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A MODEL ON MARKET EQUILIBRIUM USING A DIFFERENTIAL EQUATION WITH TIME DELAYS

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ABSTRACT

In this paper, a model on market equilibrium is proposed using a delay differential equation with discrete delays as a modified version of the one proposed by Kobayashi (1996). The price of a commodity is determined using the equation involving weighted supply and demand functions. Both supply and demand functions are considered at the current time and sometimes in the past. The delays are chosen by considering the seasonal behavior of the market. We use data on some main commodities in Indonesia from 2018 to 2024 to validate the model. We found that the implementation of our modified Kobayashi model improves the estimation given by the original one. The implementation of the method also shows some characteristics of delay equations, that is longer delay time may include more dynamics, and more fluctuation, although that means it is more prone to instabilities. However, the problem of optimal delay time is yet to be resolved.

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1. INTRODUCTION

Almost all real-life processes, whether it is natural or man-made, involve time delays. Models of these phenomena using time delay(s) make them more realistic, thus they are better to represent the phenomena. A simple example in nature is reforestation. A cut forest, after replanting, will take at least 20 years before reaching any kind of maturity. For certain species of trees (redwoods, for example) it would be much longer. Hence, any mathematical model of forest harvesting and regeneration clearly must have time delays built into it. Another example occurs due to the fact that animals must take time to digest their food before further activities and responses take place [1].

In 1908, Picard emphasized the importance of the consideration of hereditary effects in the modeling of physical systems. In 1908 and 1928, Volterra discussed the integrodifferential equations that model viscoelasticity. He also wrote a fundamental book on the role of hereditary effects on models for the interaction of species. The subject grew quite rapidly after World War II due to the consideration of meaningful models of engineering. During the last 70 years, the theory of functional differential equations has been developed extensively and has become part of the vocabulary of researchers dealing with specific applications such as viscoelasticity, mechanics, nuclear reactors, distributed networks, heat flow, neural networks, combustion, interaction of species, microbiology, learning models, epidemiology, physiology, economics, as well as many others [2], [3], [4].

This work focuses on modeling in the field of economics, particularly on the price fluctuations in a market. A market is a mechanism that facilitates economic transactions between two or more parties. Prices of commodities in a market are primarily determined by two key factors: demand and supply. Demand refers to the economic activity by consumers to fulfill their need for goods and services, while supply refers to the economic activity by sellers to provide goods and services to buyers.

Market equilibrium is the point where the quantity supplied by producers and the quantity demanded by consumers are equal. There are also some conditions in order to have a market equilibrium, such as excess supply depends on prices to the extent of their ratios, which determine exchange rates; continuity; Walras' Law, which expresses the aggregate budget constraint, by which what is bought in any transaction is paid for out of what is sold; Finite Supply Capacity, that the excess supply for any good is bounded from above; the Intercept Condition, that demand for good must exceed supply if the price is small, or that demand overtakes supply of a good as its price falls, to make excess supply negative; the Slope Condition, which requires excess supply of a good to be decreasing as a function of the other prices [5], [6], [7].

The time factor is of great importance when modeling the market as the famous economist Friedrich August von Hayek wrote in 1937. Dynamic "demand–supply" models, which take into account the time factor, describe the processes of the modern market that occur in long-term market periods more adequately than static ones. Dynamic models of economic systems help solving many problems of state economic planning, many macro- and microeconomic problems of marketing, etc [8].

The literature on the dynamic models of market equilibrium with delay is large. To name a few, Szydlowski et. al. [9] discussed nonlinear oscillation in business models with time lags. Bobalova and Novotna [10] used functional differential equations to model the meat market with time delay. Matsumoto and Szidarovszky [11] discussed in general the role of nonlinear delay in economic models. Anokye et.al. (2022) [12] studied price dynamics using the delay differential Cobweb model.

Kobayashi [13] stated that there is a time delay between demand and supply in traditional market models because market participants do not always respond instantly to changes in economic conditions.With those phenomena mentioned above in mind, we report on the analysis of our preliminary study on our proposed model of market equilibrium which is a modified version of the model discussed by K. Kobayashi [13]. Here we add some term that involves time delay, and also put some parameters as the weight of the supply and demand terms, in order to make the model more realistic. Our objective is to study the efficacy of the proposed method, in comparison to the one of Kobayashi's.

The model is applied to the market of three commodities in Indonesia in 2018 to 2024, those are eggs, chicken meat, and palm oil. Using statistical techniques on the data of supply, demand, and price, the parameters are estimated. Then, the prices obtained from the model are compared to the real data, to have a distribution of the errors. Similar procedure we apply using Kobayashi's model, as a comparison to ours. An analysis of the results is presented.

