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COMPARISON ANALYSIS OF CLAYTON, GUMBEL, AND FRANK COPULA FOR MODELING THE DEPENDENCE BETWEEN BBCA CLOSING PRICE AND INDONESIA MACROECONOMIC FACTORS

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ABSTRACT

PT. Bank Central Asia Tbk is a company in Indonesia with the biggest market capitalization. These advantages attract investors to buy PT. Bank Central Asia Tbk (BBCA) shares. However, fluctuating share prices can lead to both gains and losses, where these are not entirely caused by the company's finances, but also by the country's macroeconomic conditions. Therefore, this study aims to examine the dependency between BBCA closing price and macroeconomic indicators, which are limited on only three macroeconomic variables, consists of inflation, interest, and USD-IDR exchange rate. This study compares the Clayton, Gumbel, and Frank copula to analyze the dependence characteristics between two non-normally distributed variables based on the highest log-likelihood value. The data used are monthly data from 2021 to 2023, consists of inflation and interest rate from Bank Indonesia website, USD-IDR exchange rate from Satu Data Kementerian Perdagangan website, alongside BBCA closing price from yahoo finance website. Based on the analysis, the best copula models to describe the relationship between each macroeconomic factor (inflation, interest, exchange rate) and BBCA closing price respectively is Clayton copula with parameter 2.042, Frank copula with parameter 10.3, and Frank copula with parameter 5.891. These findings indicate that inflation shows a strong dependence with BBCA closing price when both variables are low, while exchange rate and interest rate exhibit strong dependence with BBCA closing price when these variables are high. It provides valuable insights into the asymmetric relationships between macroeconomic conditions and stock prices, offering practical relevance for investors and policymakers.

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1. INTRODUCTION

Investment is one of the important activities that affect a country's economy. As a type of investment that is considered to provide higher returns, shares are the most popular investment choice for Indonesian investors compared to other types of investment [1]. This is also marked by the continued enlargement of stock investors number, where the registered single investor identification in 2020 increased by 53.47% from the previous year [2]. Share is a type of financial instrument that can be traded on the capital market over a long period [3]. From the many issuers on Indonesia Stock Exchange, PT. Bank Central Asia is a bank that is always present in the LQ-45 and IDX-30 stock indexes, also become a company with the highest wealth-added index (ability to add value to wealth) and has always been the most active stocks, outperforming the large-cap state-owned banks [4].

Share price tends to fluctuate and change from time to time, making it has the potential to provide large profits, but also the potential for risk of loss even though the shares are large capitalization shares [5]. The condition of share prices that experience increases and decreases is not entirely caused by the company's financial activities, but can also be caused by other external factors that influence internal performance, such as the country's macroeconomic conditions [6]. Changes in macroeconomic factors can cause concern for investors because share prices adjust more quickly to the country's macroeconomic conditions compared to the company's performance [7]. Based on this, analysis is needed to see the relationship between each macroeconomic variable, including interest, inflation, and the USD-IDR exchange rate, on stock prices, which in this case is the BBCA closing price as the stock in Indonesia with the biggest market capitalization [8]. However, data in the financial sector generally has extreme values so it does not follow a normal distribution [9]. On the other hand, correlation analysis, such as Pearson correlation which is commonly used, requires that the data meet the assumption of normality [10]. Thus, analysis that can capture the relationship between macroeconomic factors and BBCA closing price without requiring the fulfillment of certain assumptions, especially the normality assumption, is needed, such as through the copula approach.

Research using the copula method was carried out by [11] to determine the relationship between climate factor and average rice production in Tabanan Regency, but the research did not formulate the copula model and visualize the simulation data. Another research was conducted by [12] to look at the dependence between the prices of Indonesia's leading export commodities. However, the research was only carried out until the model formulation and did not interpret the result. Then, [13] researched the dependency relationship between macroeconomic factors and the Composite Stock Price Index. Although the use of copula methods has been widely applied in some fields, including financial and economics, the novelty of this study lies in its specific focus on BBCA, the largest market capitalization stock in Indonesia, and its asymmetric dependence with key macroeconomic factors, those are inflation, interest, and USD-IDR exchange rate. This research conducts a dependency analysis by comparing the Clayton, Gumbel, and Frank copula as subfamilies from the Archimedean copula with a different tail-dependency structure [14]. This research aims to find the best copula model and visualize the dependency between macroeconomic factors and BBCA closing price, which can be new information for PT. Bank Central Asia and their potential investor.

