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PERFORMANCE LOSS QUANTIFICATION IN KERNEL DENSITY ESTIMATION FOR ACTUARIAL AND FINANCIAL ANALYSIS

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ABSTRACT

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Accurately estimating aggregate loss distributions is critical in actuarial and financial risk assessment, as it underpins effective risk analysis and the development of mitigation strategies. However, incorrect parametric assumptions can lead to biased risk estimates and underestimated losses. Non-parametric methods, such as Kernel Density Estimation (KDE), offer a flexible alternative by generating smooth empirical probability density functions (PDFs) directly from sample data without assuming a specific distributional form. This study examines the impact of dependence structures on risk measures by applying KDE with a Gaussian kernel to estimate aggregate loss distributions. To quantify the effects of ignoring dependence, we introduce the concept of performance loss, focusing on variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). The results show that performance loss increases with the correlation coefficient, indicating that higher dependency leads to greater underestimation of risk. Additionally, higher confidence levels amplify performance loss for VaR and TVaR, underscoring the sensitivity of these measures to tail behavior. These findings highlight the importance of incorporating dependence structures in risk modeling to avoid misleading evaluations. The implications are particularly relevant for disaster risk management in Central Asia, where overlooking interdependencies in seismic losses could result in inadequate financial and actuarial strategies.

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1. INTRODUCTION

Collective risk and dependency analysis are essential for understanding the cumulative impacts of natural disasters, particularly in earthquake-prone regions such as Central Asia [1]. Without considering the interactions between multiple risks in adjacent regions, risk models may fail to capture the potential for amplified losses. Large earthquakes, for example, often cause widespread damage not only at the epicenter but also in surrounding areas, amplifying both economic and social impacts [2], [3]. Accurate estimation of the distribution of aggregate losses from simultaneous earthquakes within a given area is crucial, as it influences the establishment of financing reserves and the development of effective risk reduction strategies [4], [5]. Estimating the distribution of aggregate losses is particularly important for actuarial science and financial risk management [6], where decisions rely on precise calculations of risk measures like Value at Risk (VaR) and Tail Value at Risk (TVaR) [7]. Additionally, the estimation of the tail behavior of loss severity distributions is vital for determining regulatory capital requirements, which are essential for maintaining the financial stability of institutions facing catastrophic events [8], [9]. However, the inherent variability and complexity of loss data present significant challenges to obtaining accurate estimations [10].

To address these challenges, the Collective Risk Model (CRM) is commonly employed to assess aggregate losses [11], [12], [13]. Despite its widespread use, CRM often relies on the assumption of independence between losses, which may not always be realistic in scenarios involving dependent risks [14]. Such dependencies are particularly relevant in seismic risk modeling, where the interactions between risks in adjacent regions can significantly alter the aggregate losses. Understanding and accurately modeling dependent losses are crucial for designing effective mitigation strategies and efficiently allocating resources to reduce the economic and social impacts of natural disasters [15]. These considerations highlight the need to focus on analyzing dependent losses to improve risk assessment and management.

Conventional parametric methods depend on predefined functional forms for loss distributions, which may not accurately capture the characteristics of real-world data. Although modern parametric models offer greater flexibility in handling complex data patterns, they still require strong assumptions about the distributional shape and parameters, which may not always hold in practice [16]. In contrast, non-parametric approaches such as Kernel Density Estimation (KDE) offer greater flexibility by estimating the distribution directly from data, without relying on restrictive assumptions about its underlying form [17], [18], [19], [20]. This is particularly advantageous when dealing with heterogeneous or multimodal loss distributions, where selecting an appropriate parametric model can be challenging. Unlike parametric methods, KDE does not require predefined assumptions about the distribution's functional form. Instead, it estimates the probability density function (PDF) directly from the data, providing a continuous and adaptable representation of the underlying distribution [21], [22], [23]. This adaptability makes KDE especially useful for capturing tail behavior, which is crucial for accurately estimating risk measures such as Value at Risk (VaR) and Tail Value at Risk (TVaR) [24]. Despite its advantages, the effectiveness of KDE depends heavily on the selection of the kernel function and bandwidth. These parameters significantly influence the accuracy of the density estimates and, consequently, the reliability of associated risk measures, including variance, VaR, and TVaR [25]. Suboptimal kernel or bandwidth choices can lead to performance losses, reducing the utility of KDE in practical risk management applications. Therefore, optimizing these parameters is essential for enhancing the accuracy of KDE-based risk assessments.

