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SIMULATION STUDIES PERFORMANCE OF EWMA-MAX MCHART BASED ON SYNTHETIC DATA

Kevin Agung Fernanda Rifki[1*], Muhammad Ahsan[2], Muhammad Mashuri[3]

[1,2,3]*Department of Statistics, Institut Teknologi Sepuluh Nopember (ITS)*
Jln. Teknik Mesin No.175, Keputih, Sukolilo, Surabaya, 60111, Indonesia

Corresponding author's e-mail: * ***fernandakevin02@gmail.com***

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ABSTRACT

Quality control has an important role in the manufacturing process. One of the statistical tools used in quality control is Statistical Process Control (SPC). The SPC product is a control chart. A control chart is a graphical tool used to determine if a process is under statistical quality control, helping to identify issues and drive quality improvements. Control charts are usually used to control variables or attribute data quality. Commonly used variable data is data with mean and variability characteristics. Various types of control charts are control charts for mean, control charts for variability, and simultaneous control charts designed to control mean and variability simultaneously. In real-field practice, manufacturing requires multivariate process control because many variables must be controlled. This research proposes a multivariate simultaneous control chart, the Exponentially Weighted Moving Average Max Multivariate (EWMA Max-Mchart). This control chart can handle multivariate process control simultaneously, both process mean and process variability. This research tests the performance of control charts with a simulation study using synthetic data with several process mean conditions and a covariance matrix. As a comparison, the development of the previous Max-M control chart was also tested. Based on the synthetic data generated, a performance comparison was made by looking at the suitability of in-control and out-of-control. The comparison results show that the EWMA Max-Mchart has better quality control performance if there is a shift than the Max-Mchart.

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1. INTRODUCTION

Statistical Process Control (SPC) is a widely used tool in quality control, helping to determine if a process is under control. It ensures compliance with specific standards, monitors measurement processes, and initiates corrective actions when necessary [1]. Control charts, a crucial element of SPC, are highly effective for process monitoring [2]. These charts track two key process parameters: mean and variance. Monitoring the mean ensures process accuracy, while variance monitoring addresses process fluctuations [3]. Control charts are categorized into univariate and multivariate types, depending on the number of observed characteristics. While univariate charts focus on monitoring a single quality characteristic, using multiple univariate charts simultaneously can be inefficient and result in inaccurate decision-making. Therefore, multivariate control charts were developed to monitor various quality characteristics simultaneously [4].

Monitoring means, and variability separately has disadvantages, such as inefficiency and the common occurrence of common and assignable causes in control charts [5]. Variability shifts can lead to shifts in mean control limits [6]. Thus, simultaneous control charts were developed to address these issues, offering efficiency and simultaneously addressing both mean and variability [7]. Designing charts that detect both large and small process shifts is challenging, as is distinguishing whether shifts are due to changes in mean, variance, or both and determining the direction of the process shift.

Previous studies proposed univariate simultaneous control charts to monitor mean and variability [8]. Max-Chart, a combination of $\overline{X}$ and S control charts, monitors mean and variability simultaneously. The Exponentially Weighted Moving Average-Max (EWMA-Max) control chart extends the Max-Chart by applying the EWMA technique [9] and developing other methods. The Maximum Exponentially Weighted Moving Average (Max-EWMA) control chart extends the EWMA Chart by applying the Max technique [10]. The Max $\overline{X} - S$ control chart monitors mean and variability stability using a single control chart, with advantages including ease of use and lower costs than variable-type inspections [11].

Several studies extended Max-Chart to multivariate forms, such as Max-Mchart, Max-MEWMA, and Max-MCUSUM. Max-MEWMA was introduced by Xie [9], while Cheng and Thaga introduced Max-MCUSUM [12]. In the case of simultaneous multivariate EWMA, it was first developed by Xie and named multivariate max-EWMA. This control chart is sensitive to small shifts in the mean vector and covariance matrix. Cheng and Thaga developed a multivariate Max-CUSUM chart for CUSUM. There is a need for further sensitivity improvement in the Max-Mchart control chart.

This research aims to develop a new control chart for individual multivariate data, the Exponentially Weighted Moving Average Max Multivariate (EWMA Max-Mchart). This chart is designed to monitor the mean and variance simultaneously, thus improving performance, and is expected to have the ability to detect small shifts. It will be tested on simulated data to evaluate its effectiveness, with the Max-Mchart used as a benchmark for comparison. The study involves applying both EWMA Max-Mchart and Max-Mchart to simulated datasets. However, there are some limitations, such as the positive correlation (ρ) used in the simulation to determine the L parameter and optimal control limit, the assumption of normal multivariate data, and the minimum specification limit applied to data in-control condition with a diagonal covariance matrix of 1.

