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DYNAMIC ANALYSIS OF THE MATHEMATICAL MODEL FOR STUNTING WITH NUTRITION AND EDUCATION INTERVENTIONS

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ABSTRACT

This study presents a mathematical model that analyzes the impact of nutrition and education interventions on stunting prevalence. Nutritional interventions are carried out on toddlers indicated to be stunted and toddlers who are healthy but susceptible to stunting. Meanwhile, education is given to the toddler's mother compartment. The model categorizes the toddler population into four compartments: susceptible, stunting-indicated, permanently stunted, and non-stunted. Similarly, the maternal population is categorized into three compartments: susceptible mothers, mothers exhibiting poor parenting practices, and educated mothers. The model's equilibrium point comprises two distinct states: a stable stunting-free equilibrium point when the basic reproduction number (R_0) is less than one and a stable stunting-endemic equilibrium point when R_0 is more significant than one. Sensitivity analysis reveals that the parameters that significantly influence the reduction or increase in stunting cases are the rate of nutritional intervention for children and the intensity of education for mothers. Numerical simulations demonstrate that implementing nutritional intervention activities and continuous education programs can effectively eliminate stunting cases in the population. The simulation results show a high number of stunting cases, reaching 161,566 cases in the population, due to poor education and poor nutritional interventions. In contrast, education programs and effective nutritional interventions eliminate stunting from the population. However, it takes longer.

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1. INTRODUCTION

Stunting, a condition of abnormal growth in children under five years of age, is characterized by physical conditions that are too short for their age. The standard used to determine the condition of young children who are short or too short (severely stunted) is based on WHO standards, namely comparing height for age with the WHO-MGRS (Multicenter Growth Reference Study) in 2006 [1][2]. The primary cause of stunting is inadequate nutrition received by the fetus or infant from conception until the first 1000 days of life, a critical window for intervention. The consequences of stunting extend beyond physical short stature. It also leads to impaired cognitive development, increased disease susceptibility, reduced productivity and performance, and imbalanced bodily functions [3]. Ultimately, stunting will reduce the economic growth of a region or country, increase poverty, and sharpen social inequalities. Stunting can also eliminate up to 11% of GDP gross domestic products (GDP), and reduce up to 20% of workers' income [1].

Symptoms of stunting are generally seen when the baby is over 2 years old. However, the process of stunting can begin while the baby is still in the womb or after the baby is born. Stunting that occurs while the baby is in the womb is caused by low nutritional intake of pregnant women, lack of access to prenatal health care, anemia of pregnant women, low maternal knowledge about nutrition, poor sanitation, and genetic factors, namely the father's or mother's height being below normal [4][5]. Meanwhile, the causes of stunting at birth are malnutrition in breastfeeding mothers, babies not exclusively breastfed, frequent infant infections, poor infant nutrition, poor parenting, lack of access to postnatal health, and a dirty environment [6][7].

Stunting can be prevented through the active role of parents and the government working together to prevent stunting in children. Stunting prevention is achieved through specific and sensitive nutrition interventions. Nutrition-specific interventions include provision of iron tablets for pregnant and lactating mothers, promotion of exclusive breastfeeding, provision of micronutrient supplements, provision of macronutrient supplements, promotion of complementary foods, promotion of iodized foods, treatment of malnutrition, provision of vitamin A and deworming of children [3]. Meanwhile, nutrition-sensitive interventions include access to clean water, access to good sanitation, access to health and family planning services, availability of national health insurance for the community, universal maternity insurance for pregnant women, education on good parenting, balanced nutrition education in the community, classes for pregnant women, and improving food security and nutrition [8][9].

Children who show symptoms of stunting can be treated. The treatment referred to here is not the administration of specific drugs to eliminate stunting, but the meaning of treatment here is to provide nutritional treatment to young children in the form of adequate nutritional intake. As with interventions to prevent stunting, efforts that can be made to treat young children diagnosed with stunting include initial treatment of young children with nutritious foods, supplementation of young children with iron, vitamins, zinc, calcium, and iodine (micronutrient supplements), and parental education and clean living practices [3].

Mathematical models can contribute significantly to efforts to control stunting. This is because mathematical models can explain the growth rate of a disease, provide accurate calculations of factors or parameters that need to be increased/decreased in order to eliminate a disease, and mathematical models can also play a role in efforts to design the right policies to overcome various health problems, such as stunting, at the lowest possible cost [10], [11]. Mathematical models used to model a disease are known as epidemiological models. A mathematical model of a health phenomenon can be obtained through a dynamical systems approach. The advantages of this approach are that changes in the system over time can be known, simulations can be performed for different scenarios of health problems that may occur, interactions between variables or groups of individuals can be described, and it is possible to model more complex systems with many interacting variables [12], [13].

Pratama and Lismayani in 2022 [14] and Winarni, et al [15] have modeled stunting using a system dynamics approach. Both researchers divided the under-five population into four compartments and did not include the mother/caregiver element in the resulting model. The groups of young children in question are compartments of young children at susceptible of stunting (S), compartments of young children with symptoms of stunting (E), permanently stunted (I), and groups of young children without stunting (R). The interventions modeled in the study were sanitation, parenting, and access to health services. The results showed that good sanitation, parenting, and health services can reduce the number of cases of stunting.

In contrast to [14] and [15], the research in this article was conducted by including mothers in the model rather than focusing only on the children. In addition, the focus of this research is to construct a mathematical model of stunting by incorporating nutrition and education interventions into the model. The mathematical model in this study was developed by dividing the toddler population into four compartments: (1) healthy but vulnerable toddlers, referred to as susceptible toddlers (S_T); (2) toddlers showing early signs of stunting (toddlers indicated stunted), referred to as exposed toddlers (E_T); (3) toddlers with permanent stunting, referred to as infected toddlers (In_T); and (4) toddlers who are not stunted due to receiving nutritional interventions (R_T). These infant compartments then become differential equation variables in the mathematical model constructed in this study. Toddlers at risk of stunting can become stunting-free if they receive adequate nutrition and proper parenting interventions. Conversely, if they experience poor nutrition and lack appropriate care, they may develop symptoms of stunting, which can eventually lead to permanent stunting. Furthermore, the population of infant caregivers (mothers) is grouped into three compartments, namely the compartment of mothers who are vulnerable to providing poor parenting to toddlers or called susceptible mother (S_m), the compartment of mothers with poor parenting (bad parenting mothers) or infected mother (In_m), and the compartment of mothers who have received education about good parenting or called educated mother (Ed_m).

Once the mathematical model of stunting is established, the study proceeds by determining the equilibrium points and their stability, calculating the basic reproduction number (R_0) using the Next Generation Matrix (NGM), analyzing the parameters that most influence the model and contribute to the increase or eliminate of stunting in the population, and finally conducting numerical simulations to determine the dynamics of changes in the number of individuals in each compartment in endemic and stunting-free states.

2. RESEARCH METHODS

The method used in this research is a literature review. Various health literature that discusses stunting, such as the causes of stunting, preventive measures, and how to deal with stunted children, is reviewed and studied. This is done so that the resulting mathematical model can approximate or represent the real stunting situation. In addition to the medical literature, literature searches and reviews of sources related to mathematical modelling were also conducted.

The mathematical model constructed in this article extends the SIR (Susceptible, Infected, and Recovered) model. The SIR model is expressed as a system of differential equations, namely [16][17]:

$$\frac{dS}{dt} = \mu N - \frac{\beta SI}{N} - \mu S$$
$$\frac{dI}{dt} = \frac{\beta SI}{N} - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

In addition to the reference to the SIR model, the model development in this research paper is also based on the SEIR mathematical model by Pratama and Lismayani [14].

Dynamic analysis of the model is carried out by checking the stability of the equilibrium point, determining the $\boldsymbol{R_0}$ number, performing parameter sensitivity analysis and numerical simulation.

Definition 1. [18] Consider the system $\frac{du}{dt} = \boldsymbol{f}(\boldsymbol{u})$, with $\boldsymbol{u} \in \mathbb{R}^{\boldsymbol{n}}$. A vector $\overline{\boldsymbol{u}}$ which satisfies $\boldsymbol{f}(\overline{\boldsymbol{u}}) = \boldsymbol{0}$ is called an equilibrium point.

The equilibrium point $\overline{\boldsymbol{u}}$ of the nonlinear system $\frac{du}{dt} = \boldsymbol{f}(\boldsymbol{u})$ is said to be asymptotically stable if the Jacobian matrix resulting from the linearisation of the system $\frac{du}{dt} = \boldsymbol{f}(\boldsymbol{u})$ produces eigenvalues whose real part is negative in its entirety. On the other hand, if there is an eigenvalue with a positive real part, the equilibrium point $\overline{\boldsymbol{u}}$ is said to be unstable. The existence and stability of equilibrium points often depend on the number $\boldsymbol{R_0}$. In this study, $\boldsymbol{R_0}$ is determined by the NGM matrix. Furthermore, a sensitivity analysis is performed based on the following definition.