2. RESEARCH METHODS

2.1 Data Sources

This research applied secondary data which were obtained from Bank Indonesia website for Indonesian inflation rate (X_1) and interest rate data (X_2), the Satu Data Kementerian Perdagangan website for USD-IDR exchange rate data (X_3), and the yahoo finance website for BBCA closing price data (Y). The data used is monthly data for 2021-2023.

2.2 Research Stages

This research is divided by seven main steps as follows:

1. Explore and describe the overview of data, including inflation rate, interest rate, USD-IDR exchange rate, and BBCA closing price.
2. Conduct a normality test using Shapiro-Wilk method
3. Calculate the Kendall's Tau correlation coefficient for each pair of variables

4. Transform data into [0,1] form using Van Der Waerden transformation
5. Estimate the copula parameter using Kendall's Tau approach
6. Determine the best copula based on the highest log-likelihood value
7. Generate 5000 new data pairs, those are X_1 and Y, X_2 and Y, also X_3 and Y according to the best copula model, then visualize and interpret the dependence of BBCA closing price on each macroeconomic factor.

Data transformation is carried out after the Kendall's Tau calculation because the Kendall's Tau does not require the data to be in a certain interval of distribution. Then, there is a limitation that at least one variable of this research is not normally distributed. This requirement aims to demonstrate the advantage of copula application in analyzing the dependency of non-normally distributed data.

2.3 Shapiro-Wilk Normality Test

The Shapiro-Wilk test is generally used to data with a limited number of samples, with a size of less than 50 samples, while still producing accurate conclusions [15]. The hypothesis for the Shapiro-Wilk test is as follows [16].

H_0: The sample comes from a normally distributed population
H_1: The sample comes from a population that is not normally distributed

H_0 is rejected if p-value is lower than alpha (0.05), where the p-value is obtained by firstly calculating the W score with the formula as in the Equation (1).

$$W = \frac{\left[\sum_{i=1}^{k} a_i (x_{n-i+1} - x_i)\right]^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \tag{1}$$

where n is number of observations, k is upper limit for the numerator (if n is odd, then $k = \frac{n}{2}$; if n is even, then $k = \frac{n-1}{2}$), $\bar{x}$ is the average of x, a_i is the test coefficient of Shapiro-Wilk for i-th data, and x_i is the i-th data for the variable X. After obtaining the W value, the p-value is obtained by comparing the value of W with the critical value in Shapiro-Wilk table

2.4 Kendall's Tau Correlation

Kendall's Tau correlation is used to measure the dependence between two variables that at least have an ordinal data scale and do not require the data to meet the normality assumption [17]. Kendall's Tau correlation is obtained from the difference between the probability's concordance and discordance, where concordance is the number of ordered pairs and discordance is the number of data pairs with an unnatural order [18]. The hypothesis and formula for calculating Kendall's Tau correlation are as follows [19].

H_0: $\tau = 0$ (no correlation between variables X and Y)
H_1: $\tau \neq 0$ (there is a correlation between variables X and Y)

When the number of data is less than or equal to 10, H_0 is rejected if $|\hat{\tau}| > \tau_{\frac{\alpha}{2},n}$, where $\hat{\tau}$ is estimated by Equation (2).

$$\hat{\tau} = \frac{2S}{n(n-1)} \tag{2}$$

If there are several data with the same value, $\hat{\tau}$ is calculated by Equation (3).

$$\hat{\tau} = \frac{2S}{\sqrt{n(n-1) - T_X}\sqrt{n(n-1) - T_Y}} \tag{3}$$

Meanwhile, when the number of data is greater than 10, the data is close to normal distribution so H_0 is rejected if $|Z| > Z_{\frac{\alpha}{2}}$, where Z is estimated by the Equation (4).

$$Z = \frac{3\hat{\tau}\sqrt{n(n-1)}}{\sqrt{2(2n+5)}} \tag{4}$$

where n is the number of objects, S is the difference between the number of pairs of natural order data (P) and the number of pairs of unnatural order data (Q), then T_X is $\sum_{i=1}^{s}(t_{i(X)}^2 - t_{i(X)})$ with $t_{i(X)}$ is the number of observation with the same value in each group of the same value in variable X. Coefficient value of Kendall's Tau can also be used to find the copula parameter with the formula in **Equation (5)**.

$$\tau = 1 + 4\int_0^1 \frac{\varphi(t)}{\varphi'(t)}\,dt \tag{5}$$

with $\varphi(t)$ is copula generator function and $\varphi'(t)$ is the first derivative of the copula generator function

2.5 Van Der Waerden Transformation

The Van Der Waerden transformation is used to scale the random variable data into a uniform domain [0,1] **[12]**. This process is needed to make the data, including BBCA closing price, inflation rate, interest rate, and USD-IDR exchange rate which are used in this research can be substituted into the copula probability density function. Data transformation to the [0,1] domain can be done by first forming a ranking and dividing it by the amount of data plus 1, as in **Equation (6) [13]**.

$$b = \frac{r}{n+1} \tag{6}$$

where n is the number of observations, r is the rank of the data, and b is the data resulting from the transformation.