In statistical terms, our data is actually a time series. For time series data, there are some purely probabilistic methods to obtain predictions, such as ARIMA which allows lagged observations, ARCH or GARCH which accommodate volatile data such as commodity data that we are using. However, in this study, we consider the deterministic dynamics of a modified Kobayashi's method via delay differential equation. The statistical part is to determine the parameters of the terms in our system.

2. RESEARCH METHOD

2.1 The Modified Kobayashi's Model

Assuming the non-instant responses, Kobayashi modified the traditional market model with the following equation.

$$\dot{p}(t) = D\big(p(t)\big) - S(p(t-h)) \tag{1}$$

where p is the price function, D is the demand function, S is the supply function: and $h > 0$ is a time delay. In this model, the supply is assumed to depend on the price in the past. It concurs with the common behaviour of the market.

In this paper, a modified model of the one proposed by Kobayashi is discussed. Our model of market equilibrium also involves the demand function that depends on the price in the past and parameters as the weight of each demand and supply function. The parameters are some functions of time. The model is presented by the following equation.

$$\dot{p}(t) = \alpha(t)D\big(p(t)\big) - \beta(t)S\big(p(t-h)\big) + \gamma(t)D(p(t-h)), \tag{2}$$

where the weight functions α, β, γ are continuous, and h is the delay time. The system in general is not linear. The well-posedness of this type of delay system is quite standard and it has been a subject of discussion in many literatures, for example, Gopalsamy [14], Liu, and Laine [15] and Mitsui and Hu [16].

2.2 Data Acquisition and Parameter Estimation

In this model, commodities require time to be acquired, produced, or distributed before customers can buy them. This phenomenon leads to a delay in the supply function. However, price changes in the market at the current time can also be influenced by past demand. For example, when a product gets good reviews on social media last month and becomes famous, the demand for the product last month can affect the price of the product today by the increase of people's interest in the product.

We use the data on price, consumption, and production of chicken meat, chicken egg, and palm oil per month in Indonesia during 2018-2024 from the Central Bureau of Statistics (BPS) to validate this model and compare the results to those obtained using Kobayashi's model. We chose those commodities for their relevance to the daily life of most Indonesians, and their market (demand-supply) situation. Relevance means that their fluctuation affects many Indonesian families from all strata. Those chosen commodities are not as highly regulated by the Indonesian government as, for example, rice, even though the price of palm oil once fluctuated wildly during early 2022, which prompted the government to impose tighter regulation temporarily [17]. Therefore, their market, largely, is still governed by the supply and demand mechanism.

To validate the model, we do the following process. For each commodity, we have monthly data with three columns: Production (Supply), Consumption (Demand), and Price. Suppose the number of rows in the data is N. We select a delay ($h = 1$) such that the model is obtained as follows:

$$p'(t) = \alpha(t)D\big(p(t)\big) - \beta(t)S\big(p(t-1)\big) + \gamma(t)D\big(p(t-1)\big)$$

In this model, we use one-month delay to represent the effect of one-month lag on the price to the current supply and demand.

For statistical methods used in this process we refer to [18], [19], [20]. For recent other parameter estimation see [21]. First, regression is performed to determine the price function p, and then p' can be calculated. To determine the parameters (α, β, γ) at $t = 2$ (the second month), we substitute the model for $t = 1, 2, 3, 4$ (the first, second, third, and fourth months of the data). Thus, we obtain a system of linear equations as follows:

$$p'(2) = \alpha D\big(p(2)\big) - \beta S\big(p(1)\big) + \gamma D\big(p(1)\big)$$
$$p'(3) = \alpha D\big(p(3)\big) - \beta S\big(p(2)\big) + \gamma D\big(p(2)\big)$$
$$p'(4) = \alpha D\big(p(4)\big) - \beta S\big(p(3)\big) + \gamma D\big(p(3)\big)$$

Next, we substitute the values of p' from the regression results, as well as the Supply and Demand values from the data for the corresponding t. Consequently,

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} D\big(p(2)\big) & -S\big(p(1)\big) & D\big(p(1)\big) \\ D\big(p(3)\big) & -S\big(p(2)\big) & D\big(p(2)\big) \\ D\big(p(4)\big) & -S\big(p(3)\big) & D\big(p(3)\big) \end{bmatrix}^{-1} \begin{bmatrix} p'(2) \\ p'(3) \\ p'(4) \end{bmatrix}$$

The process of determining the parameters continues for $t = 3$, using data from the second, third, fourth, and fifth months, and so on, until four values for each parameter are obtained. Then, regression is performed on these four values to obtain the functions $\alpha(t), \beta(t), and\ \gamma(t)$.