2. RESEARCH METHODS

This sub-chapter discusses several control chart methods used as references to propose the development of a control chart. Then, there are several simulations to see the performance of the control chart.

2.1 Exponentially Weighted Moving Average Maximum Chart (EWMA Max chart)

EWMA-Max chart is an extension control chart of Max-chart by applying the EWMA technique to Max-chart statistics. Let X represent certain characteristics of a process, μ is the process mean and σ is the process standard deviation. Given X_{ij} with $i = 1, 2, 3, \ldots, m_i$ and $j = 1, 2, \ldots, n_i$ is the measurement result of X with sample size n_i and i are the number sample index $X_{i1}, \ldots, X_{in_i}$ are a random sample from a normal distribution with mean $\mu + a$ and standard deviation $b \times \sigma$ with $a = 0$ and $b = 1$ indicating the process is in control, otherwise the process has shifted.

By using $\bar{X}_i = \frac{(X_{i1}+...+X_{in})}{n}$ is the mean of the i-th sample and $S_i^2 = \frac{\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2}{(n-1)}$ is the variance of the i-th sample, then the simultaneous control chart formulated in **Equation (1)** and **Equation (2)**,

$$U_i = \frac{(\bar{X}_i - \mu)}{\frac{\sigma}{\sqrt{n}}}, \tag{1}$$

$$V_i = \Phi^{-1}\left\{H\left(\frac{(n_i-1)S_i^2}{\sigma^2}; n-1\right)\right\}, \tag{2}$$

where $\Phi(z) = P(Z \le z)$ for $Z \sim N(0,1)$, the standard normal distribution, $\Phi^{-1}(.)$ is the inverse function of $\Phi(.)$, and $H(w,v) = P(W \le w|v)$ for $W \sim \chi_v^2$, the chi-squared distribution with v degrees of freedom.

U_i and V_i are independent because $\bar{X}_i$ and S_i^2 also independent, when $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$. The advantages of the transformation $\bar{X}_i$ to U_i and S_i^2 to V_i are: (1) the distribution of U_i and V_i independent for sample size $n_i$when $a = 0$ and $b = 1$, so that the case of variable sample sizes can be handled easily; (2) U_i and V_i have the same distribution so that a single control chart can be formed to monitor the mean and variability of the process. Specifically, statistics are defined for the max simultaneous control chart in **Equation (3)**,

$$M_i = max(|U_i|, |V_i|). \tag{3}$$

the value of the test statistic M_i becomes large when the process shifts away from μ and/or when process variability increases or decreases. On the other hand, the values M_i shrink when the process mean and variability remain close to their respective target values. Adding the EWMA technique to the Max control chart will increase performance and sensitivity. EWMA Max statistics are defined in **Equation (4) [9]**.

$$G_i = (1-\lambda)G_{i-1} + \lambda M_i \tag{4}$$

With G_0 is the initial value. U_i and V_i are independent, when $\delta = 0$ and $\gamma = 1$ so, $Ui \sim N(0,1)$ and $Vi \sim N(0,1)$ so, $\sigma_{Ui} = \sigma_{Vi} = 1$. The Cumulative Distribution Function (CDF) from statistic Max-chart M_i is as follows:

$$\begin{aligned} F(y) &= P(|U_i| \le y, |V_i| \le y), \\ &= P(|U_i| \le y)P(|V_i| \le y), \\ &= [2\boldsymbol{\Phi}\left(\frac{y}{\sigma_{Ui}}\right) - 1]^2, y \ge 0, \end{aligned} \tag{5}$$

Furthermore, the Probability Density Function (PDF) from statistics M_i is derivative of $f(y;\sigma_{Ui})$ which is shown in **Equation (6)**.

$$f(y;\sigma_{Ui}) = \frac{4}{\sigma_{Ui}}\phi\left(\frac{y}{\sigma_{Ui}}\right)[2\boldsymbol{\Phi}(\frac{y}{\sigma_{Ui}}) - 1] \tag{6}$$

Using numerical computation, the mean and variance of M_i written in **Equation (7)** and **Equation (8)**:

$$E(M_i) = \int_0^\infty y f(y;\ \sigma_{Ui}) dy, \\ = 1.128379\ \sigma_{Ui}, \tag{7}$$

$$Var(M_i) = \int_0^\infty y^2 f(y;\ \sigma_{Ui}) dy, \\ = 0.363381\ \sigma_{Ui}, \tag{8}$$

so that UCL for the EWMA-Max chart are given in **Equation (9)**.

$$UCL_i = E(G_i) + L\sqrt{\mathrm{Var}(G_i)}, \\ = E(M_i) + L\sqrt{\frac{\lambda[1-(1-\lambda)^{2i}}{2-\lambda}}\ \sqrt{\mathrm{Var}(M_i)}, \\ = 1.128379 + 0.602810L\sqrt{\frac{\lambda[1-(1-\lambda)^{2i}}{2-\lambda}}. \tag{9}$$

The Upper Control Limit (UCL) for the EWMA-Max chart when steady-state conditions with very large i is given in **Equation (10)**,

$$UCL = 1.128379 + 0.602810\ L\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{10}$$

where L and λ are parameters of EWMA-Max.