2.6 Copula Parameter Estimation

Archimedean Copula is a family of copulas that can be widely applied to describe dependencies, both linear and non-linear relationships among variables **[20]**. In this study, three types of Archimedean copula subfamilies were used, namely Clayton with lower dependency, Gumbel with upper dependency, and Frank whose dependency was seen based on parameter values. Lower dependency means that the relationship between two variables will be very close when both have low values, while upper dependency indicates that the relationship between two variables will be very close when both have high values **[9]**. Clayton Copula parameter is symbolized by θ_{Cl}, Gumbel Copula parameter is symbolized by θ_{Gu}, while Frank Copula parameter is symbolized by θ_{Fr}. The generator and bivariate copula function for each subfamily in Archimedean Copula is shown in **Table 1 [21]**.

Table 1. Generator and Bivariate Copula Function

Copula	Generator $(\varphi(t))$	Copula Function $(C(u,v))$
Clayton	$\frac{1}{\theta_{Cl}}(t^{-\theta_{Cl}}-1)$	$(u^{-\theta_{Cl}}+v^{-\theta_{Cl}}-1)^{-\frac{1}{\theta_{Cl}}}$
Gumbel	$(-\ln(t))^{\theta_{Gu}}$	$\exp\left(-\left((-\ln(u))^{\theta_{Gu}}+(-\ln(v))^{\theta_{Gu}}\right)^{\frac{1}{\theta_{Gu}}}\right)$
Frank	$\ln\left(\frac{e^{-\theta_{Fr}t}-1}{e^{-\theta_{Fr}}-1}\right)$	$-\frac{1}{\theta_{Fr}}\ln\left(1+\frac{(e^{-\theta_{Fr}u}-1)(e^{-\theta_{Fr}v}-1)}{e^{-\theta_{Fr}}-1}\right)$

Each of copula parameter can be obtained using the Kendall's Tau approach, namely based on the **Equation (5)** so that the formula for parameter calculation is shown in **Table 2**. Parameter of Frank Copula cannot be calculated analytically, but it can be calculated numerically by using the Debye function ($D_1(\theta_{Fr})$).

Table 2. Parameter Calculation Formula

Copula	Parameter
Clayton	$\theta_{Cl} = \dfrac{2\tau}{1-\tau}$, $\theta_{Cl} \in [0, \infty)$
Gumbel	$\theta_{Gu} = \dfrac{1}{1-\tau}$, $\theta_{Gu} \in [1, \infty)$
Frank	$\tau = 1 + \dfrac{4}{\theta_{Fr}}(D_1(\theta_{Fr}) - 1)$, $\theta_{Fr} \in \{\mathbb{R}\}\backslash 0$ where $D_1(\theta_{Fr}) = \frac{1}{\theta_{Fr}} \int_0^{\theta_{Fr}} \frac{t}{e^t - 1} dt$

The higher the value of Clayton copula parameter, the closer the lower dependency between the two variables. Meanwhile, the higher the value of Gumbel copula, the closer the upper dependency between two variables. For the Frank copula, the more positive the parameter value, the tighter the upper dependency between two variables. Conversely, the more negative the parameter value, the closer the lower dependency between variables [9].

2.7 Copula Parameter Selection

Log-likelihood is used to determine the most suitable copula parameters for the analyzed data. The highest log-likelihood score of the joint probability density function of copula shows the best of copula model. Log-likelihood function for bivariate copula can be written by Equation (7) [22].

$$l(\theta) = \sum_{i=1}^{n} ln\, c(u_i, v_i) \tag{7}$$

with θ is the copula parameter to be estimated and $c(u_i, v_i)$ obtained from the copula probability density function for $(u, v) \in [0,1]$ which consists of the first until n-th data. The probability density function is defined as in the Equation (8).