Before price estimation, regression is first performed on the consumption and price data over 7 months (consistent with the data used to determine the parameters). Using the price function obtained through regression, its derivative is calculated so that all the functions involved in **Equation (2)** are complete. The predicted price for the eighth month is obtained by solving the equation. This process continues until all the data is utilized.

The error determination is performed using relative error as follows:

$$\varepsilon_i = \frac{|p_i - \hat{p}_i|}{p_i}$$

where p_i is the actual price in month i, and $\hat{p}_i$ is the predicted price in month i.

For the other model, we use three-month delay. The process of obtaining parameter estimations is similar to the one-month delay model.

3. RESULTS AND DISCUSSION

We choose one-month delay and three-month delay. These time delays concur with some facts, such as: the data are available monthly, the economic report usually is made quarterly. When the market is fluctuating, a short time delay is employed to describe the phenomenon more precisely.

For each time delay (one-month and three-month), we performed statistical analysis of the difference between prediction values versus corresponding real data, that is the errors.

3.1 The First Commodity

The first commodity of concern is chicken eggs. The scatter plots below describe the error in price of chicken egg per month in case of one-month delay (M is month; D1(M) is the error in price of chicken egg at month M using one-month delay; D3(M) is the error in price of chicken egg at month M using three-month delay).

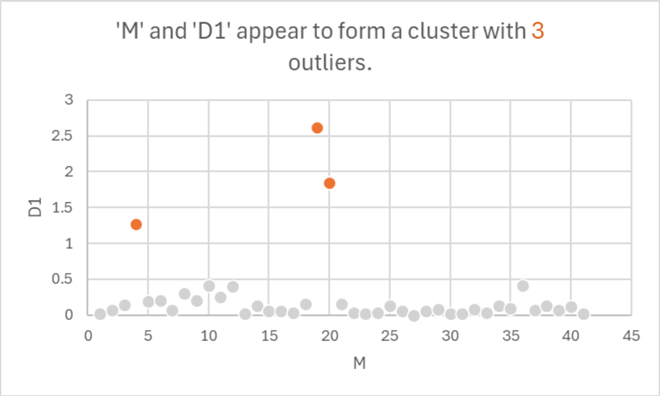


Figure 1. **Scatter Plot of Errors in Price of Chicken Egg for One-Month Delay**

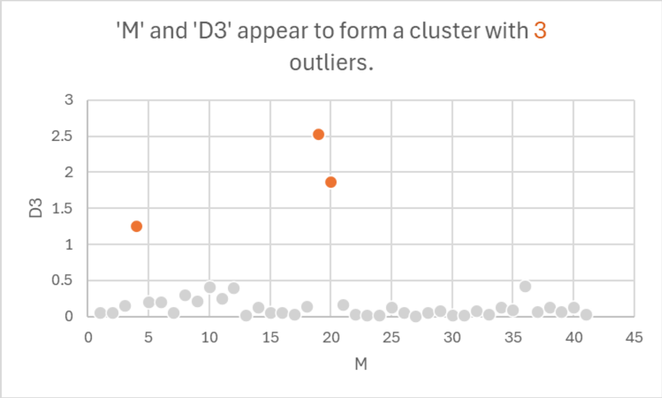


Figure 2. **Scatter Plot of Errors in Price of Chicken Egg for Three-Month Delay**

The statistical analysis is performed using Excel statistical functions. The dynamics of the error are shown in **Figure 1** (one-month delay) and **Figure 2** (three-month delay). The function identifies the 3 outliers. Those occurred in November 2020, February, and March 2022. The outliers and the fluctuation may be attributed to the abnormal fluctuation of the COVID-19 pandemic period (January 2020 to May 2023). News outlets such as Liputan 6 **[22]** on December 1, 2022, confirmed that fluctuation and the presence of extreme outliers.

Upon removing those outliers, the forecast as in the following **Figure 3** (one-month delay) and **Figure 4** (three-month delay) is obtained, with their lower and upper confidence bounds. The descriptive statistics are the following, which show that a three-month delay is slightly better in the error spread compared to one-month delay, by a very small margin.