Definition 2. [19][20] The normalized sensitivity index of a variable $\boldsymbol{R_0}$, that depends differentiated at parameter *c* is defined as

$$S_c^{R_0} = \frac{\partial R_0}{\partial c} \times \frac{c}{R_0}$$

in which case $\boldsymbol{R_0}$ is considered as the variable to be analyzed at parameter $\boldsymbol{c}$.

The data needed for numerical simulation are obtained by taking secondary data, namely data on the number of family heads, data on the number of young children, and data on the number of stunting cases in West Sumatra, which are obtained from the Ministry of Home Affairs and the West Sumatra BPS Office. The stages carried out in this study are:

1. Literature review on stunting and mathematical modelling,
2. Identify assumptions,
3. Based on the assigned compartment, create a status change diagram for each individual,
4. Construct a mathematical model of stunting with nutrition and education interventions,
5. Determine/find the equilibrium point of the resulting model,
6. Analyse the stability of the equilibrium point,
7. Determine the basic reproduction number ($\boldsymbol{R_0}$) by using the *NGM* matrix
8. Conduct a sensitivity analysis to identify the factors that have the greatest impact on increasing the number of stunted children, and
9. Perform numerical simulation of stunting cases using the generated model.

3. RESULTS AND DISCUSSION

3.1 Mathematical Model of Stunting Cases

Based on the division of compartments for the population of young children and mothers in the introduction, individuals in each compartment can change status over time. The change in status depends on the interaction of the individual with other components of the system or on the nutritional interventions for toddlers and educational interventions for mothers in a given compartment. Changes in individual status cause an individual to move from one compartment to another. This movement creates a continuous and changing dynamic in the emerging interaction system. The diagram of an individual's status change is shown below.

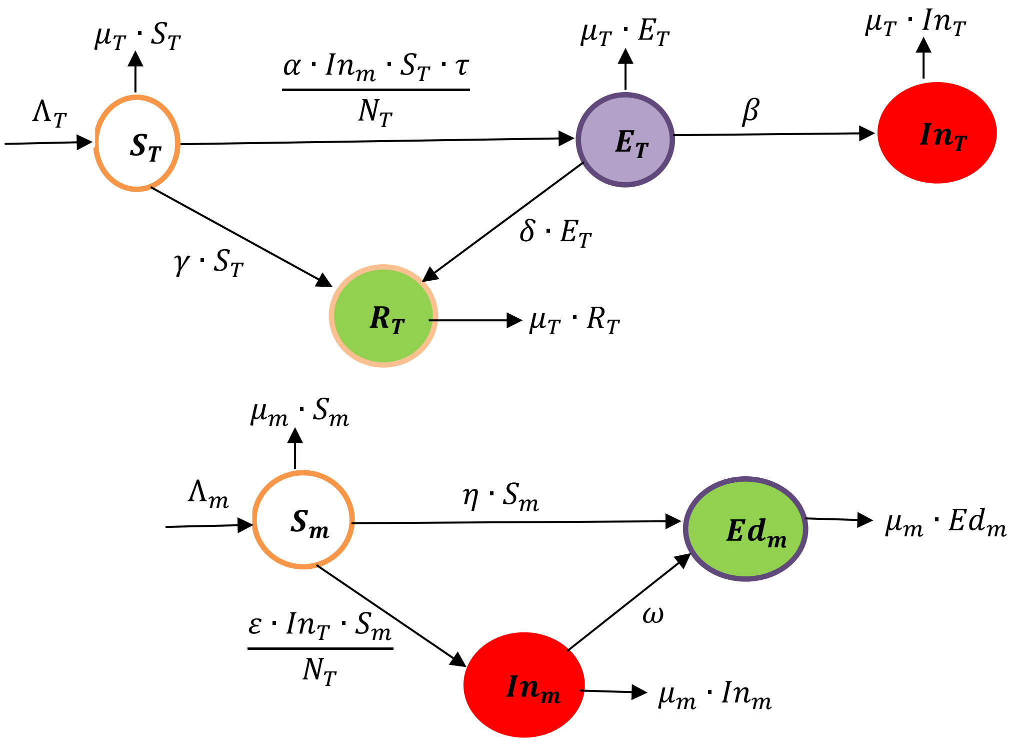


Figure 1. The Compartment Diagram and State Change

The parameters used in the compartmental diagram and mathematical model of stunting are described in **Table 1** below.

Table 1. Parameters in The Stunting Model

Parameters	Information	Unit
Λ_T	Recruitment rate of susceptible toddlers.	Toddler/Unit of time
Λ_m	Recruitment rates of susceptible mothers to poor parenting skills.	Mother/Unit of time
α	The rate of transition from S_T to E_T is due to the mother's poor parenting of the toddlers.	1/ Unit of time
τ	Number of toddlers cared for by a mother.	Toddler/Mother
β	The transition rate from E_T to In_T is due to children with symptoms of stunting not receiving nutrition interventions.	1/ Unit of time
γ	The transition rate from S_T to R_T when susceptible toddlers receive nutrition interventions.	1/Unite of time
δ	Rate of transition from E_T to R_T when toddlers with symptoms of stunting receive nutrition interventions.	1/Unite of time
ε	The rate of transition from S_m to In_m due to mothers lacking good parenting skills.	1/Unite of time
η	The rate of transition from S_m to Ed_m is due to mothers receiving education on good parenting.	1/Unite of time
ω	Transition rate from In_m to Ed_m as mothers with poor parenting receive education on good parenting.	1/Unite of time
μ_T	Mortality rate of toddlers.	1/Unite of time
μ_m	Mortality rate of mothers.	1/Unite of time
N_T	Total population of toddlers.	Toddler
N_m	Total population of mothers.	Mother

The assumptions used to construct and simplify the mathematical model of stunting cases are as follows.

1. Each child in each compartment is assumed to have a caregiver, but the concern in the model is only with mothers with poor parenting.

2. The increase in the population size of children/toddlers and mothers is constant over time.
3. The population is closed (no migration into or out of the population).
4. Susceptible toddlers ($\boldsymbol{S_T}$) can become toddlers with symptoms of stunting ($\boldsymbol{E_T}$) if raised by mothers who have poor parenting and no nutritional intervention.
5. A toddler ($\boldsymbol{S_T}$) will become a stunting free toddler ($\boldsymbol{R_T}$) only if he/she always receives a nutrition intervention at the rate of $\boldsymbol{\gamma}$ until at least two years of age.
6. Toddlers with stunting indication ($\boldsymbol{E_T}$) can turn into free of stunting ($\boldsymbol{R_T}$) if they always receive nutrition intervention at the rate of $\boldsymbol{\delta}$ until at least two years of age.
7. Toddlers with indications of stunting ($\boldsymbol{E_T}$) will turn into permanently stunted ($\boldsymbol{In_T}$) if they do not receive nutrition interventions.
8. The status change period of each individual in each compartment is assumed to be the same.
9. Mothers who take care of children with stunting are assumed to be mothers with poor parenting, causing $\boldsymbol{S_m}$ to become $\boldsymbol{In_m}$.

Based on the state transition diagram in **Figure 1** and the assumptions described above, a mathematical model for each compartment is as follows.

$$\frac{dS_T(t)}{dt} = \Lambda_T - \frac{\alpha \cdot In_m(t) \cdot S_T(t) \cdot \tau}{N_T} - \gamma S_T(t) - \mu_T S_T(t)$$

$$\frac{dE_T(t)}{dt} = \frac{\alpha In_m(t) S_T(t) \tau}{N_T} - \beta E_T(t) - \delta E_T(t) - \mu_T E_T(t)$$

$$\frac{dIn_T(t)}{dt} = \beta E_T(t) - \mu_T In_T(t)$$

$$\frac{dR_T(t)}{dt} = \gamma S_T(t) + \delta E_T(t) - \mu_T R_T(t)$$

$$\frac{dS_m(t)}{dt} = \Lambda_m - \frac{\varepsilon In_T(t) S_m(t)}{N_T} - \eta S_m(t) - \mu_m S_m(t)$$

$$\frac{dIn_m(t)}{dt} = \frac{\varepsilon In_T(t) S_m(t)}{N_T} - \omega In_m(t) - \mu_m In_m(t)$$

$$\frac{dEd_m(t)}{dt} = \eta S_m(t) + \omega In_m(t) - \mu_m Ed_m(t)$$

where all parameters in the model are non-negative, $N_T(t) = S_T(t) + E_T(t) + In_T(t) + R_T(t)$, and $N_m(t) = S_m(t) + In_m(t) + Ed_m(t)$. The dynamics for the total population $N_T(t)$ and $N_m(t)$ are given by the equation $\frac{dN_T(t)}{dt} = \Lambda_T - \mu_T(S_T(t) + E_T(t) + In_T(t) + R_T(t))$ and $\frac{dN_m(t)}{dt} = \Lambda_m - \mu_m(S_m(t) + In_m(t) + Ed_m(t))$. Thus obtained, $\lim_{t\to\infty} N_T(t) = \frac{\Lambda_T}{\mu_T}$, and $\lim_{t\to\infty} N_m(t) = \frac{\Lambda_m}{\mu_m}$.