$$c(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v} \tag{8}$$

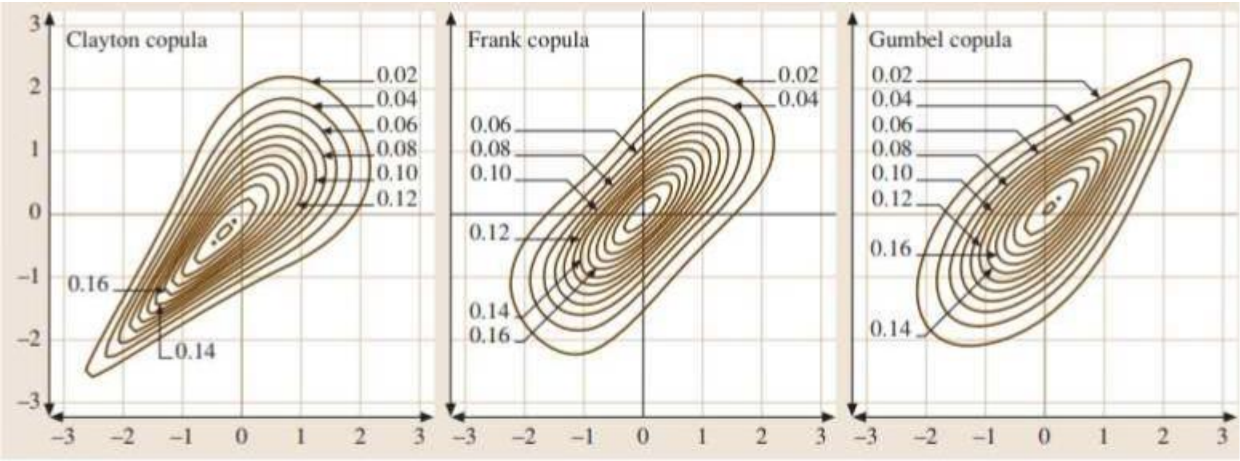
where $C(u, v)$ is the copula function for bivariate data $(u, v) \in [0,1]$. Using Equation (8), the probability density function of Clayton, Gumbel, and Frank copula is obtained as in Table 3.

Table 3. Copula Probability Density Function

Copula	Probability Density Function $(c(u, v))$
Clayton	$(uv)^{-\theta_{Cl}-1}(1+\theta_{Cl})\left(u^{-\theta_{Cl}} + v^{-\theta_{Cl}} - 1\right)^{-\frac{1}{\theta_{Cl}}-2}$
Gumbel	$\frac{1}{uv}(-\ln(u))^{\theta_{Gu}-1}(-\ln(v))^{\theta_{Gu}-1} \exp\left(-\left((-\ln(u))^{\theta_{Gu}} + (-\ln(v))^{\theta_{Gu}}\right)^{\frac{1}{\theta_{Gu}}}\right)\left(\left((-\ln(u))^{\theta_{Gu}} + (-\ln(v))^{\theta_{Gu}}\right)^{\frac{2}{\theta_{Gu}}-2} + (-1+\theta_{Gu})((-\ln(u))^{\theta_{Gu}} + (-\ln(v))^{\theta_{Gu}})^{\frac{1}{\theta_{Gu}}-2}\right)$
Frank	$\dfrac{\left(\theta_{Fr} e^{-(\theta_{Fr}u+\theta_{Fr}v)}(1-e^{-\theta_{Fr}})\right)}{\left((e^{-\theta_{Fr}u}-1)(e^{-\theta_{Fr}v}-1)+e^{-\theta_{Fr}}-1\right)^2}$

2.8 Dependency

Dependency can be interpreted as a bond or relationship [23]. In the context of analysis, the dependency method is used to solve cases related to the relationship between the independent variable and the dependent variable [24]. In analyses involving data with extreme values, dependency measures often have tails which can then be referred to as dependency tails [25]. The dependency tails for each Archimedean copula, including the Clayton, Gumbel, and Frank copula can be seen in Figure 1 [26].



(a) (b) (c)

Figure 1. Tail Dependency of Archimedean Copula, (a) Clayton, (b) Frank, (c) Gumbel

Figure 1 shows the tail dependency of Archimedean copula, which consists of Clayton, Gumbel, and Frank copula. Clayton copula has a lower dependency tail, while Gumbel copula has an upper dependency tail. Frank copula does not appear to have a dependency tail based on its visualization, but it can be interpreted as having an upper dependency tail if its parameter is positive and a lower dependency tail if its parameter is negative **[9]**.

3. RESULTS AND DISCUSSION

3.1 Data Exploration

The descriptive statistics that depict the BBCA closing price as the dependent variable and the influencing independent variables can be observed in **Table 4**.