Table 1. **Statistic Descriptions of Errors in Price of Chicken Egg**

	Delay 1 Month	Delay Three-Month
Mean	0.1185	0.1184
Max	0.4187	0.4173
Min	0.0012	0.0014
STDEV	0.1124	0.1113

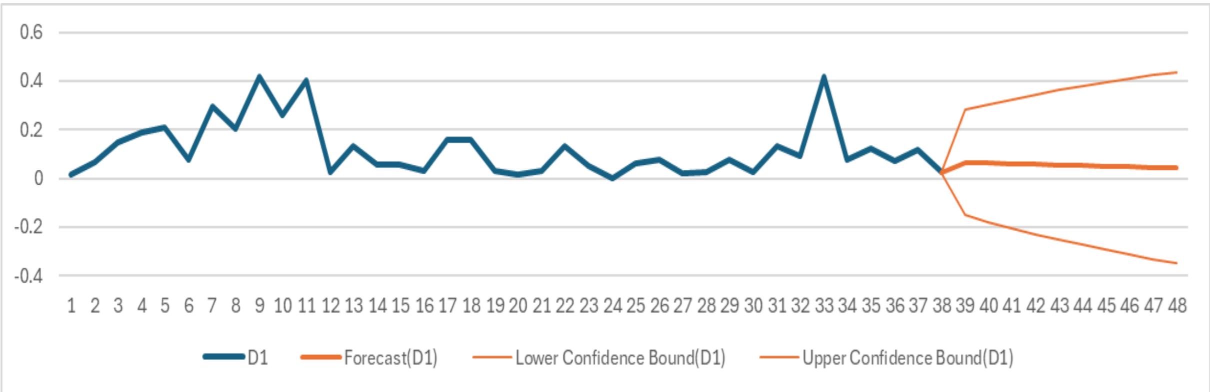


Figure 3. **Price Forecast for 10 Months for Chicken Egg with One-Month Delay**

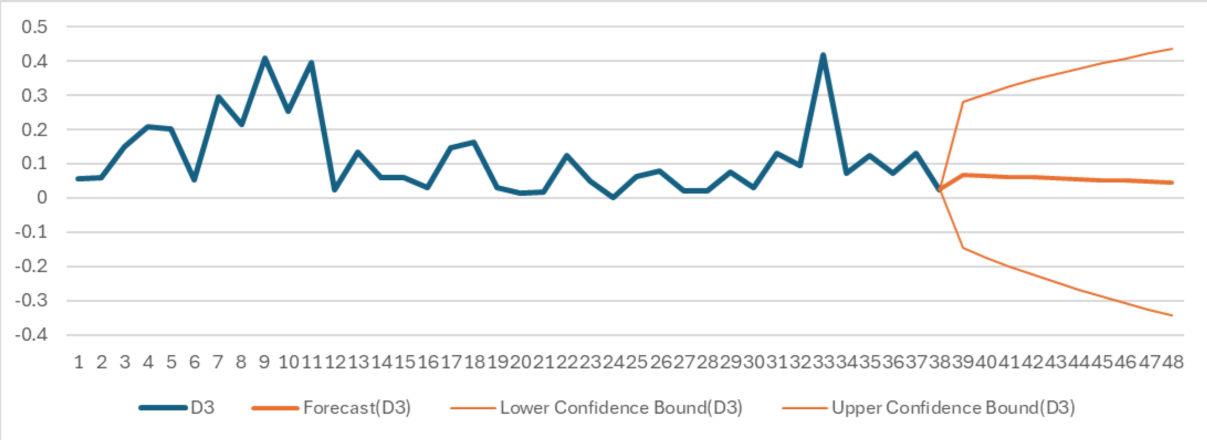
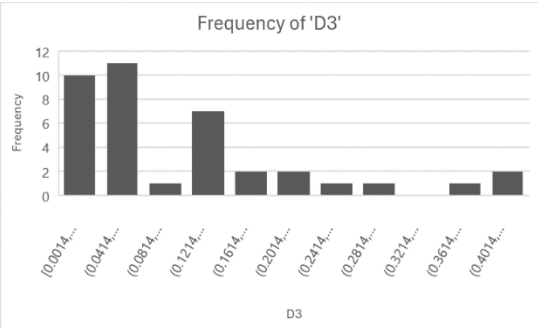