In this study, we investigate the optimization of KDE for estimating aggregate loss distributions, focusing on seismic risks in Central Asia. This region is highly seismically active due to major fault lines, such as the Ionakhsh Fault, which pose substantial risks to infrastructure and communities in countries including Kyrgyzstan, Tajikistan, and Uzbekistan [26], [27], [28]. Given the financial and economic losses associated with these earthquakes, accurate risk assessment models are essential. Traditional parametric models may not fully capture the dependency structures in seismic losses, making KDE a valuable alternative for improving aggregate loss estimations.

Specifically, we examine the effects of kernel function selection and bandwidth optimization on the precision of loss estimates and key risk measures such as VaR and TVaR. By improving the accuracy of these estimates, this research aims to enhance the reliability of disaster risk assessments, thereby providing valuable insights for actuarial and financial risk management. The findings underscore the importance of careful kernel and bandwidth selection in mitigating performance loss and advancing risk management strategies in the region.

Beyond the technical contributions, this study has broader implications for disaster risk management in Central Asia, particularly for organizations involved in regional disaster preparedness. By delivering more accurate risk estimates that account for the interdependencies between regional risks, our findings contribute to optimizing risk transfer systems, improving mitigation strategies, and enhancing disaster preparedness. These advancements aim to strengthen the resilience of communities and institutions in Central Asia against future seismic events.

Central Asia is a region highly susceptible to severe natural disasters, many of which are cross-border in nature **[29]**. Geographically situated between the Eurasian and Indian tectonic plates, the region experiences significant seismic activity due to the high levels of tectonic stress. The accumulated tectonic stress raises the probability of large earthquakes, leading to widespread destruction and significant economic losses **[30]**. One of the most prominent seismic threats in the region is posed by the Ionakhsh Fault, which spans Kyrgyzstan, Tajikistan, and eastern Uzbekistan. This fault represents a major risk for triggering simultaneous seismic disasters, potentially resulting in collective losses across these three nations **[31]**.

The seismic hazards in Central Asia are characterized by high peak ground accelerations, with a 10% probability of exceedance in 50 years reaching values as high as 9 m/s^2 **[32]**, **[33]**. Probabilistic seismic hazard assessments, which account for shallow seismicity and leverage updated earthquake catalogs, have been conducted to produce hazard maps expressed in terms of macroseismic intensity **[33]**. These assessments are crucial for disaster risk reduction, guiding loss estimation and risk management strategies. Key measures include the development of emergency response plans, infrastructure strengthening, and targeted risk reduction policies **[34]**. Furthermore, the establishment of a comprehensive fault database and interactive maps has enhanced access to critical seismic information, enabling researchers to identify data gaps and prioritize future studies **[35]**.

Despite these advancements, seismic risk management in Central Asia faces significant challenges, particularly in achieving accurate hazard evaluations. Reliable assessments rely heavily on the availability of local seismic data and direct observations, which are often limited. Collaborative efforts involving local and international scientific communities are crucial to harmonizing data collection, improving risk models, and addressing these challenges **[36]**. Future research should emphasize enhancing seismic data quality and availability, as well as developing detailed and regionally consistent risk assessments. These efforts are essential for effective policymaking and disaster resilience initiatives **[34]**, **[36]**.

The geological and seismic risks in Central Asia are further compounded by socio-economic factors that increase the region’s vulnerability. Rapid urbanization and population growth, particularly in Uzbekistan, the most populous country in the region, have heightened exposure to seismic hazards. Many buildings, especially older and self-constructed structures, are highly susceptible to seismic damage. This highlights the urgent need for measures to retrofit and strengthen such buildings to ensure safety and resilience **[37]**. Critical infrastructure, including schools, hospitals, and transportation systems, has been identified as particularly vulnerable. Economic losses from potential earthquakes have been estimated to be substantial, in some cases representing a significant percentage of the Gross Domestic Product (GDP) **[38]**. These compounded risks necessitate a combination of technical, structural, and policy-based interventions to build resilience and safeguard the region’s development.

Given these vulnerabilities, financial strategies like insurance cooperation play a key role in strengthening economic resilience. Effective disaster risk management (DRM) is essential for minimizing the devastating effects of earthquakes, protecting infrastructure, and reducing economic damage. Insurance cooperation has emerged as a key component of DRM, strengthening the region's financial resilience against natural calamities. Regional risk transfer facilities have gained attention internationally and within Central Asia as a mechanism to improve disaster preparedness and response **[39]**. Central Asian countries, many of which were part of the Soviet Union, have initiated collaborative efforts to address cross-border hazards, including seismic risks. The Central Asia Regional Economic Cooperation (CAREC) program, originally focused on economic development, has increasingly prioritized disaster risk management. CAREC’s initiatives now encompass the monitoring, planning, and prevention of natural disasters. Its proactive approach aims to anticipate risks and mitigate hazards before disasters occur, shifting focus from reactive responses to preventative measures.