2.2 Maximum Multivariate Chart (Max-Mchart)

Max-Mchart for subgroup observations using a combination T^2 Hotelling statistic and Generalized Variance (GV). GV charts are not suitable for measuring individual observations, making it necessary to adapt the Max-M chart for use with individual data. For example, $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n$ a sequence of random vectors that are independent and distributed $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. **Equation (11)** and **Equation (12)** below are transformations to monitor mean and variability simultaneously **[13]**,

$$Z_i^{MI} = \Phi^{-1}[H_p\{(\boldsymbol{x}_i - \overline{\boldsymbol{x}})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_i - \overline{\boldsymbol{x}})\}] \tag{11}$$

$$V_i^{MI} = \Phi^{-1}[H_p\left\{\frac{1}{2}(\boldsymbol{x}_i - \boldsymbol{x}_{i-1})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}_i - \boldsymbol{x}_{i-1})\right\}] \tag{12}$$

where $\Phi(.)$ is the CDF for the standard normal distribution, and $H(.)$ is the CDF for the Chi-square distribution with degrees of freedom p. In addition, $\overline{\boldsymbol{x}} = \frac{\sum_{i=1}^n x_i}{n}$ and $\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{1\,1} & \sigma_{1\,2} & \cdots & \sigma_{1\,p} \\ \sigma_{2\,1} & \sigma_{2\,2} & \cdots & \sigma_{2\,p} \\ \vdots & \ddots & \ddots & \vdots \\ \sigma_{p\,1} & \cdots & \cdots & \sigma_{p\,p} \end{bmatrix}$. Therefore, the statistics for Max-Mchart for individual data are written in **Equation (13)**.

$$M_i^{MI} = max\left(\left|Z_i^{MI}\right|, \left|V_i^{MI}\right|\right)\ ; i = 2,3, \ldots, n \tag{13}$$

If $M_i^{MI} > 0$, then Max-Mchart only has UCL. The UCL from Max-Mchart obtained bootstrap results for various parameters ρ and p **[13]**. Determination of the UCL from Max-Mchart uses a bootstrap approach as in the steps below **[13]**:

a. Determine $\alpha = 0{,}0027$.

b. Calculate the value $\boldsymbol{\mu}_0$ and $\boldsymbol{\Sigma}_0$ (for the multivariate normal distribution standard in-control condition).

c. Take steps to $l = 1$ until 1000:

1. Generates n samples that follow a multivariate normal distribution $N_p(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$.
2. Calculate the statistical mean and statistical variability to obtain simultaneous statistics M_i^{MI}, $i = 2, 3, \ldots, n$ which is the Max-Mchart of the resulting data.
3. Resample (with replacement) b times to get b values of M_i^{MI}.
4. Calculate the $\big(100(1-\alpha)\big)$-th percentile of b sample using $M^{MI}_{(100(1-\alpha)),l}$

d. Calculate $UCL = \sum_{l=1}^{1000} \frac{M^{MI}_{(100(1-\alpha)),l}}{1000}$

2.3 Performance of Control Chart

Metrics like run length are commonly used to assess the effectiveness of a control chart. Run length is a random variable that indicates the number of sample statistics plotted on the chart before an out-of-control signal appears. The expected value of this run length is referred to as the Average Run Length (ARL). The ARL signifies the average number of points plotted before detecting an out-of-control condition. A higher ARL is preferable when a process is operating correctly, as it suggests fewer false alarms. Conversely, a low ARL is preferable when a process is out of control due to shifts in average quality characteristics or variability. There are two types of ARL : ARL_0 (in-control) ARL_1 (out-of-control). This study focuses on using ARL_0.

One of the key criteria for evaluating a control chart is its ability to detect an out-of-control signal when a process shift occurs promptly. The detection speed indicates the control chart's sensitivity to such shifts. The performance measurement method employed for this purpose is the ARL [2]. ARL represents the average number of observation points until the first out-of-control signal is identified. In assessing control chart performance, two types of ARL are considered: ARL0 and ARL1. ARL0 denotes the average number of observations expected before the occurrence of the first point outside the control limits while the process is still in control [14]. The formula for ARL_0 is defined as follows in **Equation (14)**:

$$ARL_0 = \frac{1}{\alpha} \tag{14}$$

where α represents a type I error, indicating the probability of detecting an out-of-control signal even when the process is still in control. A commonly used value for α, especially when a three-sigma limit (3σ) is expected, is $\alpha = 0.0027$ where the false alarm rate is equivalent to the 3σ limit [15].