3.2 Points of Equilibrium Free and Endemic of stunting

The equilibrium point is obtained when the system (1) is in the state $\frac{dS_T(t)}{dt} = \frac{dE_T(t)}{dt} = \frac{dIn_T(t)}{dt} = \frac{dR_T(t)}{dt} = \frac{dS_m(t)}{dt} = \frac{dIn_m(t)}{dt} = \frac{dEd_m(t)}{dt} = 0$ [21][22]. System (1) has two equilibrium points, namely the stunting free equilibrium point (E_0) and the stunting endemic equilibrium point (E_1). The stunting free equilibrium point is reached when there are no cases of stunting in the population and no mothers who care for their children with poor parenting. The equilibrium point of E_0 is:

$$E_0 = \left(\frac{\Lambda_T}{\gamma+\mu_T}, 0, 0, \frac{\gamma\Lambda_T}{(\gamma+\mu_T)\mu_T}, \frac{\Lambda_m}{\eta+\mu_m}, 0, \frac{\eta\Lambda_m}{(\eta+\mu_m)\mu_m}\right) \quad (2)$$

Furthermore, the stunting endemic equilibrium point (E_1) is reached when the compartment of mothers with poor parenting and the number of stunted children under five are both positive. The equilibrium E_1 is defined as

$$E_1 = \left(S_T^*(t), E_T^*(t), In_T^*(t), R_T^*(t), S_m^*(t), In_m^*(t), Ed_m^*(t)\right) \quad (3)$$

where:

$$S_T^*(t) = \frac{L}{Y}$$

$$E_T^*(t) = \frac{(R_0^3 - 1)X}{Y}$$

$$In_T^*(t) = \frac{(R_0^3 - 1)X\beta}{\mu_T Y}$$

$$R_T^*(t) = \frac{\left((R_0^3 - 1)\mu_T + \gamma R_0^3\right)\delta J + \mu_T(H + G)}{\mu_T K}$$

$$S_m^*(t) = \frac{\Omega\mu_m}{B}$$

$$In_m^*(t) = \frac{\left((R_0^3 - 1)U - P - Q\right)\mu_T + UR_0^3\gamma}{\mu_T F} = \frac{(R_0^3 - 1)U\mu_T - (P + Q)\mu_T + UR_0^3\gamma}{\mu_T F}$$

$$Ed_m^*(t) = \frac{(R_0^3 - 1)Z + V + R_0^3 W + A}{B}$$

with:

$$R_0 = \sqrt[3]{\frac{\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha}{N_T^2(\omega + \mu_m)(\eta + \mu_m)\mu_T(\gamma + \mu_T)(\beta + \delta + \mu_T)}}$$

$$L = N_T(\mu_T(\eta + \mu_m)(\beta + \delta + \mu_T)N_T + \beta\varepsilon\Lambda_T)(\omega + \mu_m)$$

$$Y = \varepsilon\left((\gamma + \mu_T)(\omega + \mu_m)N_T + \alpha\tau\Lambda_m\right)\beta$$

$$X = \mu_T N_T^2(\omega + \mu_m)(\eta + \mu_m)(\gamma + \mu_T))$$

$$J = N_T^2\mu_T^2(\omega + \mu_m)(\beta + \delta + \mu_T)(\eta + \mu_m)$$

$$H = \gamma\mu_T(\beta + \mu_T)(\beta + \delta + \mu_T)(\omega + \mu_m)(\eta + \mu_m)N_T^2$$

$$G = \varepsilon\Lambda_T\beta\gamma(\beta + \delta + \mu_T)(\omega + \mu_m)N_T$$

$$K = \varepsilon\beta(N_T(\omega + \mu_m)\mu_T + \gamma(\omega + \mu_m)N_T + \alpha\tau\Lambda_m)(\beta + \delta + \mu_T)\mu_T$$

$$\Omega = (\beta + \delta + \mu_T)\left((\gamma + \mu_T)(\omega + \mu_m)N_T + \alpha\tau\Lambda_m\right)\mu_T N_T$$

$$B = \mu_m\tau\alpha(\mu_T(\eta + \mu_m)(\beta + \delta + \mu_T)N_T + \beta\varepsilon\Lambda_T)$$

$$U = N_T^2(\beta + \delta + \gamma)(\omega + \mu_m)(\eta + \mu_m)\mu_T^2$$

$$P = N_T^2(\omega + \mu_m)(\eta + \mu_m)\mu_T^3$$

$$Q = \gamma N_T^2(\beta + \delta)(\omega + \mu_m)(\eta + \mu_m)\mu_T$$

$$F = (\omega + \mu_m)\tau(N_T(\eta + \mu_m)\mu_T^2 + N_T(\beta + \delta)(\eta + \mu_m)\mu_T + \beta\varepsilon\Lambda_T)\alpha$$

$$Z = N_T^2\mu_T(\beta + \delta + \mu_T)\omega\mu_m(\gamma + \mu_T)$$

$$V = N_T^2\mu_T(\beta + \delta + \mu_T)\eta\mu_m(\gamma + \mu_T)$$

$$W = N_T^2\mu_T.(\beta + \delta + \mu_T)\omega\eta(\gamma + \mu_T)$$

$$A = \mu_T(\beta + \delta + \mu_T)\alpha\eta\tau\Lambda_m N_T$$

Theorem 1. (Existence of Equilibrium Points). Suppose that $R_0 = \sqrt[3]{\frac{\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha}{N_T^2(\omega+\mu_m)(\eta+\mu_m)\mu_T(\gamma+\mu_T)(\beta+\delta+\mu_T)}}$. If $R_0 < 1$, then the system (1) has exactly one equilibrium point, the stunting-free equilibrium point E_0. On the other hand, if $R_0 > 1$ and $(R_0^3 - 1)U\mu_T + UR_0^3\gamma > (P + Q)\mu_T$, then there exists a stunting-endemic equilibrium point E_1.

Proof. The stunting-free equilibrium point E_0 is obtained when $In_T(t) = 0$ and $In_m(t) = 0$. Under this condition, we also have $E_T(t) = 0$. Note that $S_T(t), R_T(t), S_m(t)$, and $Ed_m(t)$ are compartments with positive population sizes, since all model parameters are positive. Therefore, the equilibrium point E_0 does not depend on R_0, and hence, it always exists regardless of the value of R_0. Furthermore, from system (1), the stunting-endemic equilibrium point is given by $E_1 = (S_T^*(t),\ E_T^*(t), In_T^*(t), R_T^*(t), S_m^*(t), In_m^*(t), Ed_m^*(t))$ with $S_T^*(t) = \frac{L}{Y}$, $E_T^*(t) = \frac{(R_0^3-1)X}{Y}$, $In_T^*(t) = \frac{(R_0^3-1)X\beta}{\mu_T Y}$, $R_T^*(t) = \frac{((R_0^3-1)\mu_T+\gamma R_0^3)\delta J+\mu_T(H+G)}{\mu_T K}$, $S_m^*(t) = \frac{\Omega\mu_m}{B}$, $In_m^*(t) = \frac{(R_0^3-1)U\mu_T-(P+Q)\mu_T+UR_0^3\gamma}{\mu_T F}$, and $Ed_m^*(t) = \frac{(R_0^3-1)Z+V+R_0^3W+A}{B}$. Note that, $S_T^*(t)$ and $S_m^*(t)$ are always positive. The values of $E_T^*(t)$, $In_T^*(t)$, $R_T^*(t)$ and $Ed_m^*(t)$ are also positive whenever $R_0 > 1$. Meanwhile, $In_m^*(t)$ is positive if $R_0 > 1$ and $(R_0^3 - 1)U\mu_T + UR_0^3\gamma > (P+Q)\mu_T$. In other words, if $R_0 > 1$ and $(R_0^3 - 1)U\mu_T + UR_0^3\gamma > (P+Q)\mu_T$, then the stunting-endemic equilibrium point E_1 exists.■

3.3 Equilibrium Point Stability

The local stability of the equilibrium point can be determined by linearizing system (1) around the equilibrium point [18][23]. If all the eigenvalues of the characteristic polynomial of the linearized matrix around the equilibrium point are negative, then the equilibrium point is locally asymptotically stable [24]. Conversely, if any of the eigenvalues is positive, then the equilibrium point is unstable [18]. The linearization result of system (1) is expressed in the following Jacobian matrix.