Figure 4. Price Forecast for 10 Months for Chicken Egg with Three-Month Delay



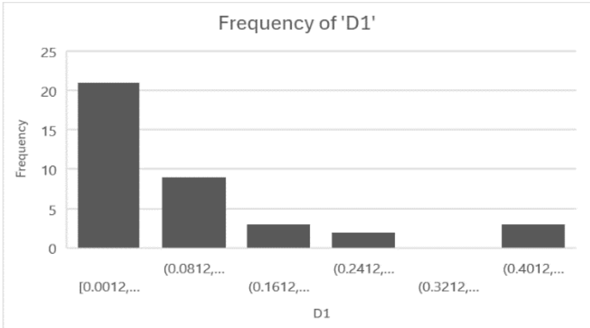


Figure 5. The Frequency of Errors in Price of Chicken Egg for One-Month Delay

Figure 6. The Frequency of Errors in Price of Chicken Egg for Three-Month Delay

Figure 3 and **Figure 4** also show the error forecast for 10 months (for the respective delays). Observing the frequencies of error fluctuations (**Figure 5** and **Figure 6**), the large fluctuation is rare, both in the case of one-month delay and three-month delay, even though in the case of three-month delay, the errors take more values than in the case of one-month delay. The means for 10 months forecasts are 0.0538 (one-month delay), and 0.0565 (three-month delay). Those numbers show that the mean of error estimate with one-month delay is better than the one with three-month delay.

3.2 The Second Commodity

For the second commodity, which is chicken meat, similar as in Section 3.1. The scatter plots below describe the error in price of chicken meat per month in case of one-month delay (M is month; D1(M) is the error in price of chicken meat at month M using one-month delay; D3(M) is the error in price of chicken meat at month M using three-month delay).

Table 2. Statistic Descriptions of Errors in Price of Chicken Meat

	Delay 1 Month	Delay Three-Month
Mean	0.1275	0.1328
Max	0.5294	0.58
Min	0.0011	0.0015
STDEV	0.1147	0.1325