In practice, computing ARL values can be complex, particularly for EWMA and CUSUM control charts, developing by various approaches. This study adopts the Markov Chain approach for ARL calculations. ARL_0 calculations, determining the parameters λ and L, along with the UCL, is essential. The ARL algorithm is outlined as follows:

Algorithm 1: Determination of the λ, *L*, and UCL using ARL with Markov Approach

Step 1. Define $\alpha = 0.0027$ and initial value $L > 0$.

Step 2. Define scenario for getting optimal L is done with various values of λ and initial value of L is 1.

Step 3. Generating distributed data $N_p(\mu_g, \Sigma_g)$ with $n = 1000$ and condition in-control is $\mu_g = 0$ and

$$\Sigma_g = \begin{bmatrix} 1 & \rho & \rho & \cdots & \rho \\ \rho & 1 & \rho & \cdots & \rho \\ \rho & \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1 \end{bmatrix}.$$

Step 4. Calculate U_i^{MI} and Q_i^{MI} with generated data.

Step 5. Calculate M_i^{MI}.

Step 6. Determine the initial value EMM_0 is 1.128379.

Step 7. Calculate statistic EMM_1 .

Step 8. Compare EMM with UCL.

Step 9. Calculate RL is observations that meet the condition $EMM_i > UCL$; UCL for a new control chart the first time, then after that stop and go back to step 3.

Step 10. Repeat steps 3–9 up to *it* = 1 - 1000 iterations to ensure a consistent ARL value. Note that a constant ARL value may be achieved before reaching the 1000th iteration

Step 11. Calculate the ARL_0 which is the average of RL from it iterations under in-control conditions, $ARL_0 = \frac{\sum_{i=1}^{it} RL_i}{it}$

Step 12. Repeat steps 3-11 until get the optimum L on $ARL_0 \cong 370$ with each iteration of the update $L_{i+1} = L_i + 0.001$.

3. RESULTS AND DISCUSSION

With some information and explanations of several methods, a new control chart method will be proposed, which is explained in the following sub-chapter. The proposed approach uses the R software package (version 4.3.1). This section presents the results of the analysis, including the optimal parameters for the EWMA Max-Mchart with UCL and a performance comparison with the Max-Mchart.

3.1 EWMA Max-Mchart

This research presents a multivariate simultaneous control chart for individual observations, named the EWMA Max-Mchart control chart. It extends the Max-Mchart by integrating the Exponentially Weighted Moving Average (EWMA) into Max statistics. The dataset consists of individual *p*-multivariate observations. The procedure starts by calculating the Max-Mchart statistics, which are then processed using the EWMA method. A transformation is applied to derive statistics for the process location (mean), and variability to monitor the process mean and variability in a single chart. This transformation is expressed in **Equation (15)** and **Equation (16)**, with subsequent calculations outlined in **Equation (18)**.

$$U_i^{MI} = \Phi^{-1}[H_p\{(x_i - \bar{x})'\Sigma^{-1}(x_i - \bar{x})\}] \tag{15}$$

and

$$Q_i^{MI} = \Phi^{-1}[H_p\left\{\frac{1}{2}(x_i - x_{i-1})'\Sigma^{-1}(x_i - x_{i-1})\right\}] \tag{16}$$

where $\Phi(.)$ CDF for the standard normal distribution and $H(.)$ is the CDF for the chi-square distribution of degrees of freedom p. Therefore, the statistics for Max-Mchart for individual data are written in **Equation (17)**.

$$Ma_i^{MI} = max(|U_i^{MI}|, |Q_i^{MI}|) \tag{17}$$

The statistics in **Equation (17)** will then be carried out using the EWMA process, similar to calculating the EWMA Max chart. EWMA Max-Mchart statistics can be calculated **Equation (18)**.

$$EMM_i = (1 - \lambda)EMM_{i-1} + \lambda Ma_i^{MI} \tag{18}$$

with EMM_0 as the initial value. U_i^{MI} and Q_i^{MI} are independent, when $\delta = 0$ and $\gamma = 1$ so $U_i^{MI} \sim N(0,1)$ and $Q_i^{MI} \sim N(0,1)$ so $\sigma_{U_i^{MI}} = \sigma_{Q_i^{MI}} = 1$. The CDF for Max M, is Ma_i^{MI} is as follows.

