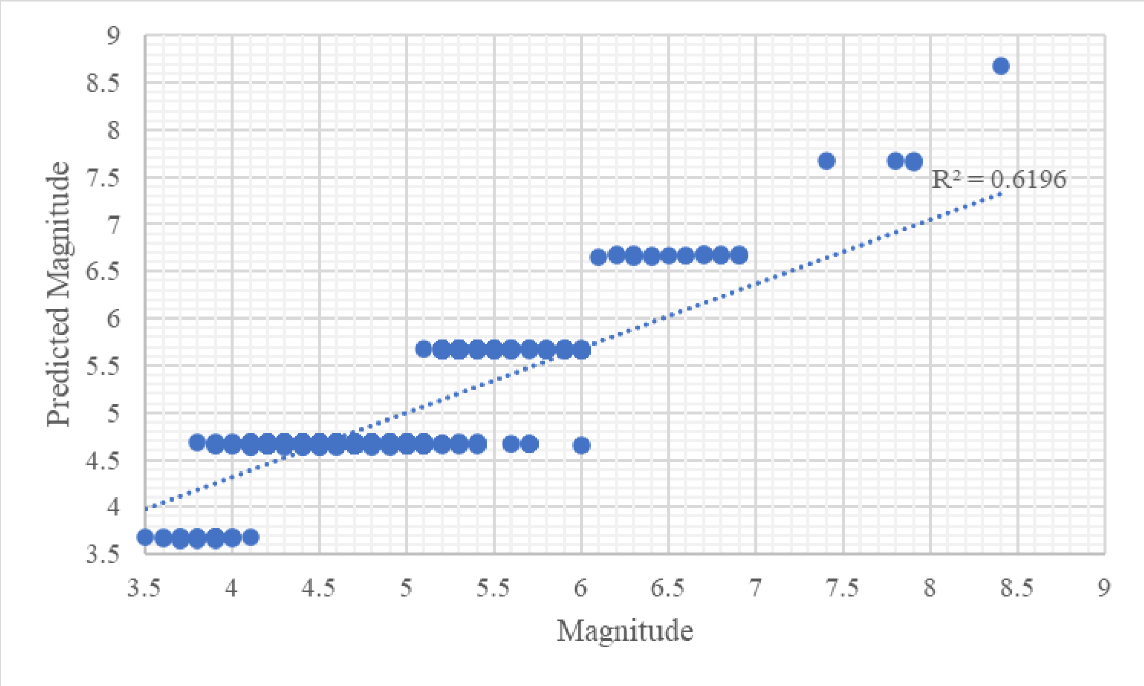

Figure 5. Scatter Plot Comparing Observed Earthquake Magnitudes (x-axis) with their Predicted Values (y-axis) using Robust Ordinary Kriging

These findings align with previous studies on seismic data analysis, which emphasize the importance of handling outliers and ensuring dataset stability. Robust statistical methods have been demonstrated to significantly enhance geostatistical predictive performance. Prior research on earthquake magnitude forecasting has shown that classical methods are highly sensitive to outliers [19][20][21]. The use of robust Kriging techniques in this study is consistent with recommendations from other researchers advocating robust methodologies for spatially correlated data. By employing robust geostatistical modeling, this study provides a more reliable approach for mapping earthquake magnitudes while mitigating the influence of extreme values.

This study encountered several limitations. The complete earthquake catalogue used in this paper for each zone was sourced from probability estimations made by the USGS Earthquake Hazards Program and regional seismic models, although not all zones may have a complete earthquake catalogue as source because of recent tectonical observation data limitations from instrumental measurements (including all-magnitude earthquakes) or ground observations. The time series plot clearly shows stationarity, but trends or seasonality would not come out because those must appear in a much longer term of the dataset similar to what we have. The efficiency of kriging significantly depends on the spread of data points in space; when instances are rare in a place, predictions will generally be less accurate.

This study specifically explores this novelty by using semivariogram modeling and kriging methods which are considered robust for the issue of earthquake magnitude prediction in Bengkulu Province, an area often hit by earthquakes events. We focus on the impact of outliers in our spatial risk estimate, thereby extending the scope of the ever-growing field of spatial statistics for disaster risk modeling through more robust approaches to geostatistics. Additionally, resilient methods integrated with geostatistical approach in earthquake research offers a strong platform for forthcoming studies to increase the predictive accuracy and policy implications pertaining to disaster preparedness can be formulated equally.

4. CONCLUSIONS

Through this research it was also successfully determined distribution of earthquake magnitudes in Bengkulu Province and surrounding area, showing that the data tends positively skewed, with most earthquakes characterized by medium magnitude level, but there is some outlier having very huge magnitude. During descriptive analysis, high kurtosis-Skewness values were found as output of results which has strong impression about the consideration of extreme scenario earthquake events for disaster risk analysis.

The chart above is a test of stationarity with data visualization; you can see that the magnitude over time does not have any significant trends or patterns and it becomes quite stable which qualify this attribute for further statistical analysis. The adoption of the powerful semivariogram model, especially the Cressie-Hawkins model, in ordinary kriging approaches results in higher-confidence and magnitude distribution predictions respecting spatial diversity that is significantly expressed in regions of considerable variation, so strengthening earthquake magnitudes forecast.

This study confirms that the choice of using a Gaussian model for semivariogram should be wise to realize accurate prediction outcomes. Nevertheless, this research has some limitations because it relies on a small data set and assumes stationarity which may not be entirely valid.

This research enhances the robustness of semivariogram and kriging approaches for predicting earthquake magnitudes in seismically sensitive zones, which is crucial for disaster risk management and mitigation measures. Findings of this study can be used as a guide for future research and the implementation of logging reduction and restoration approaches in high-risk zones.

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