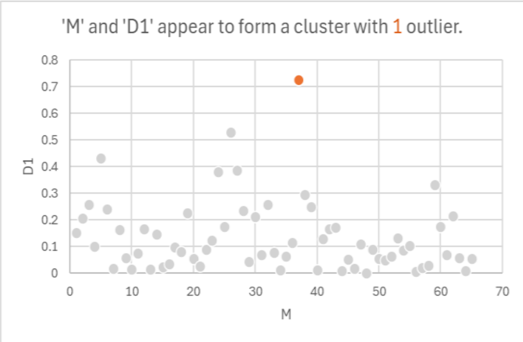


Figure 7. Scatter Plot of Errors in Price of Chicken Meat for One-Month Delay

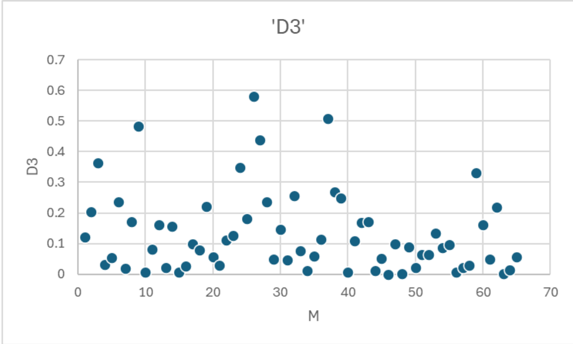


Figure 8. Scatter Plot of Errors in Price of Chicken Meat for Three-Month Delay

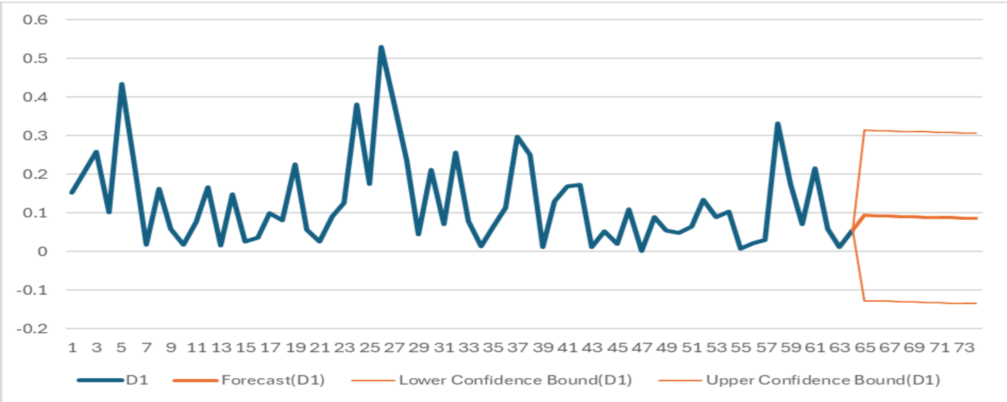


Figure 9. Price Forecast for 10 Months for Chicken Meat with One-Month Delay

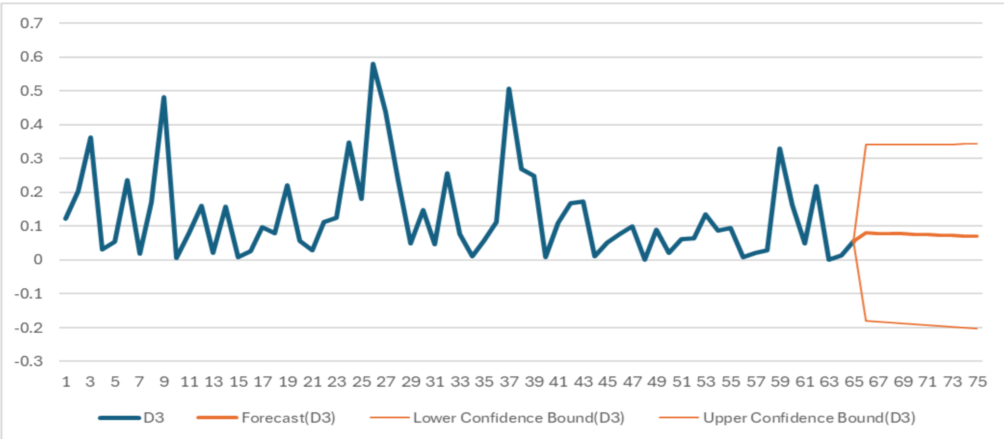


Figure 10. Price Forecast for 10 Months for Chicken Meat with Three-Month Delay

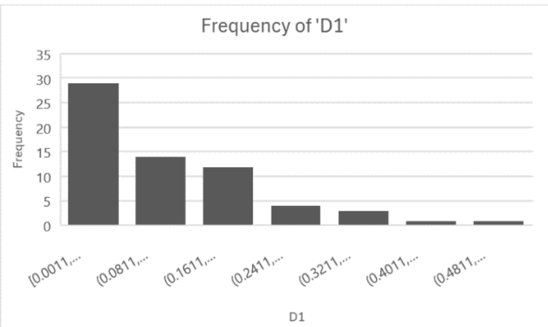


Figure 11. The Frequency of Errors in Price of Chicken Meat for One-Month Delay

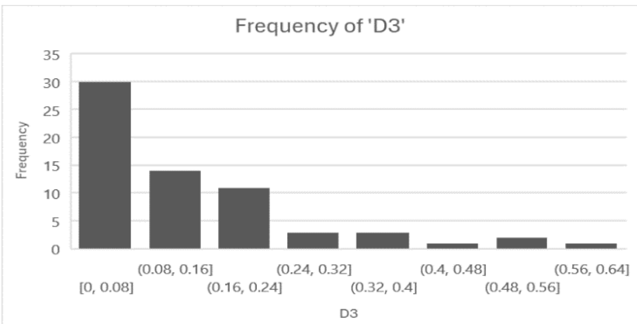


Figure 12. The Frequency of Errors in Price of Chicken Meat for Three-Month Delay

The data shows that chicken meat fluctuates highly, even past the pandemic period in 2020 -2022. Nevertheless, observing the frequencies in **Figure 11** and **Figure 12**, small fluctuations, both in one-month delay and three-month delay, actually dominate the number of oscillations. Outlier only appears in one-month delay case, and none in three-month delay case. In the case of three-month delay, the errors take more values than in the case of one-month delay.

Figure 9 and **Figure 10** also show the error forecast for 10 months (for the respective delays). The means for 10 months forecasts are 0.0894 (one-month delay), and 0.0754 (three-month delay). Those numbers show that the mean error estimates with three-month delay is better than the one with one-month delay. It seems a little bit surprising in the light of the statistical descriptions in **Table 2**, but if we look at the trend in **Figure 9** and **Figure 10**, it is quite reasonable.

3.3 The Third Commodity

The third commodity discussed is palm oil. Compared to the other two commodities, palm oil is actually the most regulated, so the dynamic is not only determined by the market mechanism but also by some external interference, which we do not include in the model actually. The scatter plots below describe the error in price of palm oil per month in case of one-month delay (M is month; D1(M) is the error in price of palm oil at month M using one-month delay; D3(M) is the error in price of palm oil at month M using three-month delay).