2. RESEARCH METHODS

In this study, we present the Research Design, Data Sources, Data Collection Techniques, and Data Analysis. Additionally, we discuss the application of Kernel Density Estimation (KDE) as a method for estimating aggregate loss distributions.

2.1 Kernel Density Estimation (KDE)

Kernel Density Estimation (KDE) is a nonparametric method for estimating the probability density function (PDF) of a continuous random variable without assuming any specific distribution. This study employs KDE to estimate the probability density function of an aggregate seismic loss simulation. The method provides flexibility and adaptability in capturing the characteristics of the data, making it a robust alternative to traditional parametric approaches. The formal definition of the KDE function is given by [16]

$$\hat{p}_n(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{X_i - x}{h}\right), \tag{1}$$

where $K(x)$ is the kernel function, a generally smooth and symmetric function, and $h > 0$ is the smoothing bandwidth, which controls the amount of smoothing applied to the data. The kernel function used in this study is the Gaussian kernel, expressed as:

$$K(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \tag{2}$$

The choice of the kernel function, while significant, is often less critical to the accuracy of KDE estimators compared to the selection of an appropriate bandwidth [40]. A commonly applied guideline for determining the optimal bandwidth is Scott's rule, defined as [41], [42]:

$$b_{opt} = 1.06 \times \sigma \times n^{-0.2} \tag{3}$$

where σ is the population standard deviation, typically estimated by the sample standard deviation, and n is the number of samples. If the sample standard deviation is zero or the sample size is less than two, a fixed bandwidth is applied to ensure stability in estimation.

KDE is widely used in probability density estimation, particularly for smoothing empirical data and modeling heavy-tailed behaviors in financial and insurance risk assessment [43]. To address asymmetry and the presence of heavy tails, log transformations have been applied, significantly enhancing KDE's performance [44]. Metrics such as Integrated Squared Error (ISE), Weighted Integrated Squared Error (WISE), and Cross-validation are frequently used to evaluate the quality of density estimates and optimize the selection of bandwidth and kernel parameters. Moreover, approaches like the shifted power transformation have demonstrated improved accuracy in modeling outcomes, particularly for skewed or heavy-tailed data.

Recent advancements have further extended the applications of KDE beyond traditional fields. For instance, in wind power generation prediction, a diffusion-based kernel density estimator (DiE) with optimal bandwidth selection has been developed to address challenges in non-stationary time series data. This method has demonstrated superior performance in generating reliable prediction intervals (PIs) compared to traditional Gaussian-based models and classical KDE approaches with fixed bandwidth selection [45]. Similarly, innovations in variable kernel density estimation (VKDE) have introduced new bandwidth selection rules based on adaptive convolutions and local function variation, enabling directional stretching of kernels. These advancements have improved the accuracy of density estimates and outperformed standard fixed and variable bandwidth methods [46]. In this study, KDE is applied to seismic risk modeling to estimate the probability density function of aggregate loss distributions. Its flexibility and robustness make it highly suitable for capturing the variability and dependencies inherent in seismic loss data.

2.2 Research Design

This study employs a Monte Carlo simulation framework combined with Kernel Density Estimation (KDE) to estimate the probability density function (PDF) of aggregate seismic losses in Central Asia. The simulation captures spatial dependencies in seismic losses across regions affected by the Ionakhsh Fault—specifically Kyrgyzstan, Tajikistan, and Uzbekistan—by incorporating a multivariate dependency structure through varying correlation coefficients. This design allows for a detailed evaluation of how interregional dependencies influence key risk measures, including variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). By integrating probabilistic modeling with KDE, the study offers a flexible and robust approach to estimating seismic financial risk and quantifying cross-regional dependencies in a non-parametric framework.

2.3 Data Sources

This study simulates aggregate seismic losses across affected regions by modeling the dependency structure using a bivariate normal distribution. Specifically, we generate two loss components, X_1 and X_2, from a bivariate normal distribution with mean vector $\mu = [0,0]$ and covariance matrix $\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, where the correlation coefficient ρ ranges from 0 to 1. The total aggregate loss is defined as $S = X_1 + X_2$. To assess the effect of dependence, we compare this dependent structure with an independent scenario by randomly permuting one of the loss components prior to aggregation.