$$\begin{aligned} F(y) &= P(Ma_i^{MI} \leq y) \\ &= P(|U_i^{MI}| \leq y, |Q_i^{MI}| \leq y), \\ &= P(|U_i^{MI}| \leq y)P(|Q_i^{MI}| \leq y), \\ &= [2\Phi\left(\frac{y}{\sigma_{U_i^{MI}}}\right) - 1]^2, y \geq 0, \end{aligned} \tag{19}$$

Furthermore, the Probability Density Function (PDF) from Ma_i^{MI} is the derivative of $f(y; \sigma_{U_i^{MI}})$ which is shown in **Equation (20)**.

$$f(y; \sigma_{U_i^{MI}}) = \frac{4}{\sigma_{U_i^{MI}}} \phi\left(\frac{y}{\sigma_{U_i^{MI}}}\right) [2\boldsymbol{\Phi}(\frac{y}{\sigma_{U_i^{MI}}}) - 1] \tag{20}$$

Using numerical computation, the mean and variance of Ma_i^{MI} are **Equation (21)** and **Equation (22)**:

$$\begin{aligned} E\,(Ma_i^{MI}) &= \int_0^\infty yf\left(\text{y};\ \sigma_{U_i^{MI}}\right) dy, \\ &= 1.128379\ \sigma_{Ui}, \end{aligned} \tag{21}$$

$$\begin{aligned} Var\,(Ma_i^{MI}) &= \int_0^\infty y^2 f\left(\text{y};\ \sigma_{U_i^{MI}}\right) dy, \\ &= 0.363381\ \sigma_{U_i^{MI}}, \end{aligned} \tag{22}$$

so that the control limits for EWMA Max-Mchart are given in **Equation (23)**,

$$\begin{aligned} UCL_i &= E(EMM_i) + L\sqrt{\text{Var}(G_i)}, \\ &= E(Ma_i^{MI}) + L\sqrt{\frac{\lambda[1-(1-\lambda)^{2i}}{2-\lambda}}\ \sqrt{\text{Var}(Ma_i^{MI})}, \\ &= 1.128379 + 0.602810L\sqrt{\frac{\lambda[1-(1-\lambda)^{2i}}{2-\lambda}}. \end{aligned} \tag{23}$$

The UCL for the EWMA Max-Mchart when steady-state conditions with very large i is given in **Equation (24)**,

$$UCL = 1.128379 + 0.602810\ L\sqrt{\frac{\lambda}{(2-\lambda)}} \tag{24}$$

where L and λ of EWMA Max-M are obtained optimally through ARL calculations.

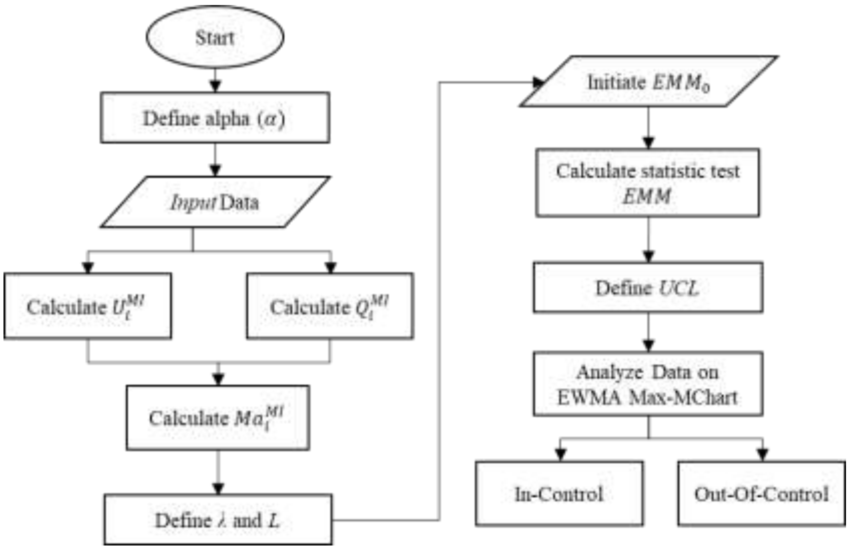


Figure 1. Detection Scheme Flowchart EWMA Max-Mchart

The EWMA Max-Mchart in this study uses the software application R. In the EWMA Max-M process flow, there are several algorithm stages. The procedure is shown as follows:

Algorithm 2: Estimate Statistics EWMA Max-M and Plot

Step 1. Prepare data.

Step 2. Calculate U_i^{MI} value with Equation (15) and Q_i^{MI} value with Equation (16).

Step 3. Calculate statistic Ma_i^{MI} with Equation (17).

Step 4. Set the value of the weighting parameters λ, *L*.

Step 5. Calculate statistic EMM with Equation (18) and the initial value EMM_0 is 1.128379.