Table 3. Statistic Descriptions of the Errors in Price of Palm Oil

	Delay 1 Month	Delay Three-Month
Mean	0.1399	0.1536
Max	0.7635	0.7157
Min	0.0034	0.0032
STDEV	0.141	0.1606

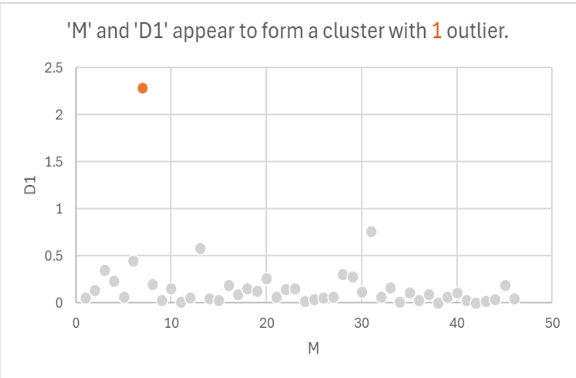


Figure 13. Scatter Plot of Errors in Price of Palm Oil for One-Month Delay

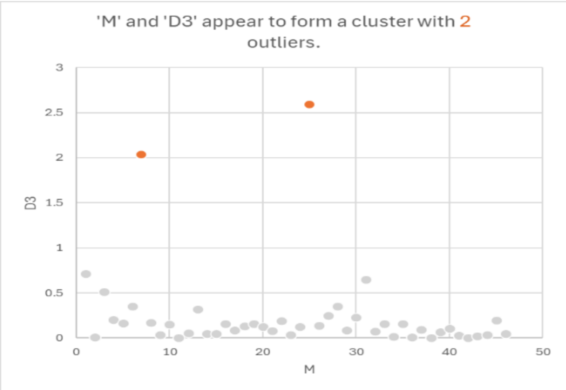


Figure 14. Scatter Plot of Errors in Price of Palm Oil for Three-Month Delay

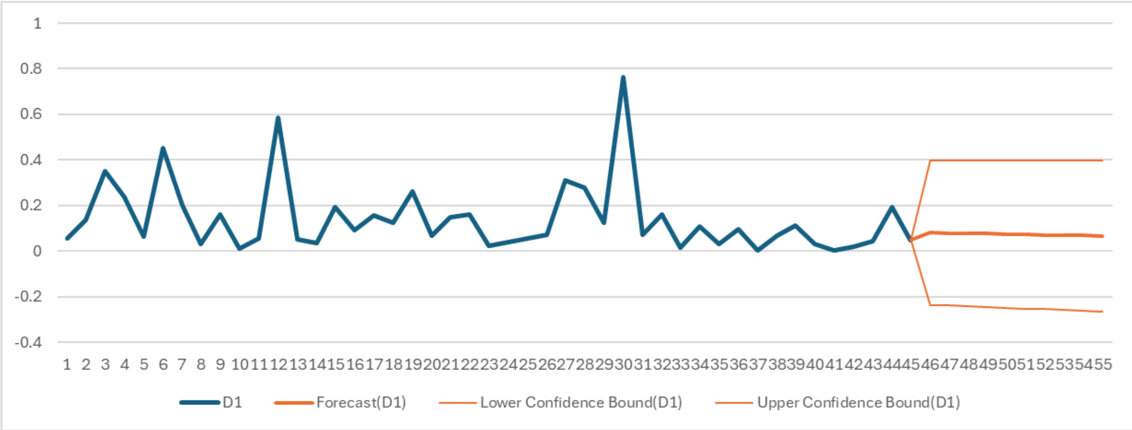


Figure 15. **Price Forecast for 10 Months for Palm Oil with One-Month Delay**

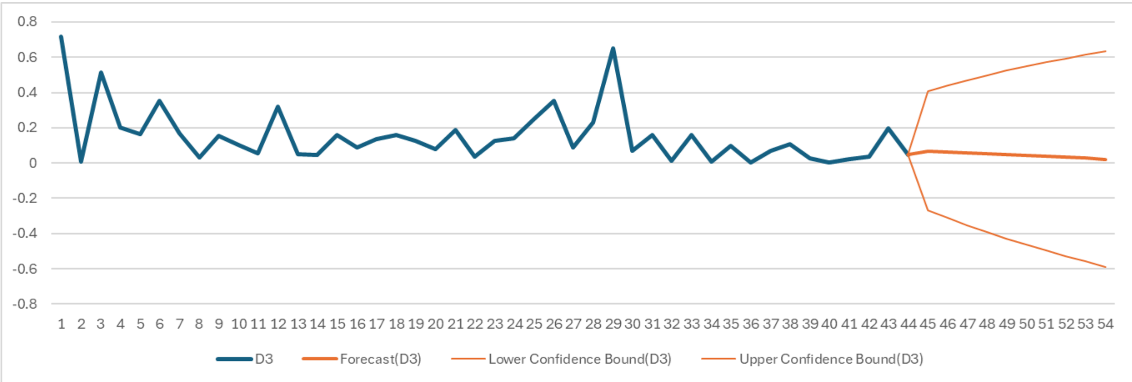


Figure 16. **Price Forecast for 10 Months for Palm Oil with Three-Month Delay**

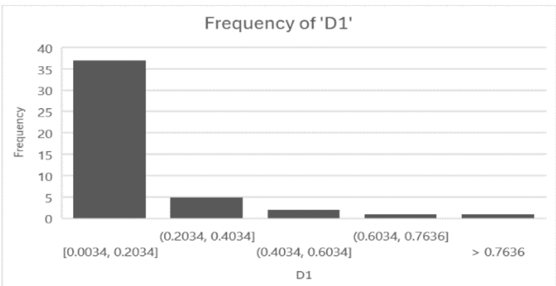


Figure 17. **The Frequency of Errors in Price of Palm Oil for One-Month Delay**

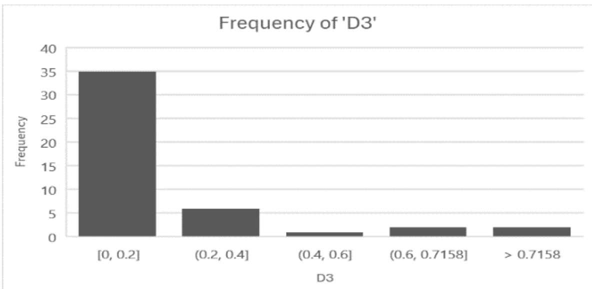


Figure 18. **The Frequency of Errors in Price of Palm Oil for Three-Month Delay**

The scatter plots in **Figure 13** and **Figure 14**, and the error distribution in **Figure 17** and **Figure 18**, show that the error of price estimates in this commodity is not as highly fluctuated as in the second one. It also shows a similar phenomenon in the error forecast for 10 months (for the respective delays), see **Figure 15** and **Figure 16**, as in the second commodity. The means for 10 months forecasts are 0.0734 (1 month delay), and 0.0455 (three-month delay). Those numbers show that the mean error estimates with three-month delay is better than the one with one-month delay.

3.4 The Estimations using Kobayashi's Model

For the second and third commodity we have the error of price estimate using Kobayashi's model as in the table below.

Table 4. Statistic Descriptions of Error using Kobayashi's Model for Chicken Meat

	Delay 1 Month	Delay Three-Month
Mean	0.3291	0.3206
Max	1.6834	1.7374
Min	0.0099	0.0149
STDEV	0.3277	0.3198

The means for 10 months forecasts for the error of price estimates of the chicken meat are 0.1061 (1 month delay) and 0.1656 (three-month delay).

Table 5. Statistic Descriptions of Error using Kobayashi's Model for Palm Oil