$$J = \begin{bmatrix} -\frac{\alpha In_m\tau}{N_T} - \gamma - \mu_T & 0 & 0 & 0 & 0 & -\frac{\alpha S_T\tau}{N_T} & 0 \\ \frac{\alpha In_m\tau}{N_T} & -\delta - \beta - \mu_T & 0 & 0 & 0 & \frac{\alpha S_T\tau}{N_T} & 0 \\ 0 & \beta & -\mu_T & 0 & 0 & 0 & 0 \\ \gamma & \delta & 0 & -\mu_T & 0 & 0 & 0 \\ 0 & 0 & -\frac{\varepsilon S_m}{N_T} & 0 & -\frac{\varepsilon In_T}{N_T} - \eta - \mu_m & 0 & 0 \\ 0 & 0 & \frac{\varepsilon S_m}{N_T} & 0 & \frac{\varepsilon In_T}{N_T} & -\omega - \mu_m & 0 \\ 0 & 0 & 0 & 0 & \eta & \omega & -\mu_m \end{bmatrix}$$

Theorem 2. Suppose that $R_0 = \sqrt[3]{\frac{\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha}{N_T^2(\omega+\mu_m)(\eta+\mu_m)\mu_T(\gamma+\mu_T)(\beta+\delta+\mu_T)}}$. The stunting free equilibrium point E_0 will be locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof. The local stability of the stunting free equilibrium point E_0 is known from the sign of the eigenvalues of the matrix J evaluated around point E_0. Eigenvalues that are all negative indicate that the system is locally asymptotically stable at this point. The matrix J at point E_0 is given by

$$J(E_0)=\begin{bmatrix} -\gamma-\mu_T & 0 & 0 & 0 & 0 & -\frac{\alpha\Lambda_T\tau}{(\gamma+\mu_T)N_T} & 0 \\ 0 & -\delta-\beta-\mu_T & 0 & 0 & 0 & \frac{\alpha\Lambda_T\tau}{(\gamma+\mu_T)N_T} & 0 \\ 0 & \beta & -\mu_T & 0 & 0 & 0 & 0 \\ \gamma & \delta & 0 & -\mu_T & 0 & 0 & 0 \\ 0 & 0 & -\frac{\varepsilon\Lambda_m}{(\eta+\mu_m)N_T} & 0 & -\eta-\mu_m & 0 & 0 \\ 0 & 0 & \frac{\varepsilon\Lambda_m}{(\eta+\mu_m)N_T} & 0 & 0 & -\omega-\mu_m & 0 \\ 0 & 0 & 0 & 0 & \eta & \omega & -\mu_m \end{bmatrix}.$$

The characteristic polynomial of the Jacobian matrix $J(E_0)$ has seven eigenvalues, four of which have a negative sign: $\lambda_1=-\mu_m$, $\lambda_2=-(\eta+\mu_m)$, $\lambda_3=-\mu_T$, and $\lambda_4=-(\gamma+\mu_T)$. Furthermore, for λ_5, λ_6, and λ_7, they are determined based on the roots of the equation $a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0$, where

$a_1=N_T^2(\gamma+\mu_T)(\eta+\mu_m)$,
$a_2=N_T^2(\gamma+\mu_T)(\beta+\delta+\omega+2\mu_T+\mu_m)(\eta+\mu_m)$,
$a_3=(\eta+\mu_m)N_T^2(\gamma+\mu_T)\left(\mu_T^2+(\beta+\delta+2\omega+2\mu_m)\mu_T+(\omega+\mu_m)(\beta+\delta)\right)$, and
$a_4=(\eta+\mu_m)N_T^2(\omega+\mu_m)\mu_T(\gamma+\mu_T)(\beta+\delta+\mu_T)-\epsilon\Lambda_T\Lambda_m\tau\beta\alpha$.

Since all parameters involved in the model are positive, the coefficients a_1, a_2, and a_3 are positive. Based on Vieta Theorem [25], the eigen values λ_5, λ_6, and λ_7 obtained from the polynomial $a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0$ will be negative if $a_4>0$ or $N_T^2(\omega+\mu_m)(\eta+\mu_m)\mu_T(\gamma+\mu_T)(\beta+\delta+\mu_T)>\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha$, which is equivalent to $\frac{\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha}{N_T^2(\omega+\mu_m)(\eta+\mu_m)\mu_T(\gamma+\mu_T)(\beta+\delta+\mu_T)}<1$. This means that $R_0^3<1$. Thus, if $R_0<1$, then all roots of the polynomial $a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0$ are negative, so the stunting free equilibrium point E_0 is locally asymptotically stable. Conversely, if $R_0>1$ then a_4 is negative, so there is a positive eigenvalue and the point E_0 is unstable.■

Furthermore, the stability of the stunting endemic equilibrium point E_1 can be determined from the sign of the eigenvalue of the characteristic polynomial of the matrix J evaluated around point E_1. The characteristic polynomial gives seven eigenvalues with two negative eigenvalues, $\lambda_1=-\mu_m$ and $\lambda_2=-\mu_T$. The next five eigenvalues are determined from the roots of the polynomial

$$b_5\lambda^5+b_4\lambda^4+b_3\lambda^3+b_2\lambda^2+b_1\lambda+b_0=0 \tag{4}$$

where:
$b_5=BN_T^2Y\mu_T^2F$,
$b_4=x_1+BN_T\beta\varepsilon X(R_0^3-1)\mu_TF+(UR_0^3-P-Q-U)\mu_T^2Y\tau\alpha+\tau\alpha Y\mu_TUR_0^3\gamma$,
$b_3=B(x_3YN_T\mu_T+x_2+3\beta\varepsilon X(R_0^3-1)\mu_T+\beta\varepsilon X(R_0^3-1)x_4)N_T\mu_TF+B\left(x_5+\tau\alpha\beta\varepsilon X(R_0^3-1)\right)\left((UR_0^3-P-Q-U)\mu_T+R_0^3U\gamma\right)$,
$b_2=x_6+x_7+x_8+x_9-\tau\beta\Omega\alpha\mu_T^2\mu_m\varepsilon LF+B\alpha((UR_0^3-P-Q-U)\mu_T+R_0^3U\gamma)(\tau\mu_Tx_{11}+\tau x_{10}+x_{12})$,
$b_1=\tau\beta\alpha\left((UR_0^3-P-Q-U)\mu_T+R_0^3U\gamma\right)(\mu_T^2x_{21}+\mu_Tx_{22}+x_{20}+x_{23})+x_{13}(YN_T\mu_T^2x_{15}+\mu_T^3x_{14}+\mu_T^2x_{16}+\mu_Tx_{17}+x_{18})N_TB\mu_TF-x_{19}$,
$b_0=\left(x_{24}-\tau\beta\Omega\alpha\mu_m\varepsilon L(\eta+\mu_m)\right)(\gamma+\mu_T)\mu_T^2F+x_{25}x_{26}$.

with:

$x_1 = 3BN_T^2\mu_T^3YF + BN_T^2Y(\beta+\delta+\gamma+\omega+2\mu_m+\eta)\mu_T^2F,$
$x_2 = 3N_T\mu_T^3Y + 2N_T\left(\beta+\delta+\gamma+\frac{3}{2}\omega+3\mu_m+\frac{3}{2}\eta\right)Y\mu_T^2,$
$x_3 = (\gamma+\omega+2\mu_m+\eta)\beta + (\delta+\omega+2\mu_m+\eta)\gamma + \mu_m^2 + (2\delta+\omega+\eta)\mu_m + \delta+\eta)\omega + \delta\eta,$
$x_4 = \beta+\delta+\gamma+\omega+\mu_m,$
$x_5 = \tau\alpha 2N_T\mu_T^2Y + \tau\alpha N_TY(\beta+\delta+\omega+2\mu_m+\eta)\mu_T,$
$x_6 = BN_T^2Y\mu_T^5F + 3BN_T^2\left(\eta+\frac{1}{3}\beta+\frac{1}{3}\delta+\frac{1}{3}\gamma+\omega+2\mu_m\right)Y\mu_T^4F\ ,$
$x_7 = (2\left(\left(\eta+\frac{1}{2}\gamma+\omega+2\mu_m\right)\beta+\frac{3}{2}\mu_m^2+\left(\frac{3}{2}\eta+2\delta+2\gamma+\frac{3}{2}\omega\right)\mu_m+\left(\frac{3}{2}\eta+\delta+\gamma\right)\omega+\left(\eta+\frac{1}{2}\delta\right)\gamma+\delta\eta\right)YN_T + 3\beta\varepsilon X(R_0^3-1)\mu_T^3N_TBF\ ,$
$x_8 = (((\mu_m^2+(\eta+2\gamma+\omega)\mu_m+(\eta+\gamma)\omega+\eta\gamma)\beta+(\delta+\gamma)\mu_m^2+((\delta+\gamma)\omega+(\eta+2\delta)\gamma+\delta\eta)\mu_m+((\delta+\eta)\gamma+\delta\eta)\omega+\eta\delta\gamma)YN_T+2(R_0^3-1)\beta\left(\beta+\delta+\gamma+\frac{3}{2}\omega+\frac{3}{2}\mu_m\right)\varepsilon X)\mu_T^2N_TBF,$
$x_9 = (R_0^3-1)\beta\big((\gamma+\omega+\mu_m)\beta+(\delta+\gamma)\mu_m+(\delta+\gamma)\omega+\delta\gamma\big)\varepsilon XB\mu_TF,$
$x_{10} = N_T\mu_T^3Y + 2N_TY\left(\eta+\frac{1}{2}\beta+\frac{1}{2}\delta+\omega+2\mu_m\right)\mu_T^2,$
$x_{11} = \big((\eta+\omega+2\mu_m)\beta+\mu_m^2+(2\delta+\omega+\eta)\mu_m+(\delta+\eta)\omega+\delta\eta\big)YN_T+2\beta\varepsilon X(R_0^3-1),$
$x_{12} = \beta\varepsilon X(R_0^3-1)(\beta+\delta+\omega+\mu_m)\tau,$
$x_{13} = N_T^2Y(\eta+\omega+2\mu_m)\mu_T^5BF,$
$x_{14} = \big((\eta+\omega+2\mu_m)\beta+3\mu_m^2+(3\eta+2\delta+2\gamma+3\omega)\mu_m+(3\eta+\delta+\gamma)\omega+\eta(\delta+\gamma)\big)YN_T+\beta\varepsilon X(R_0^3-1),$
$x_{15} = (2\mu_m^2+(2\eta+2\gamma+2\omega)\mu_m+(2\eta+\gamma)\omega+\eta\gamma)\beta+(2\delta+2\gamma)\mu_m^2+\big((2\delta+2\gamma)\omega+(2\eta+2\gamma)\delta+2\eta\gamma\big)\omega+\eta\delta\gamma,$
$x_{16} = \beta\varepsilon X(R_0^3-1)(\beta+\delta+\gamma+3\omega+3\mu_m),$
$x_{17} = (R_0^3-1)\big((\gamma+2\omega+2\mu_m)\beta+(2\delta+2\gamma)\mu_m+(2\delta+2\gamma)\omega+\delta\gamma\big)\beta\varepsilon X+\gamma Y(\omega+\mu_m)(\beta+\delta)(\eta+\mu_m)N_T,$
$x_{18} = \beta\gamma\varepsilon X(R_0^3-1)(\omega+\mu_m)(\beta+\delta),$
$x_{19} = \tau\beta\Omega\alpha\mu_T\mu_m\varepsilon L(\eta+\gamma+\mu_T+\mu_m)\mu_TF,$
$x_{20} = N_TY(\eta+\omega+2\mu_m)\mu_T^3,$
$x_{21} = Y\big((\eta+\omega+2\mu_m)\beta+2\mu_m^2+(2\eta+2\delta+2\omega)\mu_m+(2\eta+\delta)\omega+\delta\eta\big)N_T+\beta\varepsilon X(R_0^3-1),$
$x_{22} = Y(\omega+\mu_m)(\beta+\delta)(\eta+\mu_m)N_T+\beta\varepsilon X(R_0^3-1)(\beta+\delta+2\omega+2\mu_m),$
$x_{23} = \beta\varepsilon X(R_0^3-1)(\omega+\mu_m)(\beta+\delta),$
$x_{24} = (\beta+\delta+\mu_T)(\omega+\mu_m)N_T\left(N_TY(\eta+\mu_m)\mu_T+\beta\varepsilon X(R_0^3-1)\right)B,$
$x_{25} = B(\beta+\delta+\mu_T)\alpha(\omega+\mu_m)\left((UR_0^3-P-Q-U)\mu_T+R_0^3U\gamma\right),$ *and*
$x_{26} = \left(N_TY(\eta+\mu_m)\mu_T+\beta\varepsilon X(R_0^3-1)\right)\tau\mu_T.$

If the roots of **Equation (4)** are negative ($\lambda_3 < 0,\ \lambda_4 < 0,\ \lambda_5 < 0,\ \lambda_6 < 0$, and $\lambda_7 < 0$), then the stunting endemic equilibrium point E_1 is locally asymptotically stable. Based on the Vieta Theorem since $b_5 = BN_T^2Y\mu_T^2F > 0$, then **Equation (4)** will produce negative roots if $b_4 > 0,\ b_3 > 0,\ b_2 > 0,\ b_1 > 0$, and $b_0 > 0$. Thus, it must be,

1. $x_1 + BN_T\beta\varepsilon X(R_0^3-1)\mu_TF + (UR_0^3-P-Q-U)\mu_T^2Y\tau\alpha + \tau\alpha Y\mu_TUR_0^3\gamma > 0,$ which is satisfied if $R_0 > 1$ and $UR_0^3 > P+Q+U$.
2. $B(x_3YN_T\mu_T + x_2 + 3\beta\varepsilon X(R_0^3-1)\mu_T + \beta\varepsilon X(R_0^3-1)x_4)N_T\mu_TF + B\left(x_5 + \tau\alpha\beta\varepsilon X(R_0^3-1)\right)\left((UR_0^3-P-Q-U)\mu_T + R_0^3U\gamma\right) > 0$, which is satisfied if $R_0 > 1$ and $UR_0^3 > P+Q+U$.
3. $x_6+x_7+x_8+x_9-\tau\beta\Omega\alpha\mu_T\mu_m\varepsilon L\mu_TF + B\alpha\left((R_0^3U-P-Q-U)\mu_T+UR_0^3\gamma\right)(x_{10}\tau + x_{11}\mu_T\tau + x_{12}) > 0$, which is satisfied if $R_0 > 1$, $UR_0^3 > P+Q+U$, and $x_6+x_7+x_8+x_9 > \tau\beta\Omega\alpha\mu_T\mu_m\varepsilon L\mu_TF$.
4. $\tau B\alpha\left((UR_0^3-P-Q-U)\mu_T+R_0^3U\gamma\right)(\mu_T^2x_{21}+\mu_Tx_{22}+x_{20}+x_{23}) + x_{13} + (YN_T\mu_T^2x_{15}+\mu_T^3x_{14}+\mu_T^2x_{16}+\mu_Tx_{17}+x_{18})N_TB\mu_TF - x_{19} > 0$, which is satisfied if $R_0 > 1$, $UR_0^3 > P+Q+U$, and $x_{13} + (YN_T\mu_T^2x_{15}+\mu_T^3x_{14}+\mu_T^2x_{16}+\mu_Tx_{17}+x_{18})N_TB\mu_TF > x_{19}$.

5. $(x_{24} - \tau\beta\Omega\alpha\mu_m\varepsilon L(\eta + \mu_m))(\gamma + \mu_T)\mu_T^2 F + x_{25}x_{26} > 0$, which is satisfied if $R_0 > 1$ and $(\gamma + \mu_T)\mu_T^2 F x_{24} + x_{25}x_{26} > \tau\beta\Omega\alpha\mu_m\varepsilon L(\eta + \mu_m)(\gamma + \mu_T)\mu_T^2 F$.

3.4 Next Generation Matrix (NGM) and Basic Reproduction Number (R_0)

The matrix used to determine the basic reproduction number (R_0) is the next generation matrix (NGM). Based on the steps given by Diekmann and Heesterbeek [26][27][28], the NGM matrix is obtained by taking the subsystem/compartment of the population indicated as stunted (E_T), permanently stunted (In_T), and the compartment of mothers with poor parenting (In_m) from the system (1) and then linearizing around the equilibrium point free of stunting (E_0). The subsystems are written as follows

$$\begin{pmatrix} \frac{dE_T(t)}{dt} \\ \frac{dIn_T(t)}{dt} \\ \frac{dIn_m(t)}{dt} \end{pmatrix} = \begin{pmatrix} \frac{\alpha \cdot In_m(t) \cdot S_T(t) \cdot \tau}{N_T} \\ \beta \cdot E_T(t) \\ \frac{\varepsilon \cdot In_T(t) \cdot S_m(t)}{N_T} \end{pmatrix} - \begin{pmatrix} \beta \cdot E_T(t) + \delta \cdot E_T(t) + \mu_T \cdot E_T(t) \\ \mu_T \cdot In_T(t) \\ \omega \cdot In_m(t) + \mu_m \cdot In_m(t) \end{pmatrix}.$$

Suppose, $F = \begin{pmatrix} \frac{\alpha \cdot In_m(t) \cdot S_T(t) \cdot \tau}{N_T} \\ \beta E_T(t) \\ \frac{\varepsilon \cdot In_T(t) \cdot S_m(t)}{N_T} \end{pmatrix}$ is a vector representing the occurrence of new stunting cases and $V = \begin{pmatrix} \beta \cdot E_T(t) + \delta \cdot E_T(t) + \mu_T \cdot E_T(t) \\ \mu_T \cdot In_T(t) \\ \omega \cdot In_m(t) + \mu_m \cdot In_m(t) \end{pmatrix}$ is a vector representing transitions or shifts, then the Jacobian matrices F and V around the equilibrium point E_0 are given by $M = \begin{pmatrix} 0 & 0 & \frac{\alpha\Lambda_T\tau}{(\gamma+\mu_T)N_T} \\ \beta & 0 & 0 \\ 0 & \frac{\varepsilon\Lambda_m}{(\eta+\mu_m)N_T} & 0 \end{pmatrix}$ and $N = \begin{pmatrix} B + \mu_T + \delta & 0 & 0 \\ 0 & \mu_T & 0 \\ 0 & 0 & \omega + \mu_m \end{pmatrix}$, respectively. Thus, the NGM matrix is

$$NGM = MN^{-1} = \begin{pmatrix} 0 & 0 & \frac{\alpha\Lambda_T\tau}{(\gamma+\mu_T)N_T} \\ \frac{\beta}{\beta+\delta+\mu_T} & 0 & 0 \\ 0 & \frac{\varepsilon\Lambda_m}{(\eta+\mu_m)N_T\mu_T} & 0 \end{pmatrix}.$$