Step 6. Define UCL with Equation (24).

Step 7. Plot the statistic EMM dan UCL obtained from step 6.

Step 8. Compare EMM with UCL, if EMM < UCL is in-control and EMM > UCL is out-of-control.

3.2 Estimate Statistic EWMA Max-M and Control Limit

In this analysis, it will be used to get the UCL from EWMA-Max Mchart based on **Algorithm 2**. The correlation values (ρ) considered for this analysis are 0, 0.3, 0.5, and 0.7, with the number of variables (p) set at 3 and 5. The selection of the correlation values is based on the strength of the correlation, categorized as no correlation, low correlation, moderate correlation, and high correlation. Additionally, variations in the number of variables are considered for the analysis. An initial L value of 1 was used; if this value does not satisfy the ARL_0 requirements, it may be adjusted to a higher value. The weighting λ is likely to have an influence on UCL and L values based on studies on the EWMA method, so applied in this context are λ values, ranging from 0.1 to 0.9 in increments of 0.1. The ARL_0 calculation results, including the optimum L parameters and UCL, are detailed in various tables.

Table 1. Parameter L Value for $p = 3$

ρ	λ								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	2.658	3.093	3.295	3.411	3.476	3.518	3.461	3.507	3.499
0.3	2.639	3.074	3.278	3.394	3.459	3.498	3.500	3.499	3.497
0.5	2.641	3.082	3.277	3.397	3.462	3.497	3.498	3.499	3.502
0.7	2.636	3.082	3.277	3.393	3.460	3.500	3.499	3.501	3.498

Table 2. UCL for $p = 3$

ρ	λ								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	1.496	1.750	1.963	2.156	2.338	2.517	2.659	2.854	3.036
0.3	1.493	1.746	1.958	2.151	2.332	2.509	2.677	2.851	3.035
0.5	1.494	1.748	1.958	2.152	2.333	2.508	2.676	2.851	3.038
0.7	1.493	1.748	1.958	2.151	2.333	2.510	2.676	2.852	3.036

Table 3. Parameter L Value for $p = 5$

ρ	λ								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	2.665	3.098	3.293	3.401	3.468	3.523	3.513	3.518	3.453
0.3	2.637	3.084	3.279	3.400	3.458	3.498	3.499	3.500	3.496
0.5	2.639	3.075	3.279	3.399	3.462	3.498	3.500	3.499	3.497
0.7	2.637	3.079	3.282	3.403	3.46	3.499	3.497	3.502	3.501

Table 4. UCL for $p = 5$

ρ	λ								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	1.497	1.751	1.962	2.153	2.335	2.519	2.682	2.860	3.011
0.3	1.493	1.748	1.959	2.153	2.332	2.509	2.676	2.851	3.035
0.5	1.493	1.746	1.959	2.153	2.333	2.509	2.677	2.851	3.035
0.7	1.493	1.747	1.959	2.154	2.333	2.509	2.675	2.852	3.037

From the results of the various tables above, whatever the value of ρ, the difference in the L and UCL values is not much different, but the difference in λ affects the L and UCL values. The more significant λ value, the greater the L and UCL values, which means that if you want to get sensitive results, a small λ value is used.

3.3 Comparison Performance of EWMA Max-Mchart and Max-Mchart

In this section, we will discuss the performance comparison of the EWMA Max-Mchart and Max-Mchart. The performance comparison will be conducted using both synthetic and real application databases. For clarity, the comparison for synthetic data should be performed separately. The synthetic data used will be multivariate normal generation data with four scenarios, as explained in Table 5. The scenario was chosen based on previous research discussing the Max-Half-M Chart [13].