The simulation uses a sample size of 10^6 to ensure statistical robustness in estimating risk measures such as variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). While empirical seismic loss data often exhibit heavy-tailed behavior (e.g., lognormal or Pareto distributions) **[47]**, we adopt a normality assumption for computational efficiency—an approach commonly used in probabilistic seismic risk studies **[48]**, **[49]**. To enhance flexibility and better capture potential tail risk, we apply Kernel Density Estimation (KDE) to empirically estimate the loss distribution.

2.4 Data Collection Techniques

The study employs Python to generate simulated data for aggregate seismic losses across the affected regions. This data uses a multivariate normal distribution with a correlation coefficient to model the dependent risks between the areas. The simulation process reflects how dependencies in seismic losses across different regions affect overall risk assessments. The Python libraries, such as NumPy and SciPy, are utilized to perform the necessary computations, and the Gaussian kernel function is applied to estimate the probability density function of the seismic losses.

2.5 Data Analysis

The analysis is carried out using Python, leveraging numerical and statistical libraries such as NumPy and SciPy to evaluate the impact of correlation on seismic loss estimates. Key risk measures—variance, Value at Risk (VaR), and Tail Value at Risk (TVaR)—are computed for both dependent and independent loss scenarios to quantify the effect of ignoring dependence.

The data analysis consists of the following key steps:

1. Simulating Seismic Losses: Generate aggregate loss data from a bivariate normal distribution with varying correlation coefficients. The losses S_{dep} and S_{ind} are derived based on individual losses X_1 and, X_2, representing seismic losses in Kyrgyzstan and Tajikistan.
2. Applying Kernel Density Estimation (KDE): Estimate the empirical loss distribution using KDE with optimal bandwidth selection using Scott's rule of thumb.
3. Computing Risk Measures: Calculate variance, VaR, and TVaR for both dependent and independent loss scenarios, where VaR and TVaR are determined at different confidence levels $(p = 0.9, 0.95, 0.99)$.

4. Quantifying Performance Loss: Use **Equation (4)** to measure the impact of correlation on risk measures, comparing results between dependent and independent scenarios.
5. Comparing Results: Analyze the influence of correlation structures on risk measures and evaluate the KDE-estimated distribution.

3. RESULTS AND DISCUSSION

In calculating aggregate losses, the assumption of independence between individual losses is commonly employed to simplify calculations, especially in complex risk portfolios. However, this assumption can be problematic, as real-world events often exhibit dependencies. Research on competing risks has shown that assuming independence can lead to inaccurate risk estimates when dependencies between events exist **[50]**, **[51]**. To address this issue, we introduce a performance loss metric that quantifies the discrepancy between risk estimates obtained under the assumption of independence and those derived from models that incorporate dependency structures **[52]**. In this study, two types of dependency models are considered: multivariate normal and normal copula. The latter offers greater flexibility by allowing non-normal marginal distributions while capturing the dependence structure through a copula function (see, for example, **[53]** for a detailed discussion on the normal copula).

This study evaluates the effectiveness of Kernel Density Estimation (KDE) in estimating dependent variables. Specifically, we generated two aggregate losses, S, from a Normal Multivariate distribution with a correlation coefficient $\rho = 0.7$, selected based on observed dependency patterns in seismic losses in Kyrgyzstan and Tajikistan (see Subsection 2.1). Prior studies on regional seismic losses indicate that earthquake-induced damages in neighboring regions tend to exhibit moderate to high correlation due to shared fault structures and geophysical conditions. These losses were derived using different individual losses X_1 and X_2 which represent the seismic losses of Kyrgyzstan and Tajikistan, respectively. The variable S_{dep} represents the aggregate loss accounting for dependent individual risks, while S_{ind} represents the aggregate loss assuming independent individual risks. The performance loss is defined as:

$$Performance\ Loss\ =\ Risk\ Measure\left(S_{dep}\right) - Risk\ Measure(S_{ind}). \tag{4}$$

where the risk measures considered include variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). This approach quantifies how dependency structures affect risk estimation. Using **Equation (4)**, we examine the impact of the correlation coefficient ρ on the performance loss for key risk measures, including variance, Value at Risk (VaR), and Tail Value at Risk (TVaR). To illustrate this numerically, **Figure 1** depicts the performance loss for these three risk measures, where the aggregate loss follows a Normal Multivariate distribution with $\rho = 0.7$, and a confidence level $p = 0.95$.