Table 4. Data Description

Variable	n	Minimum	Maximum	Mean
X_1	36	1.33	5.95	3.15
X_2	36	3.50	6.00	4.44
X_3	36	14,084	15,916	14,826.81
Y	36	5,970	9,400	7,918.33

Based on **Table 4**, it is known that the amount of data used in the research was 36 data because the research took monthly data for three years, namely 2021-2023. For variable X_1 (Indonesian inflation rate), the lowest value of 1.33% occurred in June 2021, while the highest value of 5.95% occurred in September 2022. The very low inflation value in June 2021 could be caused by the relatively good condition of food price stability during the pandemic Covid-19 and a decrease in market demand due to the pandemic **[27]**. On the other hand, starting in 2022, even in September 2022, inflation became very high due to the increasing of public demand as pandemic conditions improved and the new normal era began **[28]**.

3.2 Normality Testing

Dependency analysis using the Archimedean copula approach does not require data to meet normality assumptions so that the data used in research can come from a normal distribution or not. However, in this study, a limit was set that there was at least one variable that was not normally distributed to be able to continue the analysis using the copula approach. Using the Shapiro-Wilk test, H_0 is rejected if the p-value < α (0.05). The p-value is obtained by first calculating the W test statistic from **Equation (1)** using data that has been sorted from the smallest value to the largest value. The calculation results for each variable can be seen in **Table 5**.

Table 5. Normality Test Results

Variable	W	p-value	Decision
X_1	0.8853	< 0.01	Reject H_0
X_2	0.7053	< 0.01	Reject H_0
X_3	0.9240	between 0.02 and 0.05	Reject H_0
Y	0.9167	between 0.01 and 0.02	Reject H_0

Based on Table 5, it can be concluded that the four variables used in this study did not come from a normally distributed population.

3.3 Kendall's Tau Correlation

Kendall's Tau correlation coefficient was used to estimate copula parameters. With a total of 36 data, then H_0 is rejected if $|Z| > Z_{\frac{\alpha}{2}}$. To find the value of Z, the formulas in Equation (3) and Equation (4) are used. Previously, the calculation began by sorting the variable data pairs X and Y based on the values from the smallest to the largest. Next, the S value calculation is carried out based on the difference between the number of reasonable ordered data (P) and the number of unreasonable ordered data (Q) of each variable pair. Two data pairs (x_1, y_1) and (x_2, y_2) can be said to have a reasonable order if $y_2 > y_1$ and $x_2 > x_1$. On the other hand, two pairs of data (x_1, y_1) and (x_2, y_2) can be said to have an unreasonable order if $y_2 < y_1$, but $x_2 > x_1$. For data consisting of 36 observations, a comparison will be made for the data pairs (x_1, y_1) and (x_2, y_2); (x_1, y_1) and (x_3, y_3); up to data pairs (x_1, y_1) and (x_{36}, y_{36}); followed by comparisons between data pairs (x_2, y_2) and (x_3, y_3); (x_2, y_2) and (x_4, y_4); until data pairs (x_2, y_2) and (x_{36}, y_{36}), and so on until the comparisons between data pairs (x_{35}, y_{35}) and (x_{36}, y_{36}). The complete results of Kendall's Tau calculation are shown in Table 6.

Table 6. Kendall's Tau Correlation Test Results

Variable	S	$\hat{\tau}$	Z	$Z_{\frac{\alpha}{2},36}$	Decision
X_1 and Y	317	0.505	4.335	1.960	Reject H_0
X_2 and Y	352	0.674	5.780	1.960	Reject H_0
X_3 and Y	319	0.508	4.362	1.960	Reject H_0

Based on Table 6, it is known that all variables X have a significant relationship with the Y variable. Thus, dependency analysis using the Archimedean Copula approach can be continued for each pair of variables.

3.4 Van Der Waerden Transformation

Data transformation can be done by first assigning a ranking, where the first ranking is intended for the smallest value in each variable. Using Equation (6), the transformation results for the four variables can be seen in Table 7.

Table 7. Van Der Waerden Transformation Results

No	X_1	X_2	X_3	Y
1	0.162	0.527	0.027	0.216
2	0.081	0.257	0.081	0.189
3	0.054	0.257	0.459	0.081
⋮	⋮	⋮	⋮	⋮
34	0.432	0.946	0.973	0.676
35	0.514	0.946	0.811	0.784
36	0.459	0.946	0.838	0.973

3.5 Copula Parameter Estimation

Based on the previously obtained Kendall's Tau correlation coefficient, the parameters of the Clayton copula, Gumbel copula, and Frank copula can be estimated. The parameter estimation results for Clayton, Gumbel, and Frank copula for each pair of variables is shown in Table 8.