The characteristic polynomial of the NGM matrix is $p(\lambda) = \lambda^3(\gamma + \mu_T)(\beta + \delta + \mu_T)(\eta + \mu_m)(\omega + \mu_m)N_T^2\mu_T - \alpha\beta\tau\varepsilon\Lambda_T\Lambda_m = 0$. The largest eigenvalue or maximum λ_i of the characteristic polynomial of the NGM matrix is defined as the basic reproduction number (R_0), namely

$$R_0 = \sqrt[3]{\frac{\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha}{N_T^2(\omega + \mu_m)(\eta + \mu_m)\mu_T(\gamma + \mu_T)(\beta + \delta + \mu_T)}} \tag{5}$$

3.5 Sensitivity Analysis

Sensitivity analysis was conducted on R_0 as a number that indicates the disappearance or endemicity of stunting in the middle of the human population. This analysis aims to identify the parameters or factors that can increase or decrease the value of R_0 [29][30]. The normalized sensitivity index of the variable R_0 differentiated at parameter c is defined as $S_c^{R_0} = \frac{\partial R_0}{\partial c} \times \frac{c}{R_0}$, in which case R_0 is considered as the variable to be analyzed at parameter c [19][20]. The parameter values used in the sensitivity analysis are based on data from West Sumatra Province. They are shown in the table below.

Table 2. Parameter Values

Parameters	Value	Information
N_T	393,475	Number of toddlers in West Sumatra Province in 2023 [31]
N_m	1,351,670	Number of family heads (assumed mothers) in West Sumatra Province in 2023 [32]
μ_T	$\frac{1}{70 \times 365}$	$\frac{1}{\text{Assumption of the average human life expectancy in the Province of West Sumatra}}$ [33]
μ_m	$\frac{1}{70 \times 365}$	$\frac{1}{\text{Assumption of the average human life expectancy in the Province of West Sumatra}}$ [33]
Λ_T	$\frac{393{,}475}{365 \times 70}$	$N_T \times \mu_T$
Λ_m	$\frac{1{,}351{,}670}{365 \times 70}$	$N_m \times \mu_m$
α	94.9	$\frac{8{,}8\,\%\ \text{stunted toddlers} \times \text{Total number of toddlers in West Sumatra}}{365\ \text{Days}}$ [31]
τ	0.3	$\frac{N_T}{N_m}$
β	$\frac{1}{1{,}000}$	$\frac{1}{\text{The first 1000 days of life}}$ [3]
γ	$\frac{1}{720}$	$\frac{1}{\text{Assumed number of days required to make the transition from } S_T \text{ to } R_T \text{ due to a low intake of nutrients}}$
δ	$\frac{1}{720}$	$\frac{1}{\text{Assumed number of days required to make the transition from } E_T \text{ to } R_T \text{ due to a low intake of nutrients}}$
ε	$\frac{1}{120}$	$\frac{1}{\text{The assumed number of days required for } S_m \text{ to become } In_m \text{ due to low education}}$
η	$\frac{1}{360}$	$\frac{1}{\text{Assumed number of days required to make the transition from } S_m \text{ to } Ed_m \text{ due to a poor training process}}$
ω	$\frac{1}{360}$	$\frac{1}{\text{Assumed number of days required to make the transition from } In_m \text{ to } Ed_m \text{ due to a poor training process}}$

Based on Equation (5) and the parameter values in Table 2, the value of $R_0 = 10.51$ is obtained. This indicates that stunting is endemic in the population. The effect of changing the parameter values by 20% on the value of $R_0 = 10.51$ is shown in the following Table 3.

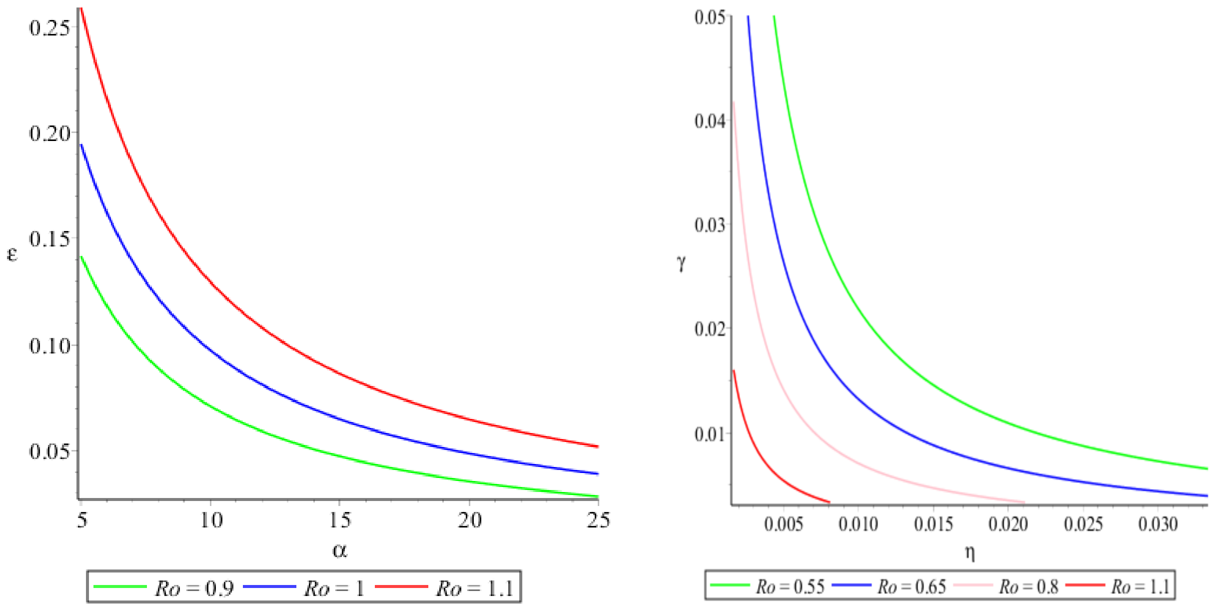
Table 3. The R_0 Value Change Due to Parameter Value Change

Para-meters	Expression of sensitivity index	Index value	Old value of R_0	Old value of the parameter	20% parameter addition creates new R_0		20% parameter reduction creates new R_0	
					Parameter + (20% x Parameter)	New R_0 Value	Parameter – (20% x Parameter)	New R_0 Value
α	$\frac{1}{3}$	0.33	10.51	94.9	113.88	11.16	75.92	9.75
τ	$\frac{1}{3}$	0.33	10.51	0.3	0.36	11.16	0.24	9.75
β	$\frac{\delta + \mu_T}{3(\beta + \delta + \mu_T)}$	0.196	10.51	0.001	0.0012	10.87	0.0008	10.04
γ	$-\frac{\gamma}{3(\gamma + \mu_T)}$	-0.32	10.51	0.00138	0.00166	9.91	0.0011	11.33
δ	$-\frac{\delta}{3(\beta + \delta + \mu_T)}$	-0.19	10.51	0.00138	0.00166	10.14	0.0011	10.96
ε	$\frac{1}{3}$	0.33	10.51	0.00833	0.01	11.16	0.00666	9.75

Para-meters	Expression of sensitivity index	Index value	Old value of R_0	Old value of the parameter	20% parameter addition creates new R_0		20% parameter reduction creates new R_0	
					Parameter + (20% x Parameter)	New R_0 Value	Parameter – (20% x Parameter)	New R_0 Value
η	$-\frac{\eta}{3(\eta+\mu_m)}$	-0.33	10.51	0.002778	0.00333	9.89	0.00222	11.31
ω	$-\frac{\omega}{3(\omega+\mu_m)}$	-0.33	10.51	0.002778	0.00333	9.89	0.00222	11.31

Based on **Table 3** above, there are eight parameters that affect the value of R_0, with 4 parameters having a positive index and 4 other parameters having a negative index. Parameters with a positive index indicate that if the value of the parameter increases, the value of R_0 will also increase. Conversely, if the value of a parameter with a positive index decrease, the value of R_0 will also decrease. Meanwhile, parameters with a negative index have the opposite relationship with R_0; if the value of a parameter with a negative index increase, the value of R_0 will decrease. Conversely, if a parameter with a negative index decrease, R_0 will increase.