	Delay 1 Month	Delay Three-month
Mean	0.8743	0.7824
Max	4.8417	3.6265
Min	0.0412	0.0542
STDEV	1.0881	0.9129

The means for 10 months forecasts for the error of price estimates of the palm oil are 0.6280 (1 month delay), and 0.4935 (three-month delay).

Table 6. Statistic Descriptions of Error using Kobayashi's Model for Chicken Egg

	Delay 1 month	Delay three-month
Mean	0.8778	0.8956
Max	17.6806	17.6732
Min	0.0057	0.0059
STDEV	2.8927	2.9146

The means for 10 months forecast for the error of price estimates of the chicken egg are 0.8778(1 month delay), and 0.8956 (three-month delay).

4. CONCLUSIONS

In this paper we proposed a modified Kobayashi model as in Section 2.2. It is quite effective to estimate the price of some basic commodities, even though they vary in nature and of different levels of regulation, whether heavily or lightly regulated. The mean of the error is around 11%-14%, while the mean of the forecast for the next ten months is around 5%-8%. Compared to the estimates using Kobayashi's model, this model produces much better estimates.

Our implementation so far has yet to be able to resolve an optimal delay time, partly due to limited availability of data, that is the length of time where data is available is not sufficiently long to attempt a longer time delay. Nevertheless, the implementation of the method shows the characteristics of delay equations; longer delay time may include more dynamics, and more fluctuation, although that means it is more prone to instabilities. To improve the predicting capability of our model, a better estimation of parameters should be employed. An attempt to get a better estimation is done by reducing data interdependence. The result is quite promising. We will follow this direction for our further studies, as well as other modification on the delay differential model. Monte Carlo-based estimation of the parameters, or some combination with time-series analysis such as [23] is also something that in our plan.

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