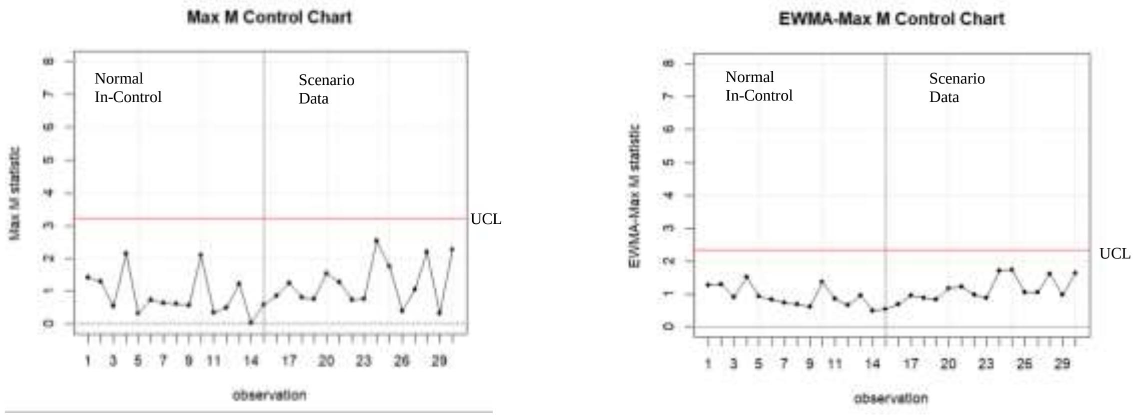
Table 5. Scenarios with Synthetic Data

Original data	a	b	**Scenario Data**	a	b
Data set 1	0.0	1.0	In control	0.0	1.0
Data set 2	0.0	1.0	Shift in the mean vector	2.2	1.0
Data set 3	0.0	1.0	Shift in the covariance matrix	0.0	2.8
Data set 4	0.0	1.0	Shift in mean vector and covariance matrix	2.2	2.8

This scenario has 2 data per row in **Table 5**, which will be combined for control with EWMA Max-Mchart and Max-Mchart. In total, there are 30 data per scenario, with 15 original data (normal in-control) and 15 additional data from the scenario (out-of-control). The data used is multivariate data with 3 variables. For UCL in Max-Mchart, a control limit 3.204 is used on the calculation and data [13]. For the EWMA Max-M chart, a λ value of 0.5 and UCL = 2.33 will be used. **Table 5** provides information on shifts in the mean vector (a) and shifts in the covariance matrix (b). The results of this analysis are presented in **Table 6**, followed by visual representations.

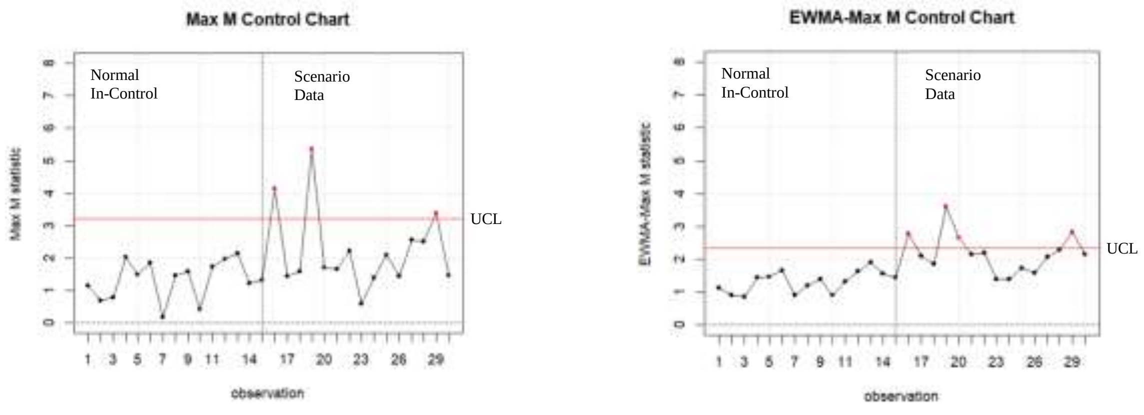
Table 6. Amount of Data Out-of-Control and in-Control on EWMA-Max Mchart and Max-Mchart for Synthetic Data

Scenario 1	Max-Mchart		Amount	Scenario 3	Max-Mchart		Amount
		out-of-control	0			out-of-control	2
		in-control	30			in-control	28
	EWMA Max-Mchart		Amount		EWMA Max-Mchart		Amount
		out-of-control	0			out-of-control	6
		in-control	30			in-control	24
Scenario 2	Max-Mchart		Amount	Scenario 4	Max-Mchart		Amount
		out-of-control	3			out-of-control	7
		in-control	27			in-control	23
	EWMA Max-Mchart		Amount		EWMA Max-Mchart		Amount
		out-of-control	4			out-of-control	14
		in-control	26			in-control	16



(a) (b)

Figure 2. Plot Chart (a) Max-M and (b) EWMA Max-M for Scenario 1



(a) (b)