Table 8. Parameter Estimation Results

Variable	Types of Archimedean Copulas	Parameter Estimation
X_1 and Y	Clayton	2.042
	Gumbel	2.021
	Frank	5.831
X_2 and Y	Clayton	4.126
	Gumbel	3.063
	Frank	10.300
X_3 and Y	Clayton	2.068
	Gumbel	2.034
	Frank	5.891

The estimated parameters show the closeness of relationship between the two variables being analyzed. However, before further interpretation, it is necessary to select the best copula type for each pair of variables.

3.6 Selection of The Best Copula

To find out the best type of copula that describes the dependency between the BBCA closing price and macroeconomic factors, a selection was made based on the highest log-likelihood score. However, before calculating the log-likelihood value, first calculate the probability density function value for each (u_i, v_i), where u_i and v_i are the i-th data for the variables u and v. For this calculation, each of θ_{Cl}, θ_{Gu}, and θ_{Fr} are substituted by each of estimated parameter, those are $\hat{\theta}_{Cl}$ for Clayton copula, $\hat{\theta}_{Gu}$ for Gumbel copula, and $\hat{\theta}_{Fr}$ for Frank copula. The results of calculating the copula density function for the first data to the last data for each type of copula can be seen in **Table 9**.

Table 9. The Calculation Results of Copula Density Function

i	X_1 and Y			X_2 and Y			X_3 and Y		
	Clayton	Gumbel	Frank	Clayton	Gumbel	Frank	Clayton	Gumbel	Frank
1	2.706	1.866	2.050	0.234	0.643	0.391	0.186	1.519	1.765
2	1.922	1.974	2.164	3.241	2.321	2.521	1.913	1.981	2.171
3	6.678	3.057	3.299	0.168	1.435	1.413	0.175	0.738	0.589
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
34	1.100	1.082	1.001	1.209	0.117	0.604	1.392	0.237	0.975
35	1.118	0.944	0.929	2.038	0.466	1.661	1.744	2.365	2.047
36	0.656	0.078	0.289	3.826	7.589	5.563	2.098	0.988	2.345

Based on this value, the log-likelihood value can be calculated for each type of copula and each pair of variables. The log-likelihood value is calculated using the formula in **Equation (7)**. The results of calculating the log-likelihood values of the Clayton copula, Gumbel copula and Frank copula parameters for each pair of variables can be seen in **Table 10**.

Table 10. Log-Likelihood Value for Each Parameter

Variable	Types of Archimedean Copula	Parameter Estimation	Log-likelihood
X_1 and Y	Clayton	2.042	15.020
	Gumbel	2.021	4.076
	Frank	5.831	9.554
X_2 and Y	Clayton	4.126	5.208
	Gumbel	3.063	14.520
	Frank	10.300	18.590
X_3 and Y	Clayton	2.068	1.020
	Gumbel	2.034	8.679
	Frank	5.891	11.258

From **Table 10**, the best type of copula can be selected for each pair of variables. The best copula is the copula that has the largest log-likelihood value. Thus, it is concluded that the best Archimedean copula to describe the relationship between X_1 and Y, X_2 and Y, also X_3 and Y respectively are Clayton copula ($\hat{\theta}_{Cl}$ = 2.042), Frank copula ($\hat{\theta}_{Fr}$ = 10.3), and Frank copula ($\hat{\theta}_{Fr}$ = 5.891)

3.7 Copula Model

The best copula to describe the relationship between inflation rate and the BBCA closing price is the Clayton copula. With parameters $\hat{\theta}_{Cl}$ = 2.042, the Clayton copula model for the inflation rate and BBCA closing price is as the following function.

$$C(u,v) = (u^{-2.042} + v^{-2.042} - 1)^{-\frac{1}{2.042}}$$

From this model, 5000 new pair of data that meets $\hat{\theta}_{Cl}$ = 2.042 were generated to visualize the relationship between the inflation rate and the BBCA closing price. The scatterplot of the 5000 data pairs generated is shown in **Figure 2**.