The results of the sensitivity index calculation in **Table 3** show that the most influential parameters with a positive index on R_0 are: the transition rate of susceptible toddlers to stunting indicated toddlers (α), the number of children raised by mothers with poor parenting (τ), and the rate at which susceptible mothers become mothers with poor parenting (ε). The index value of the three parameters is the same at 0.3. The number of children raised by a mother (τ) is not possible to change, while the α and ε parameters have the potential to be minimized. A smaller α and ε value will result in a smaller R_0 value. This can be seen in the table, where the original R_0 value of 10.51 drops to 9.75 with a 20% reduction in the alpha or epsilon parameter values. Meanwhile, the most influential parameters with negative indices are η, ω, and γ. A negative sensitivity index means that an increase in intervention in the form of education for mothers/caregivers of toddlers will result in a decrease in the value of R_0, while the lack of education can increase the value of R_0. Similarly, a high rate of nutritional intervention for toddlers will make the R_0 value smaller. This is indicated by the value of the sensitivity index γ of -0.32, where increasing the gamma parameter by 20% causes the value of R_0 to decrease from 10.51 to 9.91. The following graph shows the relationship of parameters α, ε, η, ω, δ and γ to R_0.

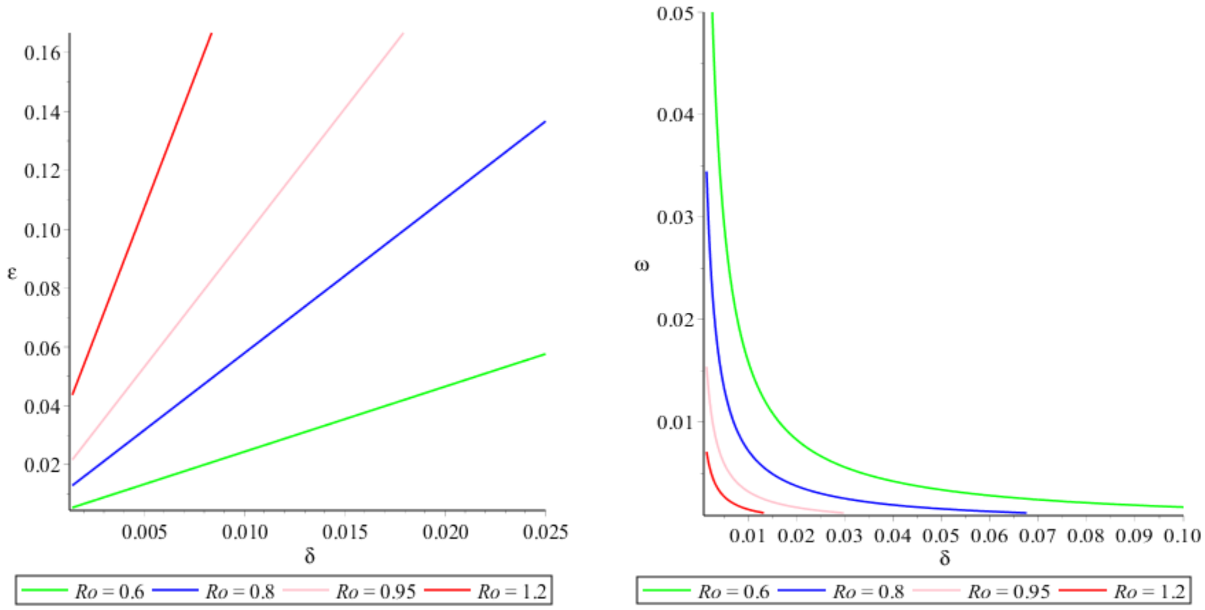


(a) Relating α and ε to R_0 **(b) Relating γ and η to R_0**

Figure 2. Sensitivity Analysis of Parameters α, ε, η, and γ against R_0

The effect of changing the α and ε parameters on R_0 is shown in **Figure 2** (a). From the figure, it appears that an increase in the rate of toddlers reported as indicated stunting (α) and the rate of increase in the number of caregivers/mothers with poor parenting (ε) will result in an increase in the value of R_0.

Conversely, a small α and ε will also minimize the value of R_0. This means that an increase in the number of mothers with poor parenting will result in an increase in the number of toddlers who will be stunted. Furthermore, Figure 2 (b) shows the results of the sensitivity analysis on the effect of maternal education (η) and nutrition intervention (γ) on increasing or decreasing the value of R_0. The figure shows that an increase in the value of η and γ parameters lead to a decrease in the value of R_0. Meanwhile, low values of η and γ will result in an increase in the value of R_0. This means that increasing educational services for mothers of toddlers and intensive nutritional interventions for toddlers will reduce the number of stunting cases and may even eliminate stunting from the population. Conversely, low levels of maternal education and low levels of nutrition intervention for young children can increase the number of stunted children and even lead to endemic stunting in the population.



(a) Relating δ and ε to R_0 **(b) Relating δ and ω to R_0**

Figure 3. Sensitivity Analysis of Parameters δ, ε, and ω against R_0

Figure 3 shows the results of the sensitivity analysis of the δ, ε, and ω parameters to R_0. Figure 3 (a) shows that an increase in the rate of caregivers/mothers with poor parenting (ε) and a decrease in the rate of nutrition interventions for toddlers indicated as stunted (δ) can increase the value of R_0. Conversely, the R_0 value will decrease if the ε value decreases and the δ value increases. This means that stunting cases can be reduced or eliminated from the population by increasing the rate of nutritional intervention and reducing the rate of poor maternal parenting by providing education. Meanwhile, Figure 3 (b) shows the state of R_0, which increases when the rate of nutritional intervention for toddlers indicated as stunted (δ) and the rate of education for mothers (ω) are both low. Conversely, the value of R_0 decreases as the values of the δ and ω parameters increase. This means that stunting will disappear from the population when the rate of nutrition and education intervention is increased (high intensity nutrition and education intervention).

3.6 Numerical Simulations

Numerical simulations are performed in the state $R_0 > 1$ and $R_0 < 1$. Numerical solutions are performed using the Runge-Kutta method [34]. Simulations were conducted with initial condition values as shown in the following Table 4.

Table 4. The Initial Values

$S_T(0)$	$E_T(0)$	$In_T(0)$	$R_T(0)$	$S_m(0)$	$In_m(0)$	$Ed_m(0)$
303,849	50,000	34,626	5,000	1,330,458	1,542	20,000

Based on the parameter values in Table 2 and Equation (5), the value of $R_0 = 10{,}51 > 1$, is obtained. Furthermore, the numerical simulation results using the initial conditions in Table 4 and the parameter values in T Table 2 are shown in the following figure.

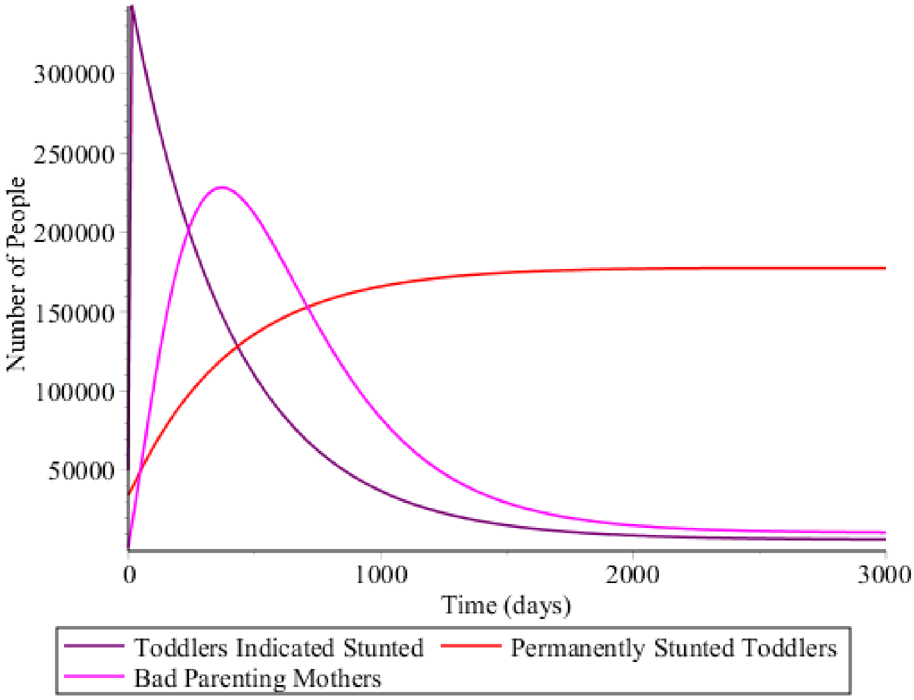
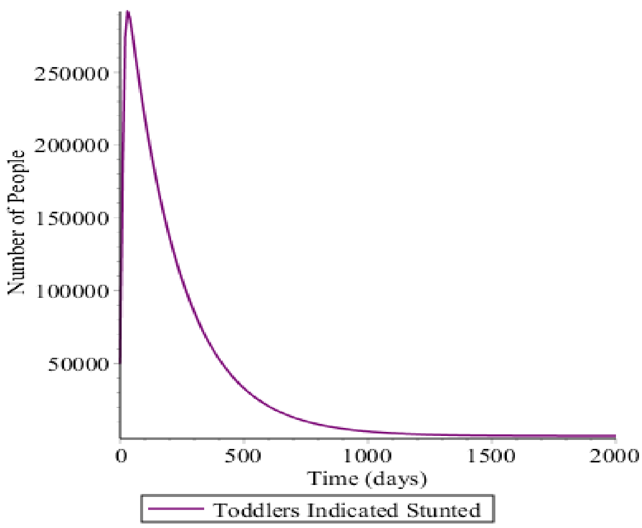