Figure 3. Plot Chart (a) Max-M and (b) EWMA Max-M for Scenario 2

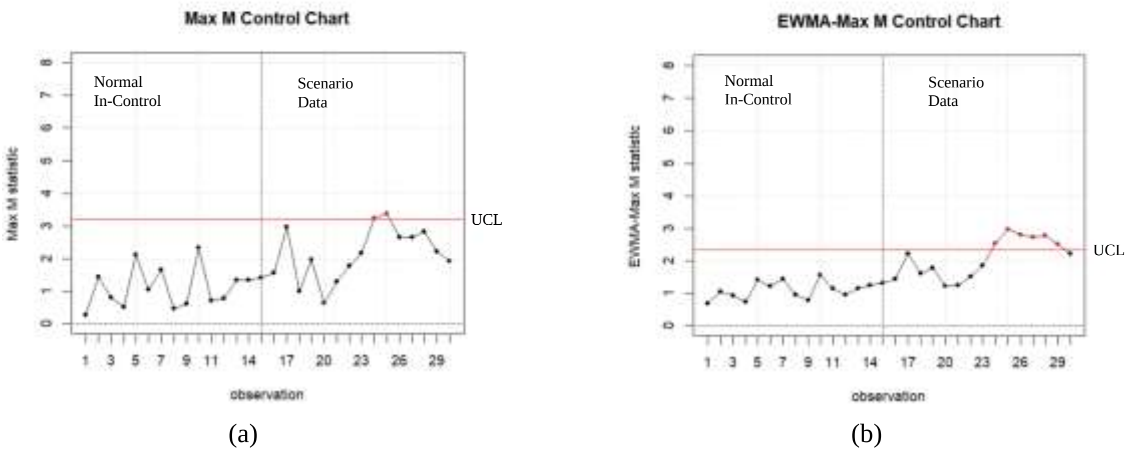


Figure 4. Plot Chart (a) Max-M and (b) EWMA Max-M for Scenario 3

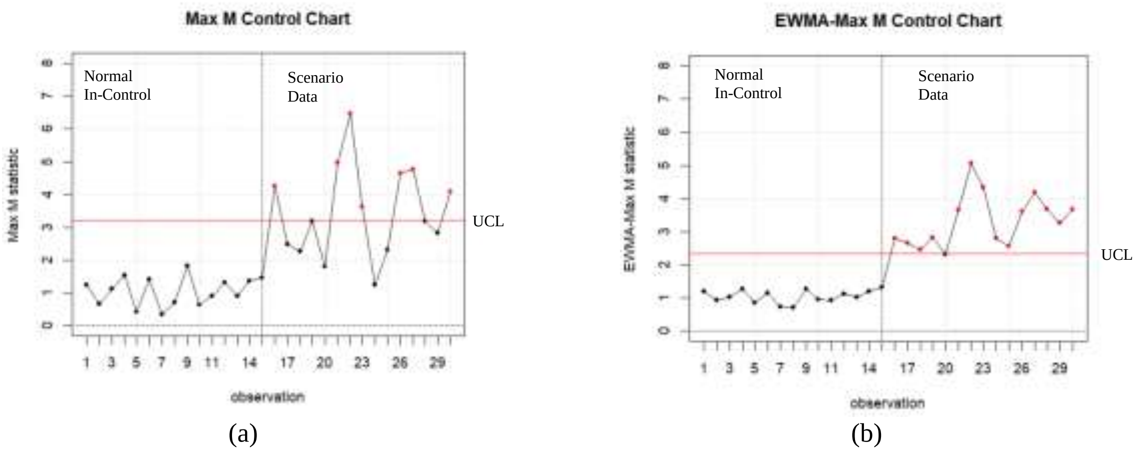


Figure 5. Plot Chart (a) Max-M and (b) EWMA Max-M for Scenario 4

The right control chart can show out-of-control data quickly and accurately based on the data that has been created in this scenario. From the results of the table and image above, for scenario 1 conditions, all control charts, both Max-Mchart and EWMA Max-Mchart, have the same or good performance for detecting multivariate normal data. The black dots on each observation indicate data in control, and the red dots are data outside the UCL or out of control. In scenario 2, both control charts are still not good enough for mean shifts because out-of-control data is read in-control conditions, but EWMA Max-Mchart detects more out-of-control than Max-Mchart. In scenario 3, the results are the same as in scenario 2, where there is still much out-of-control data that is read in the in-control condition, but EWMA Max-Mchart detects more out-of-control than Max-Mchart. The results in scenario 4 show that EWMA Max-Mchart is better monitored than Max-Mchart, where the condition of scenario 4 is a simultaneous shift in the mean vector and covariance matrix.

4. CONCLUSIONS

EWMA Max-Mchart can effectively monitor the mean and variability processes simultaneously for multivariate data with individual observations. Adding the EWMA process to the Max-Mchart can improve the performance of the control chart. A simulation study of four synthetic data scenarios shows that the EWMA Max-Mchart can detect out-of-control signals correctly when there is a shift in the mean vector and covariance matrix compared to the Max-Mchart.

For further research, it is hoped that control charts can be developed using approaches other than EWMA, such as the half-normal or Max Half-Mchart distribution approach. It is also possible to create

EWMA calculations during calculating Max statistics or the possibility of developing an Adaptive EWMA-Max Mchart or CUSUM Max-Mchart.

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