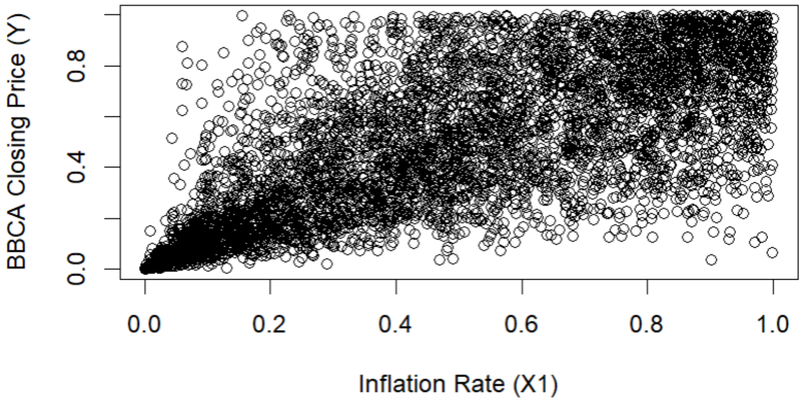


Figure 2. Scatterplot of 5000 Data Generation Based on the Best Copula Model for X_1 and Y

Based on the scatterplot in **Figure 2**, it can be seen that the relationship between the inflation rate and the BBCA closing price has a lower dependency tail. The dependency tail at the bottom indicates that the inflation rate and the BBCA closing price have a strong relationship when both variables have low values. By looking at value $\hat{\tau}$, it is known that these two variables have a positive relationship, where the higher the inflation rate, the higher the closing stock price. However, it turns out, with the Clayton copula, more information is obtained that the positive relationship between these two variables will be very strong when both variables have low values, and the relationship becomes weaker as the value of the inflation rate and stock price variables is added.

For the variable pair of interest rate and BBCA closing price, the best copula model was obtained, namely the Frank copula. With parameter $\hat{\theta}_{Fr}$ = 10.3, the Frank copula model for the interest rate and BBCA closing price is as follows.

$$C(u,v) = -\frac{1}{10.3}\ln\left(1 + \frac{(e^{-10.3\,u} - 1)(e^{-10.3\,v} - 1)}{e^{-10.3} - 1}\right)$$

From this model, 5000 new data pairs were also generated which met $\hat{\theta}_{Fr}$ = 10.3 to visualize the dependence between interest rates and the BBCA closing price. The scatterplot of the 5000 data pairs generated is shown in **Figure 3**.

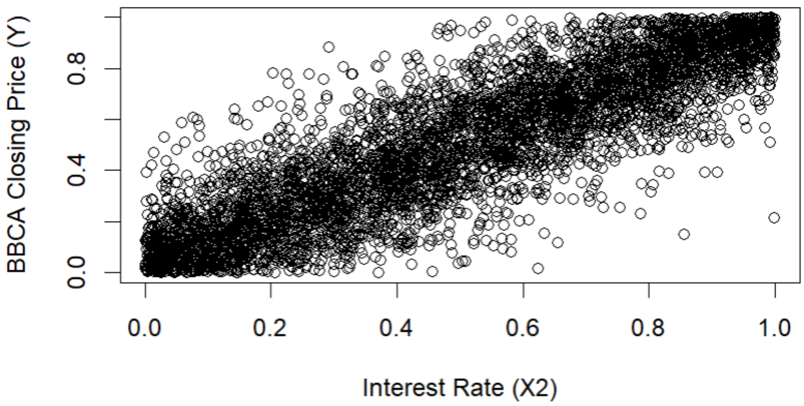


Figure 3. Scatterplot of 5000 Data Generation Based on the Best Copula Model for X_2 and Y

Based on the scatterplot in **Figure 3**, the relationship between interest rates and the BBCA closing price does not have a dependent tail. Because it does not have a dependent tail, the dependent relationship between two variables described by Frank copula can be seen based on the parameter values. If the Frank copula parameter is positive, it is concluded that there is a dependency between the two variables, and vice versa. Based on the value $\hat{\theta}_{Fr}$ = 10.3 which is positive, it is known that there is an upper dependency between the interest rate and the BBCA closing price. Previously, by looking at the $\hat{\tau}$, it is known that the higher the interest rate, the higher the BBCA closing price. Then, with the upper dependency information from Frank copula, more information is obtained that the positive relationship between these two variables will be very strong when both variables have high values, and the relationship will weaken as the value of the interest rate variable and closing stock prices decrease.

Next, the best copula to describe the relationship between the USD-IDR exchange rate and the BBCA closing price is the Frank copula. With parameter $\hat{\theta}_{Fr}$ = 5.891, the Frank copula model for the exchange rate and BBCA closing price is as follows.

$$C(u,v) = -\frac{1}{5.891}\ln\left(1+\frac{(e^{-5.891\,u}-1)(e^{-5.891\,v}-1)}{e^{-5.891}-1}\right)$$

From this model, 5000 new data pairs were also generated which met $\hat{\theta}_{Fr}$ = 5.891 to visualize the dependence between interest rates and the BBCA closing price. The scatterplot of the 5000 data pairs generated is shown in **Figure 4**.