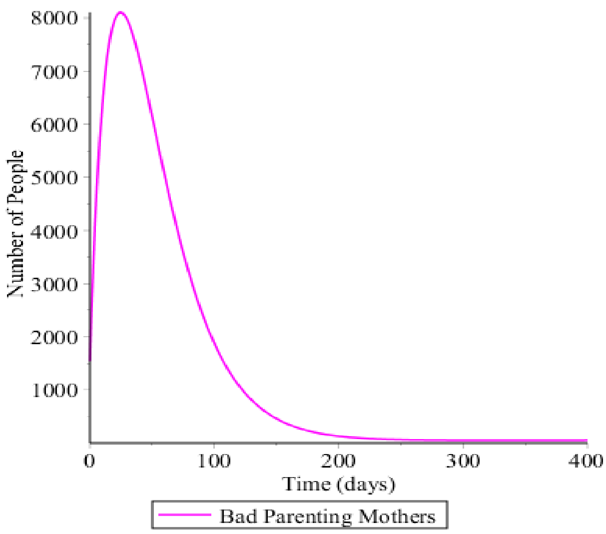
Figure 4. Dynamics of Changes in The Number of In_T, E_T, and In_m when $R_0 > 1$

The simulation shown in **Figure 4** is a simulation in the state $R_0 > 1$ with a stable stunting endemic equilibrium point. As a result, there is always a very large number of permanently stunted toddlers in the population. Based on the calculation of the equilibrium point, the system is stable with the number of permanently stunted toddlers (In_T) totaling 161,745, toddlers with stunting symptoms (E_T) totaling 6,331, and mothers practicing poor parenting/bad parenting mothers (In_m) totaling 10,306, each of which is stable after about 2,000 days.

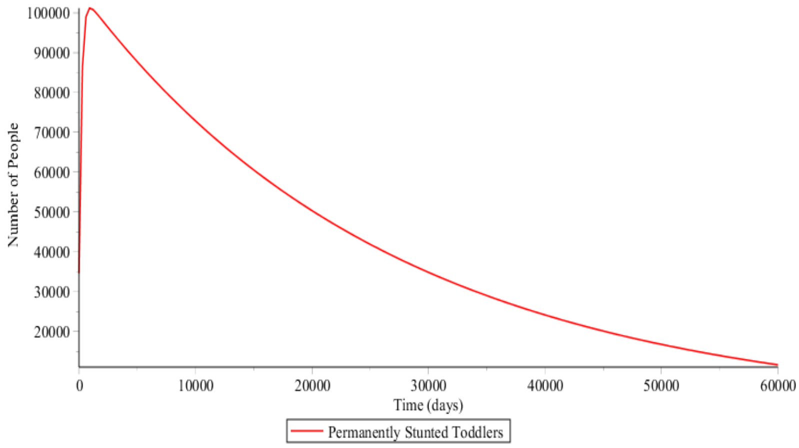
Numerical simulations for the value of $R_0 < 1$ were obtained by changing some parameter values in **Table 2**, namely the $\alpha, \delta, \eta, \gamma, \omega$, and ε parameters to $20, \frac{1}{270}, \frac{1}{21}, \frac{1}{180}, \frac{1}{30}$, and $\frac{1}{150}$, respectively. The R_0 value obtained is 0,516 < 1. Based on the parameter values in **Table 2** and their changes, as well as the initial condition values in **Table 4**, the following numerical simulation results are obtained.



(a) Toddlers Showing Symptoms of Stunting



(b) Mothers with Poor Parenting



(c) Toddlers with Permanent Stunting

Figure 5. Dynamics of Changes in The Number of In_T, E_T, and In_m when $R_0 < 1$

In **Figure 5** the population of In_T, E_T, and In_m eventually became zero. This occurs because the simulation is run in the state $R_0 < 1$, where in this state the population state is free of individuals with indicated stunting and free of toddlers with permanent stunting status. In contrast to the simulation results in **Figure 4**, where stunting cases reached 161,745 cases, the simulation results in **Figure 5** show that stunting cases can disappear from the population. The elimination of stunting is achieved through the implementation of optimal and sustainable nutrition interventions. These interventions can accelerate the transformation of S_T or E_T toddlers into permanently stunting-free toddlers (R_T). This situation is shown in the simulation by changing the parameter values γ and δ, which were originally $\gamma = \frac{1}{720}$ and $\delta = \frac{1}{720}$ as in **Table 2**, to $\gamma = \frac{1}{180}$ and $\delta = \frac{1}{270}$, resulting in a simulation as shown in **Figure 5**. The change in the parameter value γ means that an individual infant initially needs 720 days to be able to move from the S_T to the R_T compartment, but through nutritional intervention activities the time required for the move is reduced to only 180 days. Similarly, a change in the value of the parameter δ means that initially an individual toddler E_T takes 720 days to move to the R_T compartment, but with the nutritional intervention, the time required is reduced to only 270 days.

The disappearance of stunting shown in **Figure 5** is also due to the educational interventions provided to mothers who care for children aged under five. Intensive educational activities are represented by changing the parameter values $\eta = \frac{1}{360}$ and $\omega = \frac{1}{360}$ to $\eta = \frac{1}{21}$ and $\omega = \frac{1}{30}$. The change in parameter η means that with intensive education that is easily understood by mothers, the movement of mothers from the S_m compartment to the Ed_m compartment is shorter, taking only 21 days from the original 360 days. Furthermore, the change in the parameter ω means that it is easier for a mother to move from the compartment In_m to the compartment Ed_m with educational activities. This move took only 30 days, which is faster than the original 360 days. The nutritional intervention activities for young children and education for mothers are assumed to have an impact on reducing the rate of stunted children (α), namely from $\alpha = 94.9$ in simulation **Figure 4**, down to $\alpha = 20$, whose simulation results are shown in **Figure 5**. In particular, the educational activities play a role in reducing the transition rate ε, from susceptible mothers S_m to mothers with poor parenting (In_m). This decrease can be seen from the change in the value of $\varepsilon = \frac{1}{120}$ to $\varepsilon = \frac{1}{150}$.

Overall, changes in parameter values have an impact on reducing the number of stunting cases from 161,745 cases based on the simulation results in **Figure 4** to zero cases in the simulation of **Figure 5**. The disappearance of stunting in **Figure 5** occurs after 60,000 days. This is because infants who have experienced permanent stunting do not disappear from the population until they die due to natural mortality factors. Simulations like the one shown in **Figure 5** are very likely to be carried out in real life in the community. This can happen because a toddler who receives adequate nutrition from the womb can immediately become a permanently stunting-free toddler (R_T) while receiving adequate nutrition through the first 1000 days of life. Therefore, it is possible to change the parameter values to $\alpha = 20$, $\delta = \frac{1}{270}$, $\eta = \frac{1}{21}$, $\gamma = \frac{1}{180}$, $\omega = \frac{1}{30}$, and $\varepsilon = \frac{1}{150}$. These parameter values can be improved upon by implementing more optimal nutrition and education interventions, and stunting can be eliminated from the population more quickly.

4. CONCLUSION

Based on the results obtained and discussed, the following can be concluded.

1. The R_0 can be reduced through education programs for mothers of under-five children and the provision of appropriate nutritional interventions for toddlers.
2. Nutritional interventions for children under five and education for mothers are very effective in reducing and eliminating stunting in the population.
3. The resulting mathematical expression for the value of R_0 is $R_0 = \sqrt[3]{\frac{\varepsilon\Lambda_T\Lambda_m\tau\beta\alpha}{N_T^2(\omega+\mu_m)(\eta+\mu_m)\mu_T(\gamma+\mu_T)(\beta+\delta+\mu_T)}}$.
4. The mathematical model of stunting with nutrition and education interventions produces two equilibrium points, namely the stunting free equilibrium point (E_0), which is stable when $R_0 < 1$, and the stunting endemic equilibrium point (E_1), which is stable when $R_0 > 1$.

AUTHOR CONTRIBUTIONS

La Ode Sabran: Conceptualization, Formal analysis, Funding Acquisition, Validation, Software. Lathifah Annur: Project Administration, Resources, Visualization, Writing-Original Draft, Writing-Review & Editing. Athisa Ratu Laura: Conceptualization, Investigation, Methodology, Software. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest to report study.

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