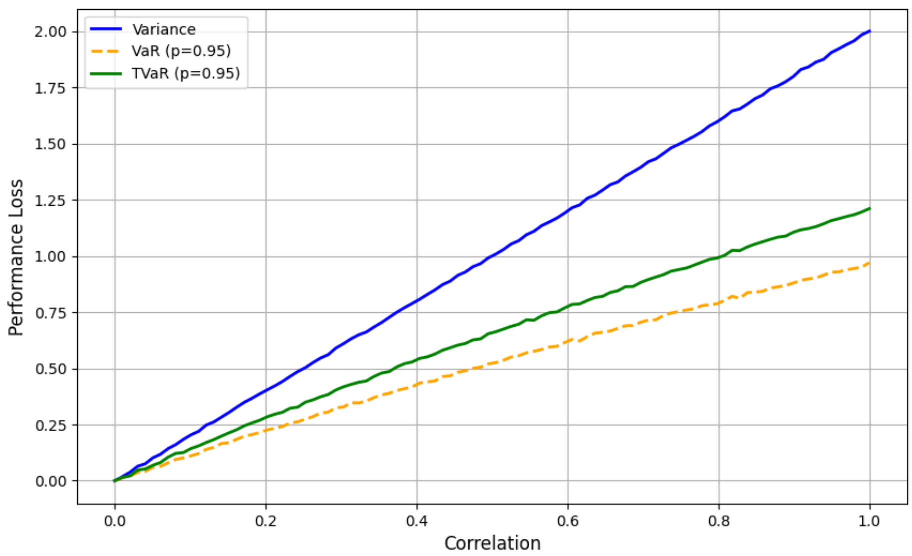
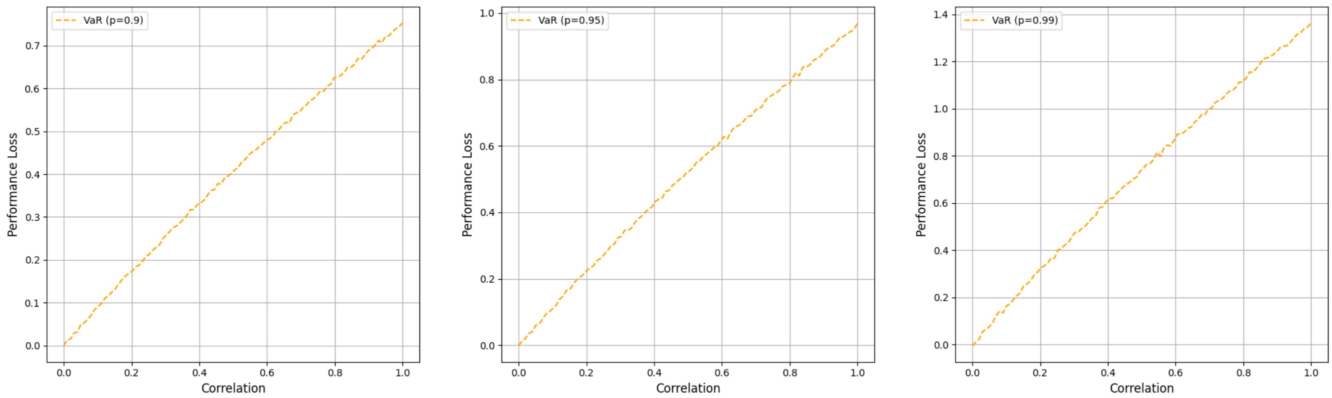


Figure 1. Performance Loss of KDE Approach

From **Figure 1**, it is evident that the correlation coefficient (ρ) significantly affects performance loss. The analysis reveals the following observations:

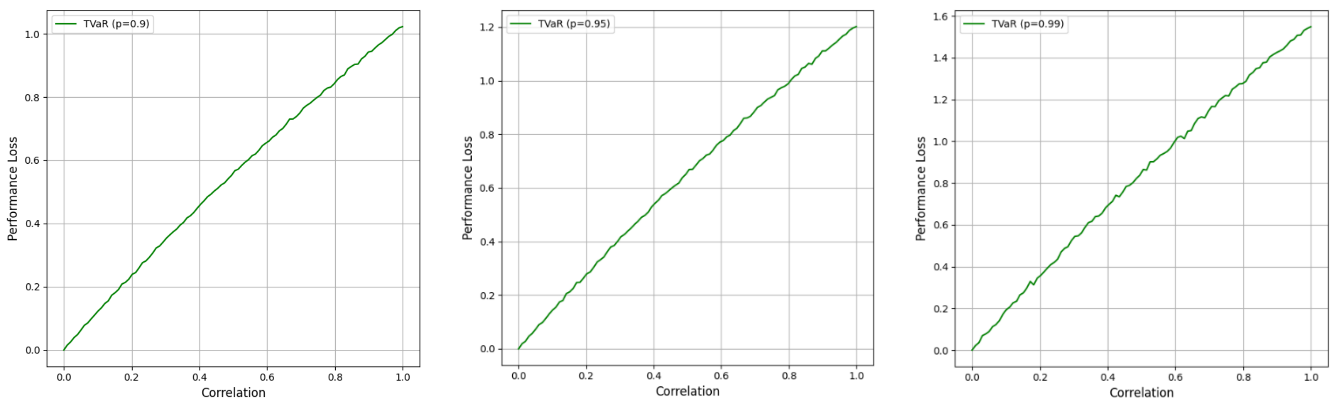
1. Increase in Correlation Coefficient: An increase in (ρ) leads to a rise in performance loss, reflecting heightened vulnerability in the portfolio.
2. Decrease in Correlation Coefficient: Conversely, a decrease in (ρ) results in a lower performance loss, indicating a reduction in overall risk. This suggests that lower correlations among risks enhance diversification and reduce the likelihood of substantial losses.

To further investigate, we analyzed the influence of the confidence level p on the performance loss for both VaR and TVaR. **Figure 2** and **Figure 3** illustrate these relationships, respectively.



(a) **(b)** **(c)**

Figure 2. Performance Loss of Value at Risk (VaR)
(a) Value at Risk (VaR) at a 90% Confidence Level ($p = 0.9$), (b) Value at Risk (VaR) at a 95% Confidence Level ($p = 0.95$), (c) Value at Risk (VaR) at a 99% confidence level ($p = 0.99$)
The Graphs were generated using Python



(a) **(b)** **(c)**

Figure 3. Performance Loss of Tail Value at Risk (TVaR)
(a) Tail Value at Risk (TVaR) at a 90% Confidence Level ($p = 0.9$), (b) Tail Value at Risk (TVaR) at a 95% Confidence Level ($p = 0.95$), (c) Value at Risk (VaR) at a 99% Confidence Level ($p = 0.99$)
The Graphs were generated using Python

As shown in **Figure 2** and **Figure 3**, a clear relationship exists between the performance loss of VaR and TVaR and the correlation coefficient ρ for each value of p. Specifically:

1. Impact of Correlation Coefficient: As the correlation coefficient ρ increases, the performance loss grows consistently for all regional pairs. The analysis confirms that higher correlations between regional losses result in greater performance losses for both VaR and TVaR.

2. Impact of Confidence Level: As the confidence level p increases, the performance loss for both VaR and TVaR also rises. This highlights the sensitivity of these risk measures to changes in p, emphasizing the importance of carefully selecting confidence levels when assessing risk.

These findings underscore the critical importance of accounting for correlations between regional losses in risk assessments and financial planning. Ignoring these dependencies can lead to significant underestimation of risk, while a thorough understanding of the relationships between losses can improve the reliability of risk measures and inform better resource allocation strategies.

4. CONCLUSIONS

This study demonstrates that Kernel Density Estimation (KDE), when properly configured with an appropriate kernel function and bandwidth selection, is an effective non-parametric method for estimating aggregate loss distributions and related risk measures under dependency. Based on simulated data, the results show that dependency structures have a significant impact on risk estimation, as evidenced by the performance loss metric. As the correlation between regional seismic losses increases, the performance loss associated with variance, Value at Risk (VaR), and Tail Value at Risk (TVaR) also increases, indicating heightened financial vulnerability. Numerical simulations confirm that assuming independence in aggregate loss estimation can lead to an underestimation of risk, particularly in high-risk seismic regions such as Kyrgyzstan and Tajikistan. The impact is even more pronounced at higher confidence levels, where tail-based risk measures are especially sensitive to dependency. These findings highlight the critical importance of incorporating dependency structures in risk models to avoid misleading financial and actuarial assessments. The insights from this study are particularly relevant for disaster risk management in Central Asia, where accounting for interregional dependencies is essential for accurate risk quantification and informed decision-making. Future research could further refine KDE techniques and extend their application to more complex, multi-hazard risk scenarios.

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