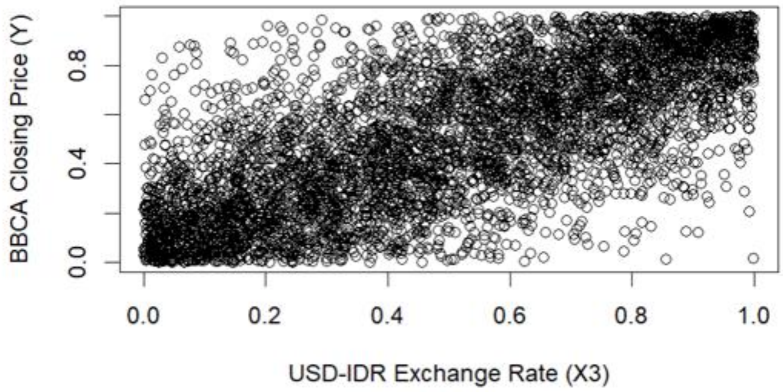


Figure 4. Scatterplot of 5000 Data Generation Based on The Best Copula Model for X_3 and Y

Based on the scatterplot in **Figure 4**, the relationship between USD-IDR exchange rate and BBCA closing price does not have a dependent tail. Because it does not have a dependent tail, the dependent relationship between two variables described by Frank copula can be seen based on the parameter values. Based on the value $\hat{\theta}_{Fr}$ = 5.891 which is positive, it is known that there is upper dependency between the exchange rate and the BBCA closing price. By looking at value $\hat{\tau}$, it is known that these two variables have a positive relationship, where the higher the USD exchange rate against the Rupiah, the higher the BBCA closing price. Using the Archimedean copula approach, it is found that the positive relationship between these two variables will be very strong when both variables have high values, and the relationship will weaken as the value of both variables decreases.

The scatterplot in **Figure 3** and **Figure 4** look similar because of the same copula model for X_2 and Y also X_3 and Y, namely Frank copula, with different parameter values. For the Frank copula, the more positive the parameter value, the tighter the upper dependency between two variables. Based on the obtained copula parameter values, the parameter value for X_2 and Y ($\hat{\theta}_{Fr}$ = 10.3) is higher than the parameter value for X_3 and Y ($\hat{\theta}_{Fr}$ = 5.891). Thus, based on the characteristics of the Frank copula, the data points in the scatterplot in **Figure 3** look closer together than the data points in the scatterplot in **Figure 4**. It means that the upper dependency between X_2 and Y is stronger than the upper dependency between X_3 and Y.

4. CONCLUSION

Among Clayton, Gumbel, and Frank, it is obtained that the best type of copula to describe the dependency between the inflation rate and the BBCA closing price is the Clayton copula with $\hat{\theta}_{Cl}$ = 2.042. Meanwhile, the best type of Archimedean copula to describe the dependency between interest rates and the BBCA closing price is the Frank copula with $\hat{\theta}_{Fr}$ = 10.3. The best type of copula to describe the dependency between the USD-IDR exchange rate and the BBCA closing price is the Frank with $\hat{\theta}_{Fr}$ = 5.891. The selection of the best copula for each variable pair was based on the highest log-likelihood value, revealing the tail dependence captured by each copula.

The Clayton copula, which captures lower tail dependence, is well suited to represent the relationship between inflation and BBCA closing price. This suggests that these variables move more closely during periods of low values, which are often associated with deflation or economic stability.

In contrast, the relationship between interest rate and BBCA closing price, as well as between the USD-IDR exchange rate and BBCA closing price, is best described by the Frank copula, whose positive parameter indicates that the dependent relationship becomes stronger as both variables increase.

Although the Gumbel copula is capable of capturing upper tail dependence, meaning dependence in high-value extremes, it was not selected as the best fit in any case. This implies that the dependency in this dataset is not dominated by co-movements in extremely high values. The choice of copula provides deeper insights into how BBCA closing price reacts to the dynamics of macroeconomic variables under different conditions.

Future research could extend this analysis by examining the dependence between BBCA closing price and multiple macroeconomic factors simultaneously.

AUTHOR CONTRIBUTIONS

Noerul Hanin: Data Curation, Methodology, Formal Analysis, Software, Visualization, Writing-Original Draft. Neva Satyahadewi: Supervision, Funding Acquisition, Writing-Review and Editing. Evy Sulistianingsih: Supervision, Funding Acquisition, Writing-Review and Editing. All authors discussed the results and contributed to the final manuscript.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest to report study.

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